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A financial market model with endogenous fundamental values through imitative behavior

Joint work with A. Naimzada

Contents:

1. Introduction

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In Naimzada and Pireddu (2014a) the agents perceive an endogenous fundamental value, but the belief biases are still exogenously determined.

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- ***follow an imitative behavior, i.e., change actions only through imitating others*;**
- ***imitate an individual that performed better with a probability that is proportional to how much better this individual performed.***

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Our updating mechanism is similar to the switching mechanism in Brock and Hommes (1997), used also by De Grauwe and Rovira Kaltwasser (2012).

Moreover, differently from the majority of the literature on the topic (see e.g. De Grauwe and Rovira Kaltwasser, 2012) and similarly to Naimzada and Pireddu (2014b), the stock price is for us determined by a nonlinear Walrasian mechanism that prevents divergence issues.

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Naimzada and Ricchiuti (2008, 2009) consider models with heterogeneous fundamentalists, perceiving different exogenous fundamental values, with switching mechanisms based on the squared errors between fundamentals and prices.

2. The model

Two crucial aspects to be described: the imitation process and the price formation mechanism (Hommes, 2013).

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Their behavior is captured by the dynamic motions of the corresponding perceived fundamental values.

The price will be determined by a Walrasian mechanism.

$$\begin{cases} X(t+1) = \underline{f}\,\frac{e^{\beta\pi_X(t+1)}}{e^{\beta\pi_X(t+1)}+e^{\beta\pi_Y(t+1)}} + F\frac{e^{\beta\pi_Y(t+1)}}{e^{\beta\pi_X(t+1)}+e^{\beta\pi_Y(t+1)}} \\ Y(t+1) = F\frac{e^{\beta\pi_X(t+1)}}{e^{\beta\pi_X(t+1)}+e^{\beta\pi_Y(t+1)}} + \overline{f}\frac{e^{\beta\pi_Y(t+1)}}{e^{\beta\pi_X(t+1)}+e^{\beta\pi_Y(t+1)}} \\ P(t+1) = P(t) + \gamma a_2 \left(\frac{a_1+a_2}{a_1 e^{-(\omega\sigma_X(X(t)-P(t))+(1-\omega)\sigma_Y(Y(t)-P(t)))}+a_2} - 1 \right) \end{cases} \quad (1)$$

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where σ_X and σ_Y are positive parameters representing the re-activities of optimistic and pessimistic agents, respectively.

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When $\beta = 0$, then $X(t+1) = \frac{1}{2}(\underline{f} + F)$ and $Y(t+1) = \frac{1}{2}(F + \overline{f}) \Rightarrow$

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When $\beta \to +\infty$, then:

$$\pi_X > \pi_Y \;\Rightarrow\; X(t+1) \to \underline{f}, \quad Y(t+1) \to F;$$
$$\pi_X < \pi_Y \;\Rightarrow\; X(t+1) \to F, \quad Y(t+1) \to \overline{f}.$$

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Y(t+1) = F\,\frac{1}{1+e^{-\beta(\pi_X(t+1)-\pi_Y(t+1))}} + \overline{f}\,\frac{1}{1+e^{\beta(\pi_X(t+1)-\pi_Y(t+1))}} \\
P(t+1) = P(t) + \gamma a_2 \left(\frac{a_1+a_2}{a_1 e^{-(\omega\sigma_X(X(t)-P(t))+(1-\omega)\sigma_Y(Y(t)-P(t)))}+a_2} - 1 \right)
\end{cases} \tag{1}
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- $\gamma > 0$ is the market maker price adjustment parameter;

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- a_1 **and** a_2 **are two positive parameters bounding the price variation;**

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- $\gamma > 0$ **is the market maker price adjustment parameter;**

- a_1 **and** a_2 **are two positive parameters bounding the price variation;**

- $\omega \in (0,1)$ **represents the fraction of the population composed by pessimists.**

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$$\underline{f} = F - \Delta \quad \textbf{and} \quad \overline{f} = F + \Delta.$$

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We may rewrite (1) **as**

$$\begin{cases} X(t+1) = F - \Delta \left(\frac{1}{1+e^{-\beta(\pi_X(t+1)-\pi_Y(t+1))}} \right) \\ Y(t+1) = F + \Delta \left(\frac{1}{1+e^{\beta(\pi_X(t+1)-\pi_Y(t+1))}} \right) \\ P(t+1) = P(t) + \gamma a_2 \left(\frac{a_1+a_2}{a_1 e^{-(\omega\sigma_X(X(t)-P(t))+(1-\omega)\sigma_Y(Y(t)-P(t)))}+a_2} - 1 \right) \end{cases} \tag{2}$$

Proposition: ***System*** (2) ***has a unique steady state in***

$$(X^*, Y^*, P^*) = \left(F - \frac{\Delta}{2}, F + \frac{\Delta}{2}, F - \frac{\Delta(\omega\sigma_X - (1-\omega)\sigma_Y)}{2(\omega\sigma_X + (1-\omega)\sigma_Y)}\right).$$

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Also when $\sigma_X = \sigma_Y$ and $\omega = \frac{1}{2}$, we find $P^* = F$, even if now $X^* \neq F \neq Y^*$.

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For $t \geq 1$, the dynamical system associated to (2) is equivalent to that associated to the two-dimensional map

$$G = (G_1, G_2) : (\underline{f}, F) \times (0, +\infty) \longrightarrow \mathbb{R}^2,$$

$$(X, P) \mapsto (G_1(X, P), G_2(X, P)),$$

defined as:

$$G_1(X,P) = F - \left(\frac{\Delta}{1 + e^{-\beta\left(\gamma a_2\left(\frac{a_1+a_2}{a_1 e^{-(\omega\sigma_X(X-P)+(1-\omega)\sigma_Y(X+\Delta-P))}+a_2}-1\right)\left(\sigma_X(X-P)-\sigma_Y(X+\Delta-P)\right)\right)}} \right)$$

$$G_2(X,P) = P + \gamma a_2 \left(\frac{a_1 + a_2}{a_1 e^{-(\omega\sigma_X(X-P)+(1-\omega)\sigma_Y(X+\Delta-P))} + a_2} - 1 \right),$$

i.e., the two systems generate the same trajectories.

3. Stability analysis

Map G has a unique fixed point in

$$(X^*, P^*) = \left(F - \frac{\Delta}{2}, F - \frac{\Delta(\omega\sigma_X - (1-\omega)\sigma_Y)}{2(\omega\sigma_X + (1-\omega)\sigma_Y)} \right).$$

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The Jacobian matrix for G computed in correspondence to it reads as

$$J_G(X^*, P^*) = \begin{bmatrix} \frac{\Delta^2\beta\widetilde{\gamma}\sigma_X\sigma_Y}{4} & -\frac{\Delta^2\beta\widetilde{\gamma}\sigma_X\sigma_Y}{4} \\ \widetilde{\gamma}(\omega\sigma_X + (1-\omega)\sigma_Y) & 1 - \widetilde{\gamma}(\omega\sigma_X + (1-\omega)\sigma_Y) \end{bmatrix},$$

where we set $\widetilde{\gamma} = \frac{\gamma a_1 a_2}{a_1 + a_2}$.

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where we set $\widetilde{\gamma} = \frac{\gamma a_1 a_2}{a_1 + a_2}$.

We use the well-known Jury conditions (see Jury, 1964):

$$\det(J) < 1, \quad 1 + \mathrm{tr}(J) + \det(J) > 0, \quad 1 - \mathrm{tr}(J) + \det(J) > 0.$$

In our framework, we have

$$\det(J) = \frac{\beta\Delta^2\widetilde{\gamma}\sigma_X\sigma_Y}{4}, \quad \mathrm{tr}(J) = \frac{\beta\Delta^2\widetilde{\gamma}\sigma_X\sigma_Y}{4} + 1 - \widetilde{\gamma}(\omega\sigma_X + (1-\omega)\sigma_Y).$$

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Stability conditions with respect to β (for $\Delta \neq 0$)

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Stability conditions with respect to β (for $\Delta \neq 0$)

Jury conditions are fulfilled if:

$$\frac{2(\widetilde{\gamma}(\omega\sigma_X + (1-\omega)\sigma_Y) - 2)}{\widetilde{\gamma}\sigma_X\sigma_Y\Delta^2} < \beta < \frac{4}{\widetilde{\gamma}\sigma_X\sigma_Y\Delta^2}.$$

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Usually, in the literature, increasing β has just a destabilizing effect, while for us it may also be stabilizing.

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or

- $\widetilde{\gamma}(\omega\sigma_X + (1-\omega)\sigma_Y) < 2$ **and**

$$\Delta < \frac{2}{\sqrt{\widetilde{\gamma}\sigma_X\sigma_Y\beta}}.$$

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Due to the nonlinearity of the Walrasian mechanism, differently from De Grauwe and Rovira Kaltwasser (2012), we do not have divergence issues when the isolated financial market is unstable.

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Due to the nonlinearity of the Walrasian mechanism, differently from De Grauwe and Rovira Kaltwasser (2012), we do not have divergence issues when the isolated financial market is unstable.

The flip bifurcation opens for us a route to chaos, not to divergence.

We will focus on the case with $\sigma_X = \sigma_Y = 1$.

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$\Rightarrow$ **the stability conditions with respect to β read as**

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if $\widetilde{\gamma} \geq 2$,

or as

$$\Delta < \frac{2}{\sqrt{\widetilde{\gamma}\beta}},$$

if $\widetilde{\gamma} < 2$.

$\Rightarrow$ **when** $\Delta = 0$, **the stability condition simply becomes** $\widetilde{\gamma} < 2$.

4. Numerical results

The role of β

We now use β as bifurcation parameter, like in De Grauwe and Rovira Kaltwasser (2012).

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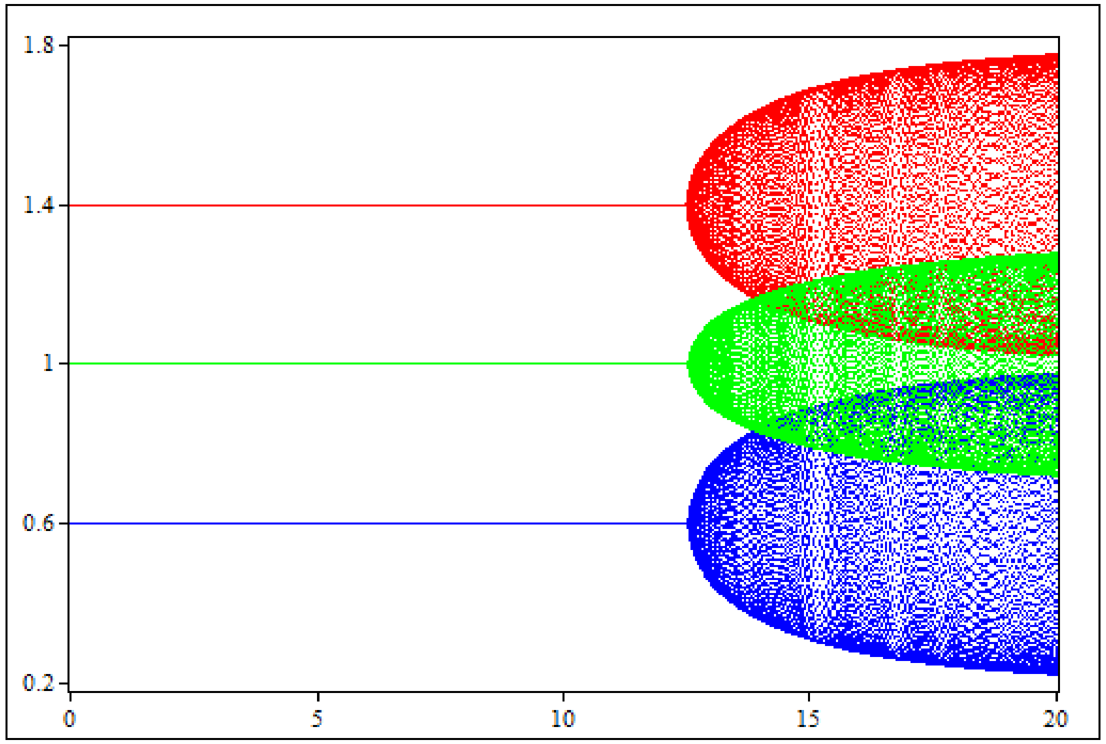

FIGURE 1: The bifurcation diagram with respect to $\beta \in [0, 20]$ for X in blue, Y in red and P in green, for $\gamma = F = 1$, $\Delta = 0.8$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 0.25$, $Y(0) = 1.2$ and $P(0) = 3$.

