Endogenous business cycles caused by nonconvex costs and interactions

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Aggregate Fluctuations

RBC(e.g. Kydland and Prescott (1982)) assume *mysterious* aggregate shocks.

- Popular economic-wide shocks, monetary shocks or oil price fluctuation, fail to explain the bulk of economic fluctuations. (Cochrane(1994))

$\Rightarrow$ The aggregate fluctuation may arise from firm-specific shocks and inherits some properties from them. (Carvalho(2010), Gabaix(2011), Acemoglu et al.(2012))

The law of Large Numbers

$\overline{X} \equiv \frac{1}{N}(X_1 + \ldots + X_N)$ converges to the expected value, $\overline{X} \longrightarrow \mu$

- Microeconomic shocks would average out at the aggregate level when the number of firms is large.

⇒ In order to explain aggregate fluctuations, shocks that affect all firms are needed, and by definition, they are aggregate shocks. (e.g. Dupor(1999))

Independent?

Assumption 1(Interaction) : The behavior and decisions of firms depends on macroeconomic condition.

- The macroscopic state of the economy not only is aggregation of the firms but also prescribe environment in which the firms engage in business activities. ``micro-macro loop”(F. Hahn(2003))

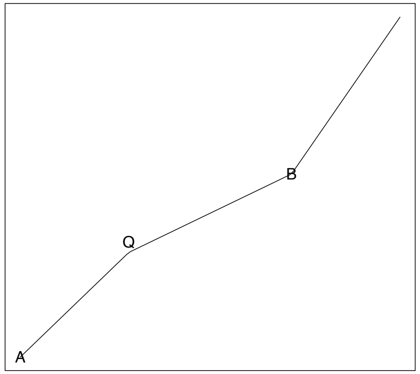
Assumption 2(Nonconvex cost function) : At the micro-level, economic activities are characterized by lumpiness and discreteness.

- Blinder and Maccini(1991) (S,s) model.

⇒The aim of this paper is to investigate how these characteristics at the firm level are related to the aggregate fluctuation.

Nonconvex cost function

We assume a nonconvex cost function.



- Ramey(1991), Cooper and Haltiwanger(1992), Bresnahan and Ramey(1994) and Hall(2000) etc.

1. A manufacturing firm has enough inventories and the demand is low.
2. The firm choose a low production state to reduce its inventories.
3. After eliminating the excess inventories, the firm waits for the improvement of the demand.
4. If it happens, the firm chooses high production.
5. Replenishing its inventories, the firm reduces its production and return to 1.

Model without Interactions

The demand of a firm fluctuate around $\overline{S}$.

$$s_t^i = \overline{S} + \xi_t^i \qquad \overline{S} = 0$$

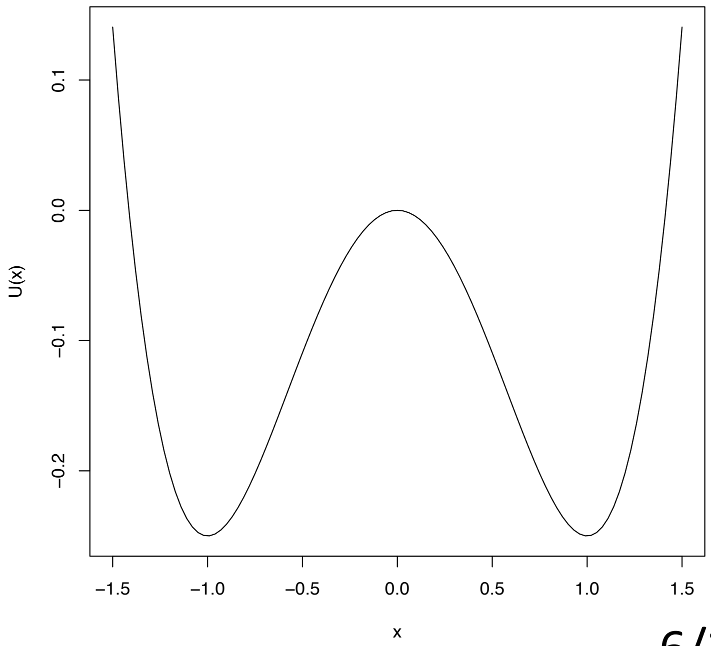
Inventories investment can be written as the difference between production and sales.

