The simplicity of optimal trading in order book markets

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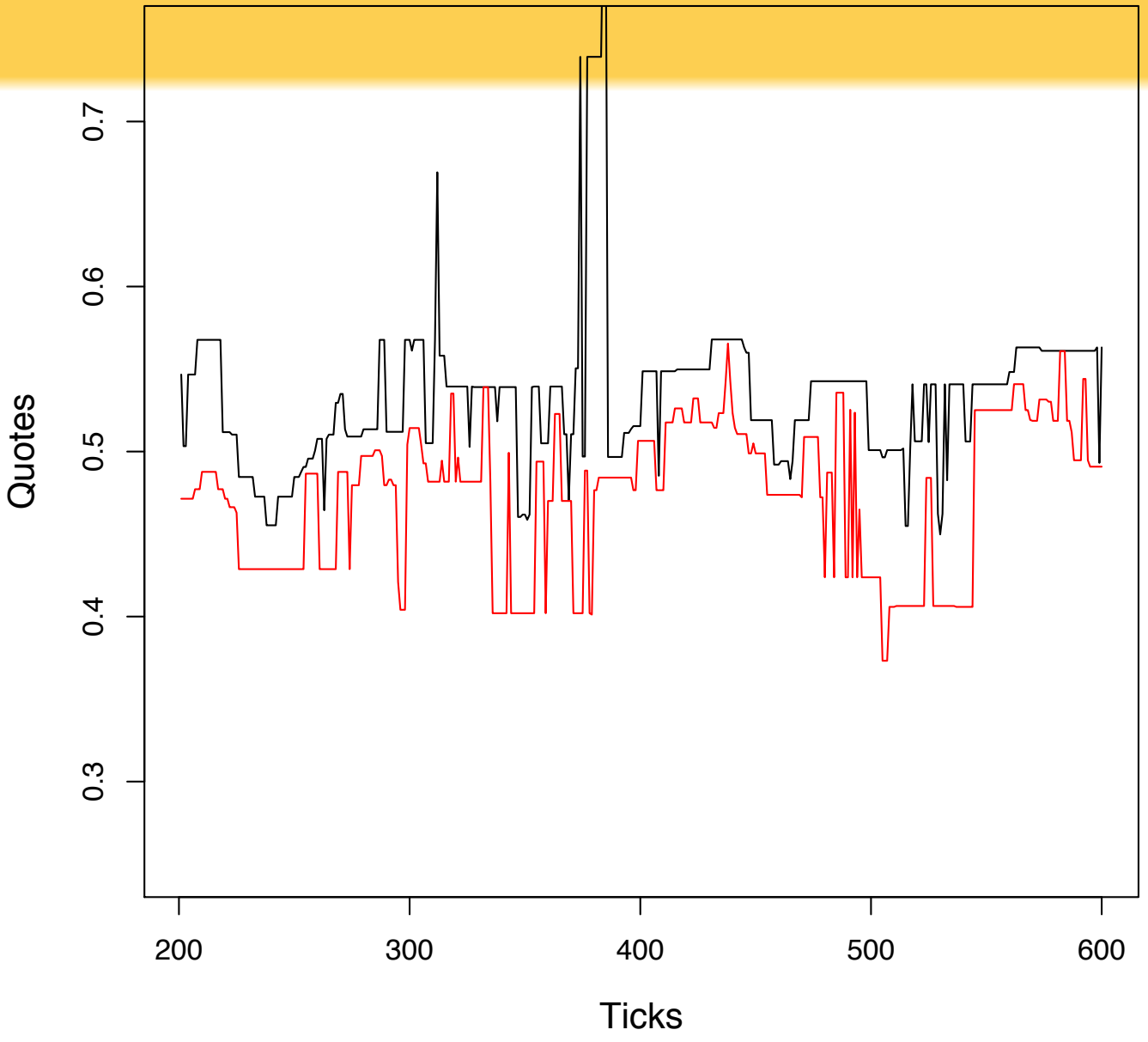
Intro



