Heterogeneity in Signal Precision and Clustering of Defaults

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Leverage Cycle

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The leverage cycle in a nutshell:

- Leverage becomes too high in boom times, and too low in bad times.
- As a result, in boom times asset prices are too high, and in crisis times they are too low.

Motivation

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Driving Questions:

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1. The link between heterogeneity and the clustering of defaults.

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Driving Questions:

1. The link between heterogeneity and the clustering of defaults.
2. Is a deterministic (non-linear) description of the default process feasible?

Agent-based Framework

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- Two kinds of traders:
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- Credit: The HFs can increase the size of their long position by borrowing from a bank using the asset as collateral.

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  1. Microscopic level: Exponentially distributed ⇒ Poisson process.
  2. After aggregation: Power-law ⇒ Scale invariance.

Consequences of the fat-tail

- The scale-free character of the power-law distribution leads to clustering of defaults.
- The statistical properties of the default process, as viewed on the aggregate level, can be accurately described by an Intermittent (type III) process.

Noise Traders

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- The price of the risky asset in the absence of the HFs is assumed to follow a mean-reverting AR(1) process.
- Thus, the demand (in cash value) $\xi_t = D^{nt} p_t$ of the NTs follows

$$\log(\xi_t^{nt}) = \rho \log(\xi_{t-1}^{nt}) + (1-\rho)\log(VN) + \sigma\chi_t, \qquad (1)$$

where $\chi_t = N(0,1)$ and $\rho \in (0,1)$ (Poledna et al., 2014).

Hedge Funds I

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- HFs are represented by risk averse agents with CRRA.

Funds

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The maximization yields

$$D_t^j = \frac{m}{\alpha \sigma_j^2} W_t^j, \quad m = V - p_t. \tag{2}$$

- Demand is capped by $\lambda^j = D_t^j p_t / W_t^j \leq \lambda_{\max}$,

$\lambda_{\max}$ the maximum allowed leverage set externally.

Price

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- The wealth of a HF evolves according to

$$W_{t+1}^{j} = W_{t}^{j} + (p_{t+1} - p_{t})D_{t}^{j} - F_{t}^{j} \tag{3}$$

- F_t^j, managerial fees following the 1/10 rule:

$$F_{t}^{j} = \gamma\left(W_{t} + 10\max\left\{W_{t}^{j} - W_{t-1}^{j}, 0\right\}\right) \tag{4}$$

- The price of the risky asset is determined by the market clearance condition

$$D_{t}^{\mathrm{nt}}(p_{t}) + \sum_{j=1}^{n} D_{t}^{j}(p_{t}) = N. \tag{5}$$

Theorem

Consider an exponential density function $P(\tau;\mu)$, parametrized by $\mu \in \mathbb{R}_+$. Assume that μ is itself a random variable with a density function $W(\mu)$. If $W(\mu)$ can be expanded in a power-series, i.e. $W(\mu) = \sum_{k=0}^{\infty} b_k \mu^k$, for $\mu \to 0^+$, then the compound probability function defined as

$$\tilde{P}(\tau) \equiv \int_0^{\infty} W(\mu) P(\tau;\mu) d\mu$$

for $\tau \gg 1$, to the leading order of $\mathcal{O}(1/\tau)$, decays as $\tilde{P}(\tau) \propto \tau^{-(n+2)}$, where n is the order of the expansion around $\mu = 0$.

Proof

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Proof.

The compound density is $\mathscr{L}\left[\phi(\mu)\right],\ \phi(\mu) \equiv \mu W(\mu)$, where $\mathscr{L}\left[.\right]$ denotes the Laplace transform with respect to μ.

Watson's Lemma: If $f(\mu)$ can be written as
$f(\mu) = \mu^a \sum_{k=0}^{m} b_k \mu^k + R_{m+1}(\mu)$, with $a > -1$, then

$$\mathscr{L}\left[f(\mu)\right](\tau) \sim \sum_{k=0}^{m} b_k \frac{\Gamma(a+k+1)}{\tau^{a+k+1}} + \mathcal{O}\left(\frac{1}{\tau^{a+m+2}}\right). \qquad (6)$$

Given that $\phi(\mu) = \mu \sum_{k=0}^{\infty} b_k \mu^k$, $\tilde{P}(\tau) \propto \tau^{-(n+2)} + \mathcal{O}\left(\frac{1}{\tau^{n+3}}\right)$.