$$dy_t^i = (x_t^i - s_t^i)dt$$

The production is described by the motion in the so-called double-well potential.

$$dx_t^i = (-V'(x_t^i) - e y_t^i)dt + \sigma_1 dW_{1,t}^i$$

$$V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$$



Model without Interactions

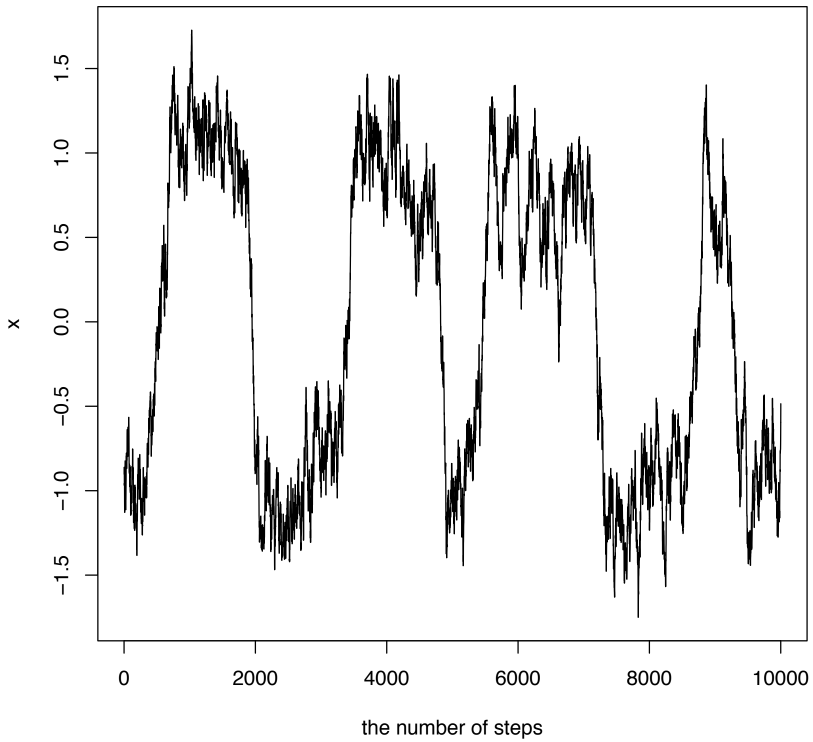
Two dimensional stochastic differential equation

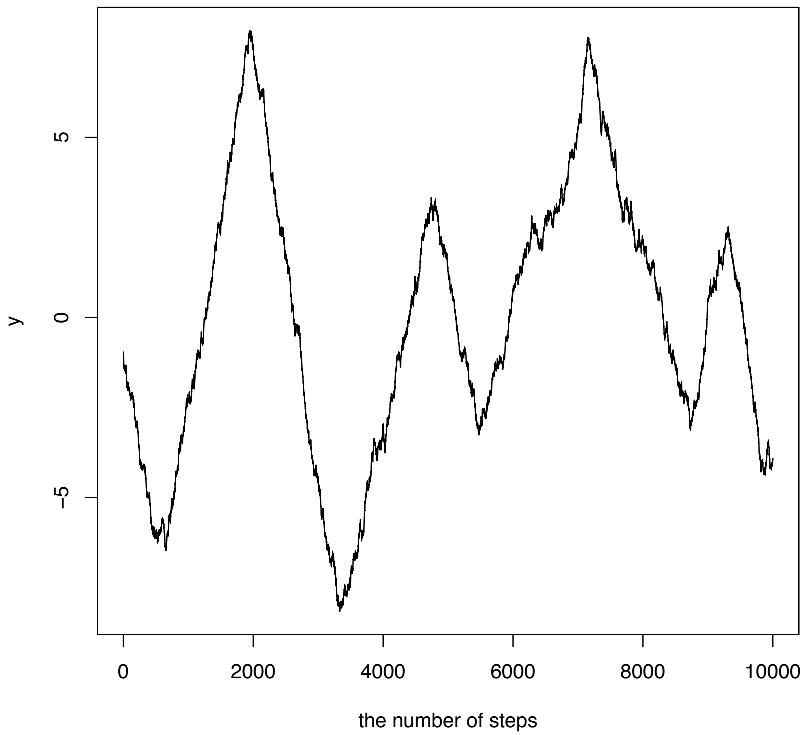
$$dx_t^i = (-V'(x_t^i) - ey_t^i)dt + \sigma_1 dW_{1,t}^i \qquad V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$$