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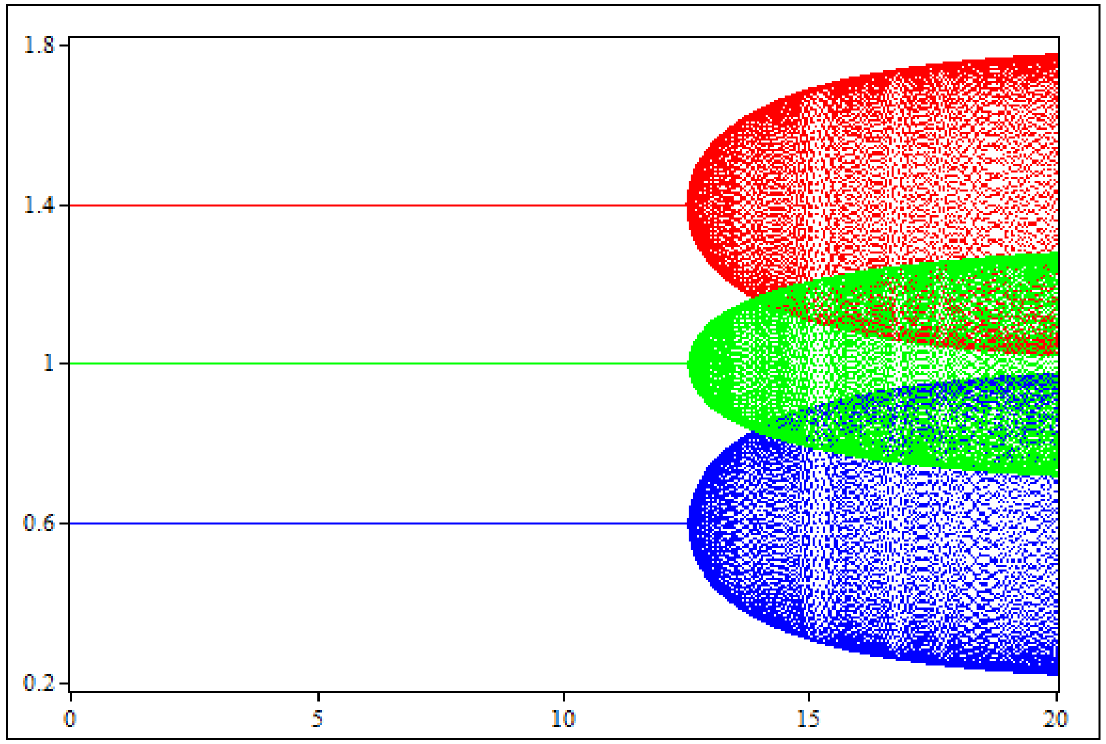

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The threshold for the flip bifurcation is $\beta = \frac{2(\widetilde{\gamma}-2)}{\widetilde{\gamma}\Delta^2} = -9.375 < 0.$

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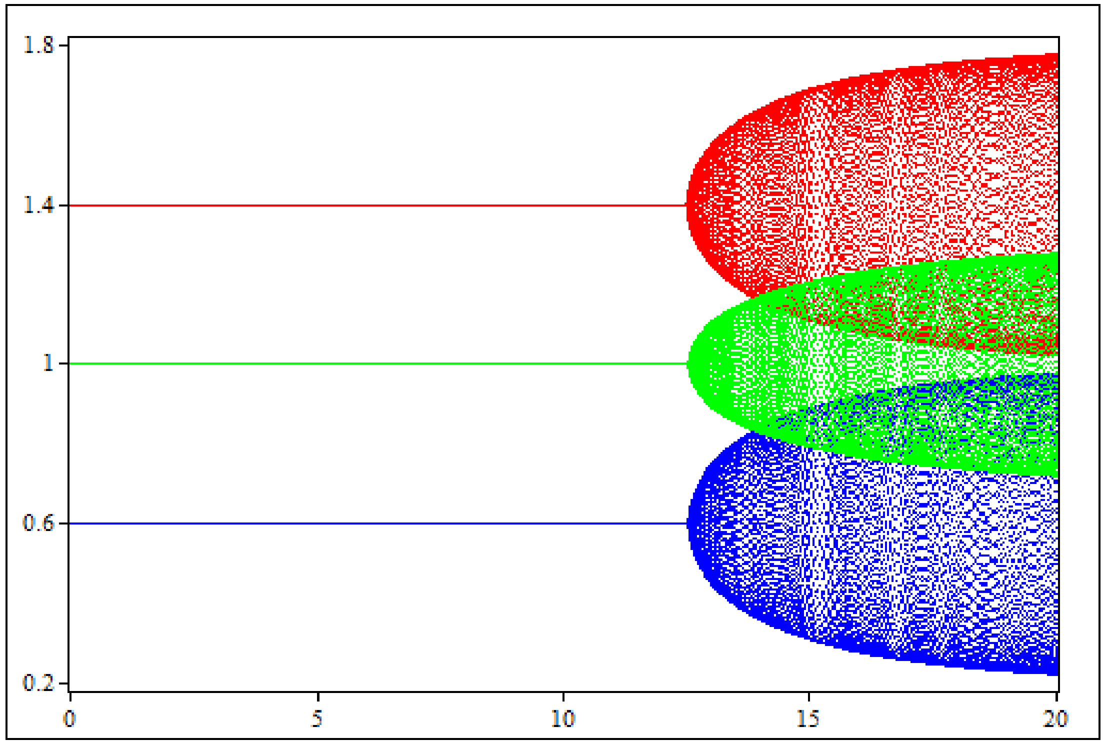

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The Hopf bifurcation occurs for $\beta = \frac{4}{\widetilde{\gamma}\Delta^2} = 12.5$.

Second scenario: mixed role of β

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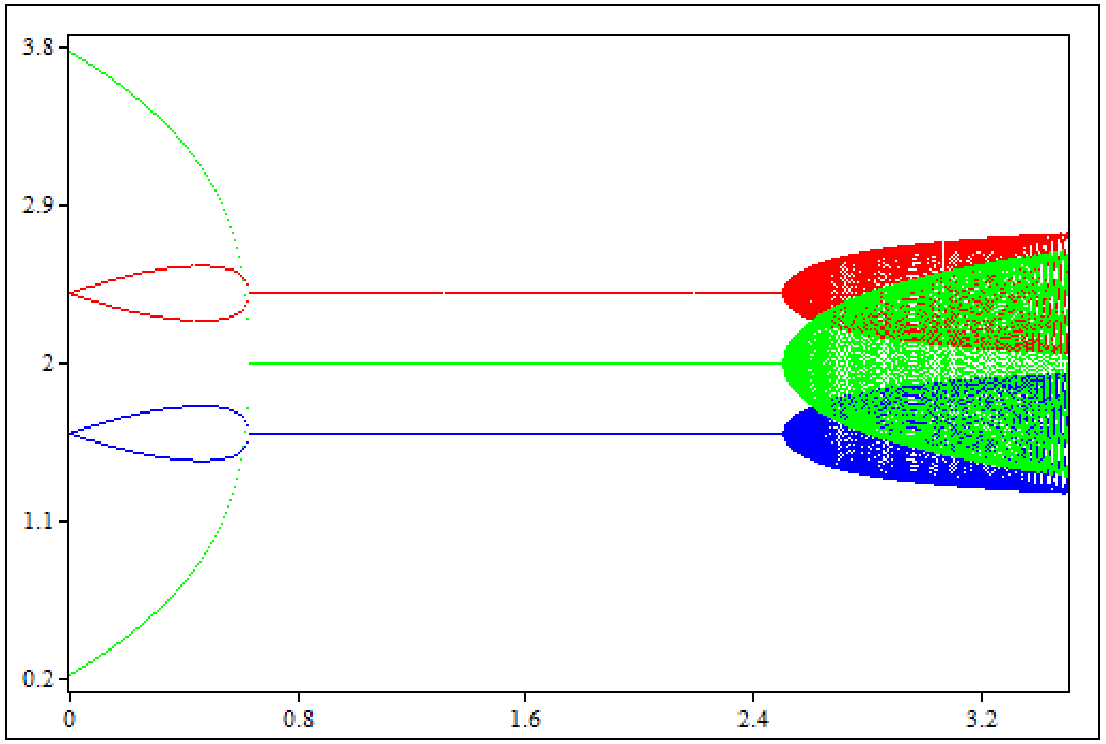
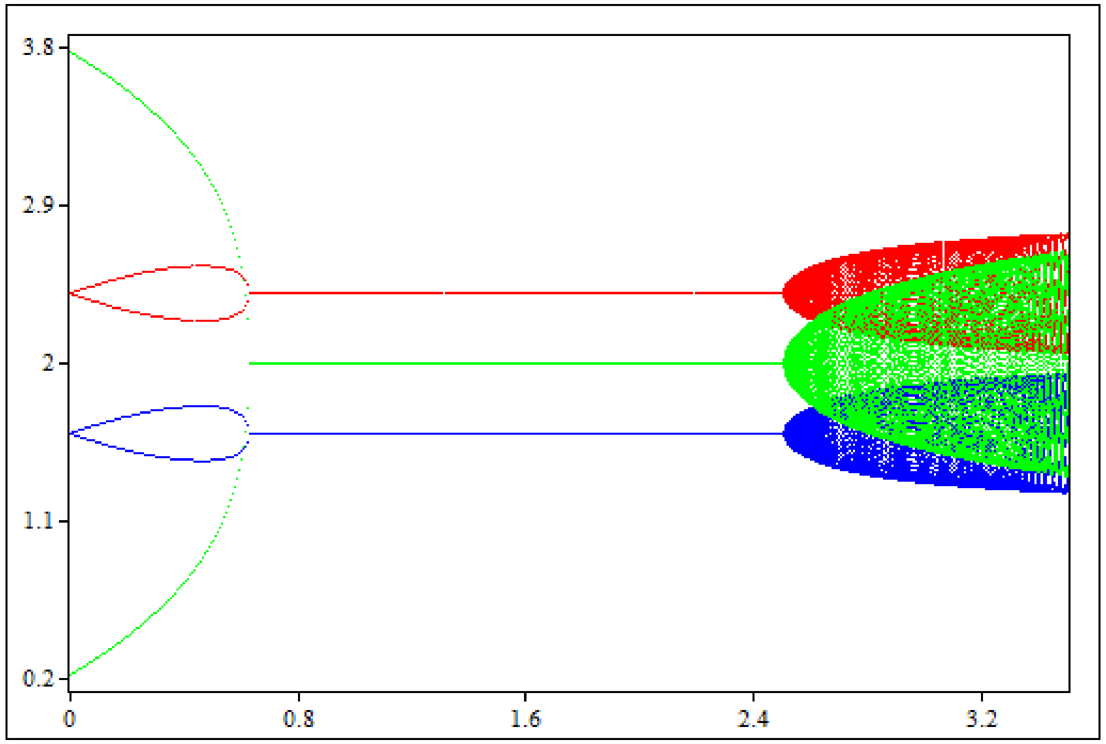


FIGURE 2: The bifurcation diagram with respect to $\beta \in [0, 3.5]$ for X in blue, Y in red and P in green, for $\gamma = 5$, $F = 2$, $\Delta = 0.8$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.5$, $Y(0) = 2.5$ and $P(0) = 3$.

