Alternative stabilization policies in a Keynes-Kaldor-Tobin model of business cycles

Hiroki MURAKAMI

Graduate School of Economics, University of Tokyo

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1. Introduction

Motivation 1

- Based upon Keynes' *General Theory* (1936), Kaldor (1940) formalized a nonlinear business cycle model by introducing the *S*-shaped investment function and
- analyzed the behaviors of aggregate income and stock of capital.
- However, **changes in prices (and inflation-deflation expectations)** are not taken into account.

Motivation 2

- On the other hand, Tobin (1975) proposed a disequilibrium Keynesian model and
- examine the behaviors of aggregate income, the price level and inflation-deflation expectations, but
- Tobin's model is a short-run model and variations in **capital stock** are not considered.

Motivation 3

- Both Kaldor (1940) and Tobin (1975) provided useful insights for understanding economic fluctuations.
- In this sense, it is worthwhile to integrate their perspectives for more complete analysis of economic fluctuations in the Keynesian world.

Purpose of presentation

- The purpose of this paper is to formalize a Keynes-Kaldor-Tobin model by unifying the basic ideas of Kaldor (1940) and Tobin (1975) and
- investigate the **stabilizing effects of monetary policies**.
- The quantity policy and the interest rate policy (a la Taylor's (1993) rule) are separately examined.
- The result is that **the interest rate policy can be more effective than the quantity policy**

2. Keynes-Kaldor-Tobin model

Model

- The model is as follows:

$$\frac{M}{p} = L(r, y, k), \tag{2.1}$$

$$\dot{y} = \alpha[I(r, y, k, \pi^e) - S(r, y, k, \pi^e)], \tag{2.2}$$

$$\dot{k} = I(r, y, k, \pi^e), \tag{2.3}$$

$$\frac{\dot{p}}{p} = H(y) + \pi^e, \tag{2.4}$$

$$\dot{\pi}^e = \beta(\frac{\dot{p}}{p} - \pi^e) \tag{2.5}$$

- y: aggregate income, r: rate of interest, k: capital stock, p: price level, π^e: expected rate of inflation, M: money supply, α: speed of quantity adjustments, β: speed of expectation formations on inflation

Assumption: partial derivatives

Assumption

I, S, L and H are of C^2 with:

$$L_r < 0, L_y > 0, L_k \geq 0, \tag{2.6}$$

$$I_r < 0, I_y > 0, I_k < 0, I_{\pi^e} > 0, \tag{2.7}$$

$$S_r \geq 0, S_y > 0, S_k \leq 0, S_{\pi^e} \leq 0, \tag{2.8}$$

$$H_y > 0. \tag{2.9}$$

- $L_k \geq 0$ reflects the positive wealth effect on money demand.

Interest rate

- Eq. (2.1) is solved for r as $(x = y, k)$:

$$r = R(y, k, p), \tag{2.10}$$

$$R_x = -\frac{L_x}{L_r}(R(y, k, p), y, k), R_p = -\frac{M}{pL_r}(R(y, k, p), y, k), \tag{2.11}$$

Model rewritten

- Substituting (2.10), the model can be rewritten as:

$$\dot{y} = \alpha X(y, k, p, \pi^e), \tag{2.12}$$

$$\dot{k} = A(y, k, p, \pi^e), \tag{2.13}$$

$$\frac{\dot{p}}{p} = H(y) + \pi^e, \tag{2.14}$$

$$\dot{\pi}^e = \beta H(y) \tag{2.15}$$

- X and A are defined as:

$$X(y, k, p, \pi^e) \equiv I(R(y, k, p), y, k, \pi^e) - S(R(y, k, p), y, k, \pi^e),$$
$$A(y, k, p, \pi^e) \equiv I(R(y, k, p), y, k, \pi^e).$$

- X is the excess demand function and A is the capital accumulation function.

Assumption: existence and uniqueness of equilibrium

Assumption

There uniquely exists an equilibrium $(y^*, k^*, p^*, 0) \in \mathbb{R}^3_{++} \times \mathbb{R}$.

Characteristic equation

- The characteristic equation associated with the Jacobian matrix is:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0, \tag{2.16}$$

- a_1-a_4 are given by:

$$a_1 \equiv -(\alpha X_y^* + A_k^*),$$
$$a_2 \equiv \alpha(X_y^* A_k^* - X_k^* A_y^* - X_p^* H_y^* p^* - \beta X_{\pi^e}^* H_y^*),$$
$$a_3 \equiv \alpha[(X_p^* A_k^* - X_k^* A_p^*)p^* - \beta(X_k^* A_{\pi^e}^* - X_{\pi^e}^* A_k^* + X_p^* p^*)]H_y^*,$$
$$a_4 \equiv \alpha\beta(X_p^* A_k^* - X_k^* A_p^*)H_y^* p^* > 0,$$

Proposition 2.3: existence of periodic orbits

Proposition

Under the assumption of Proposition 2.2 and if α and the absolute value of X_p^* are sufficiently small, there exists a positive β^* such that at least one periodic orbit exists for β sufficiently close to β^*.

