A nonlinear Cournot duopoly with advertising

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Outline

- **Motivations and existing literature**
- **The model**
- **Dynamics**
- **Conclusions**

Motivations

- **Study local and global dynamics of a nonlinear duopoly (Bischi et al., 1998; Bischi et al., 2010) with quantity setting firms and advertising investments that affect the degree of (horizontally) differentiated products (two strategic variables);**

Advertising

- **Purely informative advertising (Grossman and Shapiro, 1984; Stahl, 1994; Hamilton, 2009). It is advertising provides information about price and product characteristics, but it does not influence the consumers' willingness to pay for the advertised good. As a consequence, it tends to favour competition, for instance by reducing search costs**

- **Persuasive advertising (Dixit and Norman, 1978). It is anticompetitive because it tends to realise a spurious product differentiation, which in turn increases firms' market power. Product differentiation has become of greater importance in oligopoly markets both with profit maximizing firms (Singh and Vives, 1984) and managerial firms (Kopel and Lambertini, 2013)**

Advertising and quantity competition

- **When firms compete on multiple dimensions (e.g. quantity or price and advertising), semi-collusion may occur if economic agents choose to cooperate along some dimension(s) (advertising) while competing on another ones (price or quantity).**

- **At the present stage of the work, we have focused on noncooperative advertising with quantity-setting firms**

Advertising

- **The dynamic duopoly literature has dealt with similar topics in some works that concentrate on the relationship between advertising and goodwill (Luhta and Virtanen, 1996; Ahmed et al., 1999), and the effect of brand competition on global dynamics and multistability in models with brand competition and players with limited information (Bischi et al., 2000; Bischi and Kopel, 2003).**

- **Differently, the present model is focused on the long-term effects of advertising on product differentiation.**

The model

- **Consumers. The economy is comprised of identical consumers that have preferences towards goods q1, q2 and k captured by the separable utility function**

$$V(q_1, q_2, k) = U(q_1, q_2) + k,$$

where

$U(q_1, q_2) : R^2_+ \rightarrow R_+$ is a twice differentiable function.

The representative consumer maximises $V(q_1, q_2, k)$ subject to

$$p_1 q_1 + p_2 q_2 + k = M$$

The utility function U is defined as

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2dq_1q_2)$$

Consumer maximization programme gives the following market demands of goods 1 and 2, respectively

$$p_1 = \max\{0, a - q_1 - dq_2\} \quad \text{and} \quad p_2 = \max\{0, a - q_2 - dq_1\}$$

In order to guarantee that prices are non negative for any 0<d<1, q1 and q2 must belong to set A, where

$$A := \{q_1, q_2 : q_1 + q_2 \leq a\}$$

Firms.

- **Firm 1 and firm 2 may differentiate their products through advertising investments z1 and z2, respectively. In particular,**

$$d = d(z_1, z_2) := \frac{1}{2(1+z_1)} + \frac{1}{2(1+z_2)}$$

Firm i's cost function is

$$c_i = wq_i + gz_i$$

where w>0 is the marginal cost and g>0 is price per unit of advertising. There are constant marginal returns to labour and the production function is linear. Profits are:

$$\Pi_i = (p_i - w)q_i - gz_i$$

By considering the maximization of profits w.r.t. advertising investments

$$\max_{\{z_i\}} \Pi_i = \max_{\{z_i\}} \left\{ (\max\left[0, a - q_i - d(z_1, z_2) q_j\right] - w) q_i - g z_i \right\},$$

gives

$$z_i^{opt} := \max \left\{ 0, \sqrt{\frac{q_i q_j^e}{2g}} - 1 \right\}$$

Profits can then be written as follows

$$\tilde{\Pi}_i = \Pi_i(q_i, q_j, z_i^{opt}(q_i, q_j), z_j) = \begin{cases} \Pi_i(q_i, q_j, \sqrt{\frac{q_i q_j}{2g}} - 1, z_j), & \text{if} \quad q_i \geq \frac{2g}{q_j} \\ \Pi_i(q_i, q_j, 0, 0), & \text{if} \quad q_i < \frac{2g}{q_j} \end{cases}$$

$\tilde{\Pi}_i$ **is a continuous and differentiable function w.r.t. qi**

Marginal profits are

$$\frac{\partial \tilde{\Pi}_i}{\partial q_i} = \begin{cases} MP_i^{ADV} := (a - w + 2q_i)\sqrt{q_i} - \sqrt{\frac{2gq_j}{q_i}}, & \text{if} \quad q_i \geq \frac{2g}{q_j} \\ MP_i^{NADV} := a - w - 2q_i - q_j, & \text{if} \quad q_i < \frac{2g}{q_j} \end{cases}$$