Second scenario: mixed role of β



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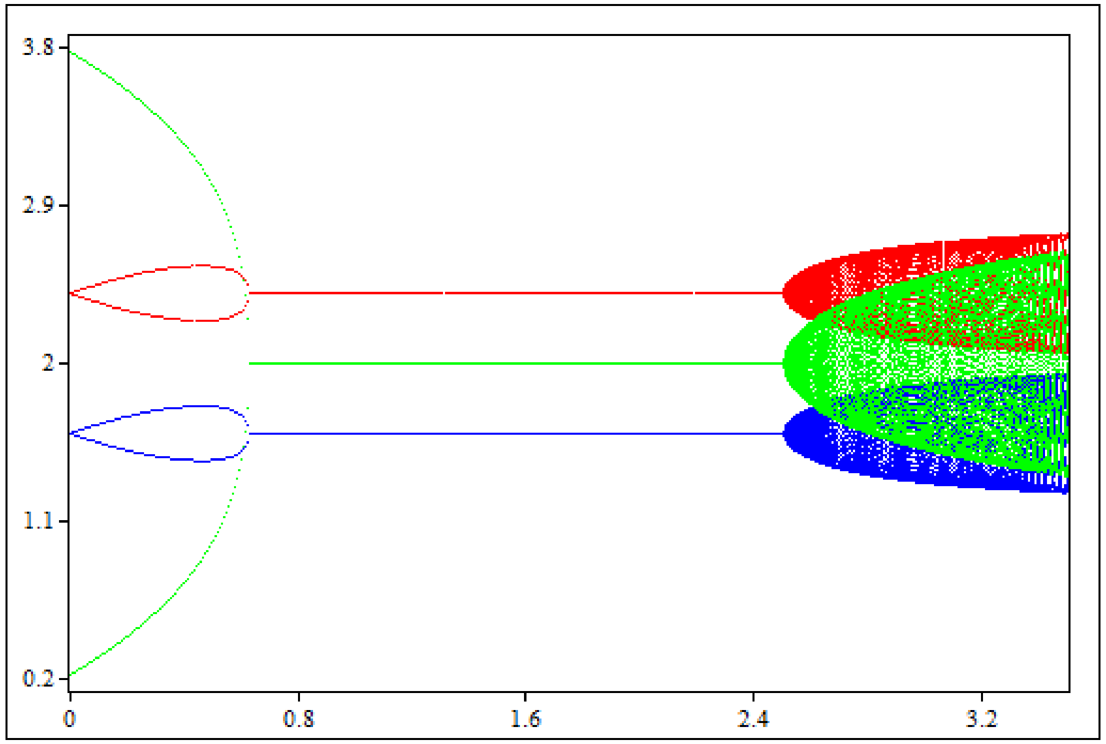
The flip bifurcation occurs for $\beta = \frac{2(\widetilde{\gamma}-2)}{\widetilde{\gamma}\Delta^2} = 0.625$.

Second scenario: mixed role of β



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With $a_1 \neq a_2$, complex dynamics can occur also for small values of β.

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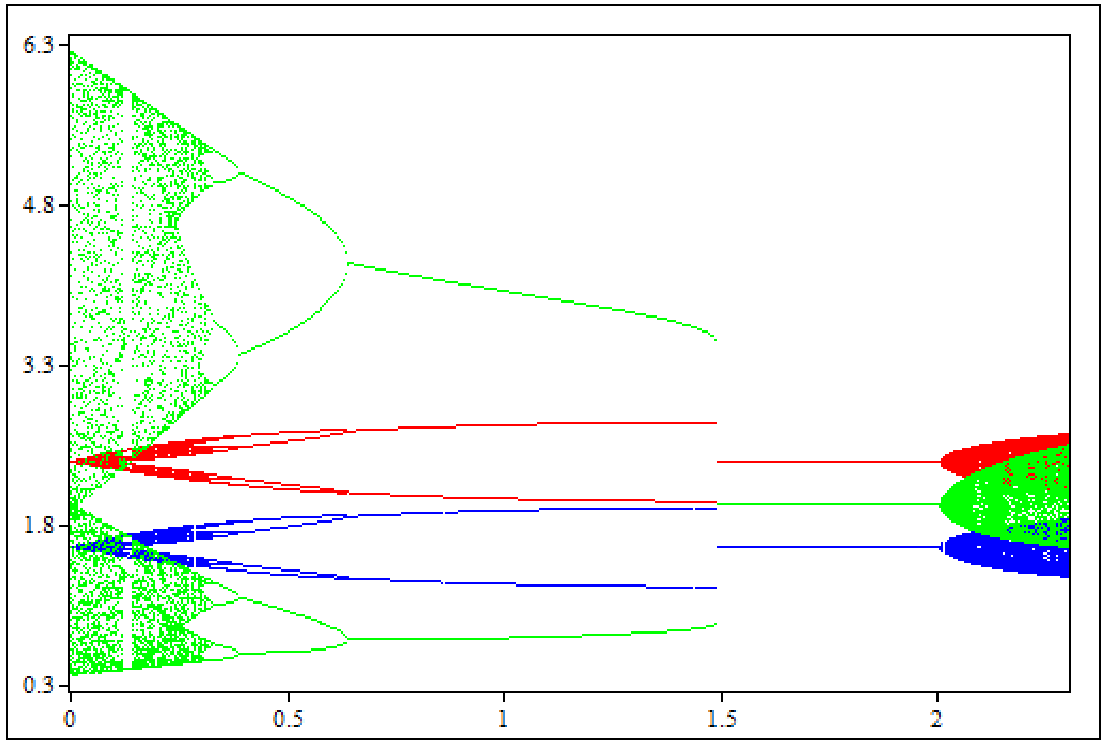

FIGURE 3: The bifurcation diagram with respect to $\beta \in [0, 2.3]$ for X in blue, Y in red and P in green, for $\gamma = 4.31$, $F = 2$, $\Delta = 0.8$, $a_1 = 2.6$, $a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.6$, $Y(0) = 2.5$ and $P(0) = 3$.

Third scenario: no stabilization with β

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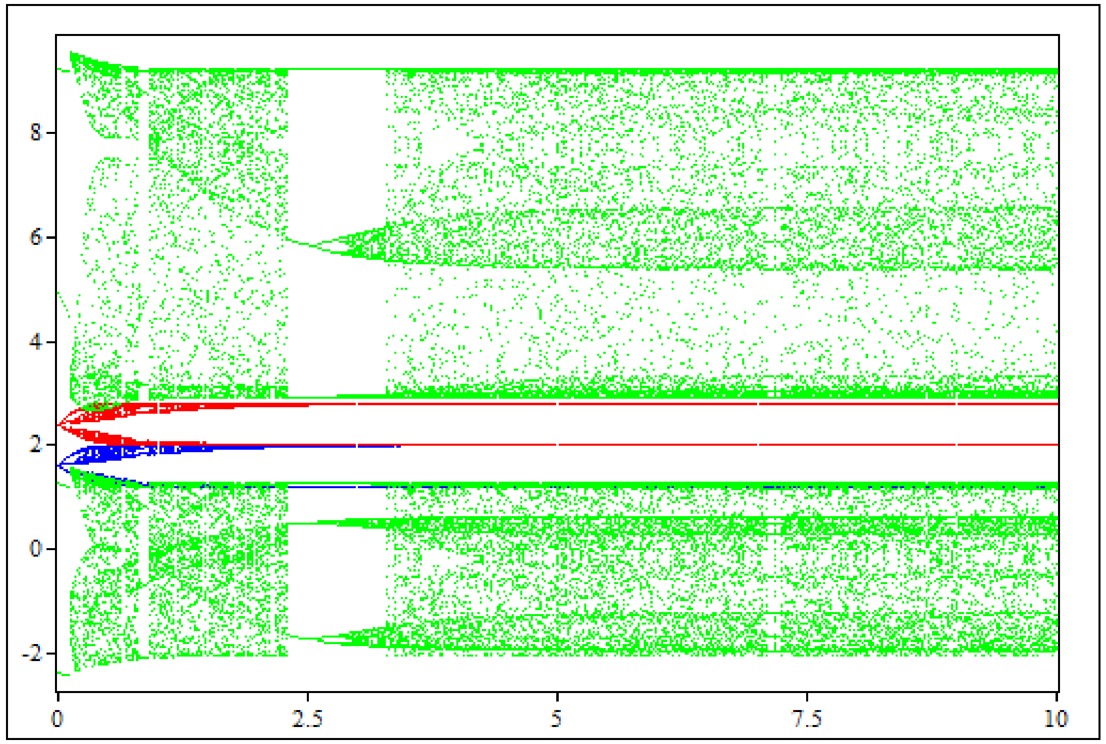
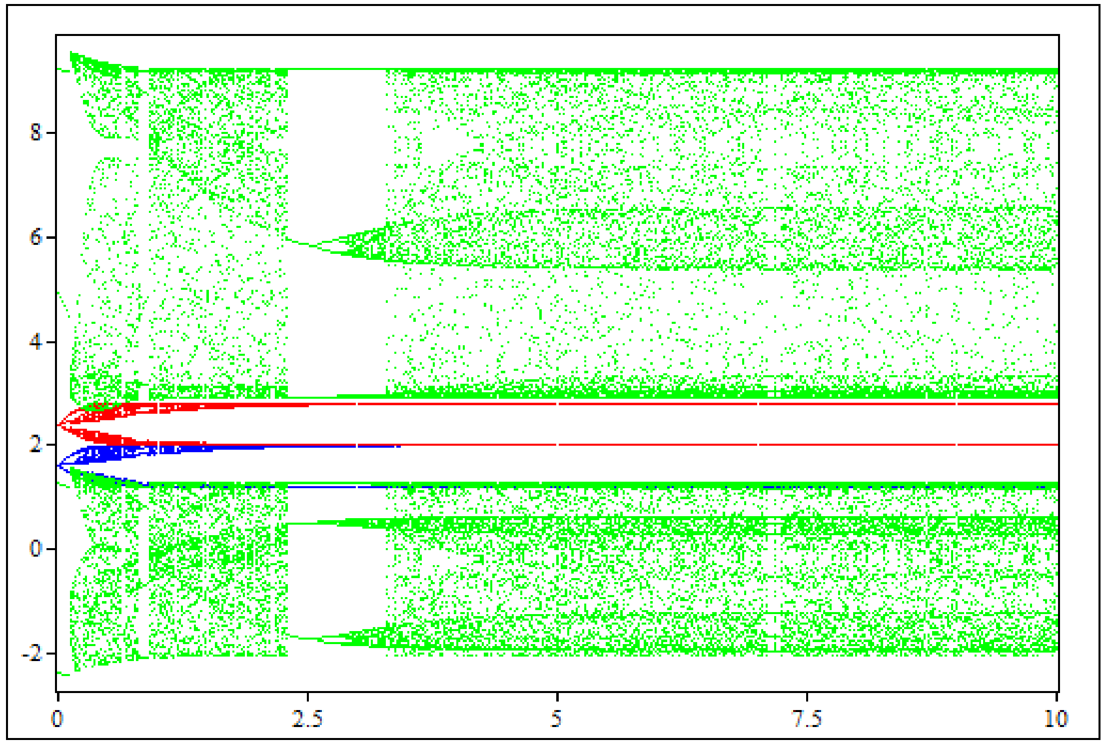


FIGURE 4: The bifurcation diagram with respect to $\beta \in [0, 10]$ for X in blue, Y in red and P in green, for $\gamma = 4$, $F = 2$, $\Delta = 0.8$, $a_1 = 3$, $a_2 = 2$, $\omega = 0.5$, and the initial conditions $X(0) = 1.3$, $Y(0) = 2.5$ and $P(0) = 2$.

Third scenario: no stabilization with β



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The stability conditions would read as

$$1.823 \simeq \frac{2(\widetilde{\gamma} - 2)}{\widetilde{\gamma}\Delta^2} < \beta < \frac{4}{\widetilde{\gamma}\Delta^2} \simeq 1.302.$$

The role of Δ

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First scenario: destabilizing role of Δ

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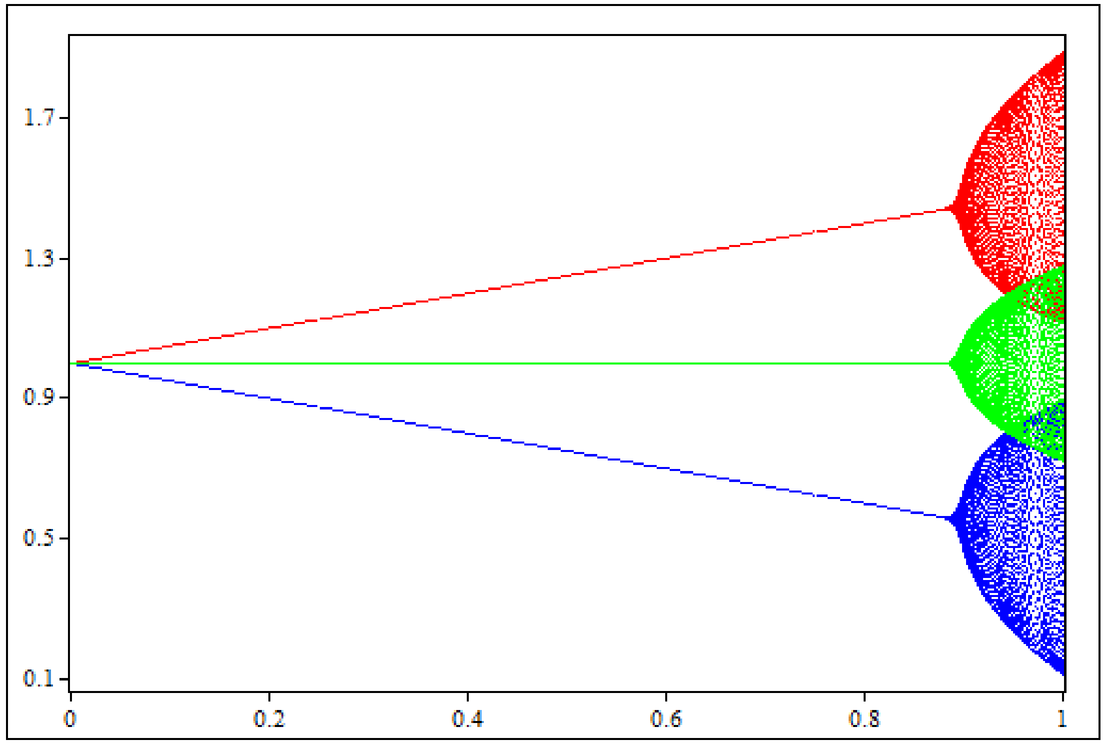

FIGURE 5: The bifurcation diagram with respect to $\Delta \in [0,1]$ for X in blue, Y in red and P in green, for $\gamma = F = 1$, $\beta = 10$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 0.25$, $Y(0) = 1.2$ and $P(0) = 3$.

