Optimality of Time Series Momentum and Reversal

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Momentum and Reversal

- **Momentum**: Tendency of assets with good (bad) recent performance to continue outperforming (underperforming) in short-run.
    - Jegadeesh and Titman (1993), predicting returns over horizons of 3-12 months by returns over the past 3-12 months in US markets;
    - Other countries, Fama and French 1998, stocks within industries, Cohen and Lou 2012, across industries, Cohen and Frazzini 2008, and global market with different asset classes, Asness at al 2013.

Momentum and Reversal

- **Reversal**: Predictability of assets that performed well (poorly) over a long period tend to subsequently underperform (outperform).
  - Poterba and Summers (1988), Jagadeesh (1991), Fame and French (1998), Lewellen (2002): reversal for holding periods more than one year;
  - Campbell and Viceira (1999), Wachter (2002) and Koijen et al (2009): Mean reversion in equity returns induce significant **market timing opportunities**.
- Short run **Momentum** and long run **Reversal** (De Bondt and Thaler 1985; Jegadeesh and Titman 1993)

Time Series Momentum and Reversal

- Cross sectional momentum;
- Time series momentum (TSM): Moskowitz, Ooi and Pedersen (2012):
  - Based purely on a securitys own past returns.
  - TSM based on the past 12 month excess returns persists for 1 to 12 months that partially reverses over longer horizons.
  - Positive auto-covariance is the main driving force for TSM and cross-sectional momentum effects, while the contribution of serial cross-correlations and variation in mean returns is small.
- **Value and momentum everywhere**, Asness, Moskowitz and Pedersen (2013)
- **Combination of momentum and reversal** outperforms the pure momentum and pure mean reversion strategies in national equity markets (Balvers and Wu 2006) and foreign exchange markets (Serban 2010);

Time Series Momentum and Reversal—Various Models

- The three-factor model of Fama and French (1996) can explain long-run reversal but not short-run momentum;
- Barberis, Shleifer and Vishny (1998)—the result of systematic errors
- Daniel, Hirshleifer and Subrahmanyam (1998) and Hong and Stein (1999)—attribute the under-reaction to overconfidence and overreaction to biased self-attribution.
- Barberis and Shleifer (2003)—style investing can explain momentum and value effects.
- Sagi and Seasholes (2007)—identify observable firm-specific attributes that drive momentum.
- Vayanos and Woolley (2013)—slow-moving capital can also generate momentum.

Contributions

- **Economically**, extend the standard GBM stock price process with a weighted average of mean reversion and moving average as the drift and show that **the combined momentum and reversal strategies are optimal**;
- **Analytically**, derive the optimal investment strategies in close form by exploring the latest development of **maximum principle for control problem of stochastic delay differential equations**.
- **Empirically**, estimate the model to the S&P 500 index and show that
  - *pure momentum and pure mean reversion* strategies cannot outperform the market but the combined optimal strategy can **outperform the market**;
  - the optimality is **immune** to the market states, investor sentiment, market volatility and short-sale constraint.

Outline

1. The Model
2. Optimal Asset Allocation
3. Model Estimation
4. Performance of the Optimal Strategies
5. Out of Sample Tests
6. Market States, Sentiment and Volatility
7. Comparison with TSM of Moskowitz, Ooi and Pedersen (2012)
8. Conclusion

The Model - Two securities

- Riskless asset:

$$dB_t/B_t = r dt.$$

- Dynamics of stock return - Non-Markovian

$$\frac{dS_t}{S_t} = \big[\phi m_t + (1-\phi)\mu_t\big]dt + \sigma_S' dZ_t, \qquad (1)$$

where Z_t is two-dimensional Brownian motions.

- **Mean reversion** process μ_t is defined by an OU process,

$$d\mu_t = \alpha(\bar{\mu} - \mu_t)dt + \sigma_\mu' dZ_t, \qquad \alpha > 0, \bar{\mu} > 0 \qquad (2)$$

where $\bar{\mu}$ is the constant long-run expected rate of return, α is the rate at which μ_t converges to $\bar{\mu}$.