□

Autocorrelation

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Theorem

Let $T_n \in \mathbb{R}_+$, $n \geq 0$, be a sequence of i.d.d. random variables. Assume that the probability density function $\tilde{P}(T_n = \tau) \propto \tau^{-\alpha}$, for $\tau \to \infty$.

Consider now the renewal process $S_n = \sum_{i=0}^{n} T_i$. Let $Y(t) = 1_{[0,t]}(S_n)$, where $1_A : \mathbb{R} \to \{0,1\}$ denotes the indicator function, satisfying

$$1_A = \begin{cases} 1 & : \quad x \in A \\ 0 & : \quad x \notin A \end{cases}$$

If $2 < \alpha \leq 3$, then the autocorrelation function of $Y(t)$, for $t \to \infty$ decays as

$$C(t') \propto {t'}^{2-\alpha} \tag{7}$$

Failure Function — Microscopic Level

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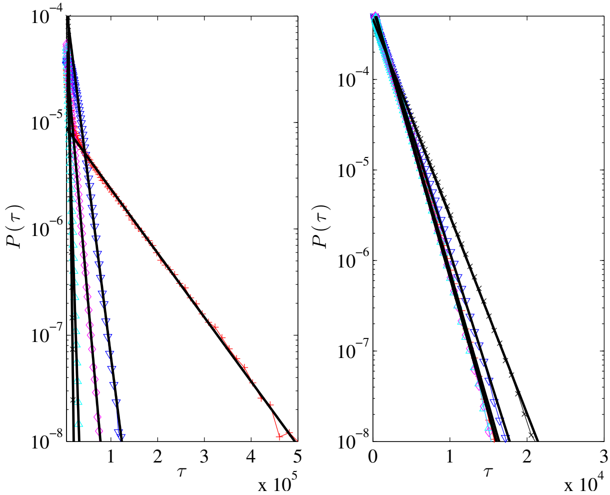

After Aggregation

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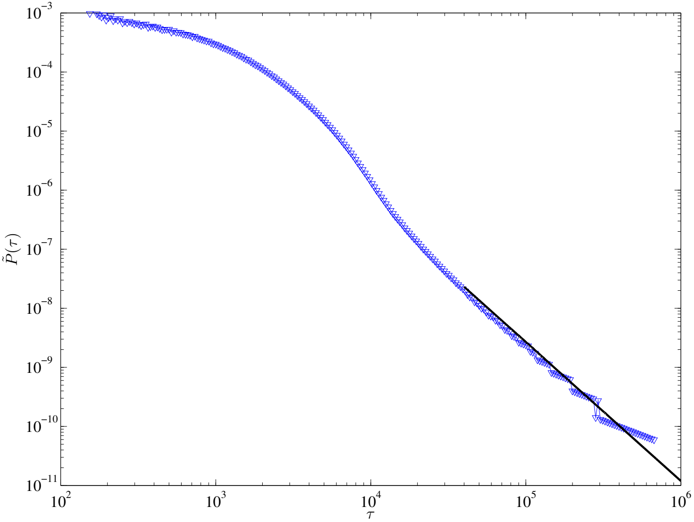

Clustering of Defaults

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Asymmetric Information Leads to Clustering of Defaults

An important effect of the emergent heavy-tail statistics stemming from the heterogeneity of the market, is the absence of a characteristic time-scale for the occurrence of defaults (scale-free asymptotic behaviour).

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An important effect of the emergent heavy-tail statistics stemming from the heterogeneity of the market, is the absence of a characteristic time-scale for the occurrence of defaults (scale-free asymptotic behaviour).

