Product Innovation Incentives by an Incumbent Firm: A Dynamic Analysis

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Agenda

- Gain new insights into intertemporally optimal innovation behavior of incumbents in dynamic oligopolies
- Present a (new) numerical method to determine boundaries of basins of attraction between locally stable steady states in dynamic optimization problems and dynamic games.

Motivation

- The evolution of many industries is characterized by the repeated emergence of new 'sub-markets', due to the introduction of products with differentiated functionality/technology relative to the established products.
    - hybrid cars, convertible minis
    - netbooks
    - flatscreen TVs
    - E-Readers
- New submarkets do not replace products in the existing product range, but these products are added to the product range of existing producers
- Firms have to invest in R&D in order to develop the new products

Research Questions

- How are the incentives to invest in product innovation influenced by the production capacities of the firm on the established market?
- Might a strong position on the established market prevent a firm from product innovation?
- How do the answers to the questions above depend on the quality of the new product and the degree of substitutability between the established and the new product?

General Model Setup

- Monopolist offers an established 'old' product (product 1).
- Standard capacity dynamics for production of old product:

$$\dot{K}_1 = I_1 - \delta_1 K_1 \quad \delta_1 > 0, K_1(0) = K_1^{ini}$$

- At $t = 0$ the firm starts an innovation project aiming at the development of a new differentiated product (product 2). Arrival time τ is stochastic. Arrival rate of the new product depends on current R&D investment $I_R(t)$ and knowledge stock $K_R(t)$:

$$\lambda(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} Prob\{\tau \in [t, t + \Delta] \mid \tau \geq t\} = \alpha I_R(t) + \beta K_R^{\psi}(t)$$

- Dynamics of the knowledge stock:

$$\dot{K}_R = I_R - \delta_R K_R \quad \delta_R > 0, K_R(0) = 0$$

General Model Setup

- 2 modes:

$$m(t) = \begin{cases} m_1 & t < \tau \\ m_2 & t \geq \tau \end{cases}$$

- Firm starts with $K_2(\tau) = 0$ at time τ when it introduces the new product.

$$\dot{K}_2 = I_2 - \delta_2 K_2 \quad \delta_2 > 0,\ t \geq \tau$$

- Capacities on both markets and knowledge stock are adapted dynamically with linear-quadratic costs.

Optimization Problem

- Objective Function:

$$J = \mathbb{E}\left[\int_0^\infty e^{-rt} \left[(1 - K_1 - \eta K_2)K_1 + (1 + \theta - \eta K_1 - K_2)K_2 \right.\right.$$
$$\left.\left. -\mu_1 I_1 - \frac{\gamma_1}{2} I_1^2 - \mu_2 I_2 - \frac{\gamma_2}{2} I_2^2 - \mu_R I_R - \frac{\gamma_R}{2} I_R^2 \right] dt \right],$$

- State and Mode Dynamics:

$$\dot{K}_i = I_i - \delta^i K_i, \qquad i \in \{1, 2, R\}$$
$$K_i(t) \geq 0 \ \forall t \geq 0, \quad i \in \{1, 2, R\}$$
$$\lim_{\Delta \to 0} \tfrac{1}{\Delta} Prob \{m(t + \Delta) = m_2 \mid m(t) = m_1\} = \alpha I_R(t) + \beta K_R^{\psi}(t)$$
$$I_2(t) = 0, \ \forall t \text{ s.t. } m(t) = m_1$$
$$K_1(0) = K_1^{ini} \geq 0, \quad K_2(0) = K_R(0) = 0, \quad m(0) = m_1$$

Mode m_2: After the innovation

- Obviously firm chooses $I_R(t) = 0$ for all $t > \tau$.
- $\Rightarrow$ In mode m_2 firms faces standard capital accumulation problem with two products.
- Linear-quadratic problem $\Rightarrow$ quadratic value function $V_{(m_2)}(K_1, K_2)$
- Under appropriate conditions unique stable positive steady state exists