The Model - Momentum

Momentum m_t is defined by a stardard moving average of past returns

$$m_t = \frac{1}{\tau} \int_{t-\tau}^{t} \frac{dS_u}{S_u}. \tag{3}$$

- τ: time horizon of momentum strategy;
- motivated by the time series momentum (TSM) strategies in Moskowitz et al. (2012) showing the average return over a past period (say, 12 months) is a positive predictor of its future returns, especially for the next month.
- different from Koijen et al. (2009),

$$M_t = \int_0^t e^{-w(t-u)} \frac{dS_u}{S_u},$$

leading to a Markovian system.

Properties of the Stock Return Process

- The system (1)-(2) has a pathwise unique solution (S, μ) for a given $\mathcal{F}_0$-measurable initial process $\varphi : \Omega \to C([-\tau, 0], R)$.
- If $\varphi_0 > 0$ a.s., then $S_t > 0$ for all $t \geq 0$ a.s..

Optimal Asset Allocation

- A long-term investor with log utility maximizes the expected utility of the terminal wealth.
- The wealth dynamics

$$\frac{dW_t}{W_t} = \Big(\pi_t[\phi m_t + (1-\phi)\mu_t - r] + r\Big)dt + \pi_t \sigma_S' dZ_t,$$

where π_t is the fraction of the wealth invested in the stock.

- The investment problem

$$J(W, m, \mu, t, T) = \sup_{(\pi_u)_{u \in [t,T]}} \mathbb{E}_t[\ln W_T],$$

where T is the terminal time of the investment.

Optimal Asset Allocation

- The optimal strategic allocation to stocks is given by

$$\pi_t^* = \frac{\phi m_t + (1-\phi)\mu_t - r}{\sigma_S' \sigma_S}. \tag{4}$$

- The remainder, $1 - \pi_t^*$, is invested in the risk-free asset.
- **A weighted average of momentum and mean-reverting strategies can be optimal**.

Optimal Asset Allocation—Two Special Cases

- **Special Case 1**: $\phi = 1$

$$\pi_t^* = \frac{m_t - r}{\sigma_S' \sigma_S}, \tag{5}$$

where $m_t = \frac{1}{\tau} \sum_{i=1}^{\tau} R_{t-i+1}$, R_t is the simple return of the index. The portfolio with $\tau = 12$ and $h = 1$ is consistent with the time series momentum strategy in Moskowitz et al (2012);

- **Special Case 2**: $\phi = 0$,

$$\pi_t^* = \frac{\mu_t - r}{\sigma_S' \sigma_S},$$

the standard optimal strategy when the drift of GBM asset pricing process is mean-reverting. In particular, when μ_t is a constant, the optimal portfolio (4) collapses to the optimal portfolio in Merton (1971).

Model Estimation

- The mean reversion variable μ_t is affine in the log dividend yield D_t:

$$\mu_t = \bar{\mu} + \nu(D_t - \mu_D) = \bar{\mu} + \nu X_t$$

with $\mathbb{E}(D_t) = \mu_D$, and $X_t = D_t - \mu_D$ denotes the de-meaned dividend yield.

- Discretizing the continuous-time model at monthly frequency results in a bivariate Gaussian VAR model for the dividend yield and simple return, which are observable,

$$\begin{cases} R_{t+1} = \dfrac{\phi}{\tau}(R_t + R_{t-1} + \cdots + R_{t-\tau+1}) \\ \qquad\qquad + (1-\phi)(\bar{\mu} + \nu X_t) + \sigma_S' \Delta Z_{t+1}, \\ X_{t+1} = (1-\alpha)X_t + \sigma_X' \Delta Z_{t+1}, \end{cases} \tag{6}$$

Model Estimation

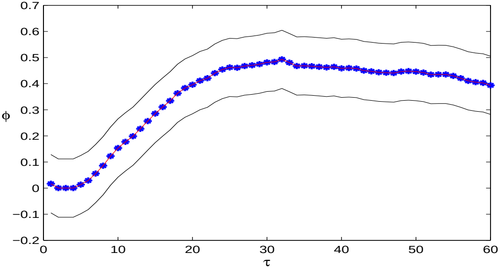
- **Method**: MLE;
- **Data**: monthly S&P 500 over the period $01/1871 - 12/2012$ obtained from the home page of Robert Shiller.
- Set the instantaneous short rate to $r = 4\%$ in annual terms.
- Parameters estimated:
  - α,
  - ϕ,
  - $\bar{\mu}$,
  - ν,
  - σ_X.

