Inflation Targeting, Recursive Inattentiveness and Heterogeneous Beliefs

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Transparency of CB decision-making

Modern monetary policy has emphasized that maintaining a stable monetary environment depends crucially on the ability of the policy regime to control inflation expectations.

- Woodford (2003): activity of modern CBs as *management of expectations*
- Policy makers develop communication strategies aimed to align expectations with policy objectives
  - Example: provision of an explicit numerical inflation target → focal point for private sector expectations
- In a world with RE and perfect information, no role for CB communication (Blinder et al. (2008), Svensson (2009))

Related literature

- Imperfect knowledge of monetary policy within the private sector and optimal degree of transparency in the context of global games (Morris and Shin (2002)) with public and private noisy signal (see, e.g., Cornand and Heinemann (2008), Cornand and Baeriswyl (2010), Walsh (2007))
- CB transparency and communication issues within New Keynesian framework with adaptive learning (see e.g., Orphanides and Williams (2005), Berardi and Duffy (2007), Eusepi and Preston (2010))

This paper (I)

Simple nonlinear model in order to investigate the dynamical consequences of monetary policy in a world with endogenously evolving heterogeneous expectations.

- CB announces the target to anchor expectations but a biased perception of the target may arise due to imperfect information flows
- Idiosyncrasies in understanding and processing information may cause heterogeneous beliefs about the true inflation target
  - Evidence for heterogeneity in inflation expectations has been provided using survey data as well as experimental data

This paper (II)

- Private sector's beliefs are revised over time as new information becomes available and the direction of change is determined by the distance between agents beliefs and actual realisations
  - Willingness to learn via continuous evaluation of individual performance: most fundamental definition of rational behaviour (De Grauwe (2012))
- Can a simple instrument rule implemented by the CB lead the economy to the targeted inflation?

NK model with heterogeneous beliefs (Kurz (2011))

Due to idiosyncrasies in the perception of the inflation target, the standard NK framework should be extended to accommodate for heterogeneous beliefs.

$$y_t = \bar{E}_t y_{t+1} - \sigma^{-1}(i_t - \bar{E}_t \pi_{t+1}) + \zeta_t \qquad \text{(IS)}$$

$$\pi_t = k y_t + \beta \bar{E}_t \pi_{t+1} + \xi_t \qquad \text{(NKPC)}$$

$$i_t = \bar{\pi} + \phi_\pi (\pi_t - \bar{\pi}) \qquad \text{(MP)}$$

with

- $\bar{E} = \int_i E_i$
- $\zeta_t \equiv \int_i (E_{i,t} c_{i,t+1} - E_{i,t} c_{t+1})$
- $\xi_t \equiv (1-\omega)\beta \int_i (E_{i,t} p_{i,t+1} - E_{i,t} p_{t+1})$

Agents' beliefs

- Private sector expectations are "anchored" to the target $\bar{\pi}$
- Biased perceptions of the target may arise to idiosyncrasies and imperfect information flows (Salle et al. (2013))

$$\bar{\pi}_i^p = \bar{\pi} + \nu_i$$

- Agents know the structural model $\rightarrow$ output gap expectations are consistent with the NK model, given $\bar{\pi}_i^p$

$$\bar{y}_i^p = (1 - \beta)\bar{\pi}_i^p / k$$

Belief dynamics

- Discrete support for noise $\nu_i \Rightarrow$ finite number (H) of biased beliefs

- Revision of beliefs as a function of distance from realisations: $U_{h,t-1} = -\sum_x (x_{t-1} - E_{h,t-2}x_{t-1})^2$, $x \in \{\pi, y\}$

$$P_t(h|U_{t-1}) = \frac{e^{\delta U_{h,t-1}}}{\sum_{h=1}^{H} e^{\delta U_{h,t-1}}}$$

- Microfoundations: random utility model (δ as *intensity of choice*, Brock and Hommes (1997)) or rational inattention ($1/\delta$ as *shadow cost of information*, Matejka and McKay (2011))

