Oligopolies with Contingent Workforce and Unemployment Insurance Systems

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MDEF 2014
Urbino, Italy, September, 18, 2014

Outline

Motivation

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- contingent workforce
- unemployment insurance systems

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The Model with Contingent Workforce

Contingent Workforce

Main features

- N firms industry
- identical product
- x_k firm k output
- $X = \sum_{k=1}^{N} x_k$
- inverse demand function: $p(X) = A - BX$
- cost function: $C_k(x_k) = c_k + d_k x_k$

The output adjustment cost at time period t

$$\bar{C}_k(x_k, x_k(t-1)) = \begin{cases} 0 & \text{if } x_k \leq x_k(t-1) \\ \gamma_k(x_k - x_k(t-1)) & \text{otherwise.} \end{cases}$$

$\gamma_k > 0$,

Contingent Workforce

The profit of firm k at time period t

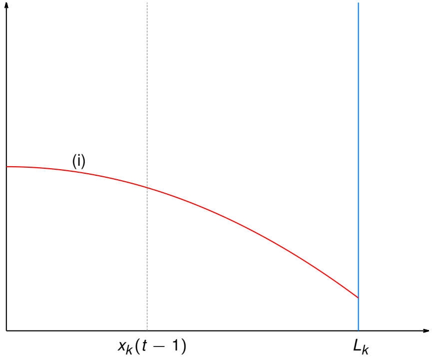
$$\Pi_k = \begin{cases} x_k(A - Bx_k - BX_k) - (c_k + d_k x_k) & \text{if } x_k \leq x_k(t-1) \\ x_k(A - Bx_k - BX_k) - (c_k + d_k x_k) - \gamma_k(x_k - x_k(t-1)) & \text{otherwise,} \end{cases}$$

where $X_k = \sum_{l \neq k} x_l$ is the output of the rest of the industry.
Some assumptions

- $A > d_k$
- L_k maximum possible output level for firm k
- $0 < x_k(t-1) < L_k$

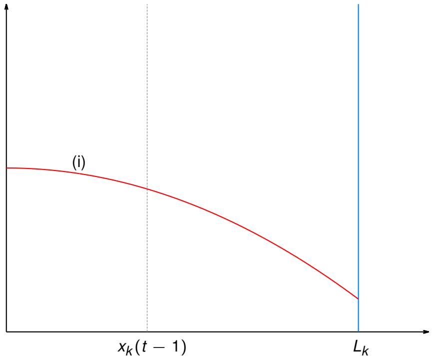
The possible shapes of the profit functions

If $\partial \Pi_k / \partial x_k \leq 0$ at $x_k = 0$



The possible shapes of the profit functions

that is, if $X_k \geq \frac{A - d_k}{B}$

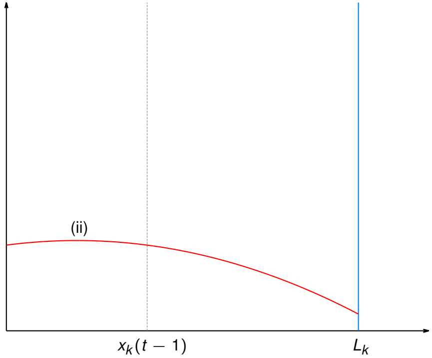


then the best response of firm k is

$$R_k\left(X_k, x_k\left(t-1\right)\right) = 0$$

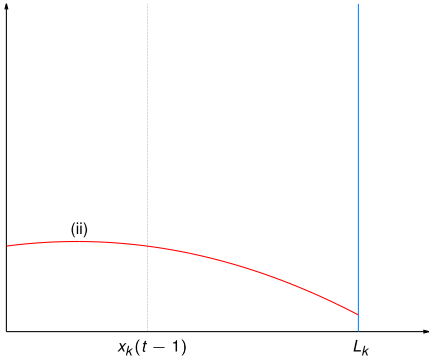
The possible shapes of the profit functions