- This proposition implies that **the equilibrium gets more likely to lose stability as the speed of inflation-deflation expectation formations β becomes greater**.
- **Processes of inflation-deflation expectation formations work as a destabilizing factor**.

3. Alternative policies and economic stability in the Keynes-Kaldor-Tobin model

3.1 Quantity policy

Model modified

- The model is modified as:

$$m = L(r, y, k), \tag{3.1}$$
$$\dot{y} = \alpha[I(r, y, k, \pi^e) - S(r, y, k, \pi^e)], \tag{3.2}$$
$$\dot{k} = I(r, y, k, \pi^e), \tag{3.3}$$
$$\frac{\dot{p}}{p} = H(y) + \pi^e, \tag{3.4}$$
$$\dot{\pi}^e = \beta(\frac{\dot{p}}{p} - \pi^e), \tag{3.5}$$
$$\frac{\dot{M}}{M} = \gamma(y^* - y), \tag{3.6}$$

- $m \equiv M/p$, y^*: natural-rate income, γ: **policy parameter**.
- Eq. (3.6) represents the counter-cyclical policy through changes in the supply of money.

Model rewritten

- Eliminating r by solving (3.1), the model is re-formalized as:

$$\dot{y} = \alpha X(y, k, m, \pi^e), \tag{3.7}$$

$$\dot{k} = A(y, k, m, \pi^e), \tag{3.8}$$

$$\frac{\dot{m}}{m} = -[H(y) + \gamma(y - y^*) + \pi^e], \tag{3.9}$$

$$\dot{\pi}^e = \beta H(y), \tag{3.10}$$

- X and A are defined as:

$$X(y, k, m, \pi^e) \equiv I(R(y, k, m), y, k, \pi^e) - S(R(y, k, m), y, k, \pi^e), \tag{3.11}$$

$$A(y, k, m, \pi^e) \equiv I(R(y, k, m), y, k, \pi^e). \tag{3.12}$$

- X is the excess demand function and A is the capital accumulation function.

Assumption: partial derivatives

Assumption

X, A and H are of C^2 with:

$$X_k < 0, X_m > 0, X_{\pi^e} > 0, \tag{3.13}$$

$$A_y > 0, A_k < 0, A_m > 0, A_{\pi^e} > 0, \tag{3.14}$$

$$H_y > 0, \tag{3.15}$$

$$X_y A_k > X_k A_y, X_k A_m > X_m A_k. \tag{3.16}$$

- The validity of the condition of $X_k A_m > X_m A_k$ can be confirmed by:

$$X_k A_m - X_m A_k = (I_r R_k + I_k)(S_r R_m + S_m) - I_r R_m (S_r R_k + S_k) > 0,$$

- This condition is fulfilled if S_r is sufficiently small.

Assumption: existence and uniqueness of equilibrium

Assumption

There uniquely exists an equilibrium $(y^*, k^*, m^*, 0) \in \mathbb{R}^3_{++} \times \mathbb{R}$.

- This assumption is introduced for the sake of simplicity only.

Characteristic equation

- The characteristic equation associated with the Jacobian matrix is calculated as:

$$\lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0, \tag{3.17}$$

- b_1-b_4 are given by:

$$\begin{aligned}
b_1 &\equiv -(\alpha X_y^* + A_k^*),\\
b_2 &\equiv \alpha(X_y^* A_k^* - X_k^* A_y^* + X_m^* H_y^* m^* - \beta X_{\pi^e}^* H_y^* + \gamma X_m^* m^*),\\
b_3 &\equiv \alpha\{[(X_k^* A_m^* - X_m^* A_k^*)m^* - \beta(X_k^* A_{\pi^e}^* - X_{\pi^e}^* A_k^* - X_m^* m^*)]H_y^*\\
&\quad + \gamma(X_k^* A_m^* - X_m^* A_k^*)m^*\},\\
b_4 &\equiv \alpha\beta(X_k^* A_m^* - X_m^* A_k^*)H_y^* m^* > 0,
\end{aligned}$$

Proposition 3.1: existence of periodic orbits

Proposition

If α, β and H_y^* are not so large, there exists a positive γ^* such that at least one periodic orbit exists, for γ sufficiently close to γ^*.

- Proposition 3.1 implies that under some reasonable assumptions, **the smallness of γ destabilizes the model.** and that
- the quantity policy may have a **negative influence** on economic stability in that **persistent economic fluctuations are yielded, if γ is not large enough.**
- Inadequate quantity policies can generate a persistent economic fluctuation (periodic orbit).

3.2 Interest rate policy

Model revised

- To examine the effects of the interest rate policy, the model is revised as:

$$\dot{y} = \alpha[I(y, k, \rho^e) - S(y, k, \rho^e)], \tag{3.18}$$

$$\dot{k} = I(y, k, \rho^e), \tag{3.19}$$

$$\dot{\pi}^e = \beta H(y), \tag{3.20}$$

$$\dot{r} = \delta(y - y^*), \tag{3.21}$$

- $\rho^e \equiv r - \pi^e$: real rate of interest, δ: **policy parameter**.
- Eq. (3.21) is an expression of the feedback policy through changes in the rate of interest.
- It is a feedback policy conducted in response to output gap (a la Taylor's rule).