Mode m_1: Before the innovation

- Non linear-quadratic problem with stochastic terminal time τ.
- Combination of Maximum Principle and Numerical Dynamic Programming (collocation with Chebyshev polynomials) used for analysis
- HJB Equation:

$$
\begin{aligned}
rV_{(m_1)}(K_1, K_R)) = \max_{I_1,\, I_R} \Big[& K_1(1-K_1) - \mu_1 I_1 - \tfrac{1}{2}\gamma_1 {I_1}^2 \\
& + \frac{\partial V_{(m_1)}}{\partial K_1}(I_1 - \delta_1 K_1) - \mu_R I_R - \tfrac{1}{2}\gamma_R {I_R}^2 + \frac{\partial V_{(m_1)}}{\partial K_R}(I_R - \delta_R K_R) \\
& + (\alpha I_R + \beta K_R^{\psi})(V_{(m_2)}(K_1, 0) - V_{(m_1)}(K_1, K_R) \Big]
\end{aligned}
$$

- Optimal Investments

$$
\begin{aligned}
I_1^* &= \frac{1}{\gamma_1}\left(\frac{\partial V_{(m_1)}}{\partial K_1} - \mu_1\right) \\
I_R^* &= \frac{1}{\gamma_R}\left(\frac{\partial V_{(m_1)}}{\partial K_R} - \mu_R + \alpha(V_{m_2}(K_1, 0) - V_{(m_1)})\right)
\end{aligned}
$$

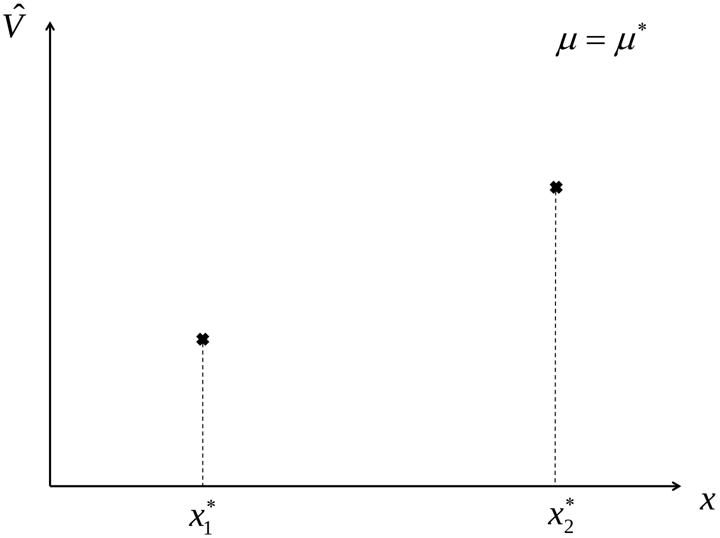
Steady states of the cannonical system in m_1 derived through the maximum principle

- Three steady states: two with positive knowledge stock, one with zero knowledge stock (and zero hazard rate).
- The first and the third are saddle points in the cannonical system (i.e. candidates for fixed points of the dynamics under opitmal investment strategies).
- Global analysis based on HJB approach is needed to find out which fixed point is reached for which initial values of (K_1, K_R)!

Collocation in Models with Multiple Stable Fixed Points

- Numerical solution of HJB often obtained through polynomial approximation of value function (collocation method).
- In dynamic optimization problems (dynamic games) where the optimized dynamics has multiple (locally) stable fixed point the optimal control typically jumps at the boundary between the basins of attraction.
- A jump in the control corresponds to a jump in the derivative of the value function (a kink).
- $\Rightarrow$ the standard collocation method based on polynomial approximation is not able to capture the qualitative features of the optimal feedback and the value function.
- Use a new method to calculate approximations of 'local value functions' around the steady state.

Combining Collocation and Homotopy..



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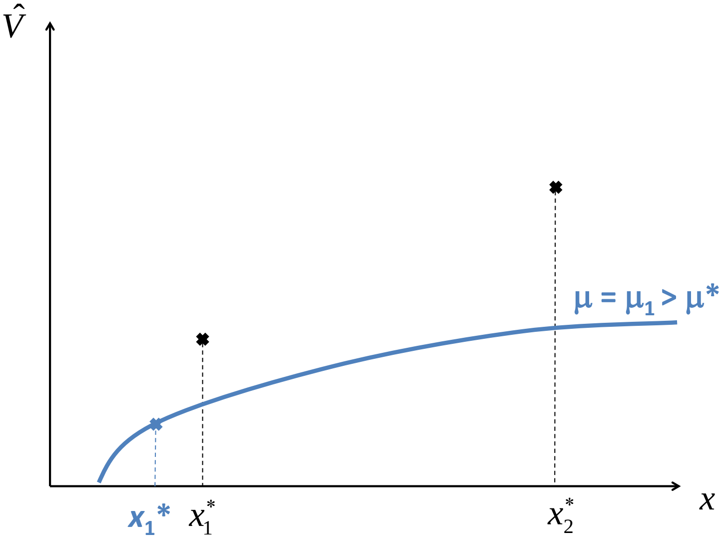