If $\partial \Pi_k / \partial x_k > 0$ at $x_k = 0$ and $\partial_{-} \Pi_k / \partial x_k \leq 0$ at $x_k = x_k\,(t-1)$



The possible shapes of the profit functions

that is, if $\frac{A-d_k}{B} - 2x_k(t-1) < X_k \leq \frac{A-d_k}{B}$

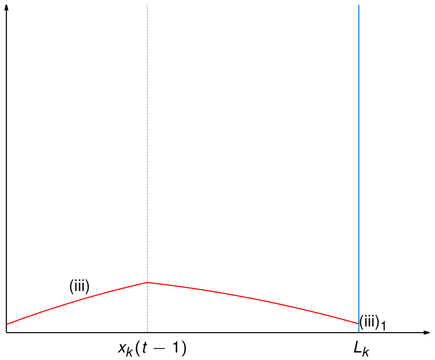


then the best response of firm k is

$$R_k(X_k, x_k(t-1)) = \frac{A - d_k - BX_k}{2B}$$

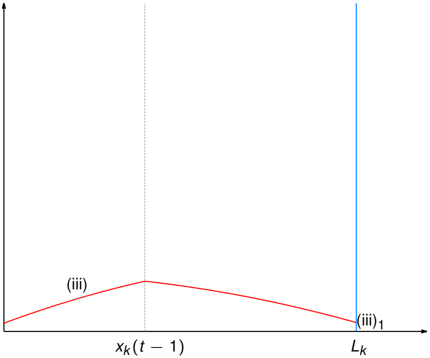
The possible shapes of the profit functions

If $\partial_{-}\Pi_k/\partial x_k > 0$ at $x_k = x_k(t-1)$ and $\partial_{+}\Pi_k/\partial x_k \leq 0$ at $x_k = x_k(t-1)$



The possible shapes of the profit functions

that is, if $\frac{A-d_k-\gamma_k}{B} - 2x_k(t-1) < X_k \leq \frac{A-d_k}{B} - 2x_k(t-1)$

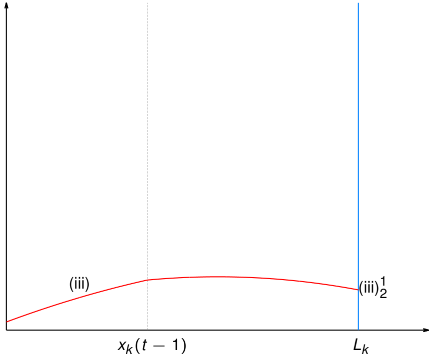


then the best response of firm k is

$$R_k(X_k, x_k(t-1)) = x_k(t-1)$$

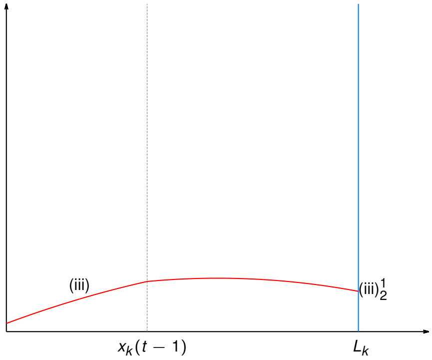
The possible shapes of the profit functions

If $\partial_{-}\Pi_k/\partial x_k > 0$ at $x_k = x_k\,(t-1)$, $\partial_{+}\Pi_k/\partial x_k > 0$ at $x_k = x_k\,(t-1)$, and $\partial\Pi_k/\partial x_k \leq 0$ at $x_k = L_k$



The possible shapes of the profit functions

That is, if $\frac{A-d_k-\gamma_k}{B} - 2L_k < X_k \leq \frac{A-d_k-\gamma_k}{B} - 2x_k\,(t-1)$