$$dy_t^i = x_t^i dt + \sigma_2 dW_{2,t}^i$$

Simulations

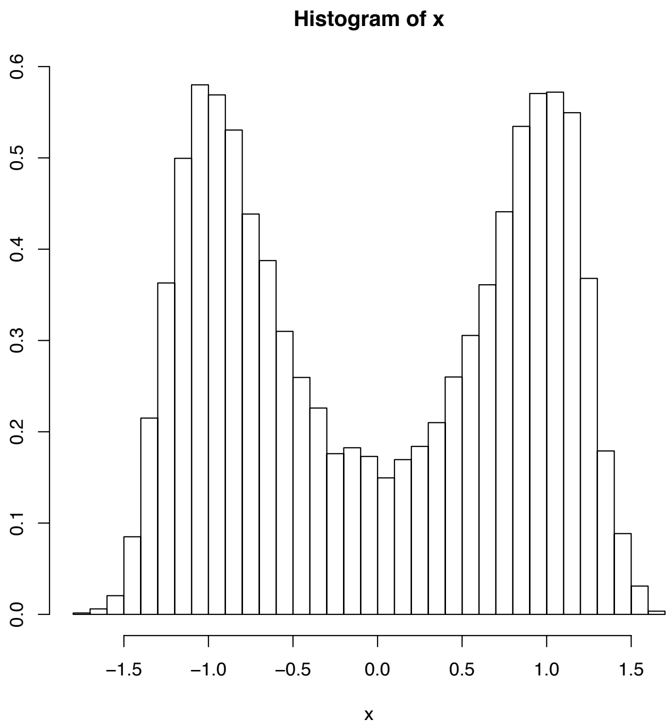
The path of production, x, alternates between the low and high production stochastically.





Simulations

The histogram of x (N=200000)



Interaction

Carvalho(2010), Acemoglu et al(2012) focus on input-output linkages.

- When significant asymmetry exists, that is, *hubs* exists, microeconomic shocks do not disappear.
  There exist sectors that have disproportinal impact on the aggregate fluctuations.
- ``Granular hypothesis"(Gabaix(2011))

Dupor (1999) says "the input-output structures in this class provide a poor amplification mechanism for sector shocks".

Interaction

We focus on a feedback mechanism : a situation where the behavior of a firm is affected by the entire economic condition while the economic condition is determined by the aggregation of the firms.

- The sale depends on GDP or average production.

$$s_t^i = h < x > + \xi_t^i \qquad < x > = \frac{1}{N} \sum_{j=1}^{N} x_t^j$$

- Firms adjust their production according to their expectation of the sale.

$$dx_t^i = (-V'(x_t^i) - e y_t^i + D(E[s_t^i] - x_t^i))dt + \sigma_1 dW_{1,t}^i$$

$$dy_t^i = (x_t^i - h < x >)dt + \sigma_2 dW_{2,t}^i$$

Propagation of Chaos

More generally, using the empirical measure, the system of equations can be written as follow,

$$dz_t^{i,N} = b(z_t^{i,N}, U_t^{(N)})dt + a(z_t^{i,N}, U_t^{(N)})dW_t^i \qquad U_t^{(N)} = \frac{1}{N}\sum_{j=1}^{N}\delta_{z_t^j}$$

where z is d dimensional variable.
The coefficients depend on the empirical measure.

We introduce the corresponding mean-field equation.

$$dz_t^i = b(z_t^i, u_t)dt + a(z_t^i, u_t)dW_t^i$$

$u_t(dz) =$ the law of z_t

Propagation of Chaos

One can show that the solution, $z_t^{i,N}$, converge to z_t^i, independent copies of the mean field equation.

- The probability distribution of $(z_t^{i_1,N},\dots,z_t^{i_k,N})$ converges to $u_t \otimes \dots \otimes u_t$ (Propagation of chaos)

- The random variable $U_t^{(N)}$ converges to the constant u_t .