According to Bischi et al. (1998) and Fanti et al. (2013b), where non-negative constraints on quantity are introduced, we have the following map:

$$T : \begin{cases} q_1' = \max\left\{0, q_1 + \alpha q_1 \frac{\partial \tilde{\Pi}_1}{\partial q_1}\right\} \\ q_2' = \max\left\{0, q_2 + \alpha q_2 \frac{\partial \tilde{\Pi}_2}{\partial q_2}\right\} \end{cases}$$

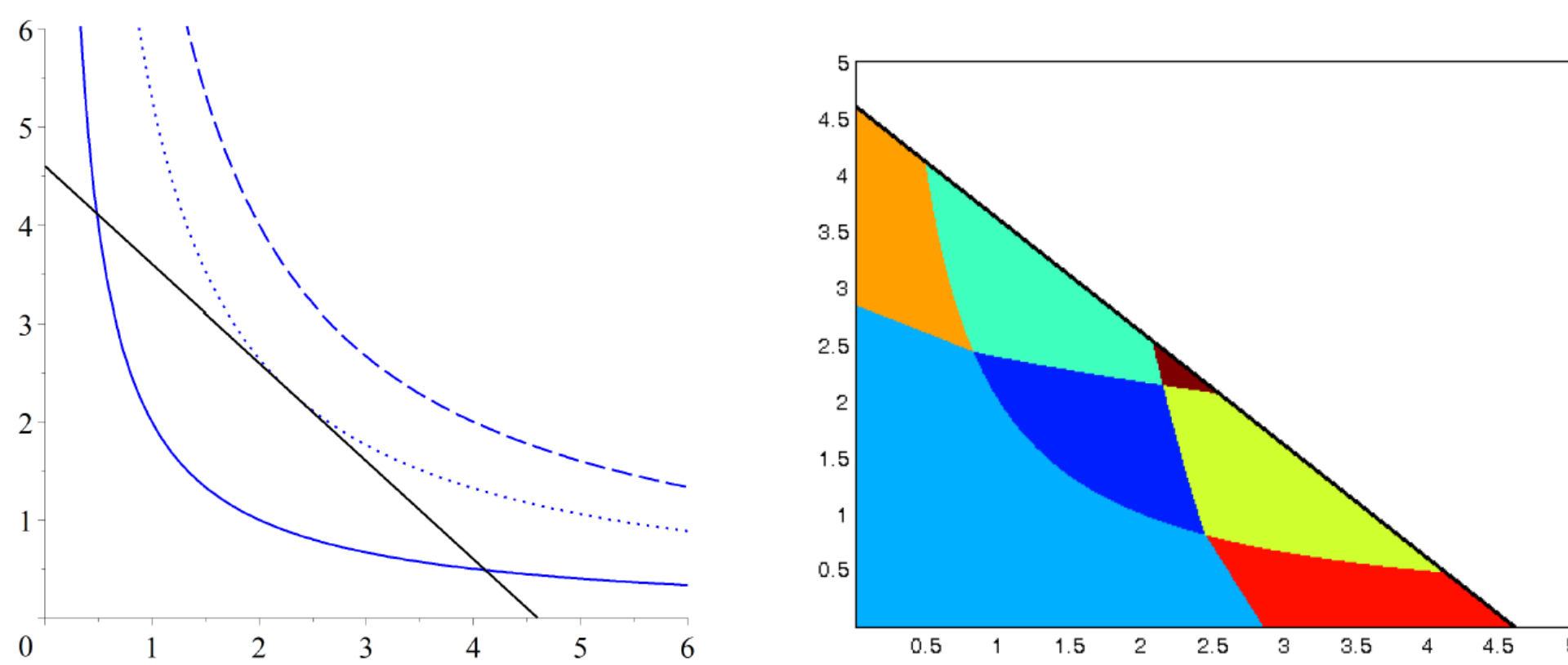
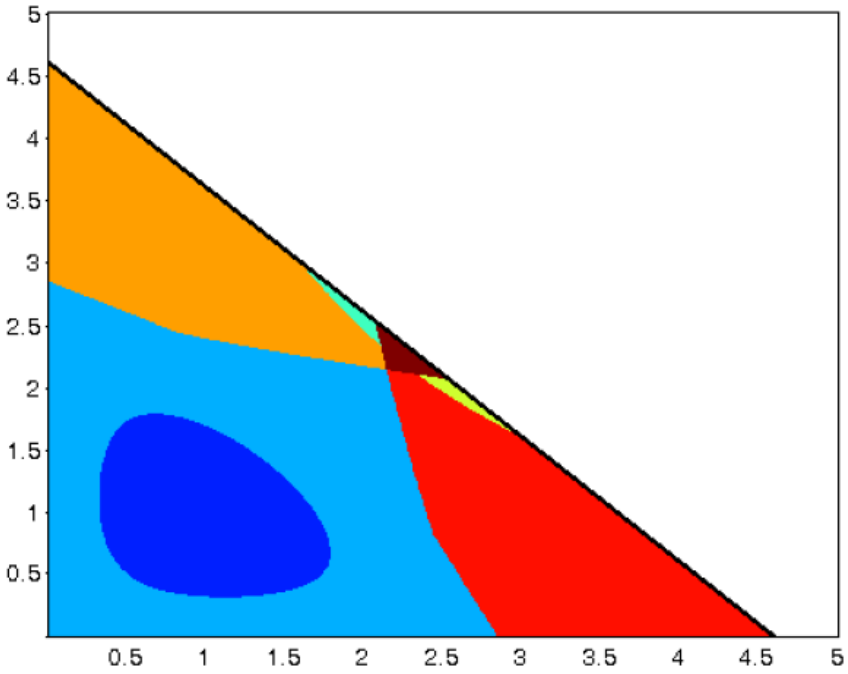


