$\boldsymbol{Dynamic\ agglomeration\ patterns}$
$\boldsymbol{in\ a\ 2-country\ 4-region\ NEG\ model}$

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Aim of the work

- Analyze the impact of regional integration

(vs. the effect of international integration)

on the location of economic activities

Contributions

- Rich structure of equilibria
- Stability and dynamics

Related Literature

Lower trade costs (internal or external) favour full agglomeration within countries

- Monfort & Nicolini (2000, JUE)

Numerical study of a 4-region NEG model with CES preferences

- Behrens, Gaigne, Ottaviano & Thisse (2007, EER)

Partial study of a 4-region NEG model with quadratic preferences

Presence of a stable asymmetric Home country distribution

The size of the outside country impacts on the structure of the Home country

- Commendatore, Kubin, Petraglia, Sushko(2014, JEDC)

3-region FE NEG Model

2-country 4-region FE NEG Model

	Home Country			Foreign Country	
	Region 1	Region 2		Region 3	Region 4
Trade	ϕ_{Ih}		ϕ_E	ϕ_{If}	
Entrepreneurs	$1-x_t$	x_t		y_t	$1-y_t$
Unskilled Labor	E/4	E/4		E/4	E/4

Short-Run Economy

Income

$$Y_{1,t} = L/4 + \pi_{1,t}(1 - x_t)E/2 \quad , \quad Y_{3,t} = L/4 + \pi_{3,t} y_t E/2$$

$$Y_{2,t} = L/4 + \pi_{2,t} x_t E/2 \quad , \quad Y_{4,t} = L/4 + \pi_{4,t}(1 - y_t)E/2$$

Entrepreneur profit

$$\pi_{1,t} = \frac{\mu}{\sigma E}\left[\frac{Y_{1,t}}{\Delta_{1,t}} + \phi_{Ih}\frac{Y_{2,t}}{\Delta_{2,t}}\right] \quad , \quad \pi_{3,t} = \frac{\mu}{\sigma E}\left[\phi_E\left(\frac{Y_{2,t}}{\Delta_{2,t}}\right) + \frac{Y_{3,t}}{\Delta_{3,t}} + \phi_{If}\frac{Y_{4,t}}{\Delta_{4,t}}\right]$$

$$\pi_{2,t} = \frac{\mu}{\sigma E}\left[\phi_{Ih}\frac{Y_{1,t}}{\Delta_{1,t}} + \frac{Y_{2,t}}{\Delta_{2,t}} + \phi_E\left(\frac{Y_{3,t}}{\Delta_{3,t}}\right)\right] \quad , \quad \pi_{4,t} = \frac{\mu}{\sigma E}\left[\phi_{If}\frac{Y_{3,t}}{\Delta_{3,t}} + \frac{Y_{4,t}}{\Delta_{4,t}}\right]$$

Price index and indirect utility

$$P_{r,t} = \beta\frac{\sigma}{\sigma - 1}E^{1/(1-\sigma)}\Delta_{r,t}^{1/(1-\sigma)} \, , \, V_{r,t} = \frac{\pi_{r,t}}{P_{r,t}^{\mu}}$$

where

$$\Delta_{1,t} = \frac{1}{2}(1 - (1 - \phi_{Ih})x_t) \quad , \quad \Delta_{3,t} = \frac{1}{2}(\phi_{If} + (1 - \phi_{If})y_t + \phi_E x_t)$$

$$\Delta_{2,t} = \phi_{Ih}(1 - x_t)/2 + x_t/2 + \phi_E y_t/2 = \frac{1}{2}(\phi_{Ih} + (1 - \phi_{Ih})x_t + \phi_E y_t) \quad , \quad \Delta_{4,t} = \frac{1}{2}(1 - (1 - \phi_{If})y_t)$$

Dynamic Map

$$Z : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x = \begin{cases} 0 & \text{if} \quad Z_h(x,y) < 0, \\ Z_h(x,y) & \text{if} \quad 0 \le Z_h(x,y) \le 1, \\ 1 & \text{if} \quad Z_h(x,y) > 1, \end{cases} \\ \\ y = \begin{cases} 0 & \text{if} \quad Z_f(x,y) < 0, \\ Z_f(x,y) & \text{if} \quad 0 \le Z_f(x,y) \le 1, \\ 1 & \text{if} \quad Z_f(x,y) > 1, \end{cases} \end{pmatrix}$$

where

$$Z_h(x,y) = x + \gamma x(1-x)\frac{\Omega_h(x,y)-1}{1+x(\Omega_h(x,y)-1)},$$

$$Z_f(x,y) = y + \gamma y(1-y)\frac{\Omega_f(x,y)-1}{1+y(\Omega_f(x,y)-1)},$$

$$\Omega_h(x,y) = \frac{V_2(x,y)}{V_1(x,y)}, \quad \Omega_f(x,y) = \frac{V_3(x,y)}{V_4(x,y)}.$$