The estimates for $\tau = 12$

Parameters	α	ϕ	$\bar{\mu}$	ν
Estimates (%)	0.46	19.85	0.36	0.20
Bounds (%)	$(0.03, 0.95)$	(8.70, 31.00)	(0.26, 0.46)	(-0.60, 1.00)
Parameters	$\sigma_{S(1)}$	$\sigma_{X(1)}$	$\sigma_{X(2)}$	
Estimates (%)	4.10	-4.09	1.34	
Bounds (%)	(3.95, 4.24)	(-4.24, -3.93)	(1.29, 1.39)	

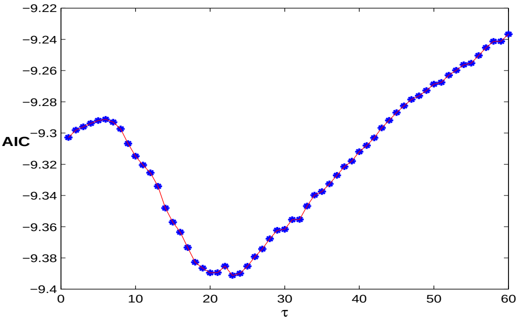
Table 1: Estimations of the full model with $\tau = 12$ and 95% confidence bounds.

The estimates of ϕ as a function of τ



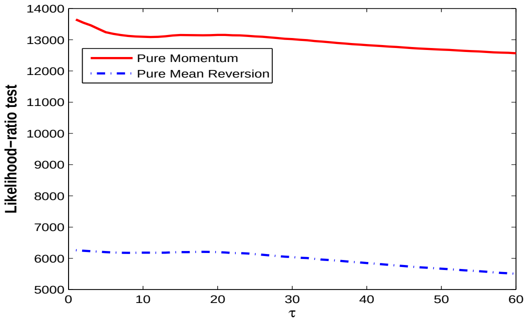
- the momentum effect ϕ is statistically different from 0 for $\tau \geq 10$;
- ϕ increases to 50% for $\tau \in [20, 30]$ and then decreases gradually when τ increases further;
- overall, $\phi \leq 0.5$.

Akaike information criteria for different τ



- the average returns over a past time period of 1.5—2 years can predict future return best.
- no momentum effect for large time horizon;

Log-likelihood ratio test



- The full model is significant better than the pure mean reversion model and pure mean reversion model.

Performance of the optimal Strategies

- **Two proxies**:
  - the utility of portfolio wealth;
  - the Sharpe ratio.
- The utility of the optimal portfolio wealth from 01/1876 to 12/2012 with $\tau \in [1, 60]$ and the passive holding portfolio.
- The optimal strategies outperform the market at the end of investment period for $\tau \in [1, 20]$