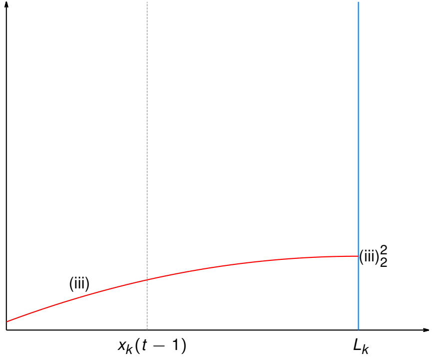


then the best response of firm k is

$$R_k\,(X_k, x_k\,(t-1)) = \frac{A - BX_k - d_k - \gamma_k}{2B}$$

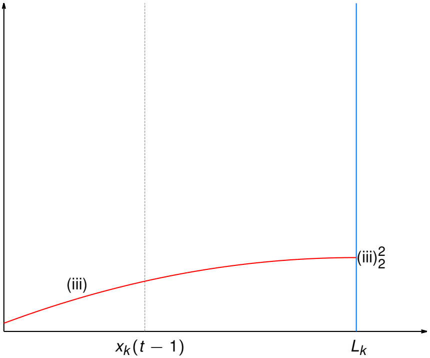
The possible shapes of the profit functions

If $\partial_{-}\Pi_k/\partial x_k > 0$ at $x_k = x_k\,(t-1)$, $\partial_{+}\Pi_k/\partial x_k > 0$ at $x_k = x_k\,(t-1)$, and $\partial\Pi_k/\partial x_k > 0$ at $x_k = L_k$



The possible shapes of the profit functions

That is, if $X_k \leq \frac{A-d_k-\gamma_k}{B} - 2L_k$



then the best response of firm k is

$$R_k\left(X_k, x_k\left(t-1\right)\right) = L_k$$

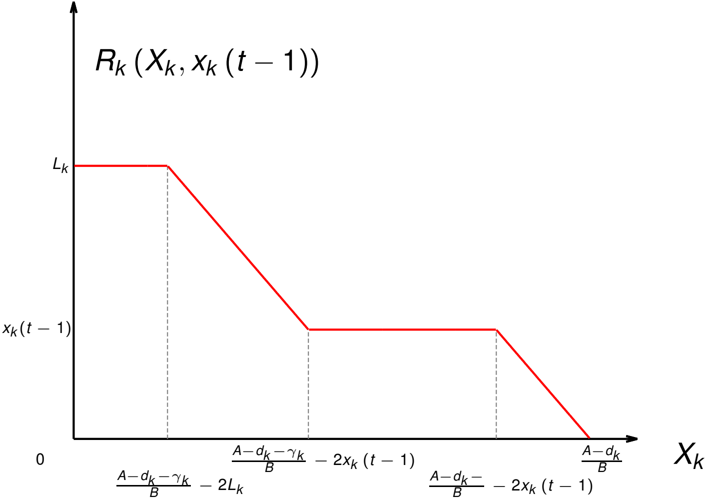
Best response of firm k as function of the output of the rest of the industry

Putting together

$$R_k\left(X_k, x_k\left(t-1\right)\right) =$$

$$= \begin{cases} L_k & \text{if} \quad X_k \leq \frac{A-d_k-\gamma_k}{B} - 2L_k \\ \frac{A-BX_k-d_k-\gamma_k}{2B} & \text{if} \quad \frac{A-d_k-\gamma_k}{B} - 2L_k < X_k \leq \frac{A-d_k-\gamma_k}{B} - 2x_k\left(t-1\right) \\ x_k\left(t-1\right) & \text{if} \quad \frac{A-d_k-\gamma_k}{B} - 2x_k\left(t-1\right) < X_k \leq \frac{A-d_k}{B} - 2x_k\left(t-1\right) \\ \frac{A-d_k-BX_k}{2B} & \text{if} \quad \frac{A-d_k}{B} - 2x_k\left(t-1\right) < X_k \leq \frac{A-d_k}{B} \\ 0 & \text{if} \quad \frac{A-d_k}{B} \leq X_k \end{cases}$$

Best response of firm k as function of the output of the rest of the industry



Dynamic extension and steady states

Discrete time dynamics

$$x_k(t) = x_k(t-1) + K_k \left(R_k \left(\sum_{l \neq k} x_l(t-1), x_k(t-1) \right) - x_k(t-1) \right)$$

where K_k denote the speed of adjustment of firm k, $k = 1, 2, \ldots, N$. As usual

- $K_k = 0 \Longrightarrow$ constant trajectories,
- $K_k = 1 \Longrightarrow$ best response dynamics.