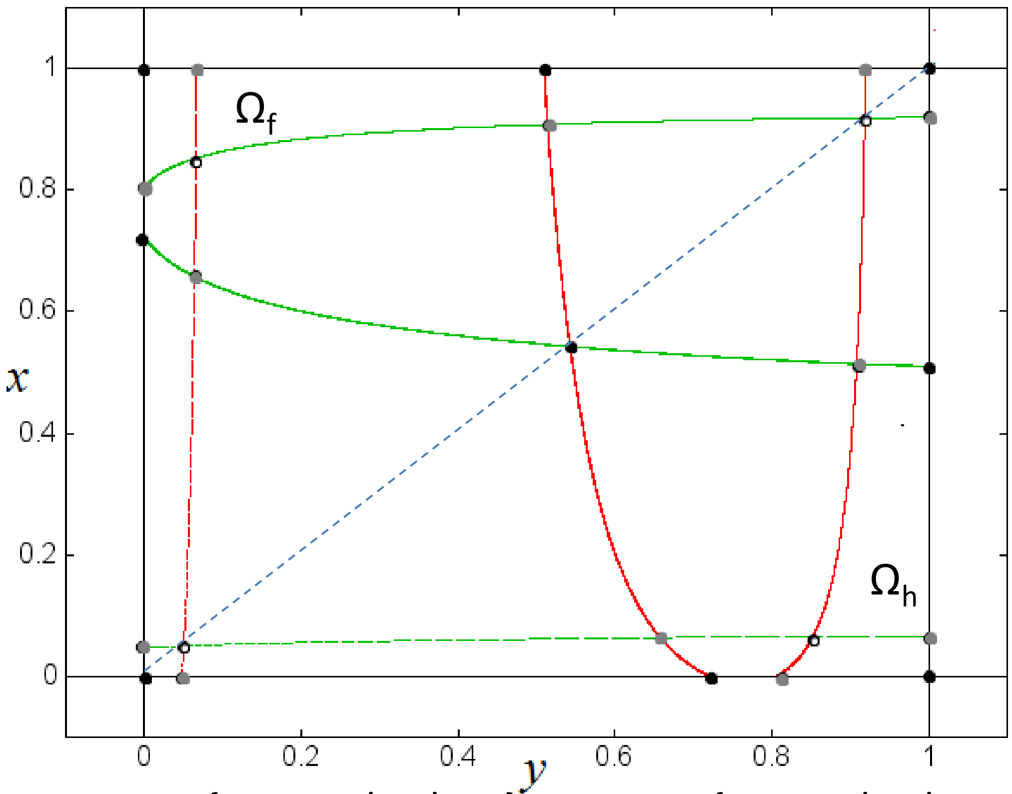
State variables: x , y

- Incomes, price indices, profits, utilities, and real wages can be expressed in terms of x and y

Dynamics

- The domain of the map is the unit square, $[0, 1] \times [0, 1]$
- The regional mobility of entrepreneurs (migration) follows the replicator dynamics equation: $Z(x, y)$.
- What happens with a simultaneous change of interior trade costs ($\phi_{lh} = \phi_{lf} = \phi$)?

Fixed points (long-run equilibra)



Ω_h = {x, y : Ω_f(x,y)= 1} Ω_f = {x,y : Ω_f(x,y)= 1

Core-Periphery fixed points

CP_{00} : (x, y) = (0, 0) , CP_{01} : (x, y) = (0, 1)
CP_{11} : (x, y) = (1, 1) , CP_{10} : (x, y) = (1, 0)

Interior symmetric fixed point

S_{aa} : (x, y) = (a, a)

Interior asymmetric fixed points

AS_{ab} : (x, y) = (a, b) , AS_{ba} : (x, y) = (b, a)