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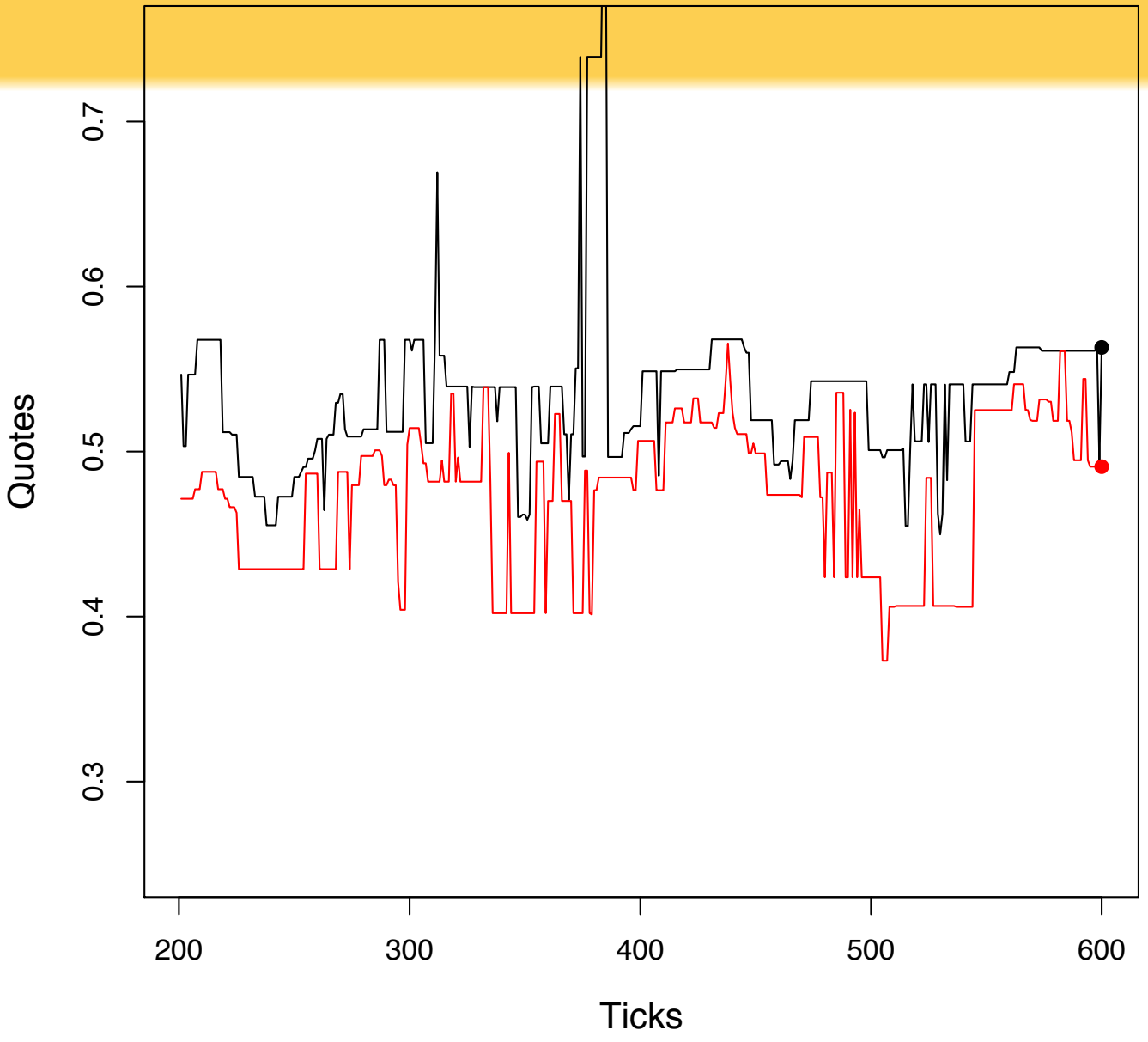

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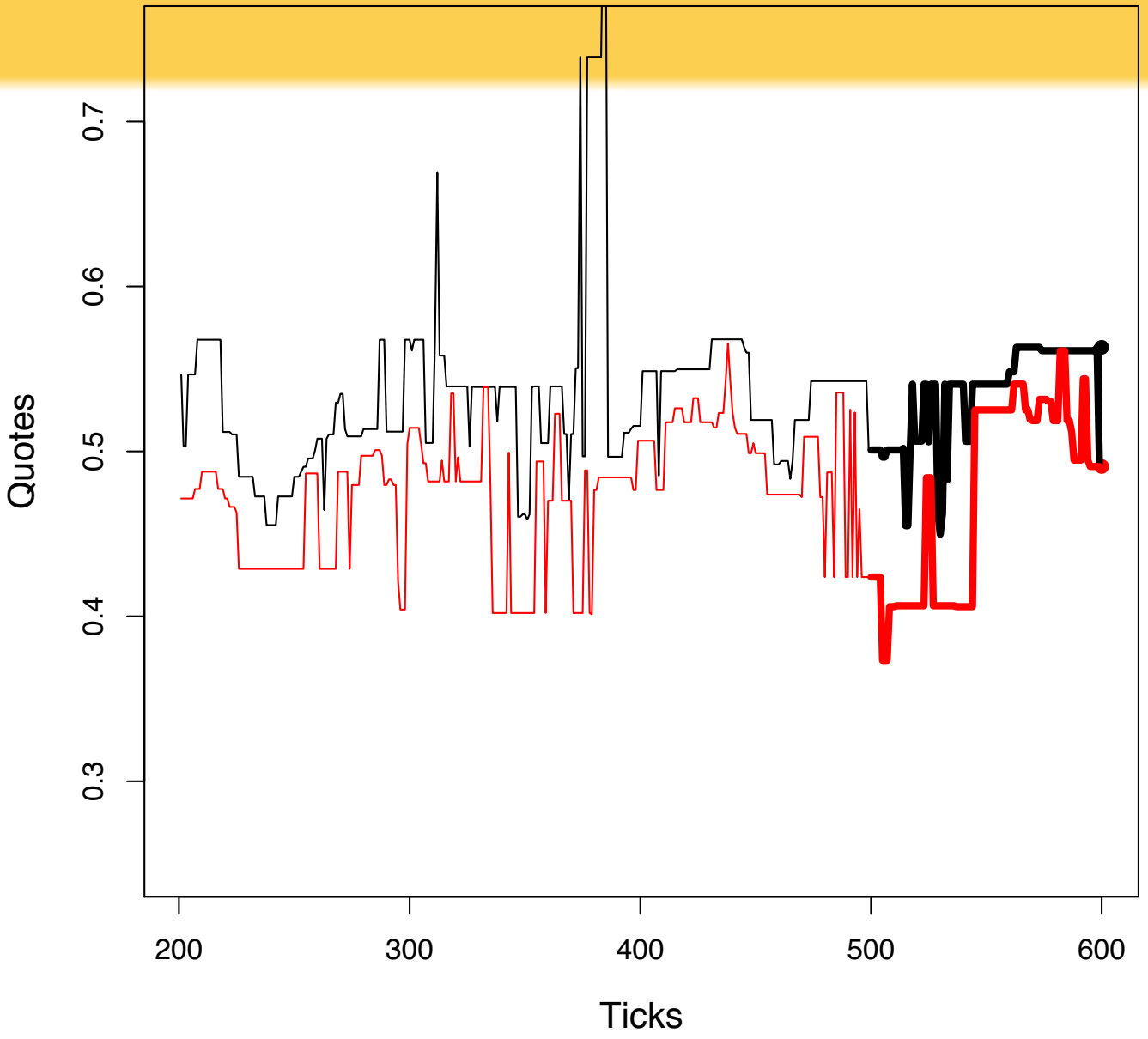