The role of Δ

First scenario: destabilizing role of Δ

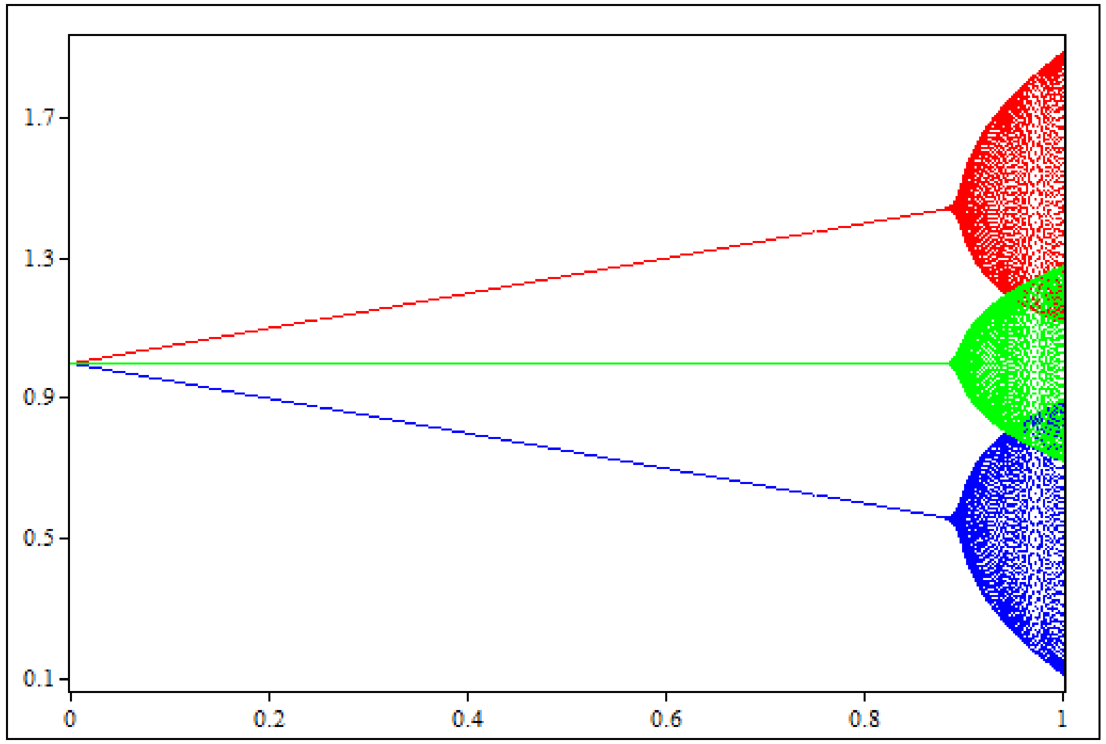

FIGURE 5: The bifurcation diagram with respect to $\Delta \in [0, 1]$ for X in blue, Y in red and P in green, for $\gamma = F = 1$, $\beta = 10$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 0.25$, $Y(0) = 1.2$ and $P(0) = 3$.

The threshold for the flip bifurcation would be $\sqrt{\frac{2(\widetilde{\gamma}-2)}{\widetilde{\gamma}\beta}} = \sqrt{-0.6}$.

The role of Δ

First scenario: destabilizing role of Δ

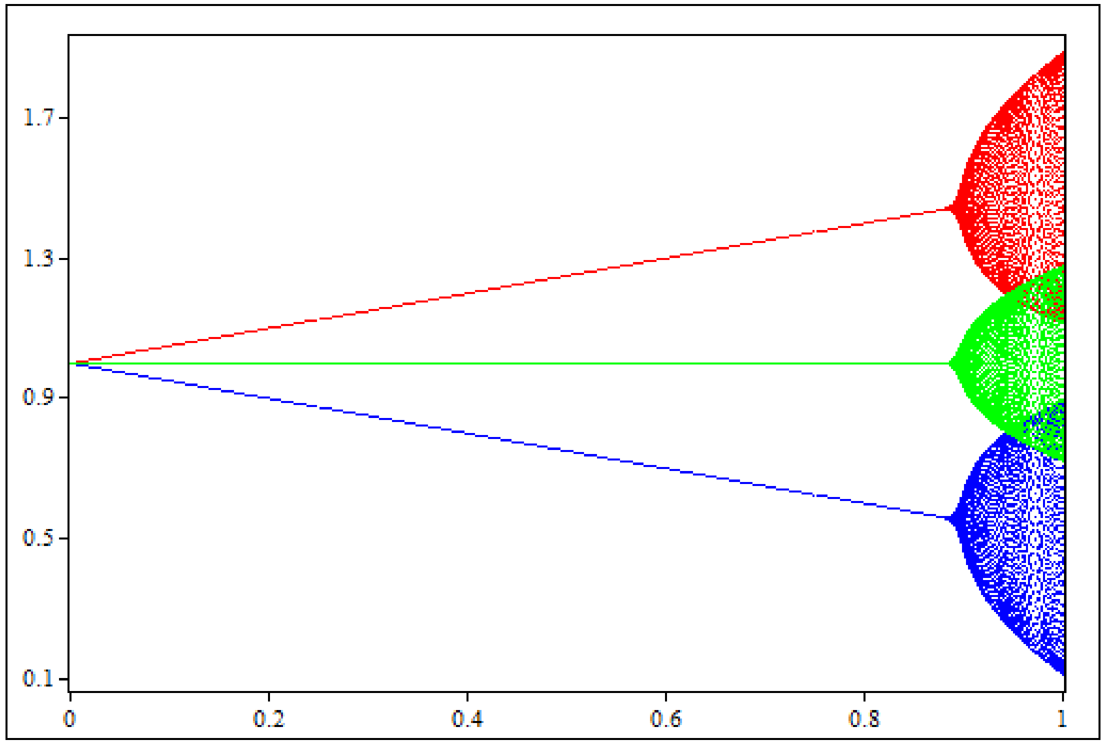

FIGURE 5: The bifurcation diagram with respect to $\Delta \in [0,1]$ for X in blue, Y in red and P in green, for $\gamma = F = 1$, $\beta = 10$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 0.25$, $Y(0) = 1.2$ and $P(0) = 3$.

The threshold for the flip bifurcation would be $\sqrt{\frac{2(\widetilde{\gamma}-2)}{\widetilde{\gamma}\beta}} = \sqrt{-0.6}$.

The Hopf bifurcation occurs for $\Delta = \frac{2}{\sqrt{\widetilde{\gamma}\beta}} \simeq 0.894$.

Second scenario: mixed role of Δ

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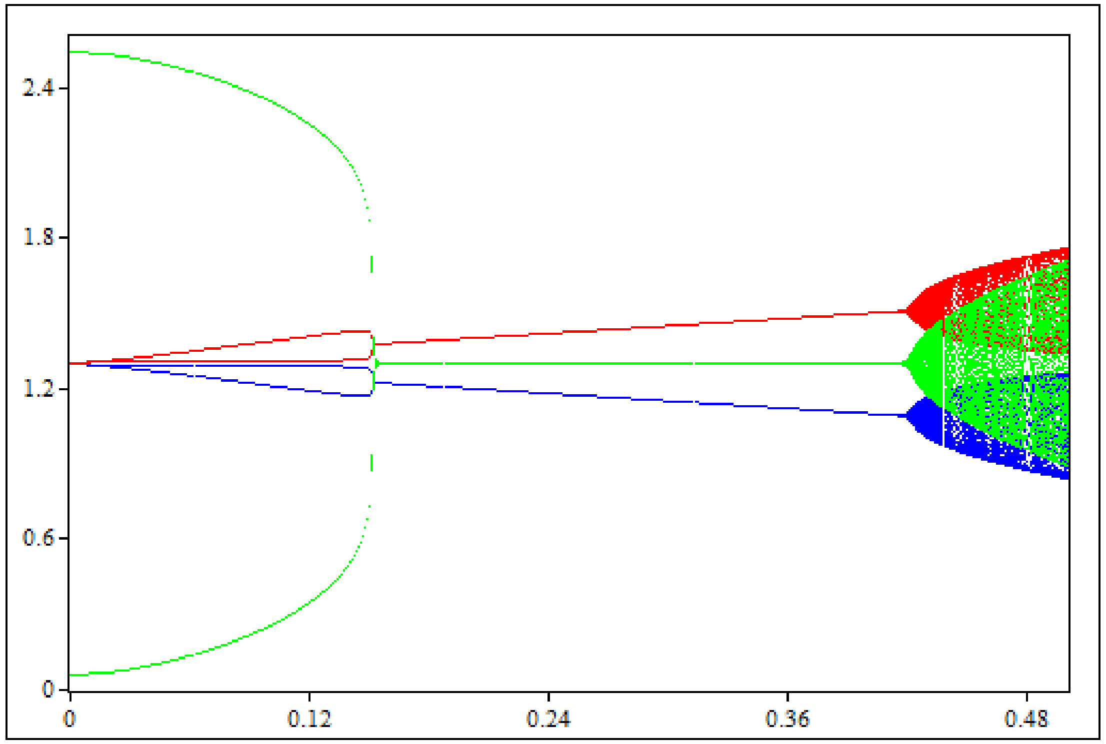

FIGURE 6: The bifurcation diagram with respect to $\Delta \in [0, 0.5]$ for X in blue, Y in red and P in green, for $\gamma = 4.5$, $F = 1.3$, $\beta = 10$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.1$, $Y(0) = 1.4$ and $P(0) = 3$.

Second scenario: mixed role of Δ

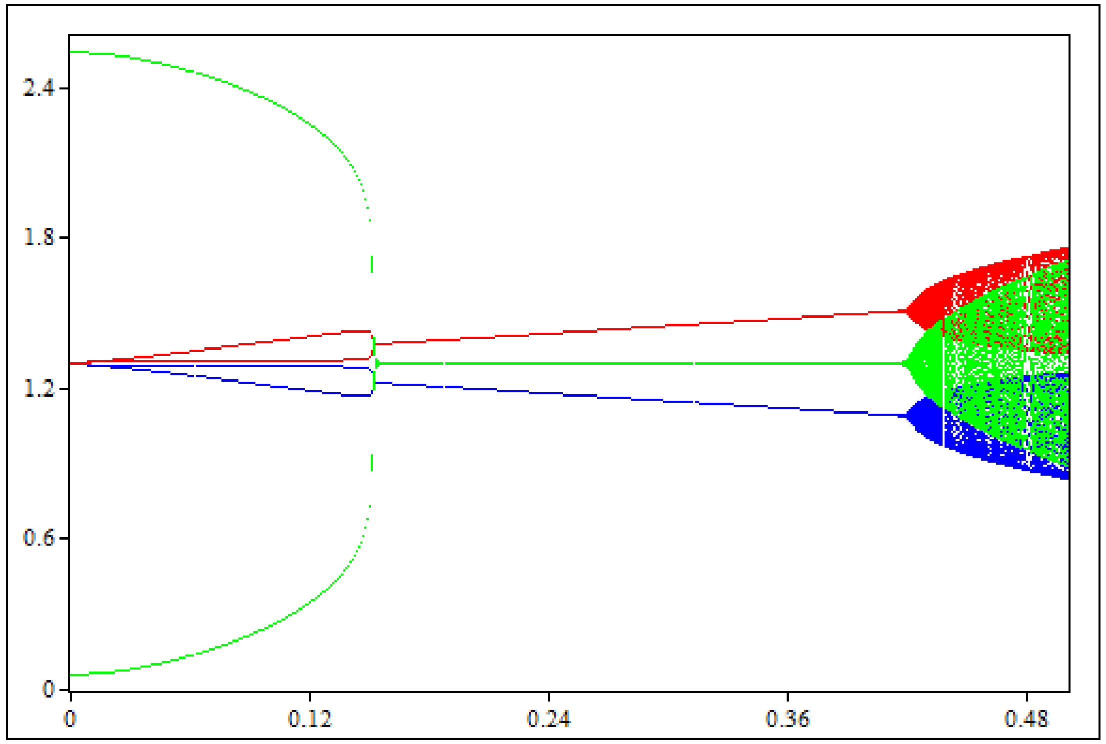

FIGURE 6: The bifurcation diagram with respect to $\Delta \in [0, 0.5]$ for X in blue, Y in red and P in green, for $\gamma = 4.5$, $F = 1.3$, $\beta = 10$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.1$, $Y(0) = 1.4$ and $P(0) = 3$.