Dynamic extension and steady states

Definition

A vector $\bar{\mathbf{x}} = (\bar{x}_k)$ is a steady state of this system if and only if for all k,

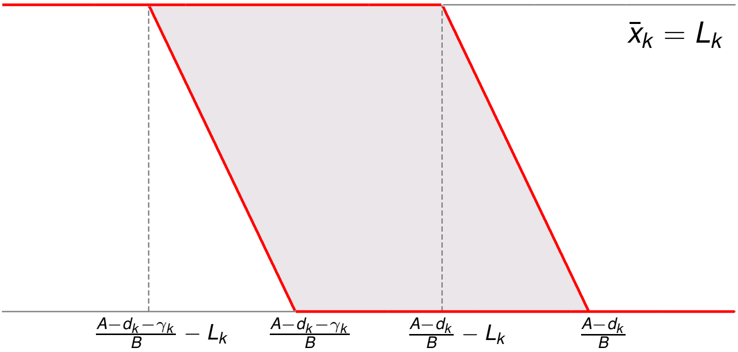
$$\bar{x}_k = R_k \left(\sum_{l \neq k} \bar{x}_l, \bar{x}_k \right)$$

Given special forms and conditions of the best response functions, for each component of the steady state we have three possibilities:

$$\begin{array}{llll} \text{(i)} & \bar{x}_k = 0, & \text{if} & \frac{A - d_k - \gamma_k}{B} \leq \bar{X}_k; \\ \text{(ii)} & 0 < \bar{x}_k < L_k, & \text{if} & \frac{A - d_k - \gamma_k}{B} - 2\bar{x}_k \leq \bar{X}_k \leq \frac{A - d_k}{B} - 2\bar{x}_k; \\ \text{(iii)} & \bar{x}_k = L_k, & \text{if} & \bar{X}_k \leq \frac{A - d_k}{B} - 2L_k, \end{array} \quad (1)$$

where $\bar{X}_k = \sum_{k \neq l} \bar{x}_l$.

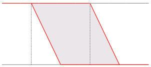
Best response of firm k as function of the total output of the industry



where

- $\bar{X}$ on the horizontal axis with domain $\left[0, \sum_{l=1}^{N} L_l\right]$,
- $\bar{x}_k$ on the vertical axis,
- the orizontal line is $\bar{x}_k = L_k$.

Steady states



For each value of $\bar{X}$,

- $\bar{x}_k$ is an interval $\left[m_k\left(\bar{X}\right), M_k\left(\bar{X}\right)\right]$ eventually 0 or L_k
- functions $m_k\left(\bar{X}\right)$ and $M_k\left(\bar{X}\right)$ are nonincreasing and continuous

Define next

- $m\left(\bar{X}\right) = \sum_{k=1}^{N} m_k\left(\bar{X}\right)$
- $M\left(\bar{X}\right) = \sum_{k=1}^{N} M_k\left(\bar{X}\right)$

We have

$$0 \leq m(0), M(0) \quad \text{and} \quad m(L), M(L) \leq L = \sum_{k=1}^{N} L_k$$

Therefore there are unique values $\bar{X}^{(1)}$ and $\bar{X}^{(2)}$ from interval $[0, L]$ such that $m\left(\bar{X}^{(1)}\right) = \bar{X}^{(1)}$ and $M\left(\bar{X}^{(2)}\right) = \bar{X}^{(2)}$.