The utility of terminal wealth

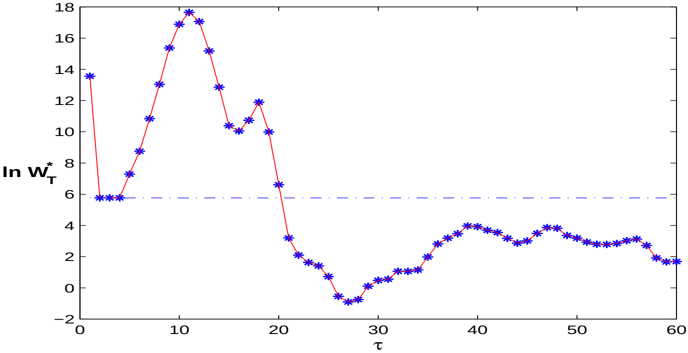


Figure 1: Terminal utility from 1/1876 to 12/2012

Results for $\tau = 12$

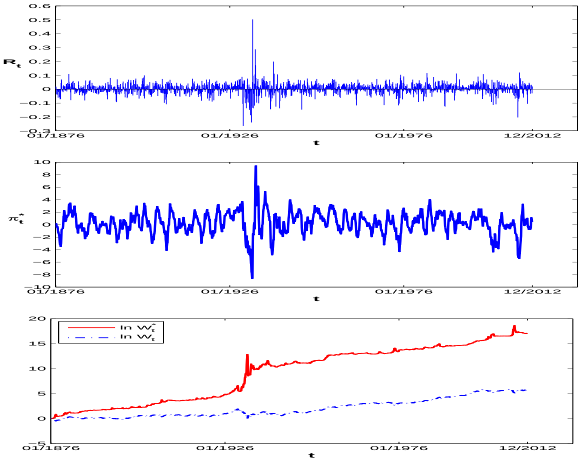


Figure 2: The simple return, optimal portfolio and utility of wealth

Pure Strategies

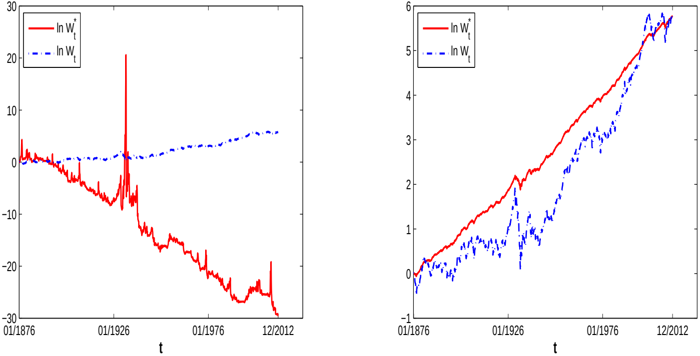


Figure 3: The time series of the utility of wealth for (a) the pure momentum model with $\tau = 12$ and (b) the pure mean reversion model from January 1876 until December 2012.

Monte Carlo Result for $\tau = 12$

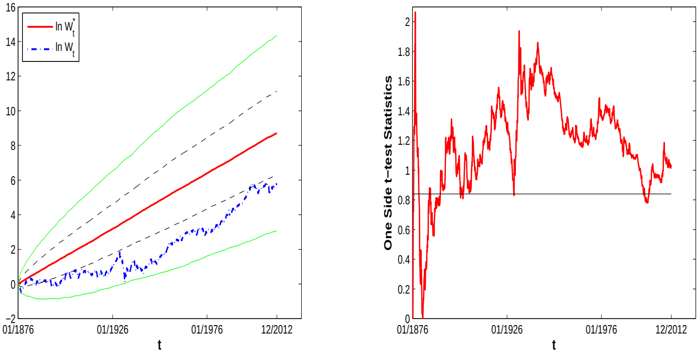


Figure 4: Average utility based on 1000 simulations with 90% and 60% confidences; one sided t-test $\ln W^* > \ln W^{SP500}$ with 80%.

Monte Carlo Result for all τ

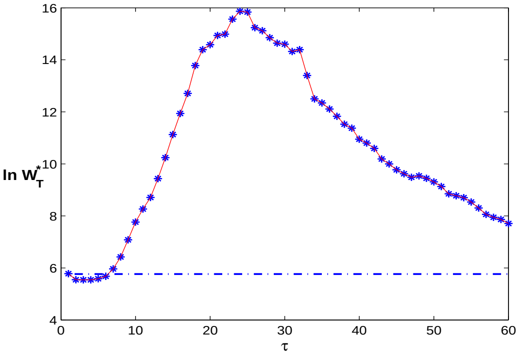


Figure 5: Average terminal utility based on 1000 simulations for $\tau \in [1, 60]$.