The flip bifurcation occurs for $\Delta = \sqrt{\frac{2(\widetilde{\gamma}-2)}{\widetilde{\gamma}\beta}} \simeq 0.149$.

Second scenario: mixed role of Δ

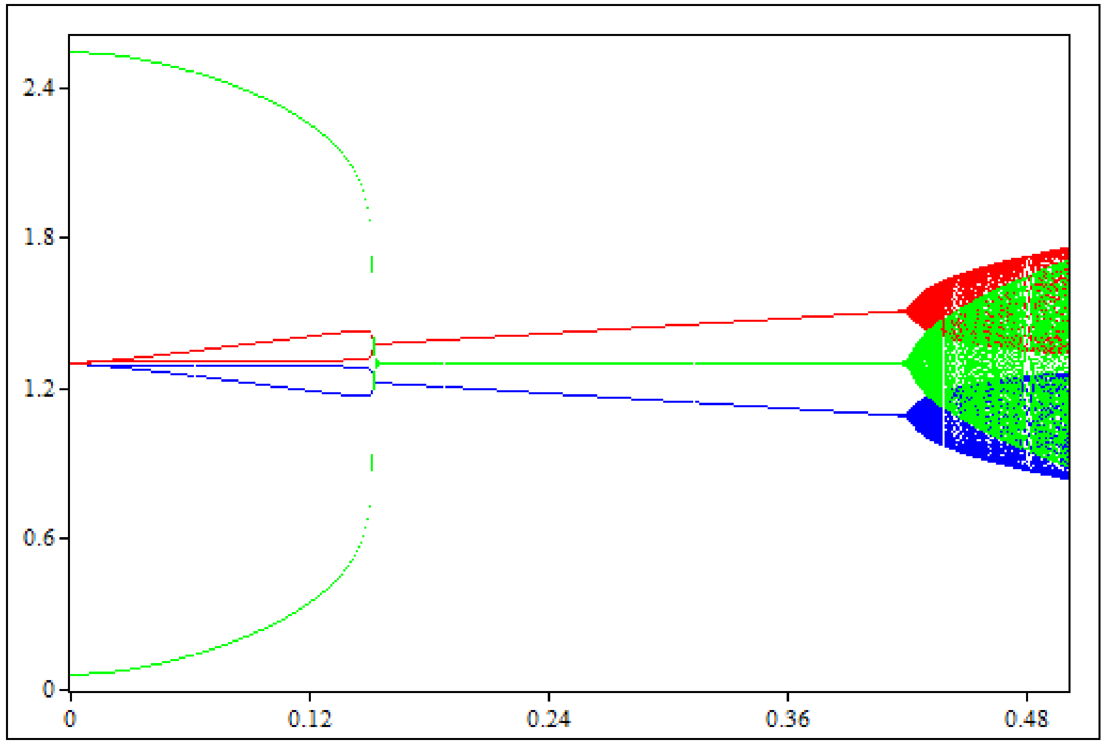

FIGURE 6: The bifurcation diagram with respect to $\Delta \in [0, 0.5]$ for X in blue, Y in red and P in green, for $\gamma = 4.5$, $F = 1.3$, $\beta = 10$, $a_1 = a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.1$, $Y(0) = 1.4$ and $P(0) = 3$.

The flip bifurcation occurs for $\Delta = \sqrt{\frac{2(\widetilde{\gamma}-2)}{\widetilde{\gamma}\beta}} \simeq 0.149.$

The Hopf bifurcation occurs for $\Delta = \frac{2}{\sqrt{\widetilde{\gamma}\beta}} \simeq 0.421.$

With $a_1 \neq a_2$, complex dynamics can occur also for small values of Δ.

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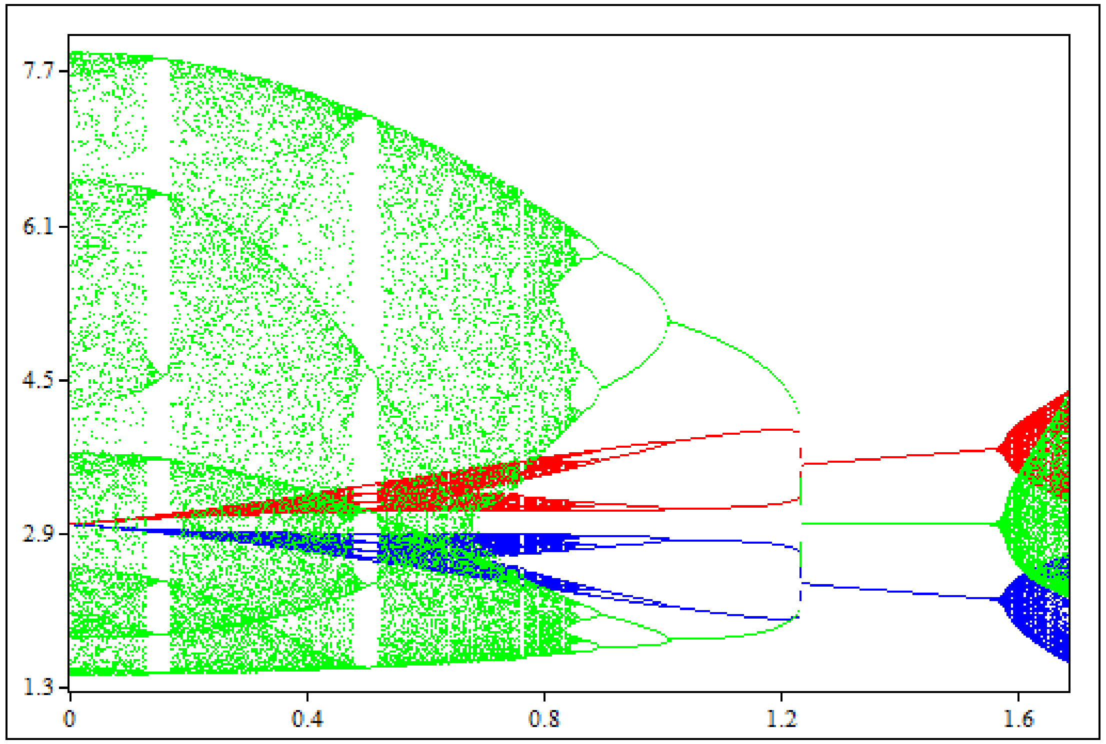

FIGURE 7: The bifurcation diagram with respect to $\Delta \in [0, 1.684]$ for X in blue, Y in red and P in green, for $\gamma = 4.2, F = 3$, $\beta = 0.5$, $a_1 = 3.3$, $a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.6$, $Y(0) = 4.5$ and $P(0) = 3$.

Third scenario: no stabilization with Δ

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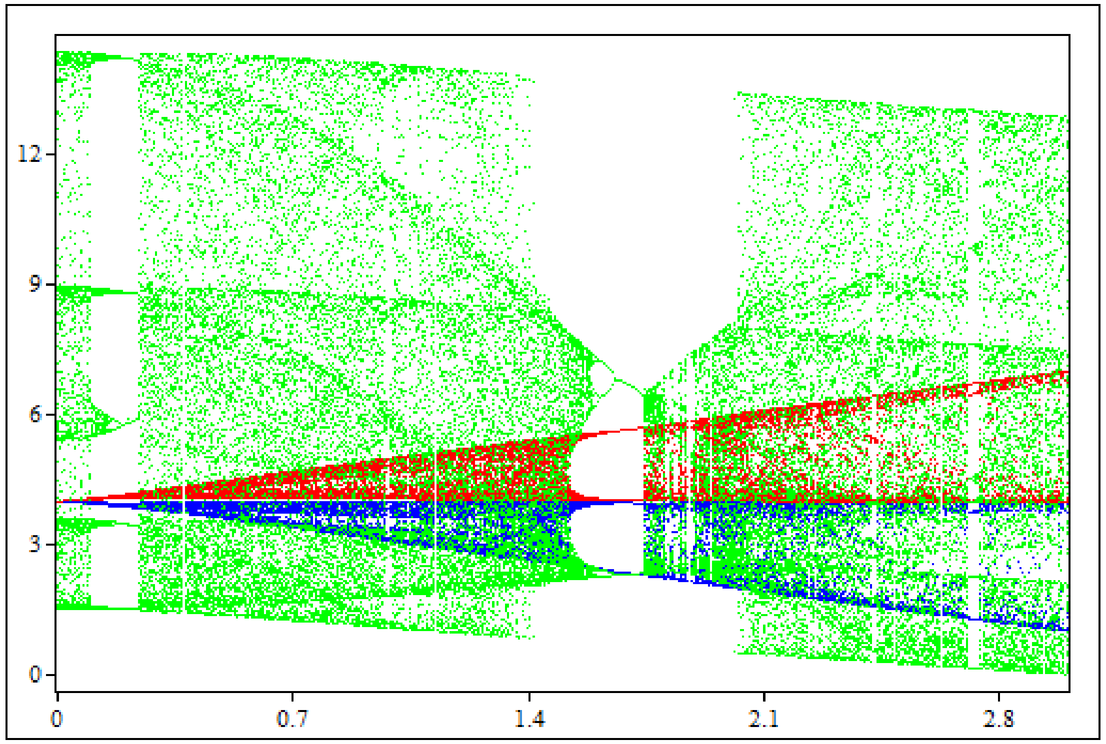


FIGURE 8: The bifurcation diagram with respect to $\Delta \in [0, 3]$ for X in blue, Y in red and P in green, for $\gamma = 5.4$, $F = 4$, $\beta = 0.5$, $a_1 = 3.3$, $a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.6$, $Y(0) = 4.5$ and $P(0) = 3$.

Third scenario: no stabilization with Δ

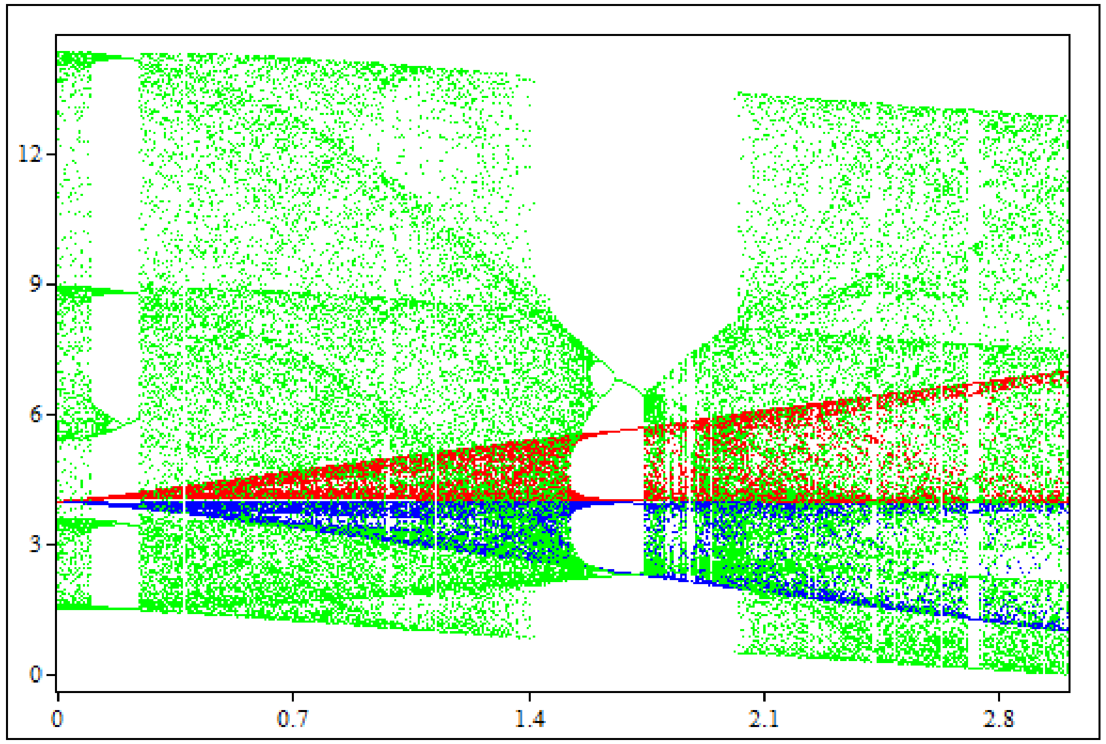


FIGURE 8: The bifurcation diagram with respect to $\Delta \in [0,3]$ for X in blue, Y in red and P in green, for $\gamma = 5.4$, $F = 4$, $\beta = 0.5$, $a_1 = 3.3$, $a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.6$, $Y(0) = 4.5$ and $P(0) = 3$.