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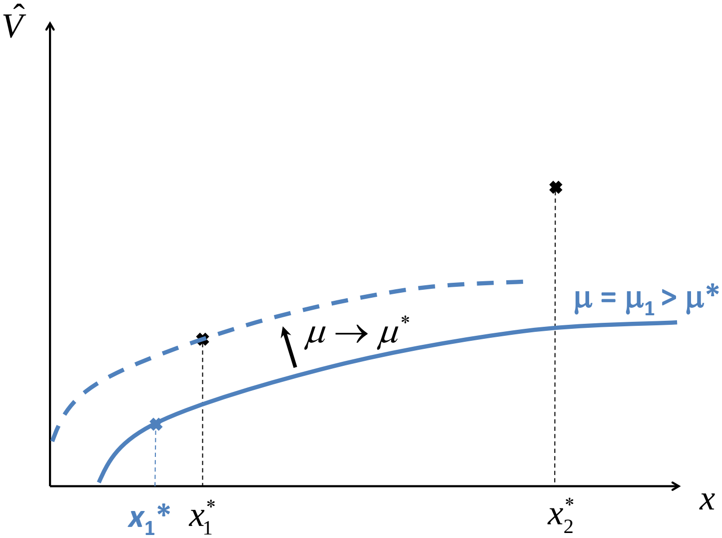

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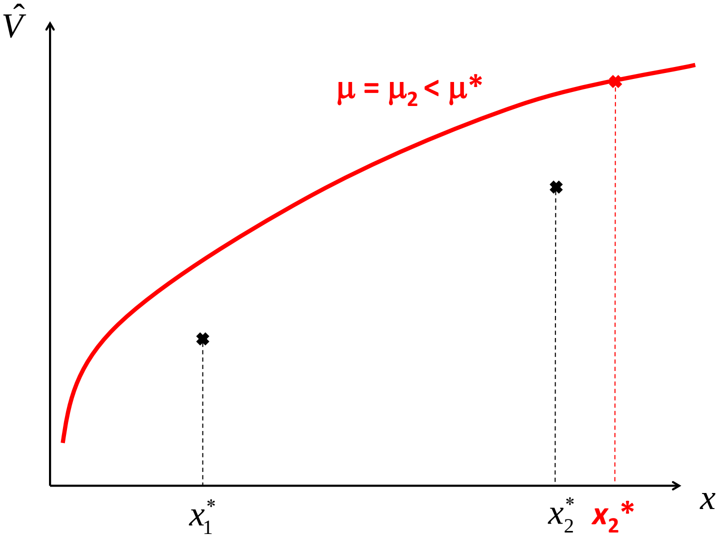

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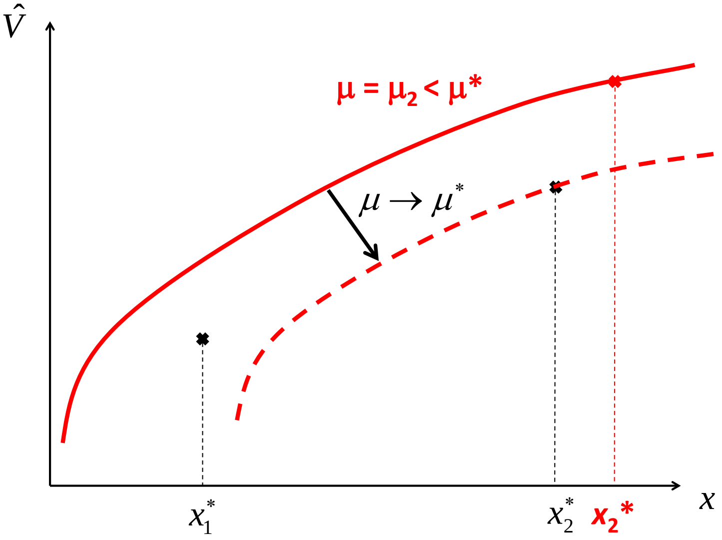