Border asymmetric fixed points

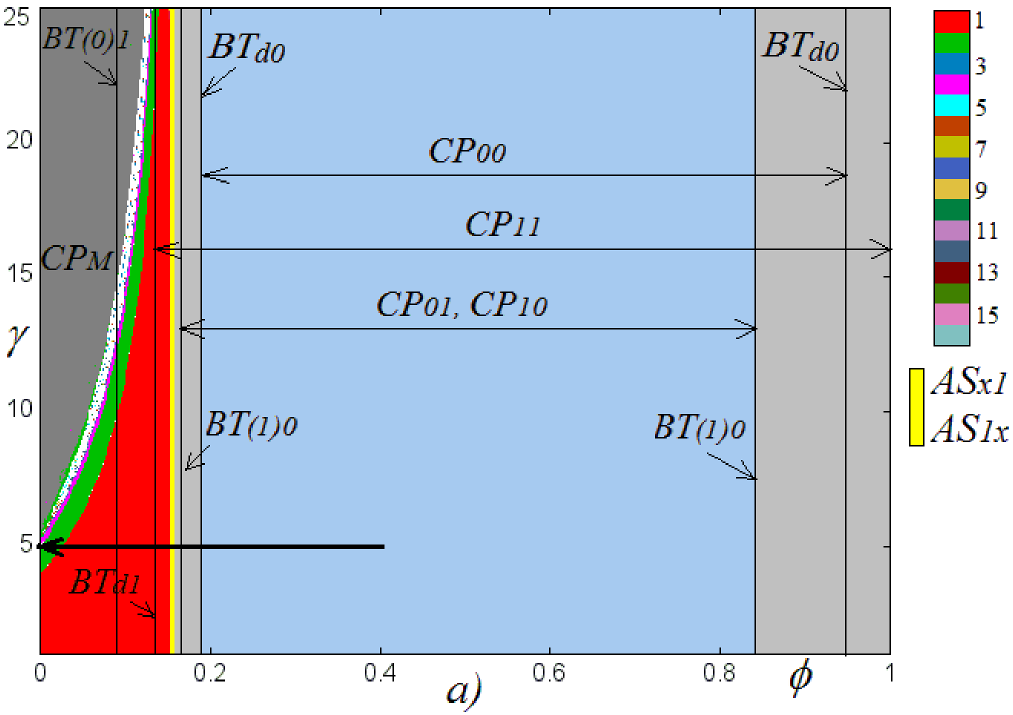
AS_{0a} : (x, y) = (0, a) , AS_{a0} : (x, y) = (a, 0)
AS_{1a} : (x, y) = (1, a) , AS_{a1} : (x, y) = (a, 1)

Interior fixed points are determined by the conditions : Ω_h(x,y)= 1 and Ω_f(x,y) = 1
Border fixed points by the condition Ω_h(x,y) = 1 and y = 0 or y = 1 (on lower side or upper side) or
by the condition Ω_f(x,y) = 1 and x = 0 or x = 1 (on left side or right side)

(Stable) long-run stationary equilibra

- CP equilibria: the manufacturing activity is agglomerated within countries
- Interior symmetric equilibrium: industry is distributed within countries (with a larger share in the border region);
- Border asymmetric fixed points: industry is agglomerated in one Country and dispersed in other country (with a larger share in the border region).

Bifurcation scenarios varying internal trade costs, ϕ



The bifurcation curves define the stability regions of fixed points

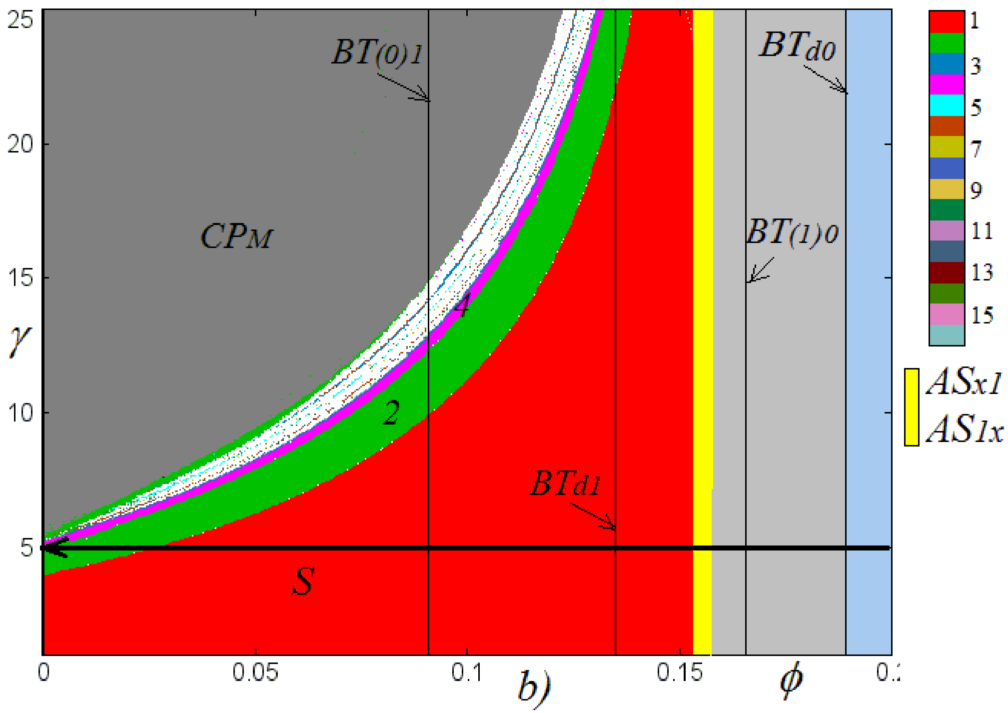
BT_{d0} : CP_{00}
BT_{d1} and 1: CP_{11}
$BT_{(1)0}$: CP_{01} and CP_{10}