Model rewritten

- Noting that $\rho^e \equiv r - \pi^e$, the model can be written as:

$$\dot{y} = \alpha[I(y, k, \rho^e) - S(y, k, \rho^e)], \tag{3.22}$$

$$\dot{k} = I(y, k, \rho^e), \tag{3.23}$$

$$\dot{\rho}^e = \delta(y - y^*) - \beta H(y). \tag{3.24}$$

Assumptions: partial derivatives

Assumption

I, S and H are of C^2 for every $(y, k, \rho^e) \in \mathbb{R}^2_{++} \times \mathbb{R}$ with:

$$I_y > 0, I_k < 0, I_{\rho^e} < 0, \tag{3.25}$$

$$S_y > 0, S_k \leq 0, S_{\rho^e} \geq 0, \tag{3.26}$$

$$H_y > 0, \tag{3.27}$$

$$I_y S_k > I_k S_y, I_k S_{\rho^e} < I_{\rho^e} S_k \tag{3.28}$$

Assumption

There uniquely exists an equilibrium (y^*, k^*, ρ^{e*}).

Characteristic equation

- The characteristic equation is calculated as follows:

$$\lambda^3 + d_1\lambda^2 + d_2\lambda + d_3 = 0, \tag{3.29}$$

- d_1-d_3 are given by:

$$\begin{aligned}
d_1 &\equiv -[\alpha(I_y^* - S_y^*) + I_k^*],\\
d_2 &\equiv \alpha[I_y^* S_k^* - I_k^* S_y^* - (\delta - \beta H_y^*)(I_{\rho^e}^* - S_{\rho^e}^*)],\\
d_3 &\equiv \alpha(I_{\rho^e}^* S_k^* - I_k^* S_{\rho^e}^*)(\delta - \beta H_y^*).
\end{aligned}$$

Local asymptotic stability

- The **necessary and sufficient condition for local asymptotic stability** of the equilibrium:

$$\alpha(I_y^* - S_y^*) + I_k^* < 0, \tag{3.30}$$

$$\delta > \beta H_y^*, \tag{3.31}$$

$$[\alpha(I_y^* - S_y^*)(I_{\rho^e}^* - S_{\rho^e}^*) + (I_k^* - S_k^*)I_{\rho^e}^*](\delta - \beta H_y^*)$$
$$> [\alpha(I_y^* - S_y^*) + I_k^*](I_y^* S_k^* - I_k^* S_y^*). \tag{3.32}$$

- Conditions (3.30)-(3.32) are satisfied if $I_y^* < S_y^*$ and $\delta > \beta H_y^*$.

Assumption for stability

Assumption

$$I_y^* < S_y^* \tag{3.33}$$

- This is the so-called Keynesian stability condition.

Proposition 3.2: condition for stability

Proposition

If δ is large enough to satisfy:

$$\delta > \beta H_y^*, \tag{3.34}$$

then the unique equilibrium (y^*, k^*, ρ^{e*}) is locally asymptotically stable.

- Proposition 3.2 implies that if the policy parameter δ is sufficiently large, the local stability can be achieved, i.e., that
- The economy can be **stabilized** when the monetary authority takes a **strong attitude** towards economic fluctuations.

Denial of a periodic orbit generated by a Hopf bifurcation

- In the case of the interest rate policy, **a periodic orbit cannot be yielded by way of a Hopf bifurcation**.
- For a Hopf bifurcation to arise, we must have:

$$I_y^* S_k^* - I_k^* S_y^* > (\delta - \beta H_y^*)(I_{\rho^e}^* - S_{\rho^e}^*), \tag{3.35}$$

$$[\alpha(I_y^* - S_y^*)(I_{\rho^e}^* - S_{\rho^e}^*) + (I_k^* - S_k^*)I_{\rho^e}^*](\delta - \beta H_y^*) \tag{3.36}$$

$$= [\alpha(I_y^* - S_y^*) + I_k^*](I_y^* S_k^* - I_k^* S_y^*).$$

- By (3.35) and (3.36), we obtain the following expression:

$$\frac{(I_y^* S_k^* - I_k^* S_y^*)(I_k^* S_{\rho^e}^* - I_{\rho^e}^* S_k^*)}{\alpha(I_y^* - S_y^*)(I_{\rho^e}^* - S_{\rho^e}^*) + (I_k^* - S_k^*)I_{\rho^e}^*} > 0. \tag{3.37}$$

- Condition (3.37) contradicts Assumptions made so far.

Conclusion: quantity policy vs. interest rate policy

- Our analysis implies that **the interest rate policy can be more effective than the quantity policy** in that
- **inadequate quantity policies can give rise to a persistent business cycle** (by way of a Hopf bifurcation), but
- **interest rate policies cannot yield a persistent economic fluctuation** (by way of a Hopf bifurcation).

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Thank you for listening to my presentation !!