The Sharpe ratios

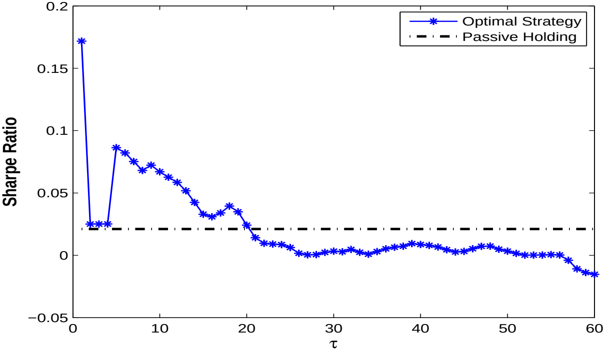


Figure 6: The average Sharpe ratio for the optimal portfolio and the passive holding portfolio

Monte Carlo Result on Sharpe ratios

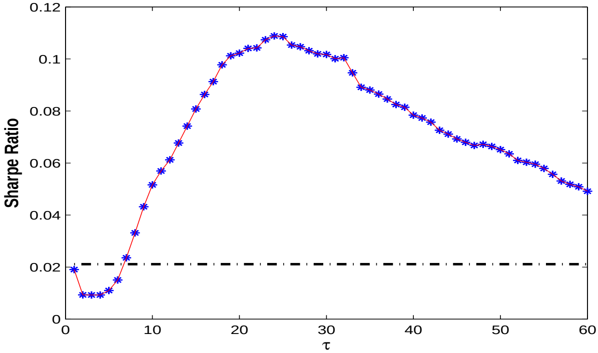


Figure 7: Average Sharpe ratio based on 1000 simulations for $\tau \in [1, 60]$.

Monte Carlo Result on Sharpe ratios of Pure Momentum

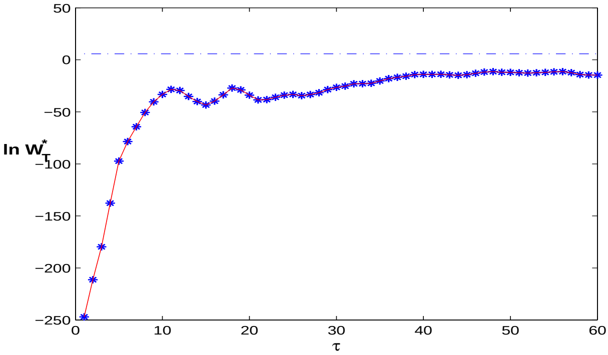


Figure 8: The utility of terminal wealth of the pure momentum model for $\tau \in [1, 60]$.

Out of Sample Tests over the last 5 years (1/08-12/12)

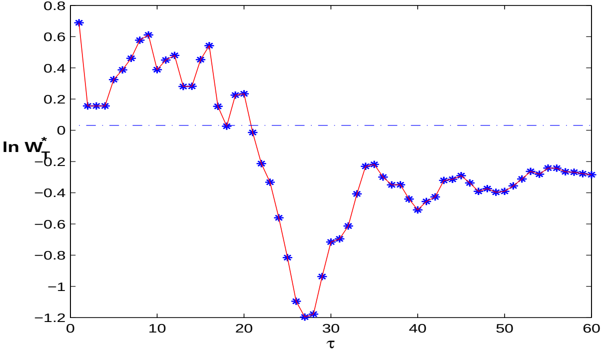


Figure 9: The utility of terminal wealth for the optimal portfolio with $\tau \in [1, 60]$ and the passive holding portfolio with out of sample data of the last 5 years.

Results $\tau = 12$

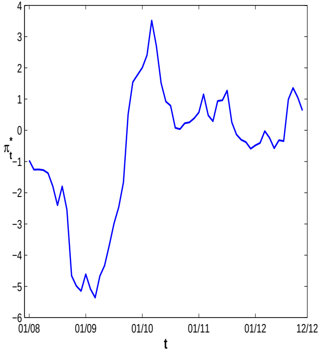


Figure 10: The time series of (a) the optimal portfolio and (b) the utility of wealth from January 2008 until December 2012 for $\tau = 12$ with out of sample data of last 5 years.