Multistability:
Blue : 4 attracting fixed CP
Red : attracting symmetric equilibrium

γ is the migration speed

$\sigma = 2$, ϕE = 0.1

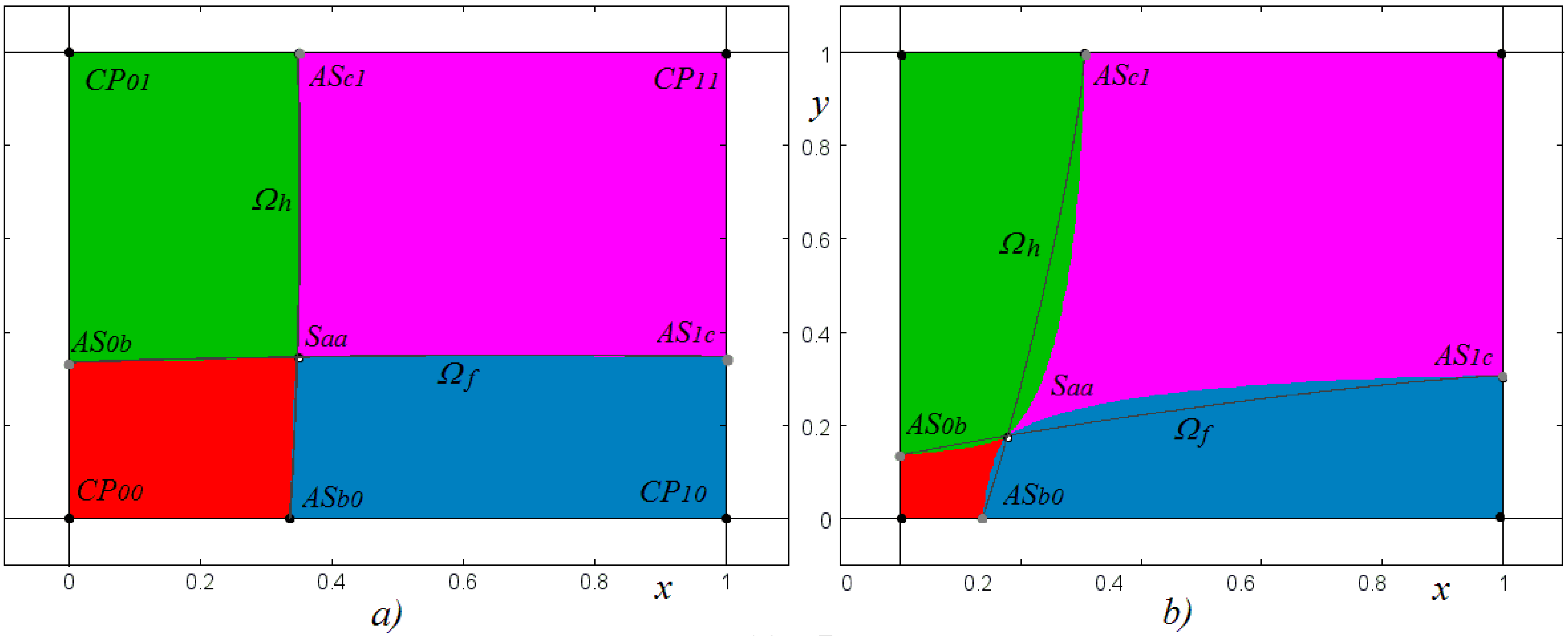
Bifurcation scenarios varying internal trade costs, ϕ