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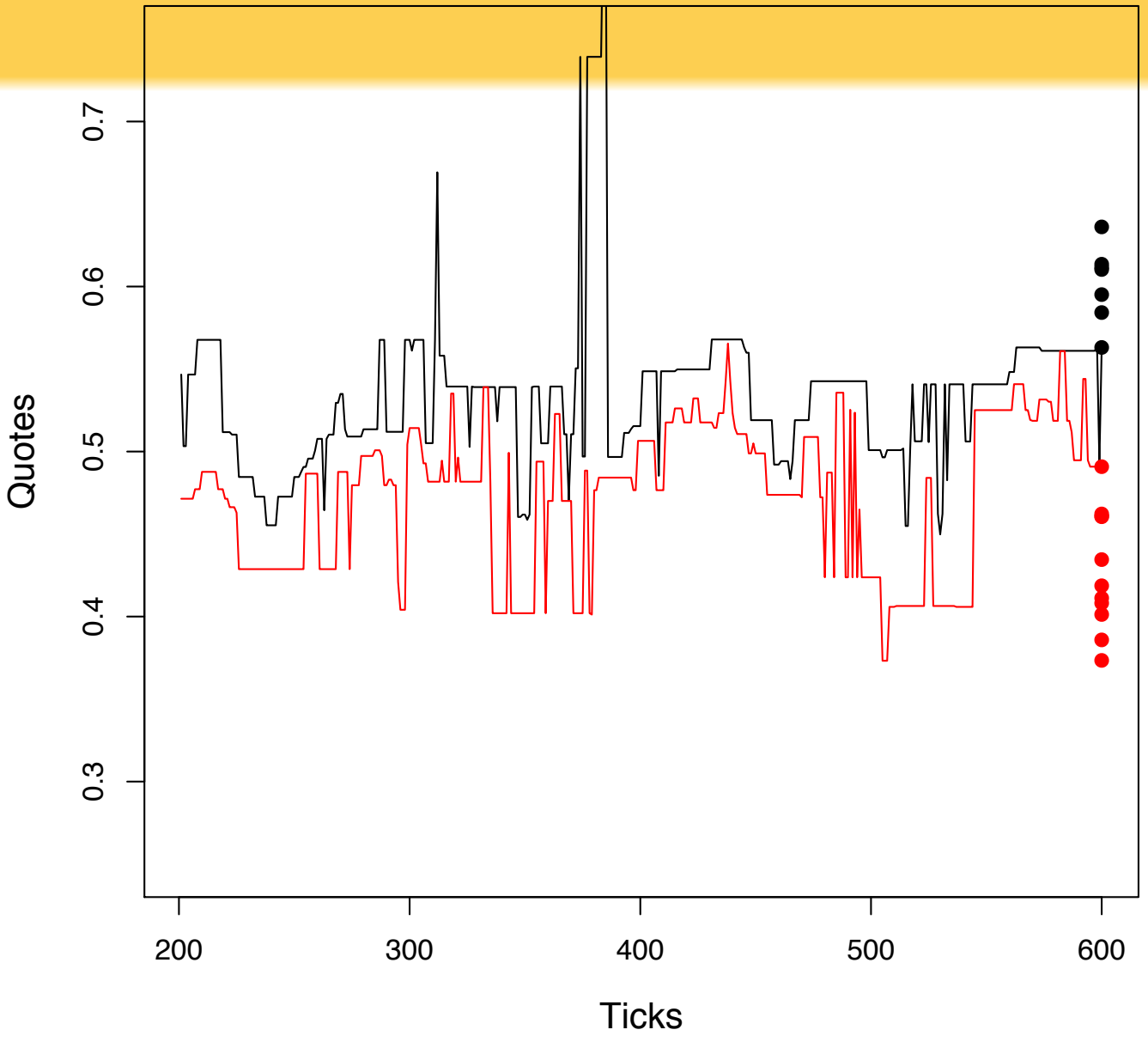

Intro

- A trader's execution strategy has a large effect on profits but identifying an optimal strategy is complex.
- Law of *many* prices: it is believed that in Continuous Double Auctions (CDA) information has a role.
- Information resolves the clash: *Should I stay or should I go?*
  1. Trade now (certain execution but low gain)
  2. Delayed trade (uncertain but higher gain)
- This is potentially of huge importance: algorithmic trading, 60 to 73% of orders are machine generated and 45% are left on the market for less than 1 second (source: SEC).