The full model

$$\begin{aligned}
y_t &= \sum_{h=1}^{H} n_{h,t} E_{h,t} y_{t+1} - \sigma^{-1} \left(i_t - \sum_{h=1}^{H} n_{h,t} E_{h,t} \pi_{t+1} \right) \\
\pi_t &= k y_t + \beta \sum_{h=1}^{H} n_{h,t} E_{h,t} \pi_{t+1} \\
i_t &= \bar{\pi} + \phi_\pi (\pi_t - \bar{\pi}) \\
n_{h,t} &= \frac{e^{\delta U_{h,t-1}}}{\sum_{h=1}^{H} e^{\delta U_{h,t-1}}} \\
U_{h,t-1} &= -\sum_{x} \left(x_{t-1} - E_{h,t-2} x_{t-1} \right)^2 \ ,
\end{aligned}$$

with $x \in \{\pi, y\}$ and the set of predictors $h = 1, ..., H$ is composed by pairs of beliefs

Reduction to 1D map

- The model can be re-written as

$$T : \begin{cases} y_t = b_\pi \, \Lambda \, z\,(\pi_{t-1}, y_{t-1}) \\ \pi_t = b_\pi \, \Gamma \, z\,(\pi_{t-1}, y_{t-1}) \end{cases}$$

  where Λ and Γ are structural parameter collections.
- The Jacobian matrix J of T has $\det J(y, \pi) = 0$ and in any point of the phase space one eigenvalue is equal to zero.
- There ought to exist a one-dimensional invariant plane on which dynamics take place. Indeed we can state

Proposition

The straight line $y = \frac{\Lambda}{\Gamma}\pi$ *is invariant.*

Therefore, dynamics is described by the restriction of the map T to the invariant line.

Simple(st) example

We assume agents may overestimate the target by an amount b_π, underestimate by an amount $-b_\pi$, or have correct beliefs about the target.

- Three types of beliefs

Zero inflation target:	$E_{1,t}\pi_{t+1} = 0$	$E_{1,t}y_{t+1} = 0$
Positive bias:	$E_{2,t}\pi_{t+1} = b_\pi$	$E_{2,t}y_{t+1} = \frac{(1-\beta)}{k} b_\pi$
Negative bias:	$E_{3,t}\pi_{t+1} = -b_\pi$	$E_{3,t}y_{t+1} = -\frac{(1-\beta)}{k} b_\pi$

- The 1D map under study takes the form

$$m_t = f_{\delta,\phi_\pi}(m_{t-1}) = \frac{e^{-\delta[M - N m_{t-1}]} - e^{-\delta[M + N m_{t-1}]}}{1 + e^{-\delta[M - N m_{t-1}]} + e^{-\delta[M + N m_{t-1}]}}$$

where $m_t = n_{2,t} - n_{3,t}$ and M, N parameter collections.

Inflation target stability

- Let $\theta = [\beta, k, \sigma]$ and introduce the positive constants

$$\phi_\pi^w(\theta) < \phi_\pi^m(\theta) < \phi_\pi^a(\theta) < \phi_\pi^o(\theta)$$

defining different monetary policy regimes

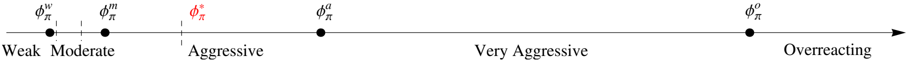


Figure : Policy regimes, CGG calibration

Example: *Moderate* policy

Let $\phi_\pi^w < \phi_\pi < \phi_\pi^m$. Then values $0 < \delta_1^* \leq \delta_2^*$ exist such that

- for $\delta < \delta_1^*$ the target steady state is unique and globally stable;
- for $\delta > \delta_2^*$ five steady states exist, three steady states are locally stable and two other steady states are unstable.

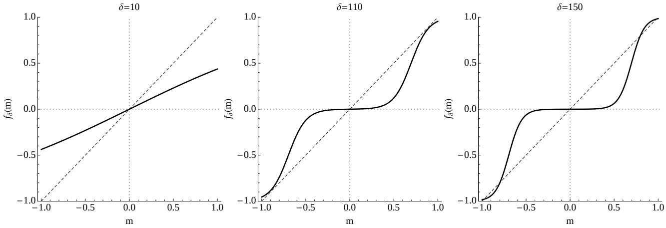


Figure : Map $f_\delta(m)$ for different values of δ in the *moderate* monetary policy.

Example: *Aggressive* policy

When the nominal interest rate reacts aggressively to inflation, the CB avoids multiplicity of equilibria and the target equilibrium is globally stable.