Steady states

The set of all steady states can be described as follows. Let $\bar{X}$ be an arbitrary value from interval $\left[\bar{X}^{(1)}, \bar{X}^{(2)}\right]$, then the corresponding steady state coordinates form the set

$$S(\bar{X}) = \left\{ (\bar{x}_1, \ldots, \bar{x}_N) \mid \sum_{k=1}^{N} \bar{x}_k = \bar{X}, m_k(\bar{X}) \leq \bar{x}_k \leq M_k(\bar{X}), k = 1, 2 \ldots, N \right\}$$

Example

Example: symmetric duopoly

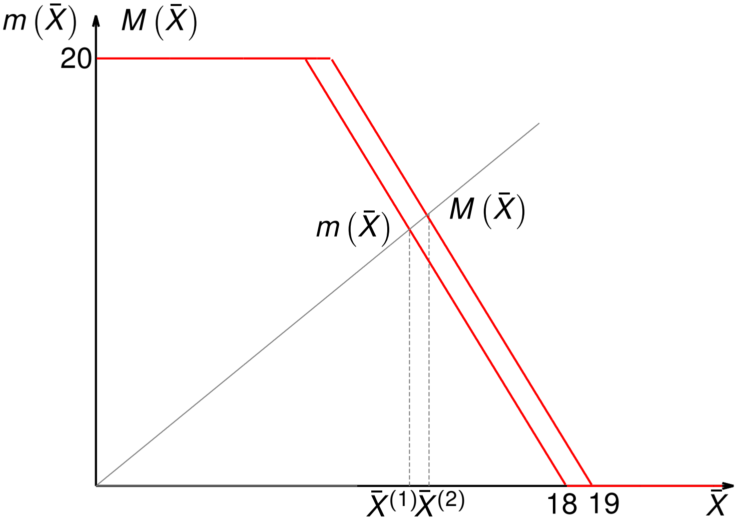
$A = 20$, $B = 1$, $c_1 = c_2 = 0$, $d_1 = d_2 = \gamma_1 = \gamma_2 = 1$ and $L_1 = L_2 = 10$
In this case

$$m_k(\bar{X}) = \begin{cases} 10 & \text{if} \quad \bar{X} \leq 8 \\ 18 - \bar{X} & \text{if} \quad 8 \leq \bar{X} \leq 18 \\ 0 & \text{if} \quad \bar{X} \geq 18 \end{cases} \quad M_k(\bar{X}) = \begin{cases} 10 & \text{if} \quad \bar{X} \leq 9 \\ 19 - \bar{X} & \text{if} \quad 9 \leq \bar{X} \leq 19 \\ 0 & \text{if} \quad \bar{X} \geq 19 \end{cases}$$

$k = 1, 2$

Example: symmetric duopoly

By symmetry, $m(\bar{X}) = 2m_1(\bar{X})$ and $M(\bar{X}) = 2M_1(\bar{X})$



Example: symmetric duopoly

General duopoly

$$
\begin{array}{lll}
\text{(i)} & \bar{x}_k = 0, & \text{if} \quad \frac{A-d_k-\gamma_k}{B} \leq \bar{x}_l; \\
\text{(ii)} & 0 < \bar{x}_k < L_k, & \text{if} \quad \frac{A-d_k-\gamma_k}{B} - 2\bar{x}_k \leq \bar{x}_l \leq \frac{A-d_k}{B} - 2\bar{x}_k; \\
\text{(iii)} & \bar{x}_k = L_k, & \text{if} \quad \bar{x}_l \leq \frac{A-d_k}{B} - 2L_k
\end{array}
$$

with $k = 1, 2$ and $l \neq k$.