Research question

- I do not have any research question
- but have a research claim:

  Trading is simple and you should disregard nearly all the information available on the book.

- (Under some restrictive assumptions) most of the information is irrelevant at the time of submission.
- This supports the following facts
  1. The book is only dynamically complete, Bouchaud et al. (2009), as opposed to statically complete;
  2. Trading may be less cognitively complex than we thought (simple linear strategies are as good as complex Markov Perfect Equilibrium strategies).
  3. Speed is better than depth (of reasoning).

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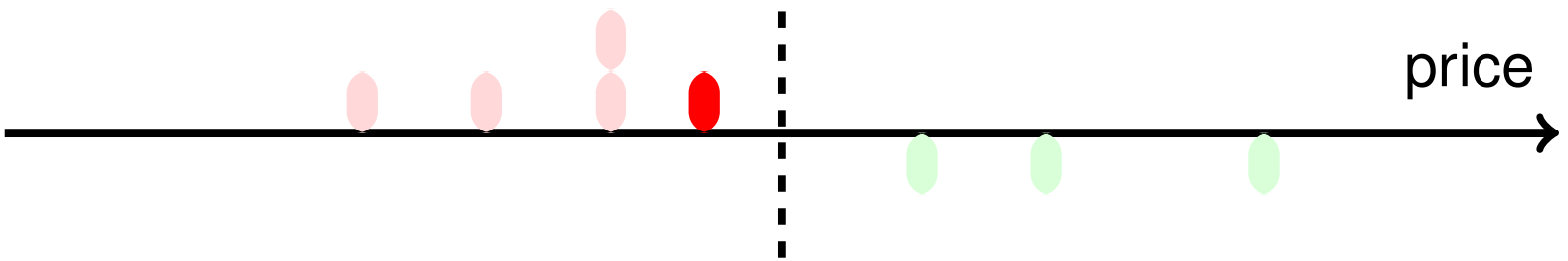
Continuos Double Auction

- Continuos Double Auction
  1. Agents place orders on separate buying and selling books
  2. Bids (asks) are sorted in decreasing (increasing) order according to price-time priority.
  3. Orders are canceled only when a counterpart is found (after execution) or with a smal exogenous probability P_c (unexecuted).



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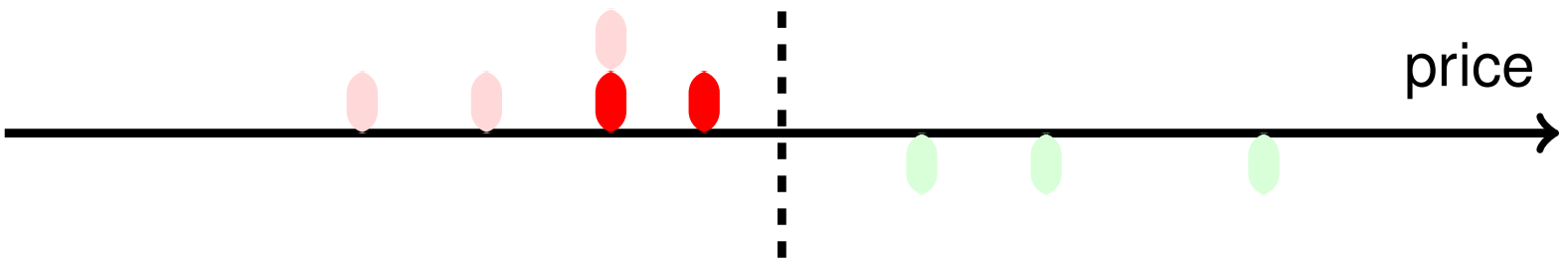

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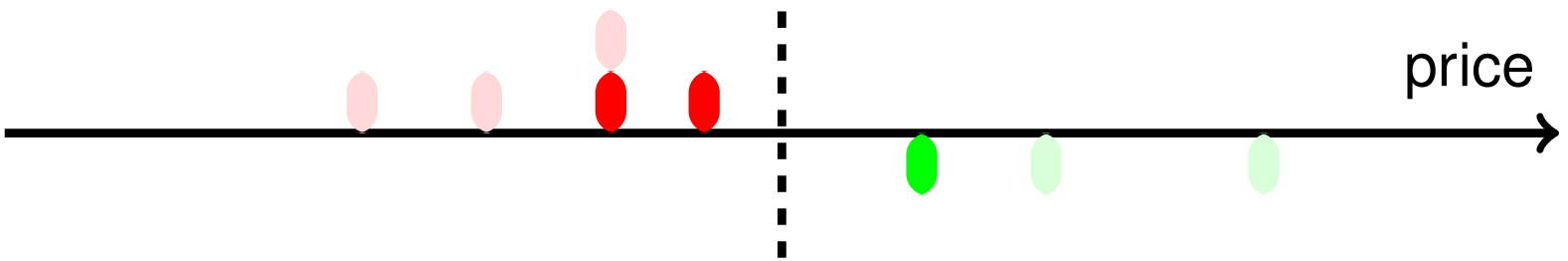