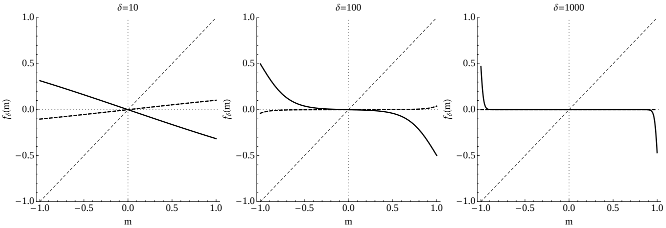
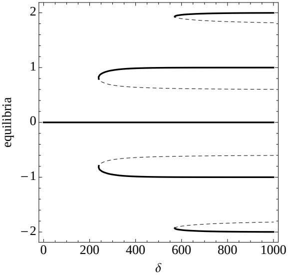


Figure : Map $f_\delta(m)$ (solid line) and second iterate $f_\delta^2(m)$ (thick dashed line) for different values of δ in the *aggressive* monetary policy.

Adding beliefs

- Bifurcation diagram with five beliefs types. Solid (dashed) lines indicate stable (unstable) equilibria.



- Hard to obtain analytical results with many belief types
- If the intensity of choice is high enough, more and more agents will adopt the belief yielding the most precise forecast, causing dynamics to lock into a self-fulfilling non-fundamental equilibrium.

The *Large Type Limit* (LTL) (Brock et al. (2005))

- Average (inflation) expectations with H belief types drawn from $\psi(b)$, with $a \equiv (1-\beta)/k$

$$\bar{E}_t \pi_{t+1} = \frac{\sum_{h=1}^{H} b_h \exp\left(-\delta\left((b_h - \pi_{t-1})^2 + (ab_h - y_{t-1})^2\right)\right)}{\sum_{h=1}^{H} \exp\left(-\delta\left((b_h - \pi_{t-1})^2 + (ab_h - y_{t-1})^2\right)\right)}$$

- Replace sample mean with population mean

$$\bar{E}_t \pi_{t+1} = \frac{\int b \exp\left(-\delta((b - \pi_{t-1})^2 + (ab - y_{t-1})^2)\right) \psi(b) db}{\int \exp\left(-\delta((b - \pi_{t-1})^2 + (ab - y_{t-1})^2)\right) \psi(b) db}$$

- For suitable distribution of beliefs (e.g., $\psi(b) \sim N(0, s^2)$), the LTL can be computed explicitly

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LTL results

The general policy implications carry over to the case of an arbitrarily large number of heterogeneous beliefs.

- Taylor principle does not hold: $\phi_\pi < 1$
  - $\delta > (<) \delta^* \Rightarrow$ target unstable (stable)
- Taylor principle does hold: $\phi_\pi > 1$
  - $1 < \phi_\pi < \phi_\pi^{**} \Rightarrow$ target stable $\forall\ \delta$
  - $\phi_\pi > \phi_\pi^{**} \Rightarrow$ target stable (unstable) if $\delta < (>) \delta^*$

The Taylor principle is not sufficient to guarantee convergence to the target. Monetary policy may overreact to deviations of inflation from the target, causing different dynamics.

Conclusions

- Inflation target implementability via simple instrument rules
- CB announces the target to anchor expectations but biased perception may arise due to imperfect information flows
- Recursive evaluation of beliefs as new info becomes available $\Rightarrow$ dynamical system in which macro variables and private sector beliefs co-evolve over time
- Taylor principle may not be sufficient to reach the target
- Monetary policy should be fine-tuned to ensure that signal sent by realisations of aggregate variables corrects wrong agents' beliefs

Example: *Weak* policy

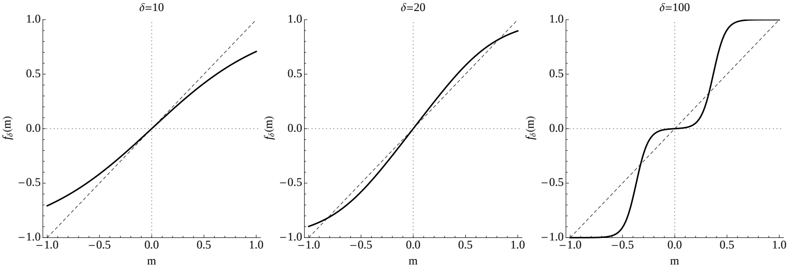

Figure : Map $f_\delta(m)$ for different values of δ in the *weak* monetary policy scenario.