For reviews, see Sznitman(1991) and Gartner(1988).

Stability Analysis

Rather than seeking for u_t directly, we study the dynamics of lower moments of u_t, mean, variance and covariance.

$$<\dot{x}>=<x>-<x>^3-3\mu_{3,0}-e<y>-D(1-h)<x>$$

$$<\dot{y}>=(1-h)<x>$$

$$\dot{\mu}_{2,0}=-2D\mu_{2,0}-2e\mu_{1,1}+2(1-3<x>^2)\mu_{2,0}-6<x>\mu_{3,0}-2\mu_{4,0}+\sigma_1^2$$

$$\dot{\mu}_{1,1}=-D\mu_{1,1}-e\mu_{0,2}+(1-h)\mu_{2,0}+(1-3<x>^2)\mu_{1,1}-3<x>\mu_{2,1}-\mu_{3,1}$$

$$\dot{\mu}_{0,2}=2(1-h)\mu_{1,1}+\sigma_2^2$$

$$\mu_{n,m}=<(x-<x>)^n(y-<y>)^m>$$

Gaussian Approximation

We focus on a state $\langle x \rangle = \langle y \rangle = 0$ (disordered state)

We approximate the system by the Gaussian distribution with time-varying parameters.

The behavior of <x> and <y> around the disordered state can be determined by the Jacobian matrix A.

$$A = \begin{pmatrix} 1 - 3\mu_{2,0}^{*} - D(1-h) & -e \\ 1-h & 0 \end{pmatrix}$$

Gaussian Approximation

The eigenvalues of A is given by

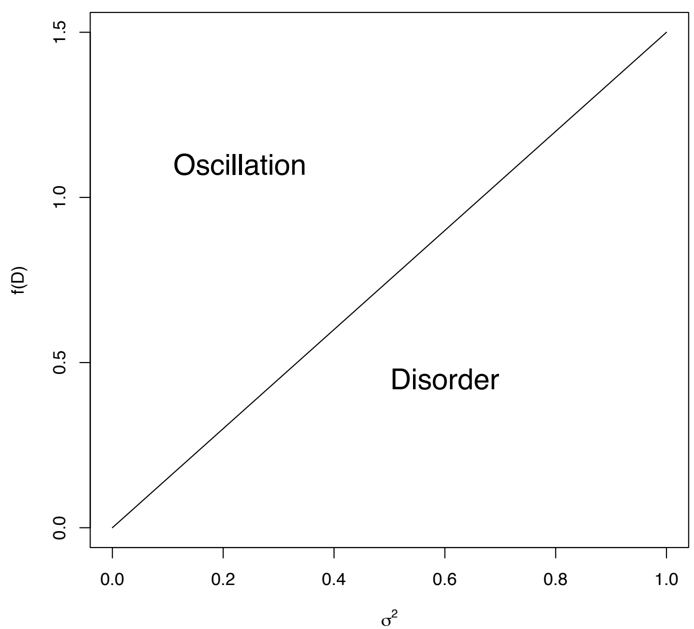
$$\lambda_{\pm} = \frac{1}{2}(1 - 3\mu_{2,0}^{*} - D(1-h) \pm \sqrt{(1 - 3\mu_{2,0}^{*} - D(1-h))^2 - 4(1-h)e})$$

⇒ We examine when the stability of the disordered state is lost,
i.e., the condition that the real part of the eigenvalues becomes 0.

Bifurcation

The condition is given by

$$f_h(D) \equiv Dh(1 - D + Dh) = \frac{3}{2}(\sigma_1^2 + \frac{e\sigma_2^2}{1-h}) \equiv \frac{3}{2}\sigma^2$$