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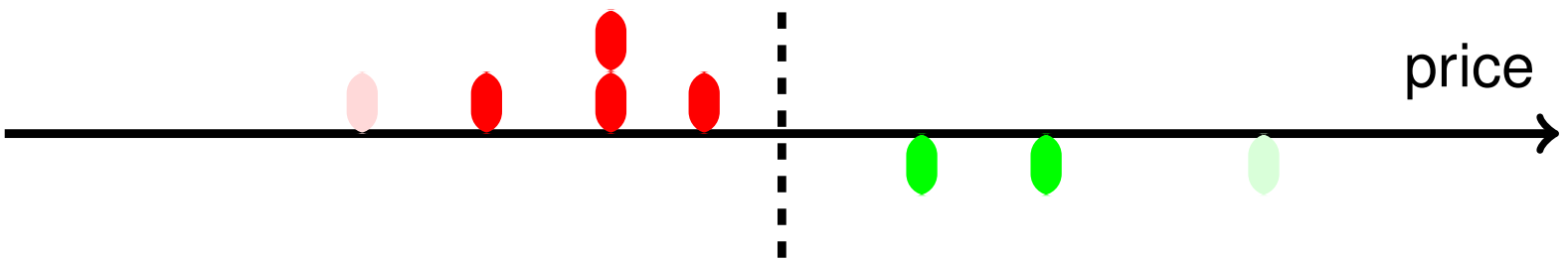

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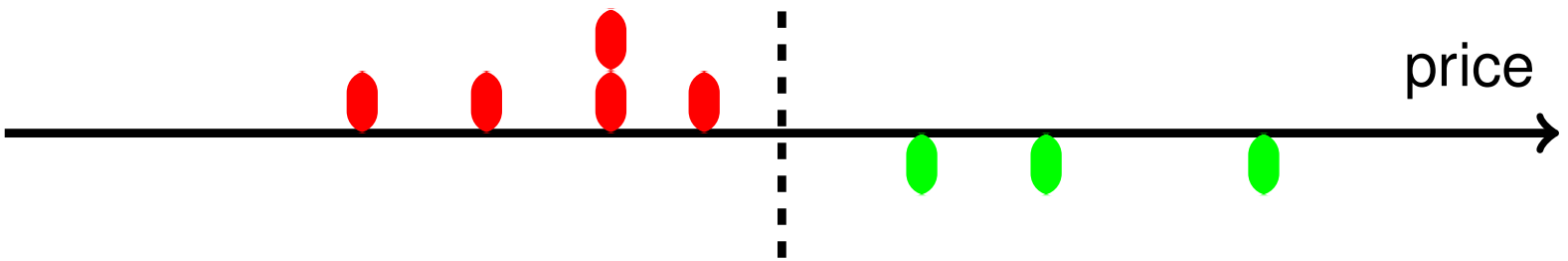

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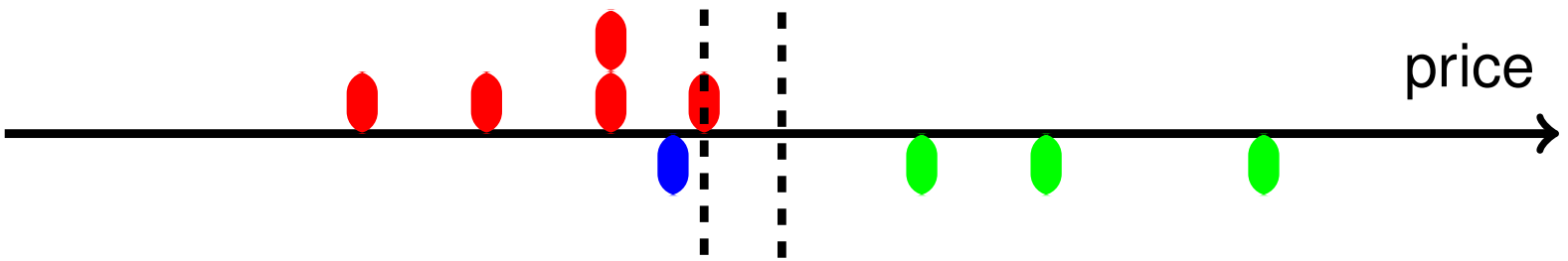

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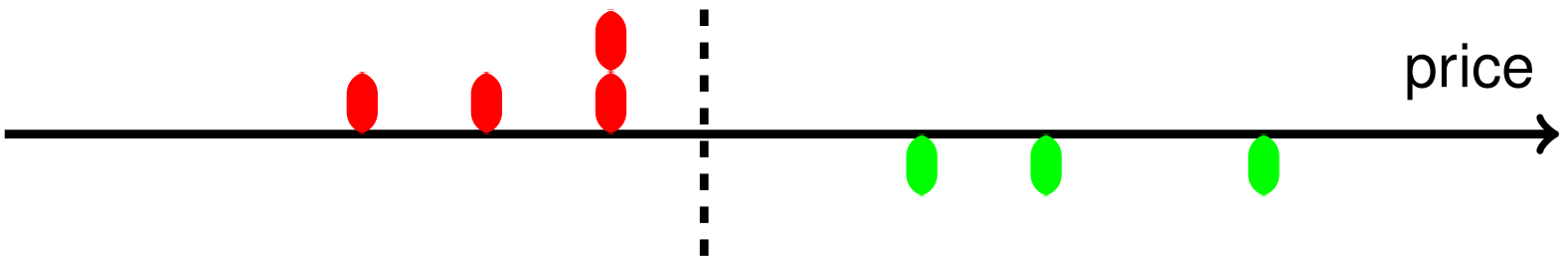