Green region : stable period-2 cycle
Magenta region: stable period-4 cycle
White area : higher periodicity, chaos
Dark Grey : CP as Milnor attractors
Yellow : attracting border asymmetric fixed points

γ is the migration speed

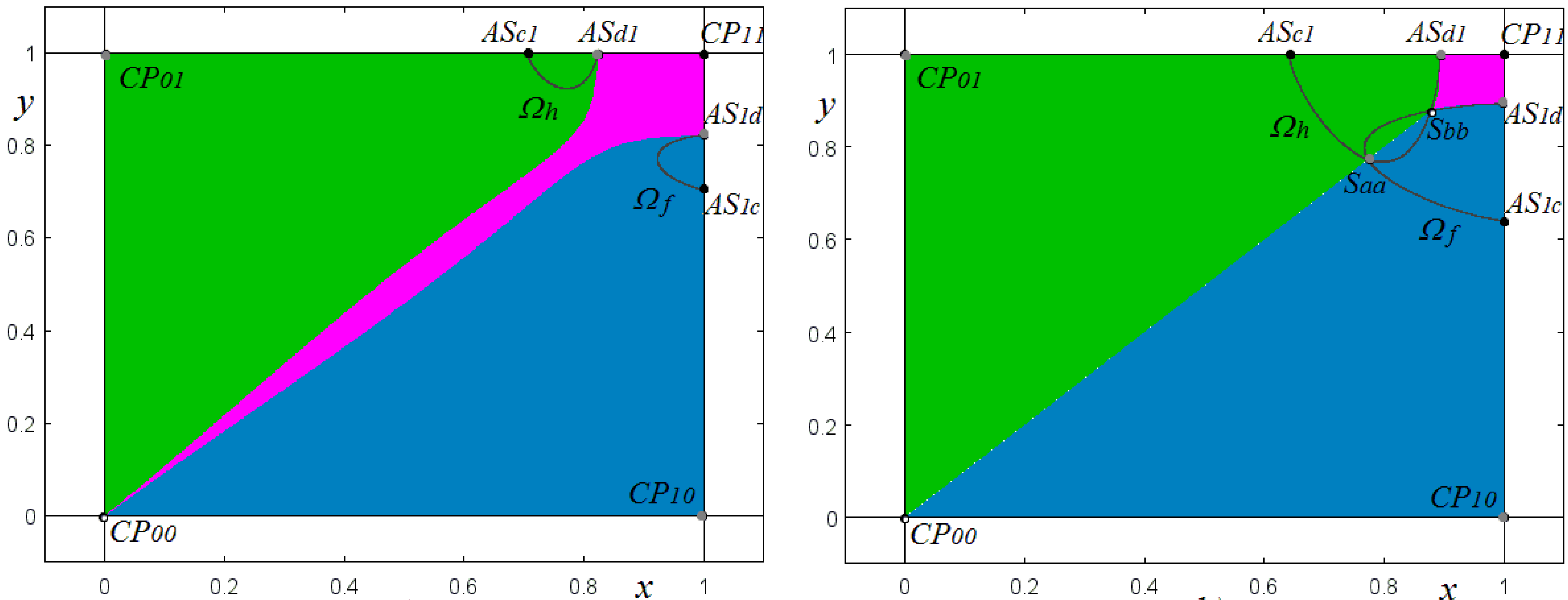
Bifurcation scenarios varying internal trade costs, ϕ



$\Phi = 0.4$ $\Upsilon = 5$ $\Phi = 0.25$

Basins of attraction of four CP stable fixed points -
separated by stable sets of border saddle fixed points, AS_{b0}, AS_{0b}, AS_{1c}, AS_{c1}
As ϕ is decreased the basin of CP_{11} enlarges and the basins of CP_{00}, CP_{01} and CP_{10} shrink

Bifurcation scenarios varying internal trade costs, ϕ



a) $\Phi = 0.16$

$\Upsilon = 5$

b) $\Phi = 0.155$

We are in the yellow region:

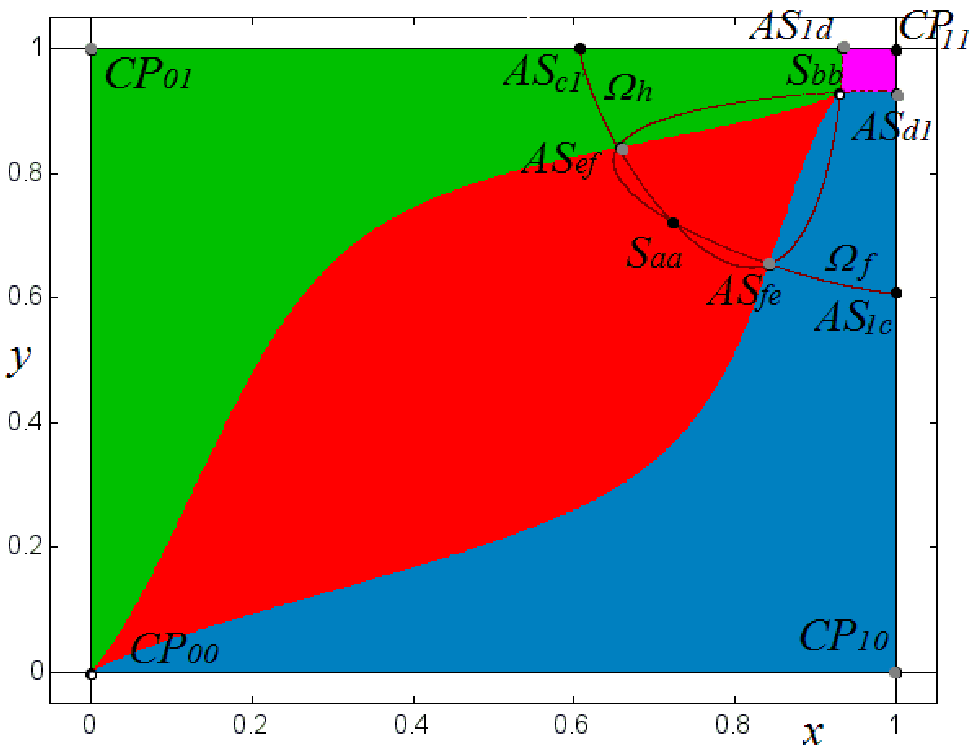
a) CP_{00} (after crossing BT_{d0}) has lost stability and CP_{10} and CP_{10} (after crossing $BT_{(1)0}$) are saddle;

Via fold bif.: two (asymmetric) stable border fixed poits AS_{c1} and AS_{1c} and 2 saddle (AS_{d1} and AS_{1d})

b) Decreasing ϕ the basin of CP_{11} shrinks;

Via fold bif: 2 interior fixed points emerge S_{aa} (saddle), S_{bb} (repelling)