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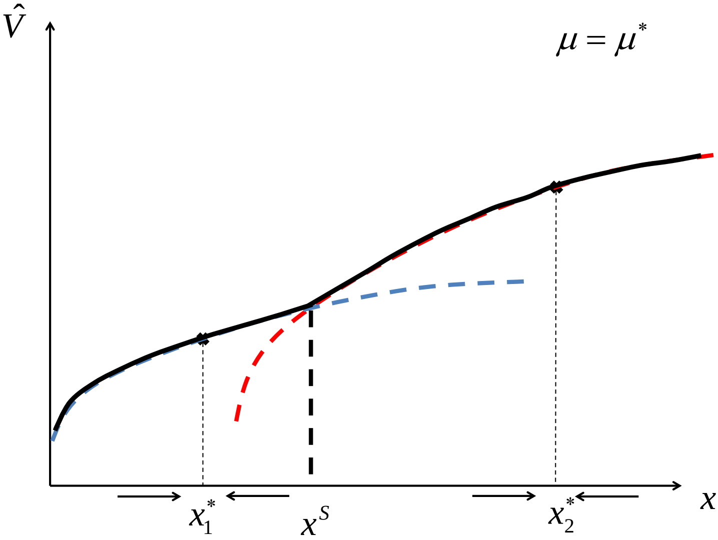
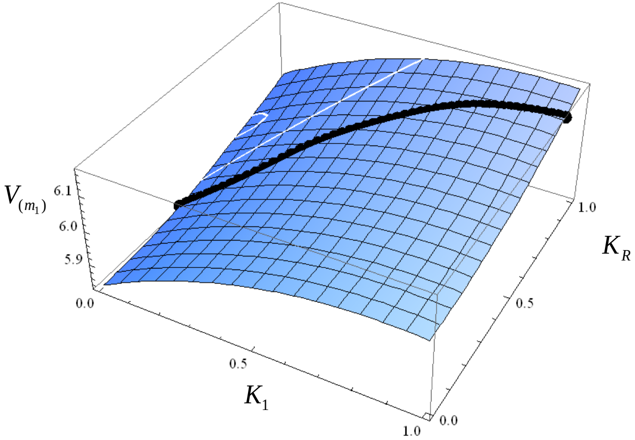
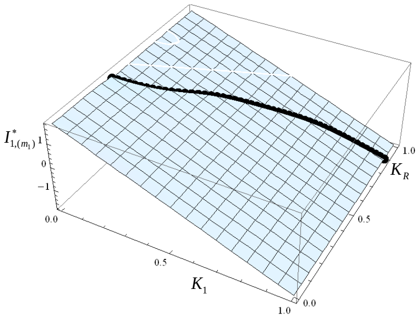


Value function (black line is the boundary of the basins)