Short-sales Constraints

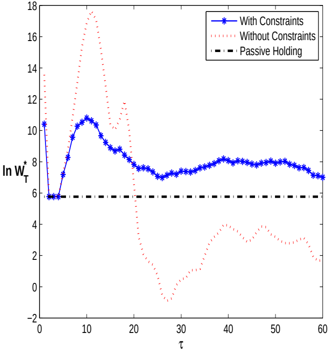


Figure 11: The terminal utility of wealth and Sharpe ratio

Short-sales Constraints

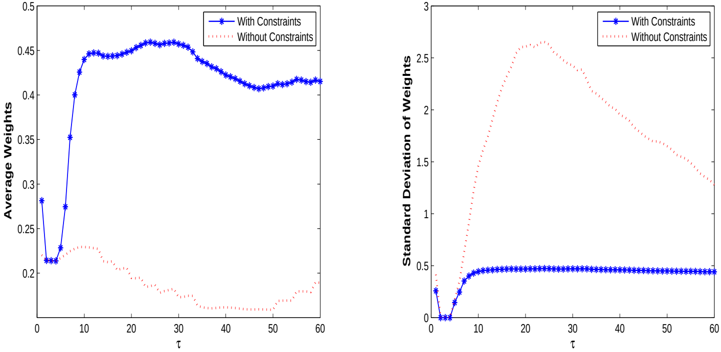


Figure 12: The mean values and standard deviations of the weights for the optimal portfolio

Market states

Market states (Cooper, Gutierrez and Hameed 2004; Griffin, Ji and Martin 2003)

$$\begin{aligned} R_t^* - r = & 0.0094 + 0.0005 I_t(UP) - 1.0523(R_t - r) + \epsilon_t; \\ & (1.46) \qquad (0.06) \qquad\qquad (-12.48) \end{aligned}$$

$$\begin{aligned} R_t^* - r = & 0.0086 - 0.0008 I_t(UP) + 0.1994(R_t - r) I_t(UP) \\ & (1.44) \quad (-0.11) \qquad\qquad (1.90) \\ & - 2.5326(R_t - r) I_t(DOWN) + \epsilon_t; \\ & (-22.16) \end{aligned}$$

$$\begin{aligned} R_t^* - r = & 0.0083 + 0.0006 I_{t-1}(UP) + \epsilon_t; \\ & (1.22) \qquad (0.07) \end{aligned}$$

Investor Sentiment and Market Volatility

Investor sentiment (Baker and Wurgler 2006)

$$\begin{aligned} R_t^* - r = & 0.0059 + 0.0040 T_{t-1} + \epsilon; \\ & (1.77) \qquad (1.20) \end{aligned}$$

Market volatility (Wang and Wu 2012)

$$\begin{aligned} R_t^* - r = & 0.0037 + 0.0138 \hat{\sigma}_{S,t-1} + \epsilon_t; \\ & (1.06) \qquad (0.25) \end{aligned}$$

$$\begin{aligned} R_t^* - r = & -0.002 + 0.1043 \hat{\sigma}^+_{S,t-1} + 0.1026 \hat{\sigma}^-_{S,t-1} + \epsilon_t; \\ & (-0.27) \qquad (1.34) \qquad\qquad (1.80) \end{aligned}$$