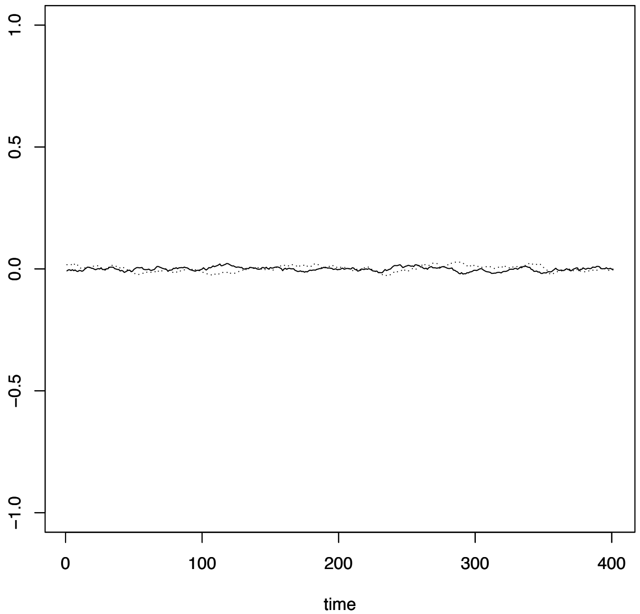


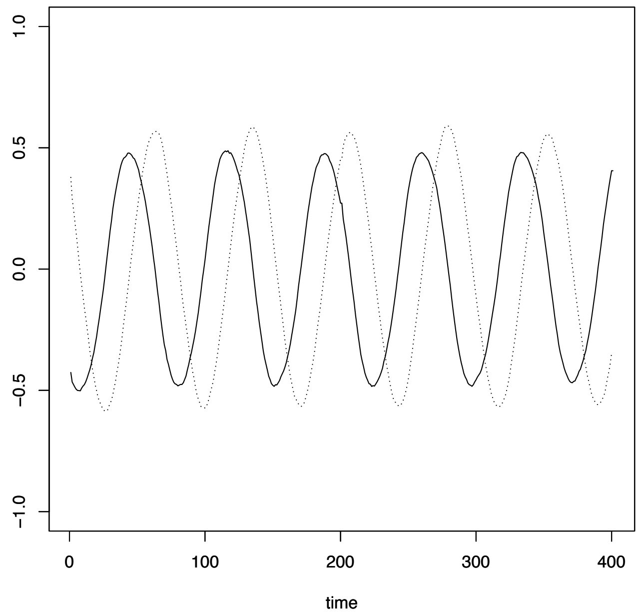
When the two parameter, σ and D, satisfy this relation, the disordered state loses its stability and the bifurcation occurs.

⇒This suggests the possibility of cooperative or collective behavior in the system.

Simulations

D=0.1(Left) and 0.32(Right)