In the case of the previous example

$$
\begin{array}{ll}
\bar{x}_k = 0, & \text{if} \quad 18 \leq \bar{x}_l; \\
0 < \bar{x}_k < 10, & \text{if} \quad 18 - 2\bar{x}_k \leq \bar{x}_l \leq 19 - 2\bar{x}_k; \\
\bar{x}_k = 10, & \text{if} \quad \bar{x}_l \leq -1
\end{array}
$$

Example: symmetric duopoly

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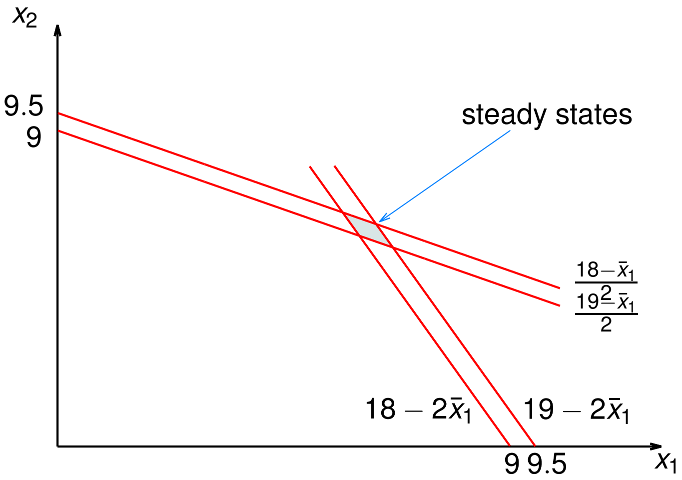
\end{array}
$$

Set of steady states for Example 1

It can be proved that all the steady states are internal



Asymptotic behavior

Asymptotic behavior

Nonempty simplex with usually infinitely many points $\longrightarrow$ there is no reason to examine analytically local or global asymptotical stability:

If $\bar{\mathbf{x}}$ is a steady state and the initial state of the system is selected in its neighborhood as another steady state, then the trajectory will stay there for all $t > 0$, so it does not converge back to $\bar{\mathbf{x}}$.

The asymptotic properties of the system are therefore examined by using computer simulation.

- semisymmetric case of N firms ($N > 1$)
- $p(X) = 20 - 2X$,
- cost functions:
  - $C_k(x_k) = x_k$, for $k = 1, 2, \ldots, N-1$)
  - $C_N(x_N) = 2x_N$, for N-th firm
- $\gamma_k = 1$
- $L_k = 10$
- identical initial output quantities. for firms $k = 1, 2, \ldots, N-1$

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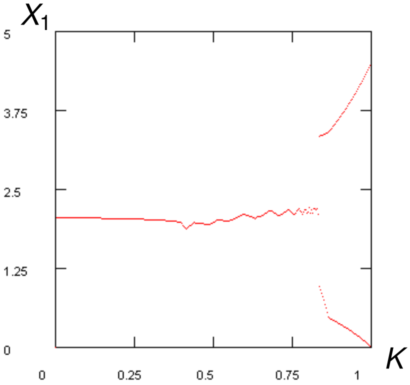
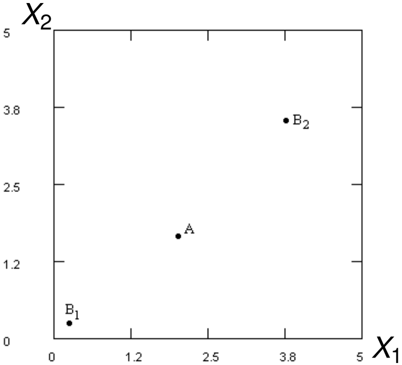
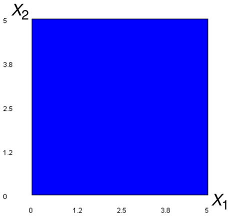
The case of $N = 4$



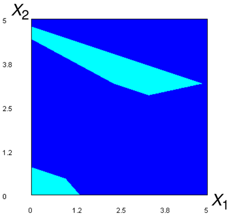


- $K = 0.2$: steady state A
- $K = 0.92$: 2-cycle $B_1 - B_2$

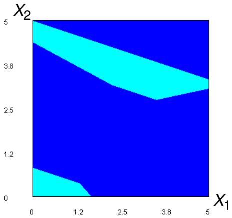
Basins with different values of parameter K