Figure 1. Panel (a). The sets Ω_1, Ω_3 and Ω_4 lie at north-east with respect to curve $q_1 q_2 = 2g$. For sufficiently high values of g, that is $g > a^2/8$ (the price of advertising is relatively high), curve $q_1 q_2 = 2g$ (dashed line) completely lies outside of triangle A and then maps defined at least by one M_i are not involved in the dynamics of the model. In this case the dynamics are described only by maps T_2, T_5 and T_6. The solid line describes case $g < a^2/8$ (the price of advertising is relatively low) for which sets Ω_i are non-empty. The dotted line ($g = a^2/8$) represents the separating element between the two cases. Panel (b). Case $g < a^2/8$. Regions Ω_i $(i = 1, ..., 7)$ of the phase plane are evidenced by different colors. The dark-blue region is Ω_1, the light-blue region is Ω_2, the aqua green region is Ω_3, the green region is Ω_4, the orange region is Ω_5, the red region is Ω_6 and the blood-red region is Ω_7.

With regard to the dynamic analysis at time t = 0 (the time in which both firms enter the market), it is assumed that each firm does not know the quantity produced by the rival and then it cannot decide the optimal value of z through equation (6). Therefore, at t = 0 firm i has to simultaneously choose both the quantity qi and advertising investment zi.

According to the partition of the phase plane and depending on expectations that firm i has on the behaviour of firm j, at time t = 1 the system will be in one of the region Ωi.