Book / buyers

- The book is

$$S_t = \{0 \leq ... \leq b_{3t} \leq b_{2t} \leq b_{1t} < a_{1t} \leq a_{2t} \leq a_{3t} \leq ...\},$$

- The highest bid b_1 and lowest ask a_1 are referred as best bid and best ask, respectively. The spread is $a_1 - b_1$.
- Single unit order when entering the market.
- Buyer: the bidding function (or strategy) is

$$B_{it} = f_i(a_{1t}, b_{1t}, I_{it})$$

- The submission of B_{it} changes the book and
  1. If $B_{it} \geq a_{1t}$: *marketable order*, the i-th agents gains $v_i - a_{1t}$. The best ask is updated.
  2. If instead $B_{it} < a_{1t}$, the new order is inserted in the book (for possible future trades).
  3. if $b_{1t} < B_{1t} < a_{1t}$: the order is *price improving*.

Book / sellers

- Seller: the asking function (or strategy) is

$$A_{jt} = g_j(a_{1t}, b_{1t}, J_{jt}),$$

- The submission of A_{jt} changes the book and
  1. If $A_{it} \leq b_{1t}$: *marketable order*, the j-th agents gains $b_{1t} - c_j$. The best bid is updated.
  2. If instead $A_{jt} > b_{1t}$, the new order is inserted in the book (for possible future trades).
  3. if $b_{1t} < A_{jt} < a_{1t}$: the order is *price improving*.
- Symmetry: all costs $c_j \in C$, exactly as all values $v_i \in V$.

Linear strategies

▸ Evolution strategies

Bid/ask to be submitted by traders at time t are given by

$$B_{it} = f_i(a_{1t}, b_{1t}, I_i) = \min(\overline{B}, \alpha_i a_{1t} + \beta_i b_{1t} + \gamma_i), \quad (1)$$

for buyers and

$$A_{jt} = g_j(a_{1t}, b_{1t}, J_j) = \max(\overline{A}, \delta_i a_{1t} + \phi_i b_{1t} + \eta_i), \quad (2)$$

for sellers, where $\alpha_i, \beta_i, \gamma_i, \delta_j, \phi_j, \eta_j$ are real constant to be determined and $I_i = J_j = \emptyset, \forall i, j$.

Observe that no execution probability is estimated.

Markov Perfect Equilibrium Strategies

- Information level l: number of consecutive quotes on the bid and ask side.
- Beliefs on the probabilities of order execution are explicitly calculated for each state of the market.
- Such an assignment P of probabilities is consistent and

$$f_i : V^{2l} \longrightarrow V, \quad (b_1, ..., b_l, a_1, ..., a_l) \mapsto B_{it},$$

where the bid B_{it} maximises

$$P(B_{it}|S_t)pay_{it}.$$

- Probabilities are iteratively found as outlined in Pakes and McGuire (2001).

Results: parameters

Variable	Description	Value
V	Buyer valuations	$\{0.05, 0.10, ..., 0.90, 0.95\}$
C	Seller Valuations	$\{0.05, 0.10, ..., 0.90, 0.95\}$
P_c	Probability of cancellation	0.01
$\overline{B}$	Maximum Bid	1.0
$\overline{A}$	Minimum Ask	0.0
P_R	Prob. random order	0.01
X	Convergence period	1,000,000
T	Optimisation period	1,000,000,000

Summary statistics

Model	$l = 1$	$l = 2$	$l = 3$	Linear
Best bid	0.466	0.465	0.464	0.472
Best ask	0.534	0.534	0.536	0.524
Spread	0.068	0.069	0.071	0.051
Quantity at best bid	2.39	2.40	2.32	-
Quantity at best ask	2.42	2.44	2.40	-