Example: *Very Aggressive* policy

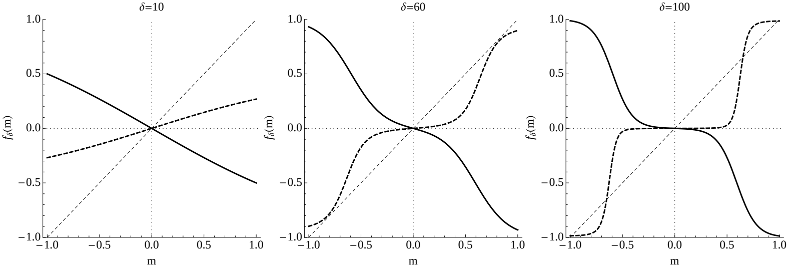

Figure : Map $f_\delta(m)$ (solid line) and second iterate $f_\delta^2(m)$ (thick dashed line) for different values of δ in the *very aggressive* monetary policy scenario.

Example: *Overreacting* policy

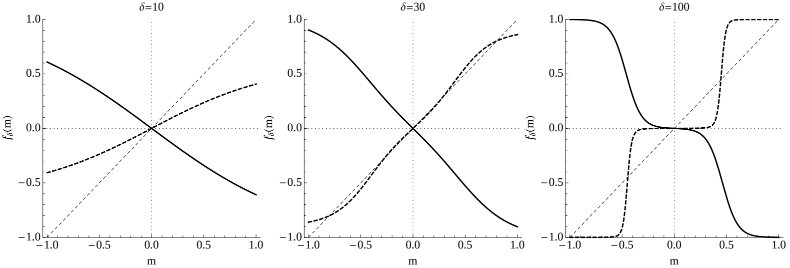

Figure : Map $f_\delta(m)$ (solid line) and second iterate $f_\delta^2(m)$ (thick dashed line) for different values of δ in the *overreacting* monetary policy scenario.

The *Rational Inattention* problem (I)

- The DM is presented with a group of H options from which she must choose one
- Let U_h denote the *value* of option $h \in \{1, \ldots, H\}$
- The DM has prior knowledge of the available option described by $g(U)$, where $U = (U_1, \ldots, U_H)$
- The info about the H options are available to the DM, but processing the information is costly

The *Rational Inattention* problem (II)

- The DM's strategy is described by $P(h|U)$ and it is the solution to the problem

$$\max_{\{P(h|U)\}_{h=1}^{H}} \left(\sum_{h=1}^{H} \int_U U_h P(h|U) g(dU) - \text{cost of info processing} \right)$$

subject to:

$$\begin{aligned} P(h|U) &\geq 0 \; \forall \; h \\ \sum_{h=1}^{H} P(h|U) &= 1 \end{aligned}$$

The *Rational Inattention* problem (III)

- Entropy : measure of uncertainty associated with a random variable (X with pdf $p(x)$)

$$F(X) = -E[\log p(x)]$$

- Unit cost of information: λ
- Amount of information processed, s, measured by the expected reduction in entropy of U, i.e., difference between the *prior* entropy of U and the expectation of the *posterior* entropy of U conditional on the chosen option h

$$s = F(U) - E_h[F(U|h)]$$

The *Rational Inattention* problem (IV)

- The strategy of the rationally inattentive DM is the collection of conditional probabilities $\{P(h|U)\}_{h=1}^{H}$ that solves

$$\max_{\{P(h|U)\}_{h=1}^{H}} \left(\sum_{h=1}^{H} \int_U U_h P(h|U) g(dU) - \lambda\, s \right)$$

- If $\lambda > 0$, then the DM forms his strategy such that

$$P(h|U) = \frac{P_h^0 e^{U_h/\lambda}}{\sum_{h=1}^{H} P_h^0 e^{U_h/\lambda}} \qquad \text{with } P_h^0 = \int_U P(h|U) g(dU)$$

- If $\lambda = 0$ then the DM selects the option with the highest value with prob. 1
- $P_h^0 = 1/H \ \forall h$ $\Rightarrow$ MNL formula with $\delta = 1/\lambda$