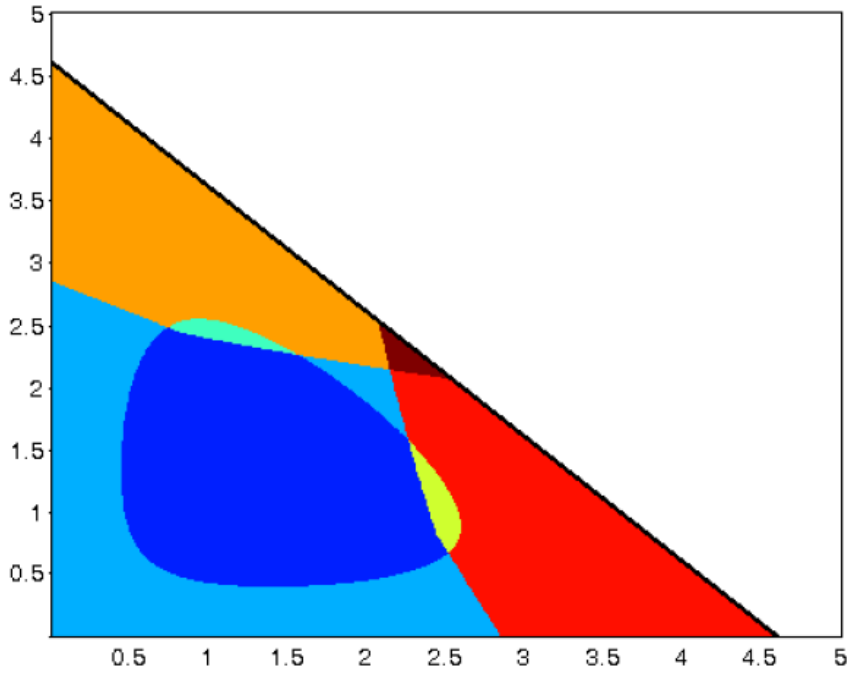


Figure 2. Panel (a). At time $t = 0$, each firm chooses quantity and advertising investment by expecting no advertising investments by the rival. Panel (b). At time $t = 0$, each firm chooses quantity and advertising investment by expecting positive advertising investments by the rival. Different colours evidence different behaviours of map T at $t = 0$. Colours in Figures 2.a and 2.b correspond to colours in Figure 1.b. Specifically, an initial condition $(q_1(0), q_2(0))$ that starts from the dark-blue region in both figures is mapped into Ω_1 at time $t = 1$ (a similar behaviour holds for other colours).

Fixed points

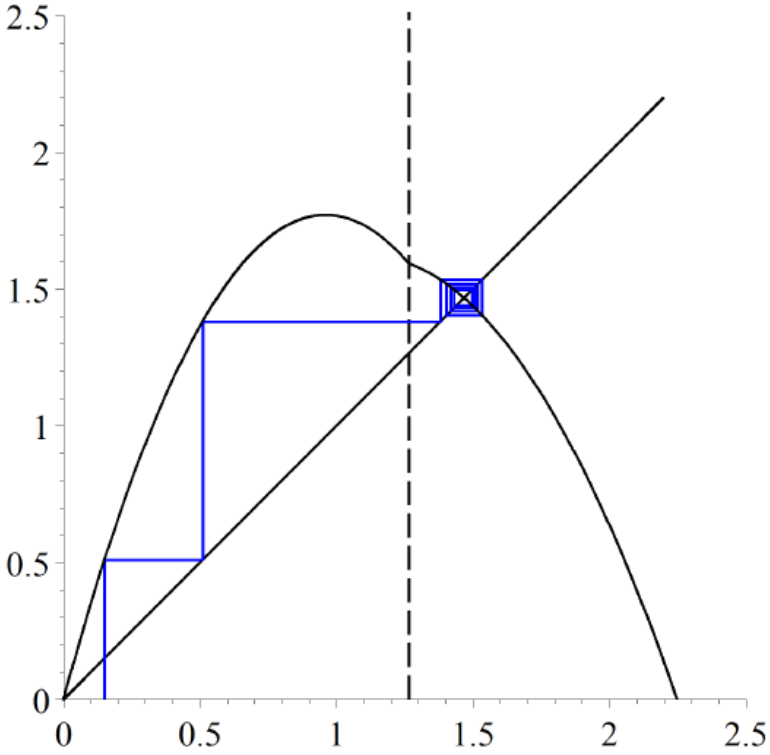
$$E_0 = (0,0), \quad E_1 = \left(\frac{a-w}{2}, 0\right), \quad E_2 = \left(0, \frac{a-w}{2}\right)$$