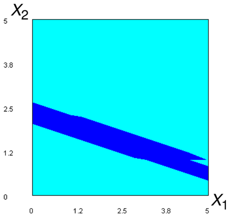
(a) $K \simeq 0.8352941$



(b) $K \simeq 0.835295$

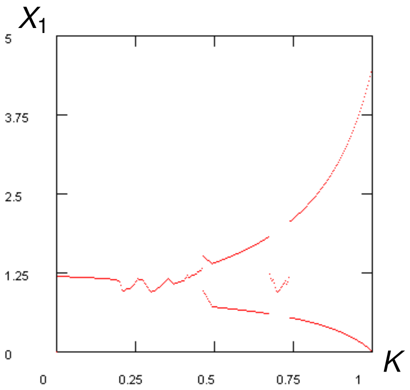


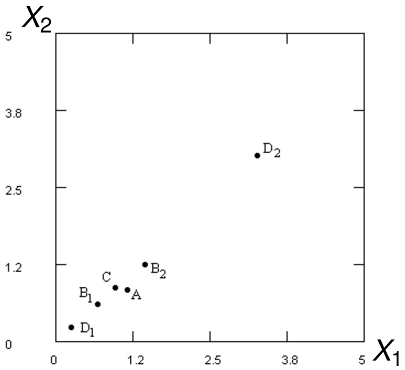
(c) $K \simeq 0.86$



(d) $K \simeq 1.0$

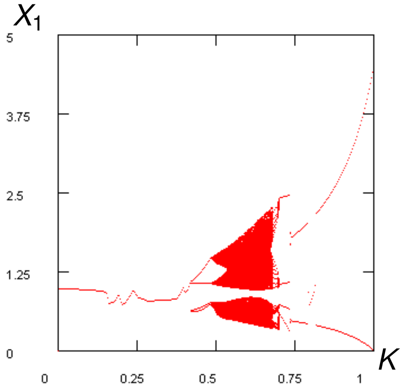
The case of $N = 9$

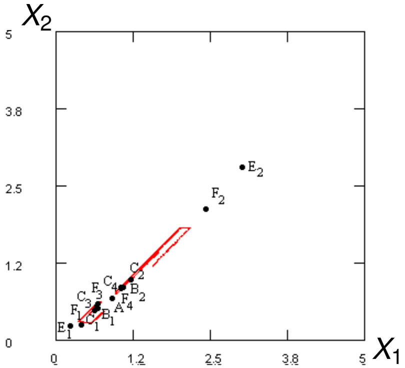




- $K = 0.07$: steady state A
- $K = 0.52$: 2-cycle $B_1 - B_2$
- $K = 0.696$: steady state C
- $K = 0.92$: 2-cycle $D_1 - D_2$