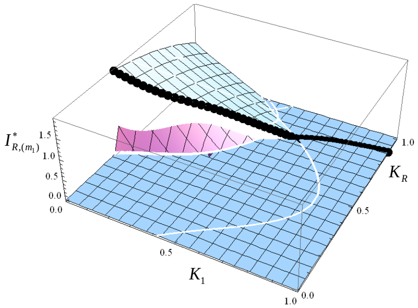


Optimal Investment Functions

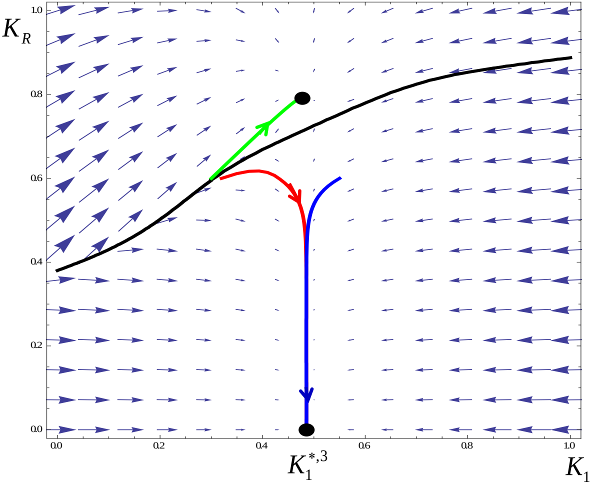
Physical Captial



R&D

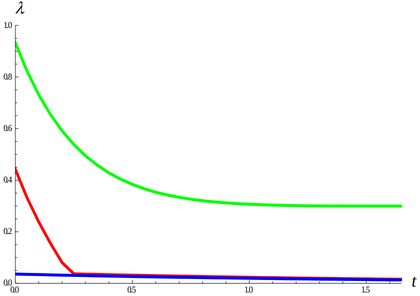


Optimal Dynamics

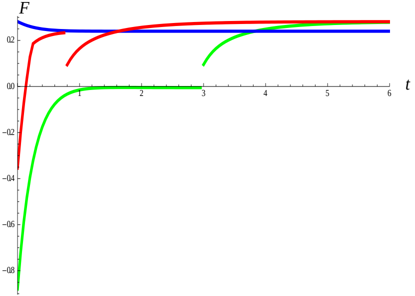


Optimal Dynamics

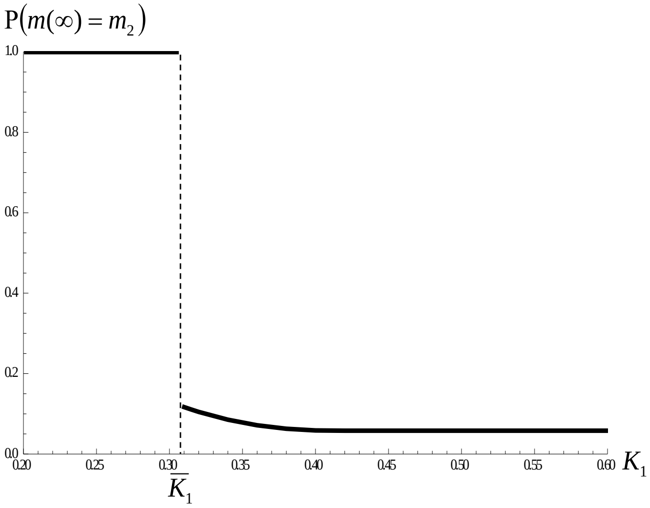
Hazard Rate



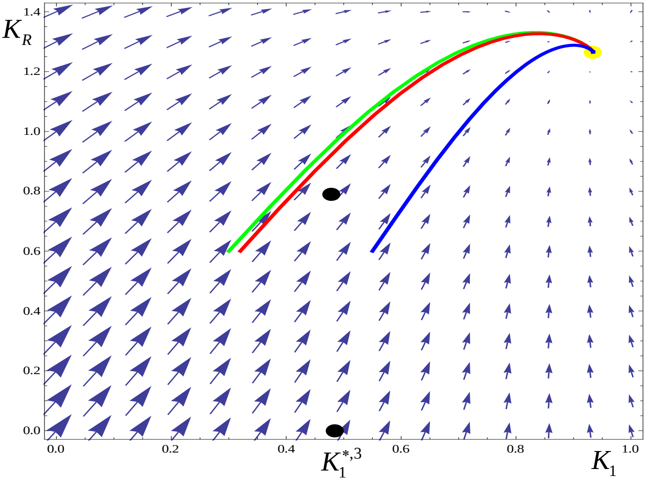
Inst. Payoff



Innovation Probability

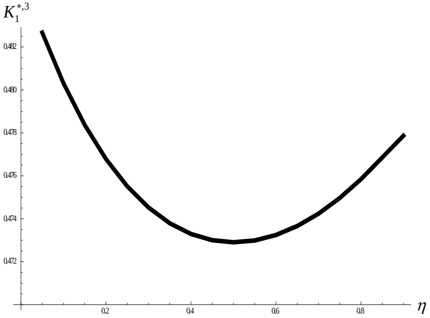


Welfare Optimizing Dynamics

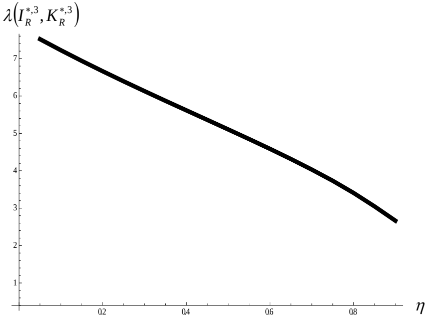


Sensitivity: Horizontal Differentiation Parameter (positive Steady State)

Physical Capital



Hazard Rate



Conclusions

- Initial capacity on established markets negatively (for substitutes) influences R&D invesmtents and innovation rate.

- In scenarios with two steady states long run state and mode depend on initial conditions: large initial capacities may prevent innovation in the long run.

- 'Skiba area' of initial conditions is characterized where different long run states and modes have positive probability.

Extensions

- Oligopolistic Competition
- Anticipatory build-up of capacities for the new product
- Delays between R&D investment and effect on innovation rate
  $\Rightarrow$ 'time-to-build'-type problem

Thank you for your attention !