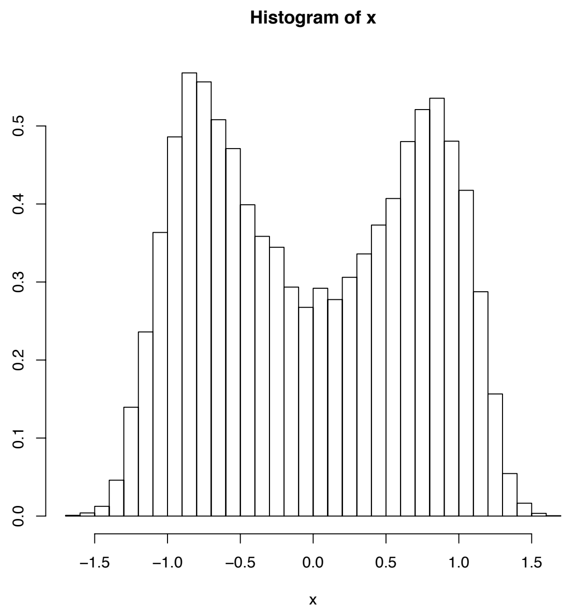


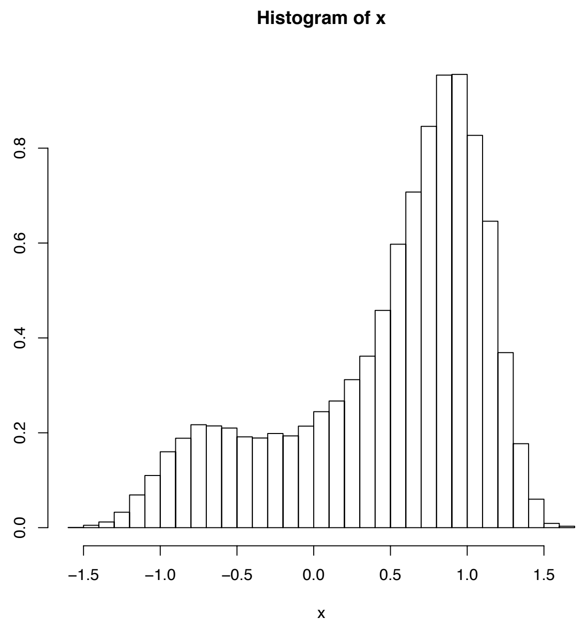


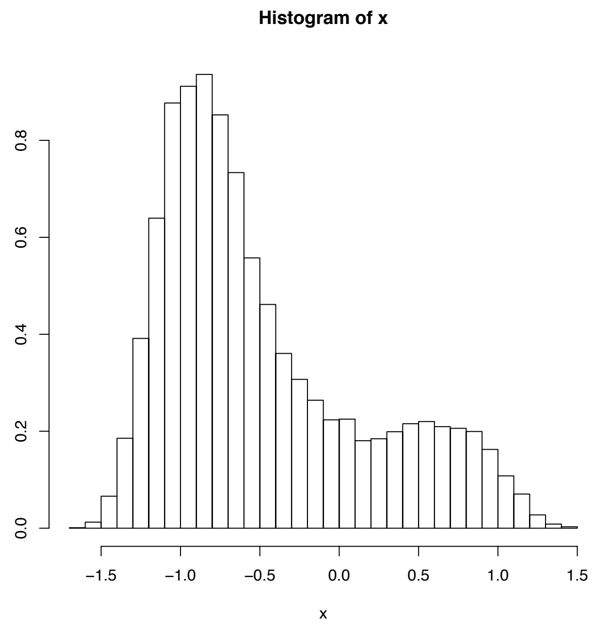
- In the left figure, there is no observable aggregate behavior. Only small variation around 0 is observed.
- In the right figure, an endogenous cyclical movement is observed. This is consistent with the fact that the eigenvalues has an imaginary part different from 0 near the bifurcation point.

Simulations

Histograms with D=0.32





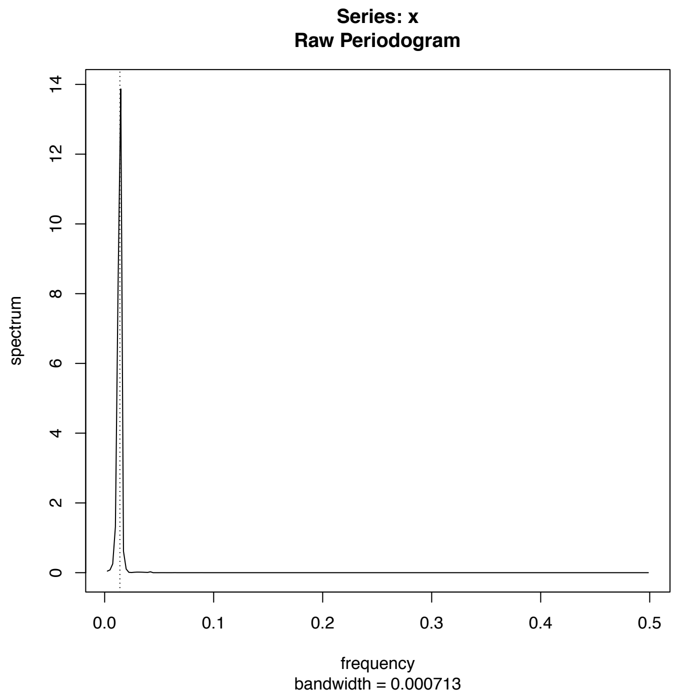


- Even when the economy is bad, there are firms which choose high production, depending on their states.
- There is no representative firm corresponding to the motion of $\langle x \rangle$ and $\langle y \rangle$.

Frequency

We also estimate the frequency of the cycle and compare that value with those predicted from stability analysis.

⇒ At least, the qualitative feature can be captured.



$$\omega_{es} = 0.014, T_{es} = 71$$

$$\omega_{st} = 0.016, T_{st} = 63$$

Concluding Remarks

- The economic fluctuation, especially the inventory cycle, is a consequence of nonconvex costs function and interactions.
- The key element is that the behavior of a firm is affected by the entire economic condition while the economic condition is determined by the aggregation of the firms.
- This feedback mechanism generates a collective behavior, which is different from microeconomic behavior.

Thank you for your attention!!