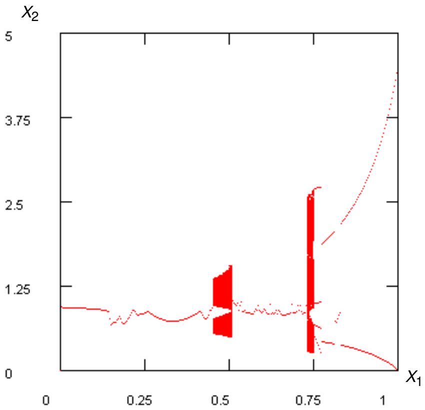
The case of $N = 12$



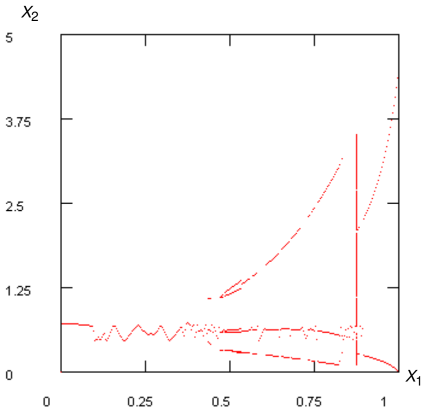


- $K = 0.30$, steady state: A
- $K = 0.40$, 2-cycle: $B_1 - B_2$
- $K = 0.45$, 4-cycle: $C_1 - C_4$
- $K = 0.66$, chaotic trajectory
- $K = 0.72$, 2-cycle: $E_1 - E_2$
- $K = 0.92$, 4-cycle: $F_1 - F_4$

Bifurcation diagrams with $N = 13, 20$



(a) $N = 13$



(b) $N = 20$

Unemployment Insurance Systems

Unemployment Insurance Systems

We assume that each firm k pays a certain proportion of the lost wages to the unemployed workers:

- with all the workers employed, each firm would be able to produce the maximum amount L_k
- the number of unemployed workers is proportional to the output difference $L_k - x_k$
- the total amount of unemployed compensation is also proportional to $L_k - x_k$.

So the profit of firm k can be formulated as

$$\Pi_k = x_k (A - Bx_k - BX_k) - (c_k + d_k x_k) - s_k (L_k - x_k).$$

Unemployment Insurance Systems

Again semisymmetric case

- firms $1, 2, \ldots, N-1$

$$c_k \equiv c,\ d_k \equiv d,\ s_k \equiv s,\ L_k \equiv L,\ K_k \equiv K$$

- firm N

$$c_N = \bar{c},\ d_N = \bar{d},\ s_N = \bar{s},\ L_N = \bar{L},\ K_N = \bar{K}$$

In this case the dynamic behavior of the firms can be described by the two-dimensional system

$$\begin{cases} x(t+1) = x(t) + K\left(-\frac{1}{2}(y(t) + (N-2)x(t)) + \frac{A-d+s}{2B} - x(t)\right) \\ y(t+1) = y(t) + \bar{K}\left(-\frac{1}{2}(N-1)x(t) + \frac{A-\bar{d}+\bar{s}}{2B} - y(t)\right) \end{cases}$$

by assuming interior best responses.

This model is equivalent to the well known semisymmetric linear oligopoly model.

Unemployment Insurance Systems

Again semisymmetric case

- firms $1, 2, \ldots, N-1$

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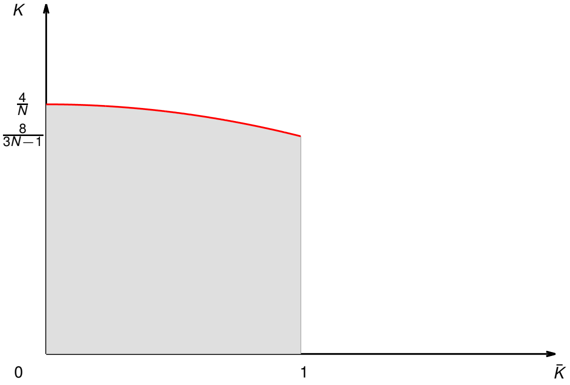
Unemployment Insurance Systems

- $N = 2$: the system is asymptotically stable
- $N = 3$: $0 < K, \bar{K} \leq 1$ the system is asymptotically stable (if $K = \overline{K} = 1$, then the steady state is marginally stable)
- $N \geq 4$, the condition for asymptotical stability is

$$K < \frac{16 - 8\bar{K}}{4N - \bar{K}\,(N + 1)}$$

Unemployment Insurance Systems

Stability region



since the system is linear the asymptotical stability is global

Conclusion

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  - more complex dynamics for higher values of adjustment speeds
  - may be unstable
  - the dynamics becomes complex with increasing adjustment costs, since the flexibility given by the contingent workforce is damped by the searching and training costs
- unemployment insurance system
  - simpler dynamics
  - no cycles

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