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Quotes

		0.45-0.55	0.40-0.60	0.35-0.65
Linear	0.45-0.55	0.50	0.69	0.74
	0.40-0.60	0.68	0.89	0.95
	0.35-0.65	0.72	0.94	0.99
$l = 1$	0.45-0.55	0.61	0.79	0.80
	0.40-0.60	0.78	0.97	0.98
	0.35-0.65	0.80	0.98	1.00
$l = 2$	0.45-0.55	0.62	0.79	0.81
	0.40-0.60	0.78	0.95	0.97
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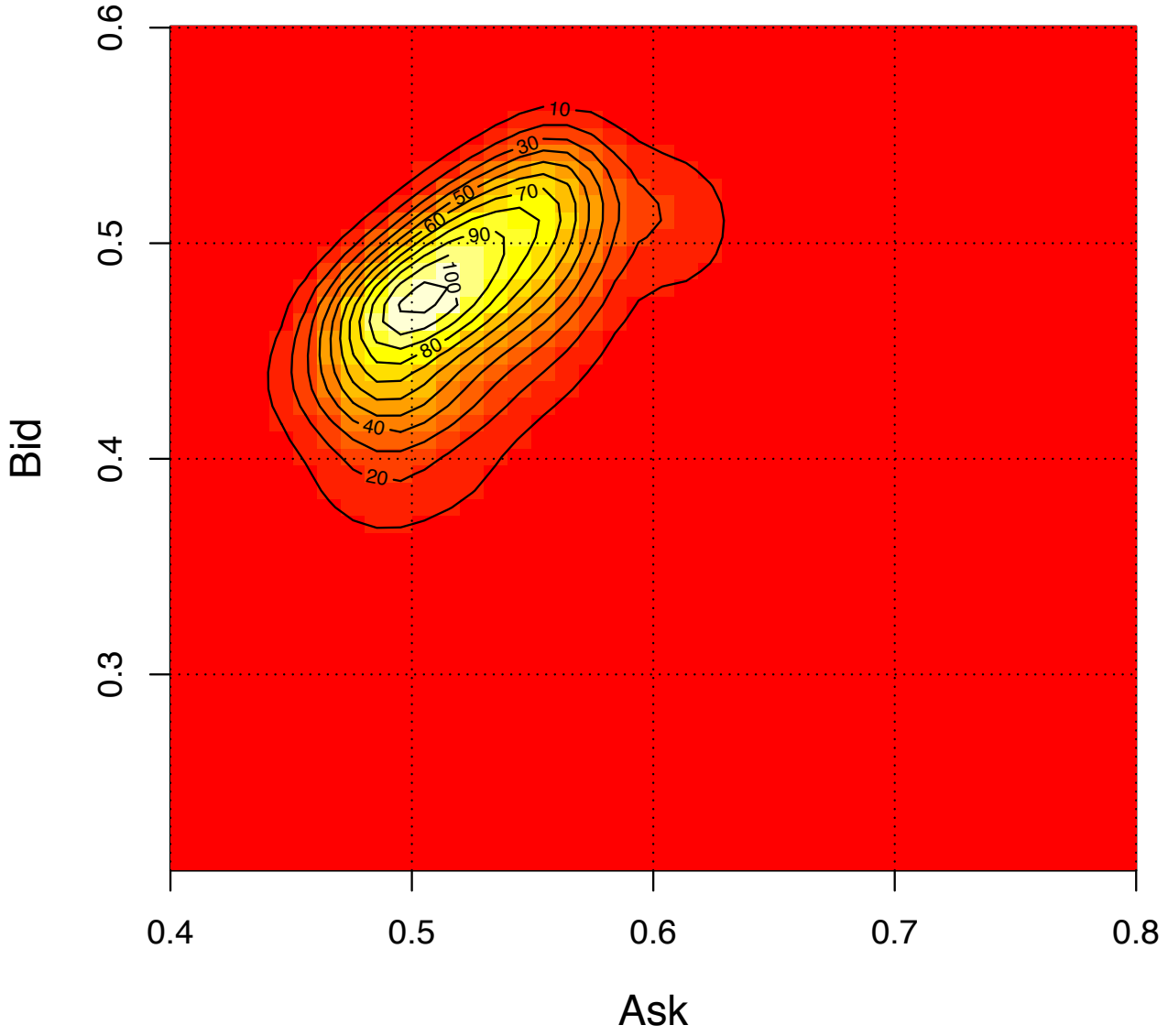
Equilibrium states



Orders' aggressiveness

Model	$l = 1$	$l = 2$	$l = 3$	Linear
Market Orders	0.233	0.233	0.233	0.257
Price Improving LO	0.108	0.104	0.109	0.181
LO at Best quotes	0.162	0.167	0.161	-
LO Behind Best Quote	0.497	0.496	0.498	0.563

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Conclusion

- We have used two models of order book markets to investigate the importance of information and strategic sophistication.
- Market and traders' behaviour differed little across levels of information.
- The crucial piece of information are the best quotes: the book may be dynamically complete even when quotes far from the best ones add little information or don't convey useful trading signals.

Conclusion (2)

- In equilibrium only a relatively small number of order book states occurr.
- Hence, the possible situations that traders must develop optimal responses for are small in number. Traders strategies may therefore be relatively simple and easily learnt.
- Markov strategies and linear approximations are similar: optimal trading may be achieved by a simple functional form, further easing the cognitive burden placed on traders.