Comparison with TSM

$(\tau \setminus h)$	1	3	6	9	12	24	36	48	60
1	0.1337	0.1387	0.1874	0.1573	0.0998	0.0222	0.0328	0.0479	0.0362
	(1.28)	(1.84)	(**3.29**)	(**2.83**)	(1.84)	(0.42)	(0.63)	(0.90)	(0.66)
3	0.0972	0.0972	0.0972	0.0972	0.0972	0.0972	0.0972	0.0972	0.0972
	(0.93)	(0.93)	(0.93)	(0.93)	(0.93)	(0.93)	(0.93)	(0.93)	(0.93)
6	0.2022	0.2173	0.2315	0.1462	0.0700	-0.0414	0.0199	0.0304	0.0014
	(1.93)	(**2.28**)	(**2.60**)	(1.75)	(0.88)	(-0.58)	(0.32)	(0.53)	(0.02)
9	**0.3413**	0.3067	0.2106	0.1242	0.0333	-0.0777	-0.0095	0.0000	-0.0450
	(**3.27**)	(**3.12**)	(**2.28**)	(1.45)	(0.41)	(-1.16)	(-0.17)	(0.00)	(-1.11)
12	0.1941	0.1369	0.0756	-0.0041	-0.0647	-0.0931	-0.0234	-0.0137	-0.0587
	(1.85)	(1.40)	(0.80)	(-0.04)	(-0.76)	(-1.30)	(-0.41)	(-0.30)	(-1.46)
24	-0.0029	-0.0513	-0.0776	-0.0591	-0.0557	-0.0271	0.0261	-0.0020	-0.0082
	(-0.03)	(-0.51)	(-0.79)	(-0.62)	(-0.61)	(-0.34)	(0.40)	(-0.03)	(-0.15)
36	0.0369	0.0602	0.0517	0.0419	0.0416	0.0657	0.0351	0.0273	0.0406
	(0.35)	(0.59)	(0.52)	(0.43)	(0.44)	(0.81)	(0.49)	(0.42)	(0.64)
48	0.1819	0.1307	0.1035	0.0895	0.0407	-0.0172	0.0179	0.0500	0.0595
	(1.74)	(1.30)	(1.06)	(0.93)	(0.43)	(-0.21)	(0.24)	(0.70)	(0.86)
60	-0.0049	-0.0263	-0.0800	-0.1160	-0.1289	-0.0396	0.0424	0.0518	0.0680
	(-0.05)	(-0.26)	(-0.81)	(-1.20)	(-1.41)	(-0.49)	(0.55)	(0.69)	(0.92)

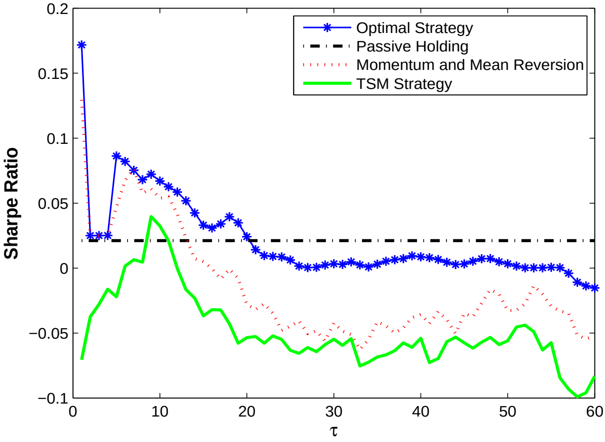
Table 2: The average excess return (%) based on the optimal strategy for different look back period τ (different row) and different holding period h (different column).

Comparison with TSM

$(\tau \setminus h)$	1	3	6	9	12	24	36	48	60
1	-0.0144	0.0652	0.0714	0.0689	0.0568	-0.0040	0.0006	0.0010	-0.0133
	(-0.14)	(0.89)	(1.34)	(1.52)	(1.37)	(-0.12)	(0.02)	(0.04)	(-0.57)
3	0.1683	0.1915	0.1460	0.1536	0.0764	-0.0360	-0.0290	-0.0143	-0.0395
	(1.61)	(**2.16**)	(1.91)	(**2.20**)	(1.17)	(-0.69)	(-0.72)	(-0.45)	(-1.38)
6	0.2906	0.2633	0.2635	0.1884	0.1031	-0.0484	-0.0130	0.0157	-0.0281
	(**2.78**)	(**2.79**)	(**3.01**)	(**2.29**)	(1.34)	(-0.75)	(-0.26)	(0.40)	(-0.77)
9	**0.4075**	0.3779	0.2422	0.1538	0.0545	-0.0735	-0.0217	-0.0047	-0.0460
	(**3.91**)	(**3.78**)	(**2.62**)	(1.76)	(0.66)	(-1.05)	(-0.38)	(-0.10)	(-1.12)
12	0.2453	0.1660	0.0904	0.0122	-0.0748	-0.1195	-0.0602	-0.0454	-0.0798
	(**2.35**)	(1.67)	(0.94)	(0.13)	(-0.86)	(-1.63)	(-1.02)	(-0.95)	(-1.88)
24	0.0092	-0.0242	-0.0800	-0.0962	-0.0955	-0.0682	-0.0081	-0.0140	-0.0211
	(0.09)	(-0.24)	(-0.81)	(-1.03)	(-1.06)	(-0.88)	(-0.13)	(-0.24)	(-0.39)
36	-0.0005	0.0194	0.0219	0.0212	0.0113	0.0030	0.0127	0.0241	0.0206
	(-0.01)	(0.19)	(0.23)	(0.22)	(0.12)	(0.04)	(0.18)	(0.37)	(0.33)
48	0.0779	0.0733	0.0231	0.0019	-0.0392	-0.0676	-0.0004	0.0435	0.0382
	(0.74)	(0.73)	(0.24)	(0.02)	(-0.42)	(-0.83)	(-0.01)	(0.61)	(0.55)
60	-0.0568	-0.0852	-0.1403	-0.1706	-0.1986	-0.1091	-0.0043	0.0157	0.0239
	(-0.54)	(-0.84)	(-1.41)	(-1.77)	(-2.15)	(-1.36)	(-0.06)	(0.22)	(0.34)