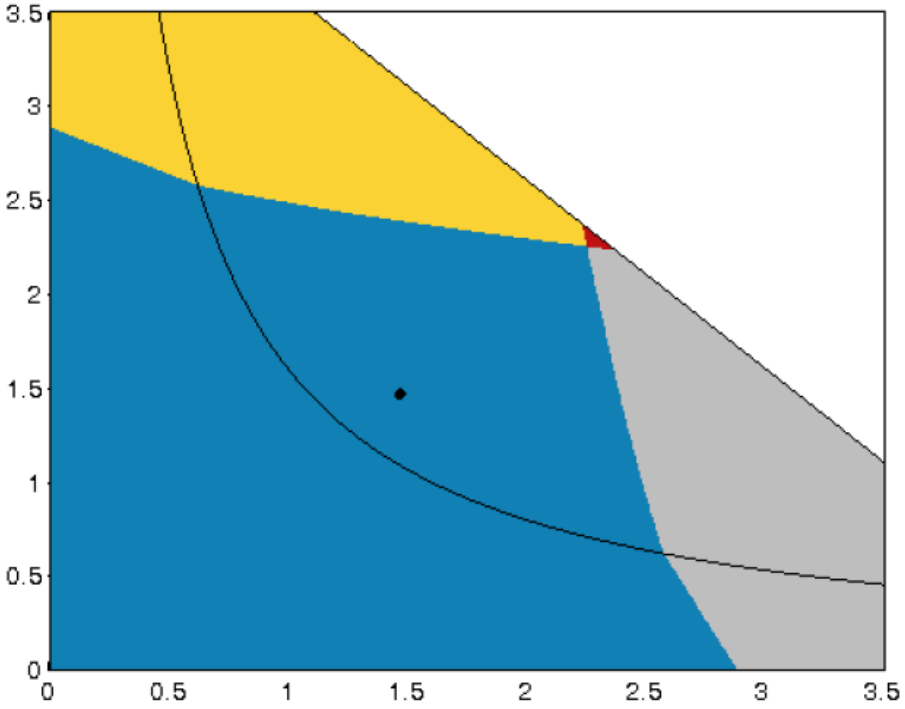
E0 is a source, E1 and E2 are saddles

$$E^* = \begin{cases} \left(\frac{a-w-\sqrt{2g}}{2}, \frac{a-w-\sqrt{2g}}{2}\right), & \text{if} \quad g \leq \frac{(a-w)^2}{18} \\ \left(\frac{a-w}{3}, \frac{a-w}{3}\right), & \text{if} \quad g > \frac{(a-w)^2}{18} \end{cases}$$

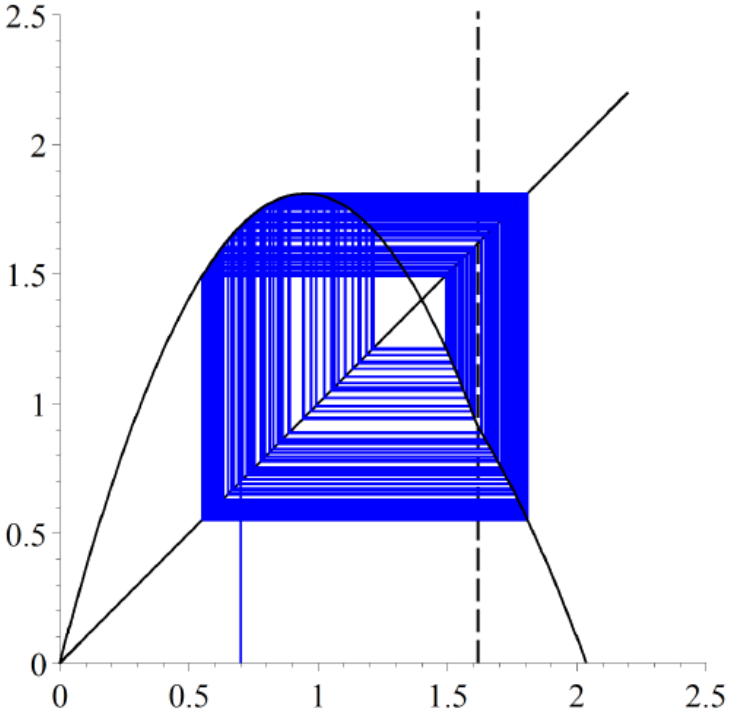
E* can be locally asymptotically stable. It can undergoes a supercritical flip bifurcation or a border collision bifurcation. This depends on parameter g (the unitary cost of advertising)