Bifurcation scenarios varying internal trade costs, ϕ

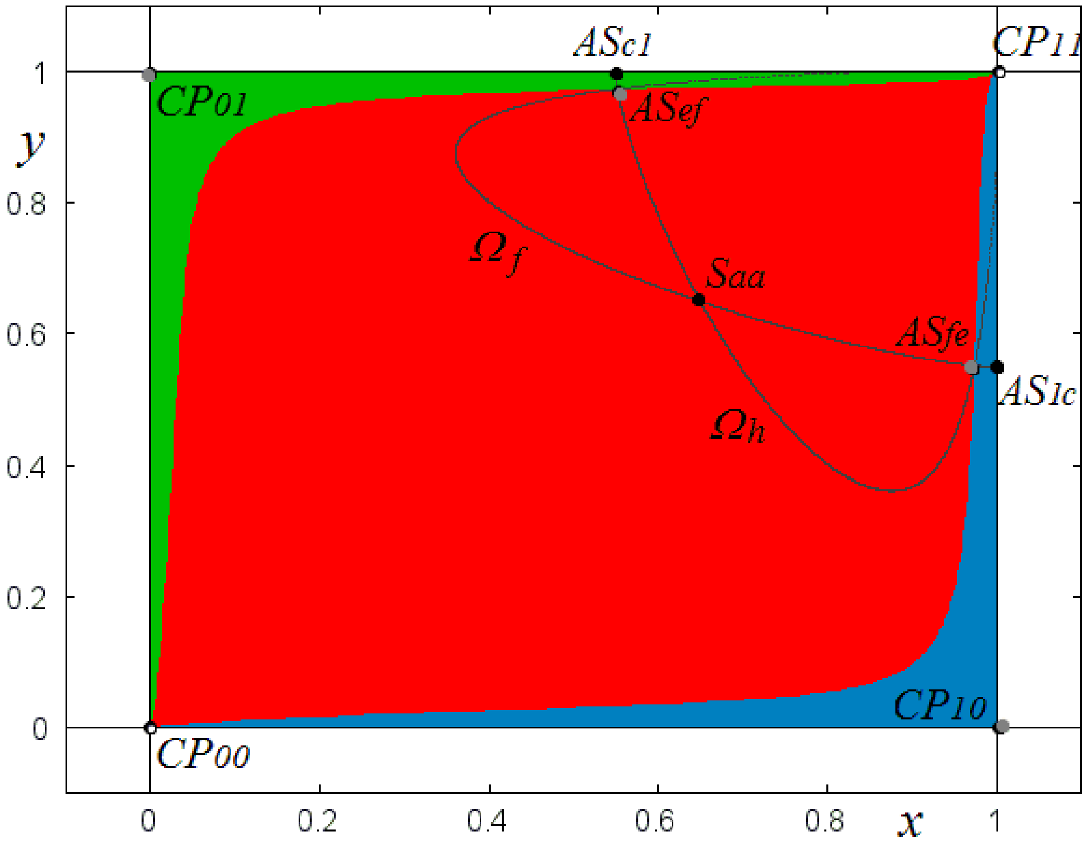


We are in the (red) region :

- Via (subcritical) pitchfork bif.:
  1. the stable symmetric interior fixed point S_{aa};
  2. 2 saddles interior asymmetric fixed points: AS_{ef} and AS_{fe};
- Four co-existing stable equilibria: CP_{11}, S_{aa}, AS_{c1}, AS_{1c}

$\Phi = 0.15$

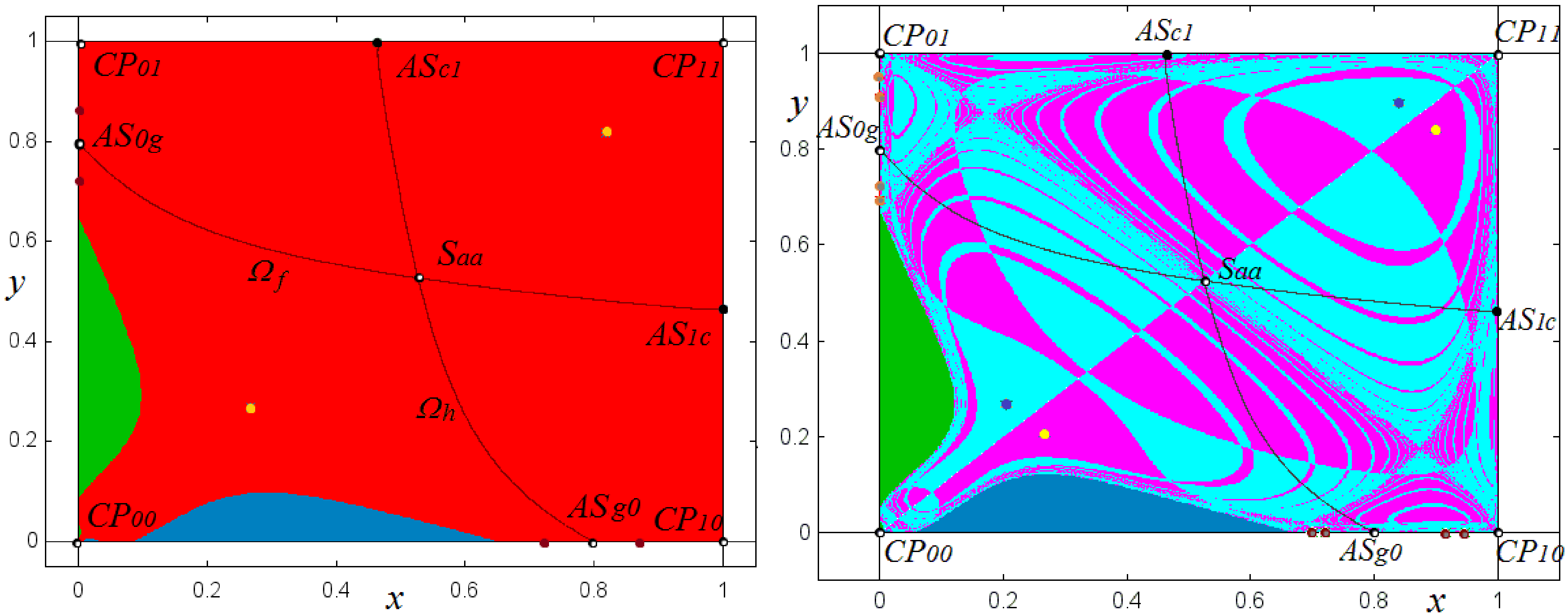
Bifurcation scenarios varying internal trade costs, ϕ