The stability conditions would read as

$$1.438 \simeq \sqrt{\frac{2(\widetilde{\gamma} - 2)}{\widetilde{\gamma}\beta}} < \Delta < \frac{2}{\sqrt{\widetilde{\gamma}\beta}} \simeq 1.389.$$

Economic interpretation

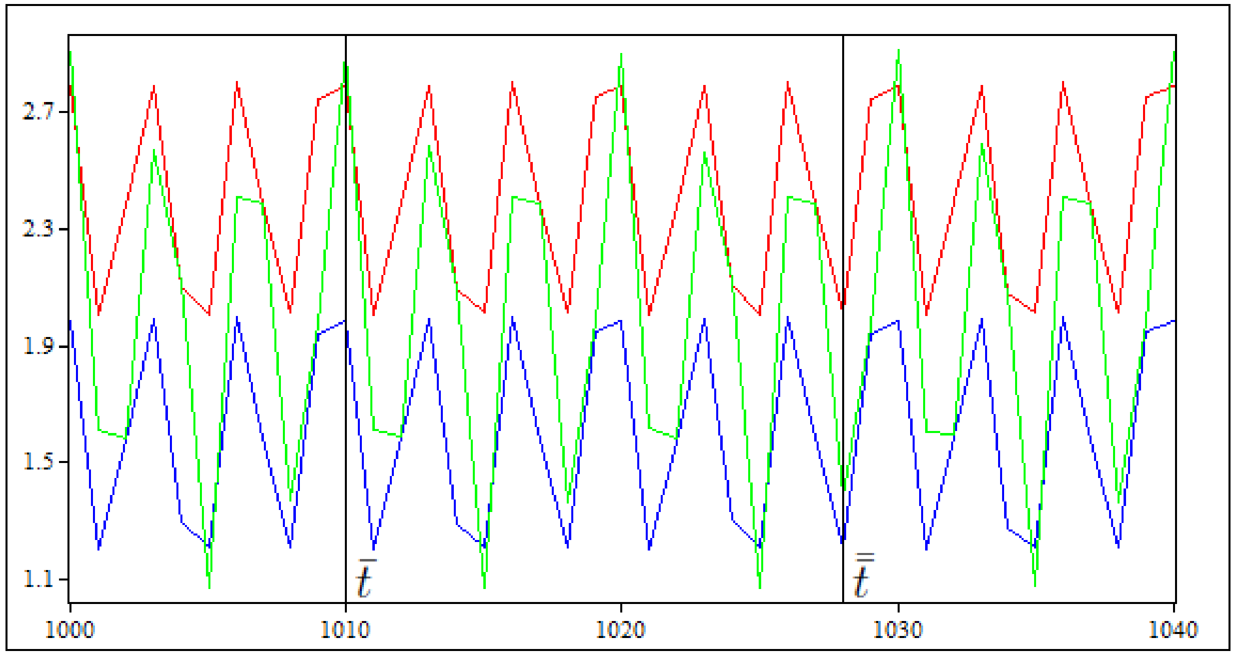


FIGURE 9: The time series for X in blue, Y in red and P in green, respectively, for $\gamma = 5.1$, $F = 2$, $\beta = 5.35$, $a_1 = a_2 = 1$, $\omega = 0.5$, $\Delta = 0.8$, and the initial conditions $X(0) = 1.5$, $Y(0) = 2.5$ and $P(0) = 2.1$.

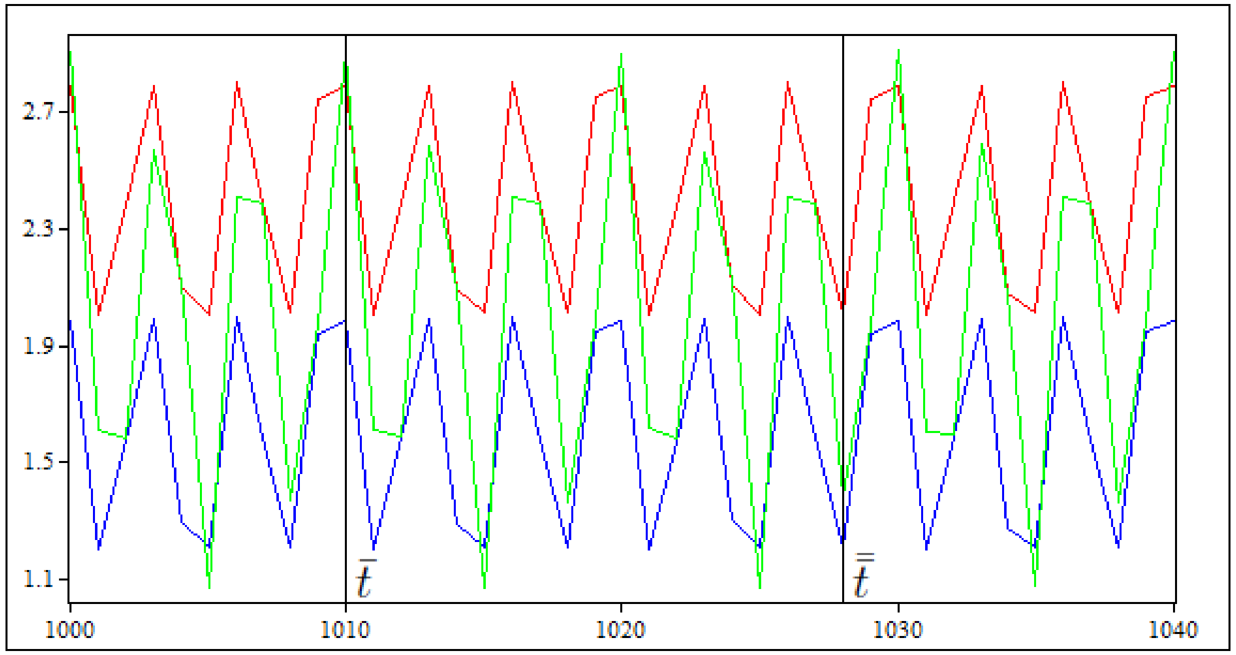


FIGURE 9: The time series for X in blue, Y in red and P in green, respectively, for $\gamma = 5.1$, $F = 2$, $\beta = 5.35$, $a_1 = a_2 = 1$, $\omega = 0.5$, $\Delta = 0.8$, and the initial conditions $X(0) = 1.5$, $Y(0) = 2.5$ and $P(0) = 2.1$.

The dynamics of price:

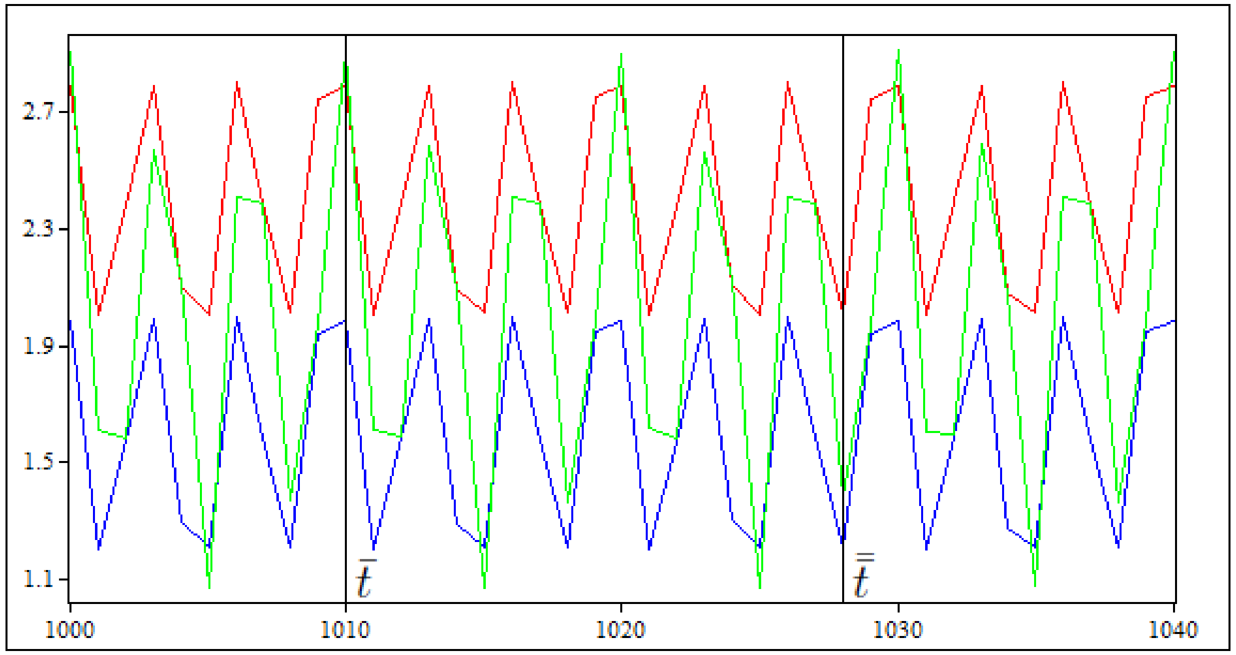
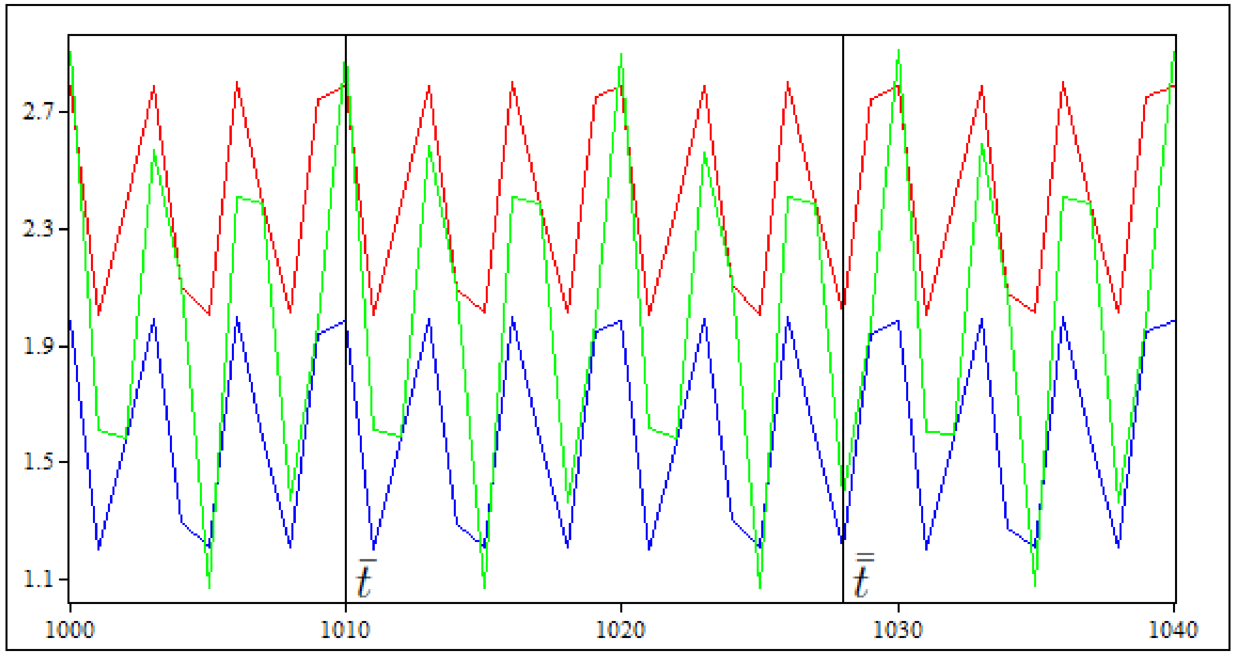


FIGURE 9: The time series for X in blue, Y in red and P in green, respectively, for $\gamma = 5.1$, $F = 2$, $\beta = 5.35$, $a_1 = a_2 = 1$, $\omega = 0.5$, $\Delta = 0.8$, and the initial conditions $X(0) = 1.5$, $Y(0) = 2.5$ and $P(0) = 2.1$.