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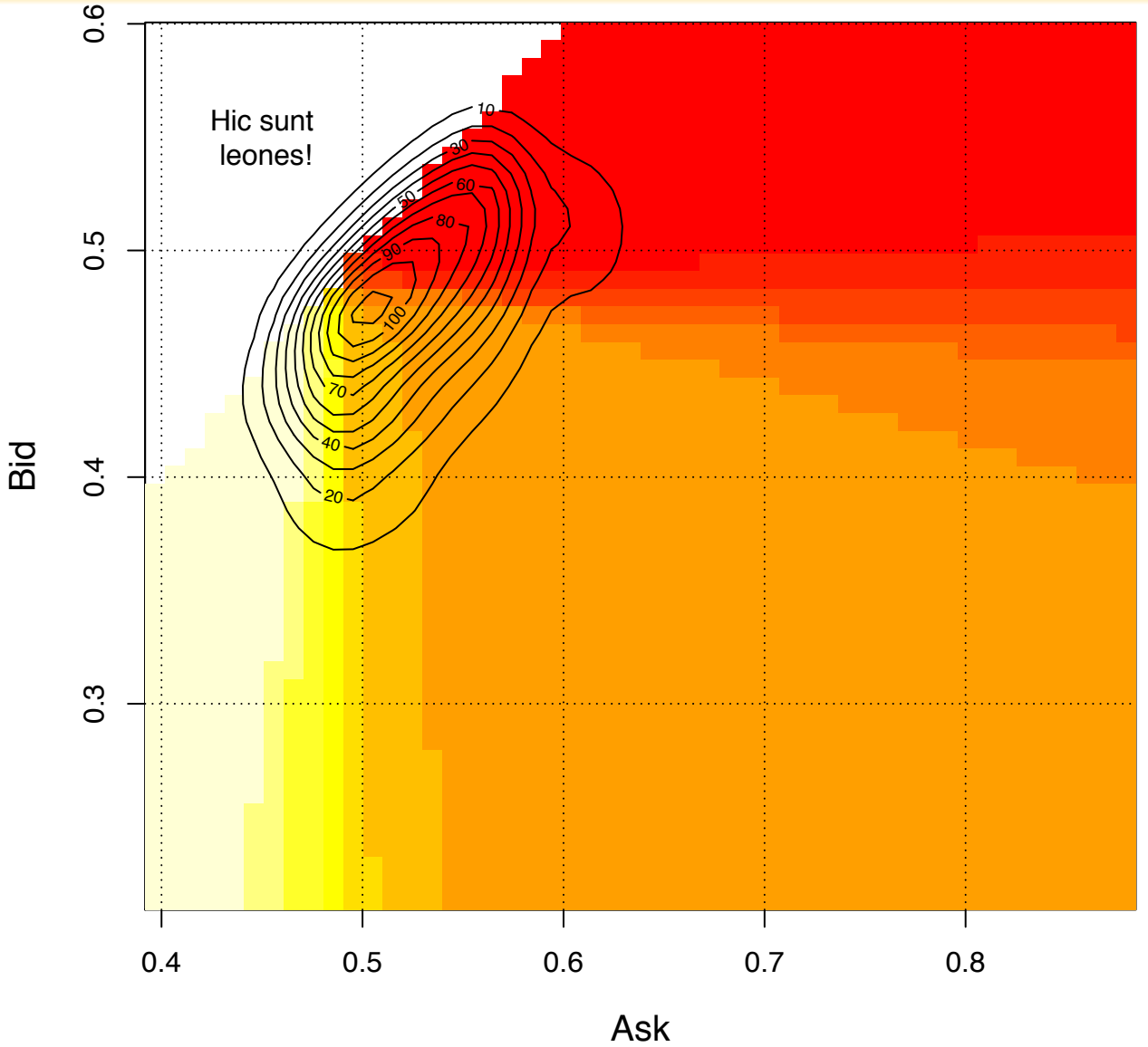
One more thing ($v_i = 0.55$)



Thank you

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http://virgo.unive.it/paolop/

The paper is also on IDEAS or SSRN

Evolution strategies - ES

Return

Each population independently maximizes the gain from trade over $\tau = 200$ of sessions, given the behavior of the other populations:

$$\max_{\alpha,\beta,\gamma} \sum_{t=0}^{\tau} \pi_t(\alpha, \beta, \gamma | \text{Other types}).$$

1. Set $g = 0$ and initialize the population $\mathcal{P}^{(0)}$ with $y_m^{(0)} = (\alpha_m^{(0)}, \beta_m^{(0)}, \gamma_m^{(0)}, A_m^{(0)}, B_m^{(0)}, C_m^{(0)})$, $m = 1, \ldots, \lambda$;
2. Repeat
   1. sample without replacement $n + n$ agents and trade;
   2. cumulate profit $F_m^{(g)}$ for τ sessions;
   3. select the best μ agents out of λ according to $F_m^{(g)}$. Let the selected agents form the population $\mathcal{Q}^{(g)}$;
   4. s-mutation, y-mutation.

Evolution strategies - ES, II

Return

for $l = 1, \ldots, \lambda$ do

1. sample with replacement one agent $(\alpha_k, \beta_k, \gamma_k, A_k, B_k, C_k) \in \mathcal{Q}^{(g)}, k \in \{1, \ldots, \mu\}$
2. let

$$\begin{array}{rclrcl} A_l^{(g+1)} & = & \exp(\upsilon\tilde{z})A_k^{(g)}; & \alpha_l^{(g+1)} & = & \alpha_k^{(g)} + \tilde{z}A_l^{(g+1)} \\ B_l^{(g+1)} & = & \exp(\upsilon\tilde{z})B_k^{(g)}; & \beta_l^{(g+1)} & = & \beta_k^{(g)} + \tilde{z}B_l^{(g+1)} \\ C_l^{(g+1)} & = & \exp(\upsilon\tilde{z})C_k^{(g)}; & \gamma_l^{(g+1)} & = & \gamma_k^{(g)} + \tilde{z}C_l^{(g+1)} \end{array}$$

let the new individuals $(\alpha_l^{(g+1)}, \beta_l^{(g+1)}, \gamma_l^{(g+1)}, A_l^{(g+1)}, B_l^{(g+1)}, C_l^{(g+1)}), l = 1, \ldots, \lambda$ form the population $\mathcal{P}^{(g+1)}$.

$g = g + 1$

Gauging convergence

Return