- Fitting the aggregate distribution we obtain $\tilde{P}(\tau) \sim \tau^{-(7/3)}$.
- According to Theorem 2, the autocorrelation function decays as,

$$C(t') \sim t'^{-1/3}. \tag{8}$$

Autocorrelation Function

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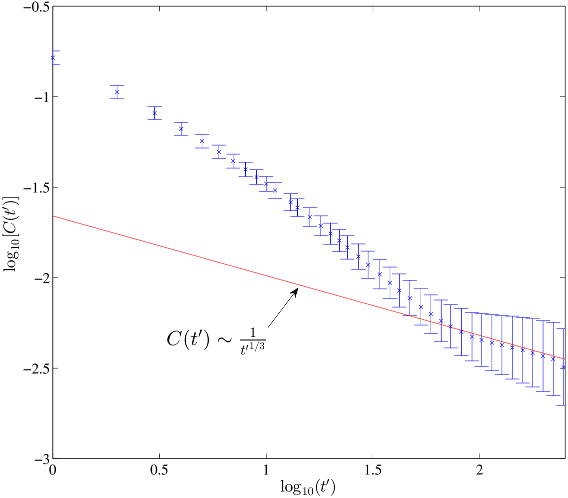

Deterministic Description I

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- All statistical properties of default events can be replicated by a very simple deterministic map.

$$x_{t+1} = x_t + u x_t^z \mod 1, \; z > 1. \tag{9}$$

- Characteristic behaviour: The evolution of x_t is regular close to the vicinity of 0 (marginally unstable fixed point) and chaotic away from it $\Rightarrow$ Random alternation between almost regular and chaotic dynamics.
  - Regular motion $\rightarrow$ Laminar phase.
  - Chaotic motion $\rightarrow$ Turbulent phase.

Deterministic Description II

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- The distribution of waiting times between transition from the laminar to the turbulent phase follows a power-law (Schuster and Just, 2006).

$$\rho(\tau) \propto \tau^{-\frac{z}{z-1}}, \tag{10}$$

- Also, the autocorrelation function of x_t decays algebraically

$$C(t') \propto t'^{\frac{z-2}{z-1}},\ 3/2 \leq z < 2. \tag{11}$$

Setting $z = \frac{7}{4}$, and mapping the:

- HFs Active $\rightarrow$ Laminar phase.
- Default events $\rightarrow$ Turbulent phase.

$$\rho(t) \sim \tau^{-7/3},\ C(t') = t'^{-1/3} \tag{12}$$

- We assume that the heterogeneity of the agents stems from the HFs' different quality of the mispricing signals they receive.
- We show that the failure function of the HFs is qualitatively different when observed on the micro and the aggregate level.
- We also show that the scale-free property of the emergent statistics on the aggregate level is directly connected with the clustering of defaults.

Which is the Real Cause?

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. . . A crucial part of my story is heterogeneity between investors. . . But an important difference is that I do not invoke any asymmetric information. . . Of course, the asymmetric information revolution in economics was a tremendous advance, and asymmetric information plays a critical role in many lender-borrower relationships; sometimes, however, the profession becomes obsessed with it. . . (Geanakoplos, 2010)

References I

Geanakoplos, J., Dec. 1996. Promises promises. Cowles Foundation Discussion Papers 1143, Cowles Foundation for Research in Economics, Yale University.
URL http://ideas.repec.org/p/cwl/cwldpp/1143.html

Geanakoplos, J., 2010. The leverage cycle. In: NBER Macroeconomics Annual 2009, Volume 24. University of Chicago Press, pp. 1–65.

Poledna, S., Thurner, S., Farmer, J. D., Geanakoplos, J., 2014. Leverage-induced systemic risk under Basle II and other credit risk policies. Journal of Banking & Finance 42 (C), 199–212.
URL http://ideas.repec.org/a/eee/jbfina/v42y2014icp199-212.html

Schuster, H., Just, W., 2006. Deterministic Chaos: An Introduction. Wiley.
URL http://books.google.co.uk/books?id=9y2qSGpQR7QC

Thurner, S., Farmer, J. D., Geanakoplos, J., 2012. Leverage causes fat tails and clustered volatility. Quantitative Finance 12, 695,707.