The dynamics of price:

Since $\sigma_X = \sigma_Y = 1$ **and** $\omega = 0.5$, **then**

$$ED(t) = 0.5(X(t) - P(t)) + 0.5(Y(t) - P(t)).$$



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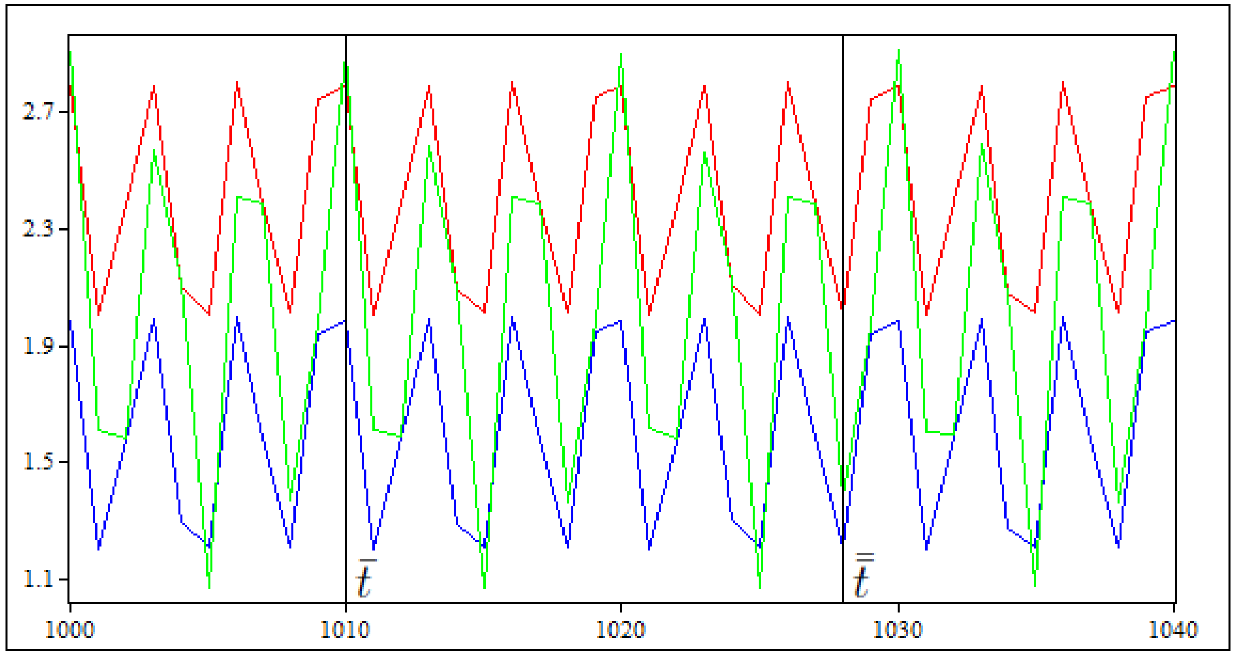
The dynamics of price:

Since $\sigma_X = \sigma_Y = 1$ **and** $\omega = 0.5$, **then**

$$ED(t) = 0.5(X(t) - P(t)) + 0.5(Y(t) - P(t)).$$

For $t = \bar{t} : P(t) > Y(t) > X(t) \Rightarrow ED(t) < 0 \Rightarrow P(t+1) < P(t)$.



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The dynamics of price:

Since $\sigma_X = \sigma_Y = 1$ **and** $\omega = 0.5$, **then**

$$ED(t) = 0.5(X(t) - P(t)) + 0.5(Y(t) - P(t)).$$

For $t = \bar{t}$: $P(t) > Y(t) > X(t) \Rightarrow ED(t) < 0 \Rightarrow P(t+1) < P(t)$.

For $t = \bar{\bar{t}}$: $Y(t) > P(t) > X(t)$ **and** $|Y(t) - P(t)| > |X(t) - P(t)| \Rightarrow ED(t) > 0 \Rightarrow P(t+1) > P(t)$.

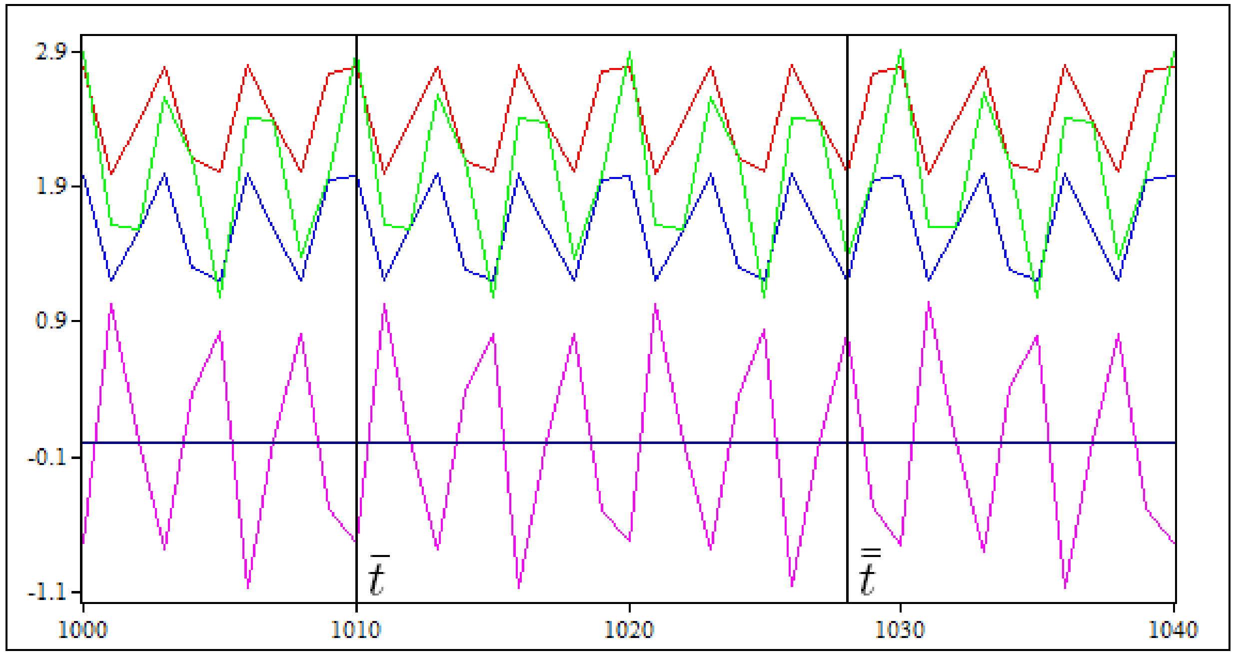
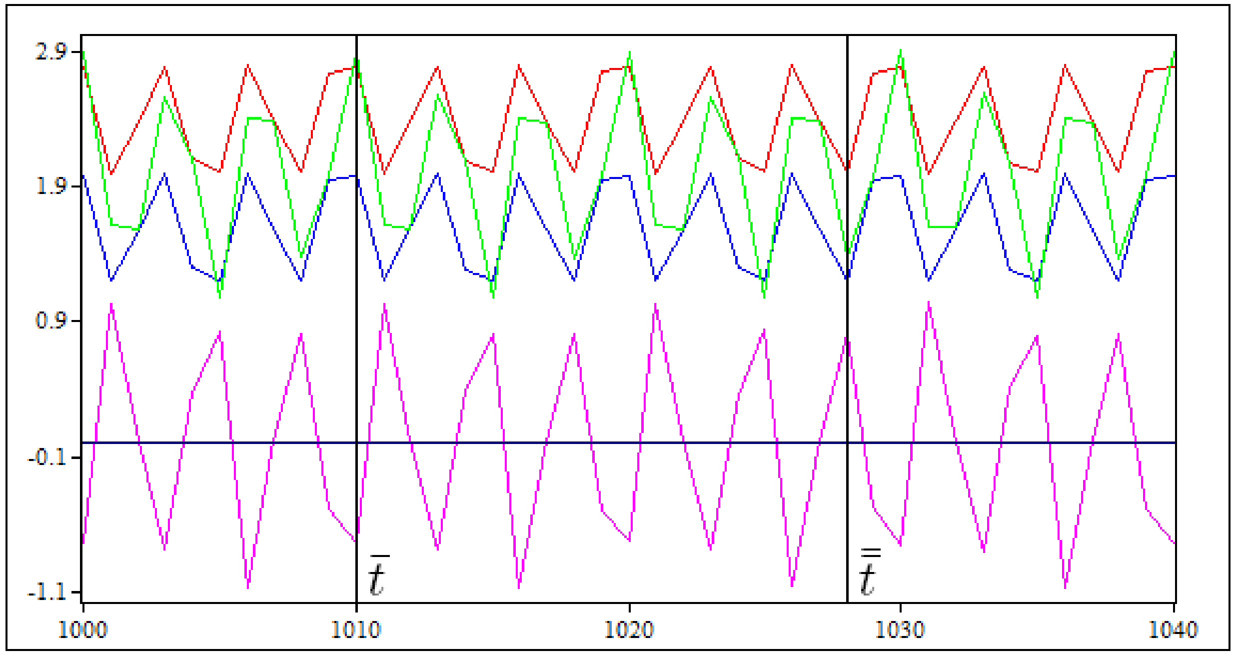


FIGURE 10: The time series for X in blue, Y in red, P in green, and $\pi_X - \pi_Y$ in pink, for $\gamma = 5.1$, $F = 2$, $\beta = 5.35$, $a_1 = a_2 = 1$, $\omega = 0.5$, $\Delta = 0.8$, and the initial conditions $X(0) = 1.5$, $Y(0) = 2.5$ and $P(0) = 2.1$.



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The dynamics of fundamental values:

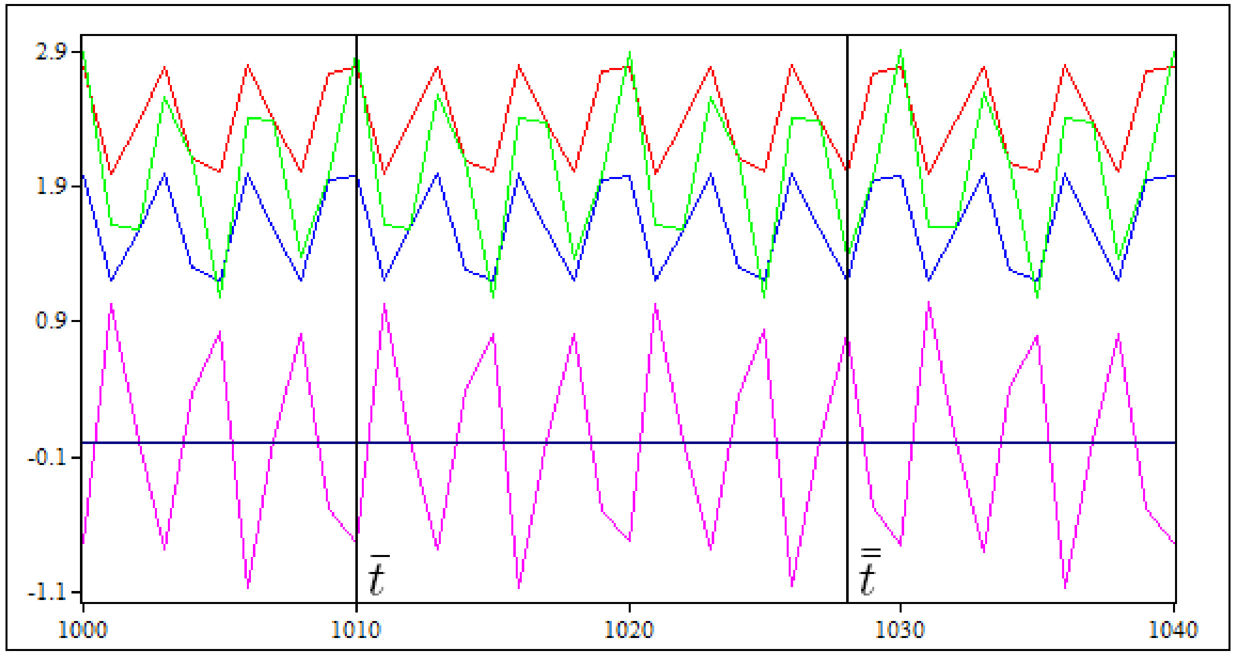

FIGURE 10: The time series for X in blue, Y in red, P in green, and $\pi_X - \pi_Y$ in pink, for $\gamma = 5.1$, $F = 2$, $\beta = 5.35$, $a_1 = a_2 = 1$, $\omega = 0.5$, $\Delta = 0.8$, and the initial conditions $X(0) = 1.5$, $Y(0) = 2.5$ and $P(0) = 2.1$.