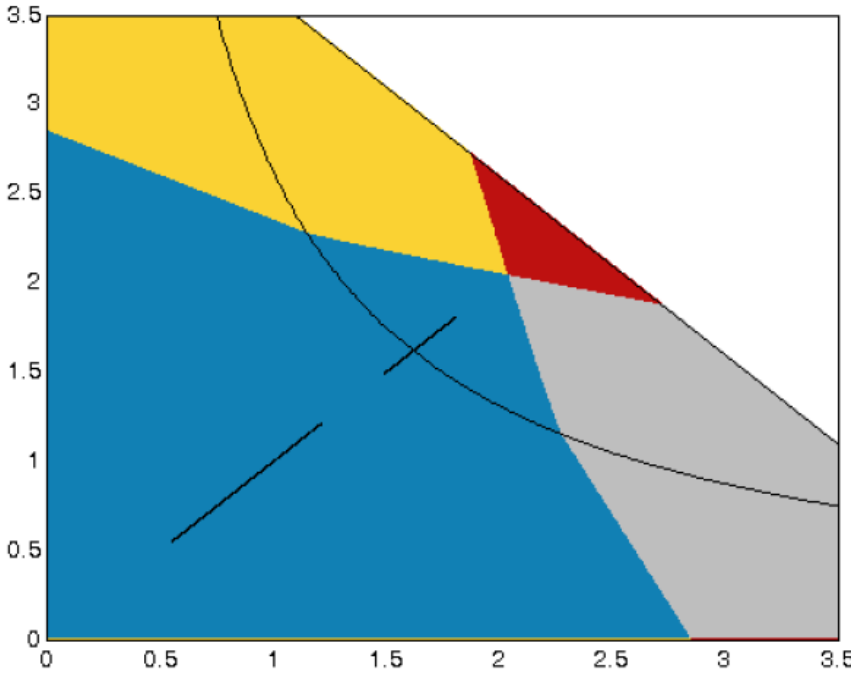
Case 1



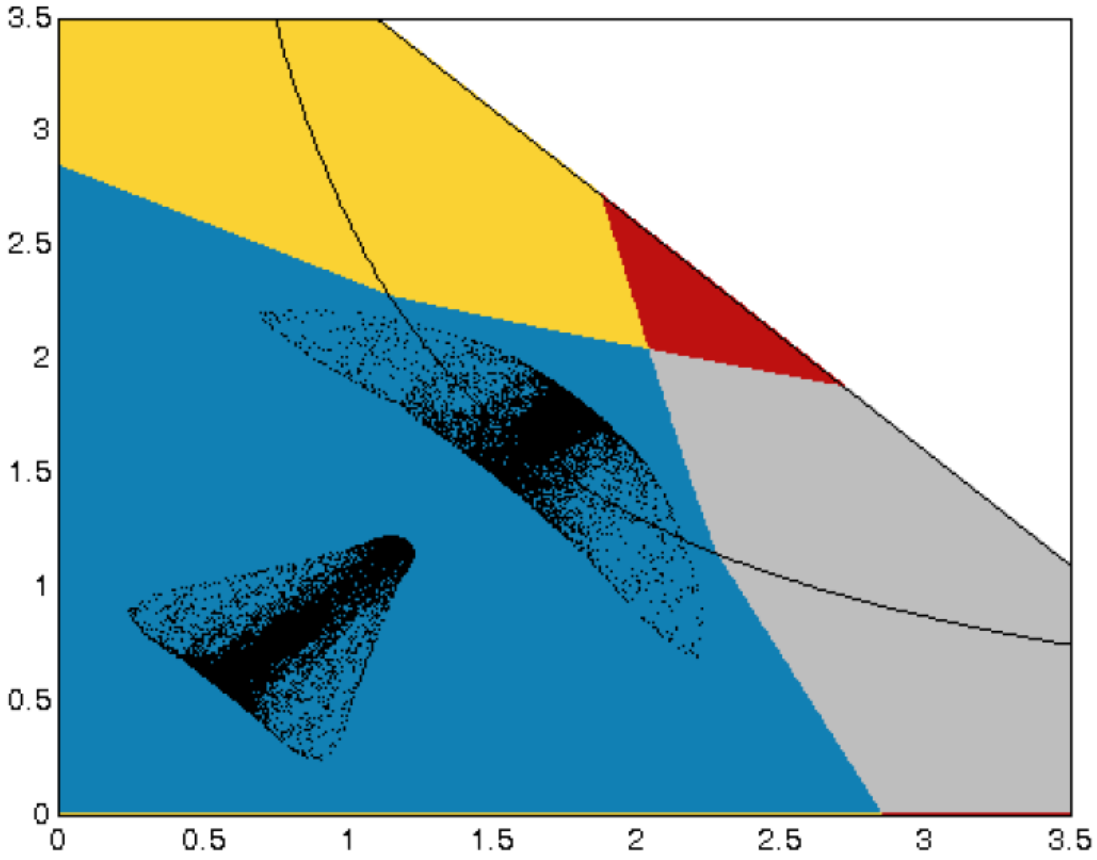


Case 2

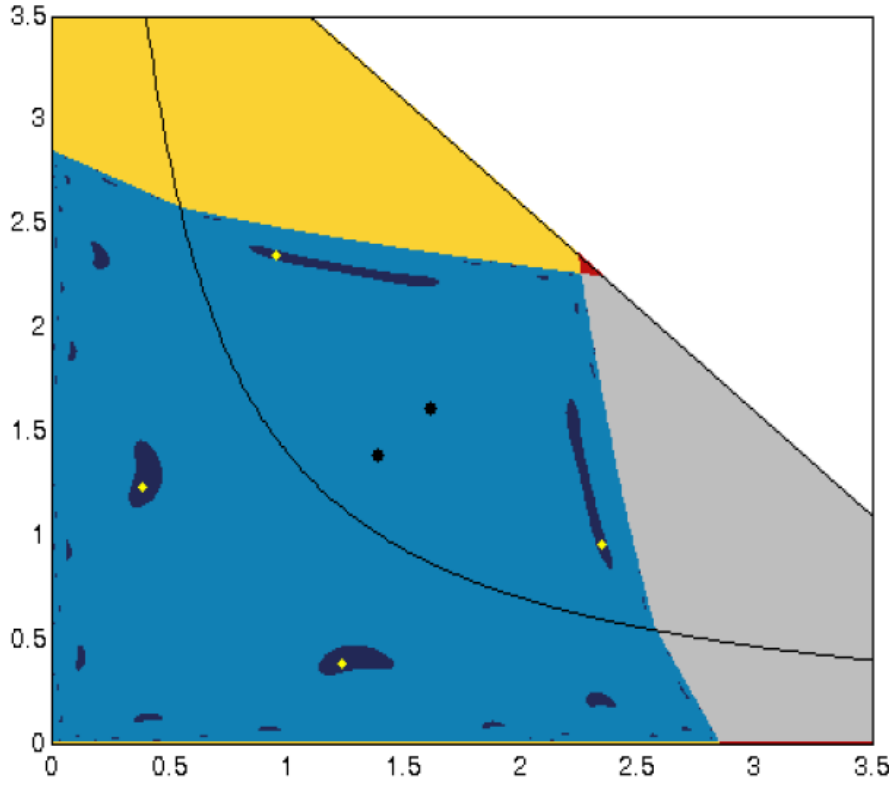


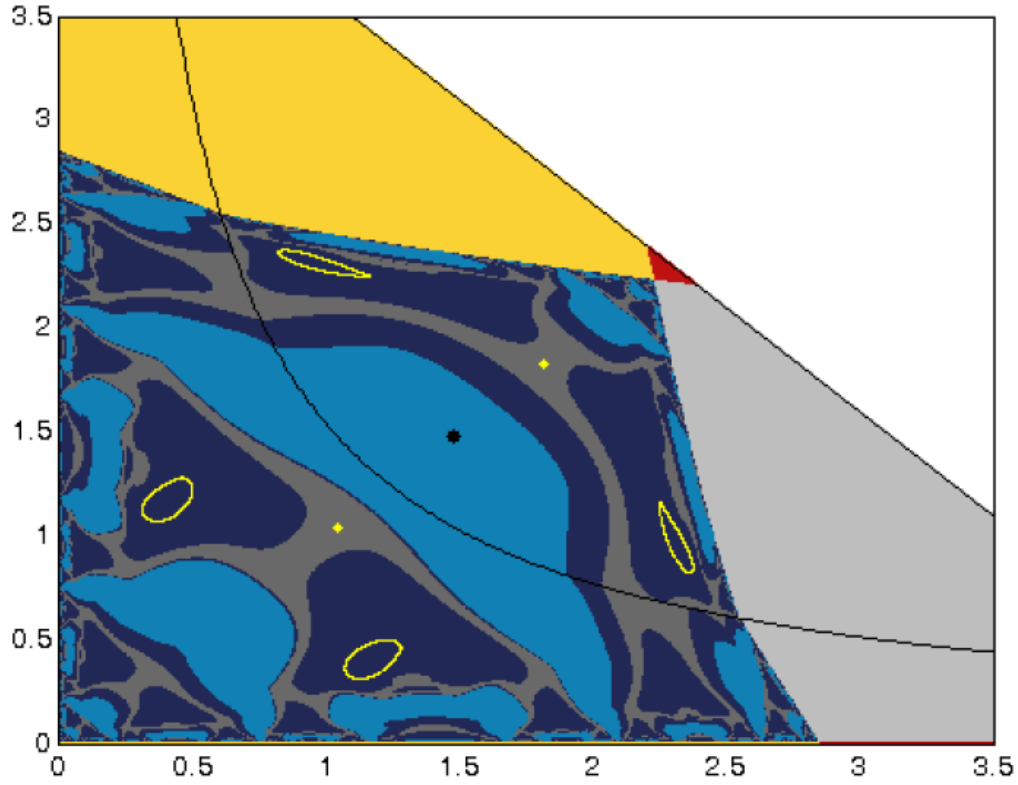


Case 3

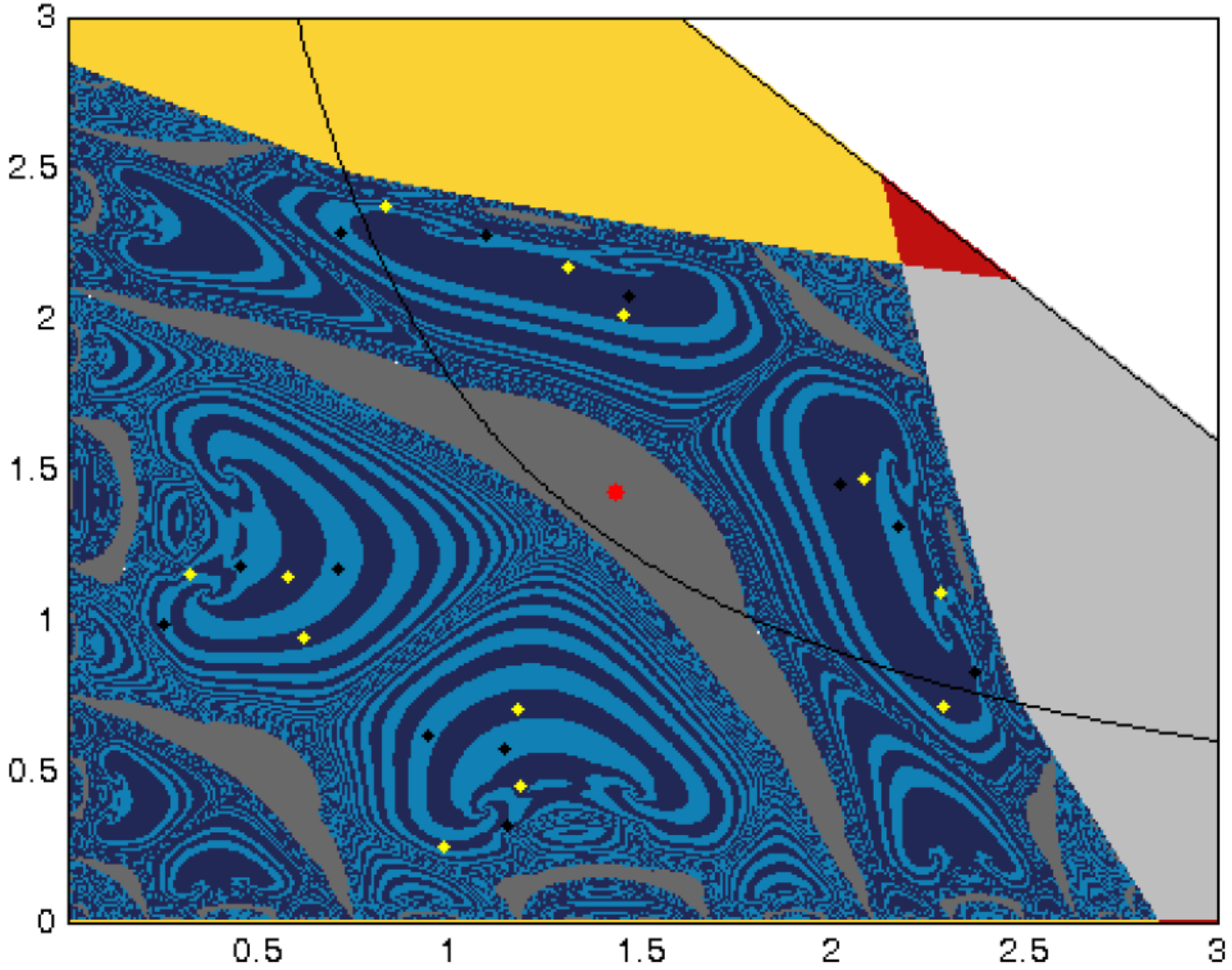


Case 4





Case 4 (multiple attractors, tangled basins of attraction)



Conclusions

This paper has studied local and global dynamics of a nonlinear duopoly with quantity setting firms, limited information and non-cooperative (persuasive) advertising that affects the degree of (horizontal) products differentiation. As stressed by Bagwell (2007), advertising represents a relevant instrument for firms to make their own product different for consumers and get a competitive advantage on the market. In this context, we have found that the long-term behaviour of the economy is compatible with the existence of attractors such that firms continuously alternates between a state where there positive advertising investments to a state where there are no advertising investments.

Dramatic changes in the dynamics and in the basins of attraction may be caused by variations in g.