Table 3: The average excess return (%) of the optimal strategy for different look back period τ (different row) and different holding period h (different column) for the pure momentum model.

The average Sharpe ratio



Performance based on the cumulative excess return

$$\hat{R}_{t+1} = sign(\pi_t^*)\frac{0.1424}{\hat{\sigma}_{S,t}}R_{t+1}, \tag{7}$$

where 0.1424 is the sample standard deviation of the total return index.

$$\hat{\sigma}_{S,t}^2 = 12\sum_{i=0}^{\infty}(1-\delta)\delta^i(R_{t-1-i} - \bar{R}_t)^2. \tag{8}$$

where $\bar{R}_t$ is the exponentially weighted average return based on the weights $(1-\delta)\delta^i$, $\sum_{i=1}^{\infty}(1-\delta)\delta^i = \delta/(1-\delta) = 2$.

Performance based on the log cumulative excess return

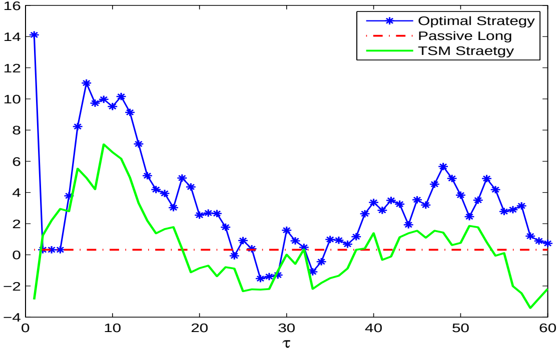


Figure 13: Terminal log cumulative excess return of the optimal strategies (4) and TSM strategies with $\tau \in [1, 60]$ and passive long strategy from January 1876 until December 2012.

Log cumulative excess return with $\tau = 12$

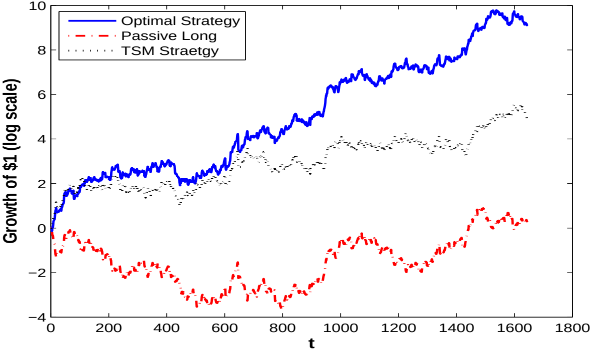


Figure 14: Log cumulative excess return of the optimal strategy (4) and momentum strategy with $\tau = 12$ and passive long strategy from January 1876 until December 2012.

Conclusion

- We derive *the optimal strategies* analytically by applying the *maximum principle for control problem of SDDE*.
- We find that *pure momentum and pure mean reversion strategies cannot outperform the market, however, a combination of them can outperform the market* by taking the timing opportunity with respect to the trend in return and the market volatility.
- We show that the optimal strategy is *immune to the market states, investor sentiment, market volatility and short-sale constraint*.