After crossing BT_{d1} , CP_{11} loses stability (merging with Sbb, AS1d and ASd1);
three co-existing stable equilibria: S_{aa}, AS_{c1}, AS_{1c}
Further decreasing ϕ, AS_{c1} and AS_{1c} lose stability, S_{aa} only stable equilibrium

Φ = 0.134

Bifurcation scenarios varying internal trade costs, ϕ



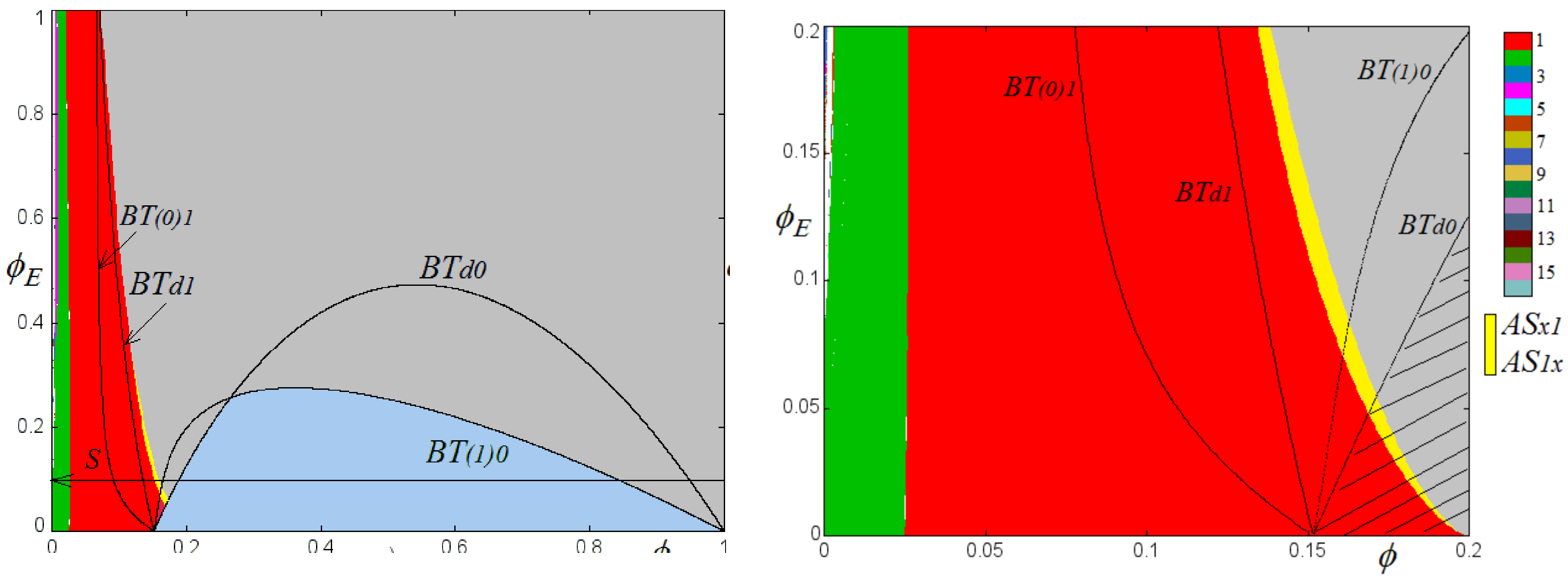
a) Φ = 0.01 ϒ = 5 b) Φ = 0.004

a) After Flip bifurcation of S_{aa} : attracting 2-cycle (on the diagonal); 2 saddle period-2 cycles (on y = 0 and x = 0);

b) After supercritical pitchfork bifurcation of period-2 cycle: 2 stable period-2 cycles (symmetric along the diagonal) ; 2 Saddle period-4 cycles (on y=0 and x=0);

a) and b) AS_{c1} and AS_{1c} Milnor attractors.

Bifurcation scenarios varying external trade costs, ϕ_E



ϕ_E is the international trade freeness

Conclusions

1. Increasing internal (and external) trade freeness enhances the likelihood of industrial agglomeration in the two countries
2. Reducing internal (and external) trade freeness increases the likelihood of industrial dispersion within the two countries (but also the emergence of complex behaviour).
3. Increasing the mobility speed could also make more likely agglometarion of industrial activities but this occurence is much less predictable.
4. Rich multistability scenarios.