The dynamics of fundamental values:

$$\sigma_X = \sigma_Y = 1 \;\Rightarrow\; \pi_X(t+1) - \pi_Y(t+1) = (P(t+1) - P(t))(X(t) - Y(t)).$$

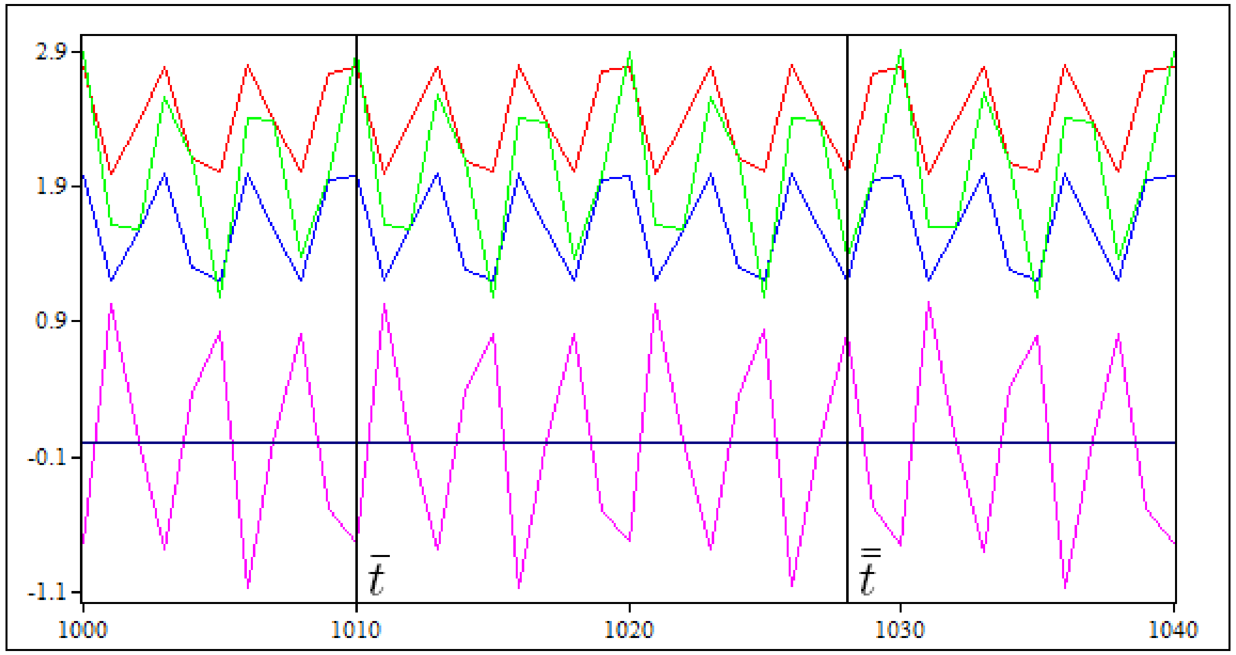

FIGURE 10: The time series for X in blue, Y in red, P in green, and $\pi_X - \pi_Y$ in pink, for $\gamma = 5.1$, $F = 2$, $\beta = 5.35$, $a_1 = a_2 = 1$, $\omega = 0.5$, $\Delta = 0.8$, and the initial conditions $X(0) = 1.5$, $Y(0) = 2.5$ and $P(0) = 2.1$.

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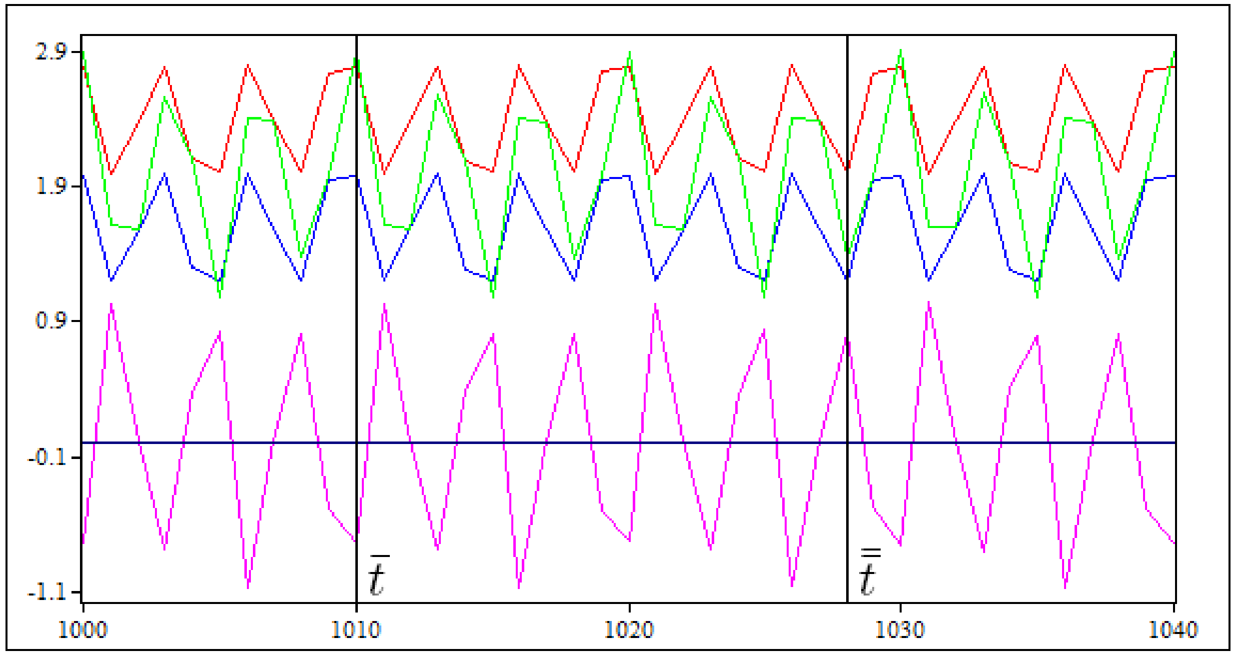
$$\sigma_X = \sigma_Y = 1 \;\Rightarrow\; \pi_X(t+1) - \pi_Y(t+1) = (P(t+1) - P(t))(X(t) - Y(t)).$$

For $t = \bar{t}$: $P(t+1) < P(t) \Rightarrow \pi_X(t+1) - \pi_Y(t+1) > 0 \Rightarrow$ **more pessimism** $\Rightarrow X(t+1) < X(t)$ **and** $Y(t+1) < Y(t)$.



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The dynamics of fundamental values:

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For $t = \bar{t}$: $P(t+1) < P(t) \Rightarrow \pi_X(t+1) - \pi_Y(t+1) > 0 \Rightarrow$ **more pessimism** $\Rightarrow X(t+1) < X(t)$ **and** $Y(t+1) < Y(t)$.

For $t = \bar{\bar{t}}$: $P(t+1) > P(t) \Rightarrow \pi_X(t+1) - \pi_Y(t+1) < 0 \Rightarrow$ **more optimism** $\Rightarrow X(t+1) > X(t)$ **and** $Y(t+1) > Y(t)$.

Some multistability phenomena

Some multistability phenomena

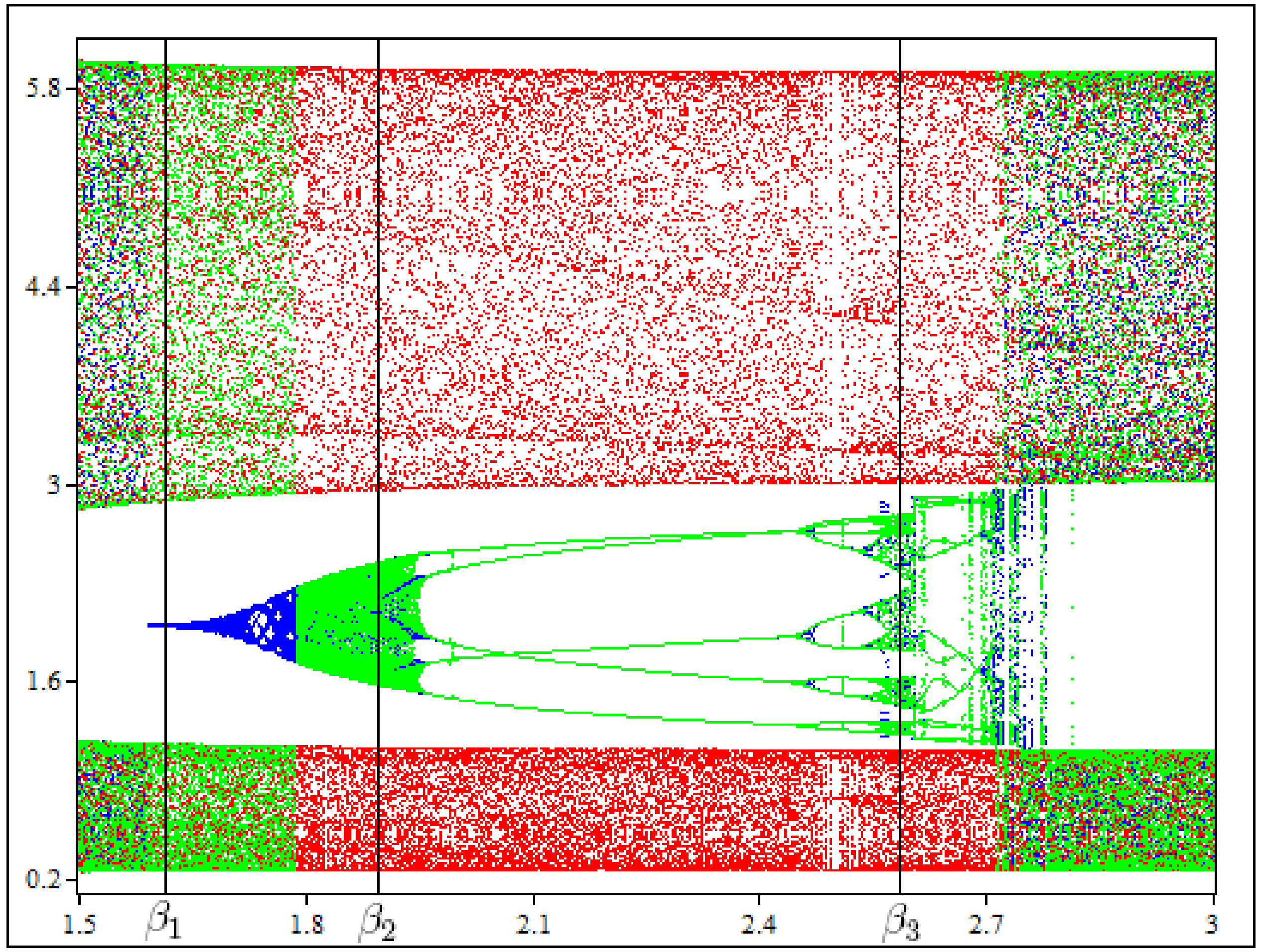

FIGURE 11: The bifurcation diagram with respect to $\beta \in [1.5, 3]$ for P with $\gamma = 5$, $F = 2$, $\Delta = 0.8$, $a_1 = 2.6$, $a_2 = 1$, $\omega = 0.5$, and the initial conditions $X(0) = 1.3$, $Y(0) = 2.5$, and $P(0) = 2$ for the blue points, $P(0) = 3$ for the red points, and $P(0) = 2.1$ for the green points, respectively.

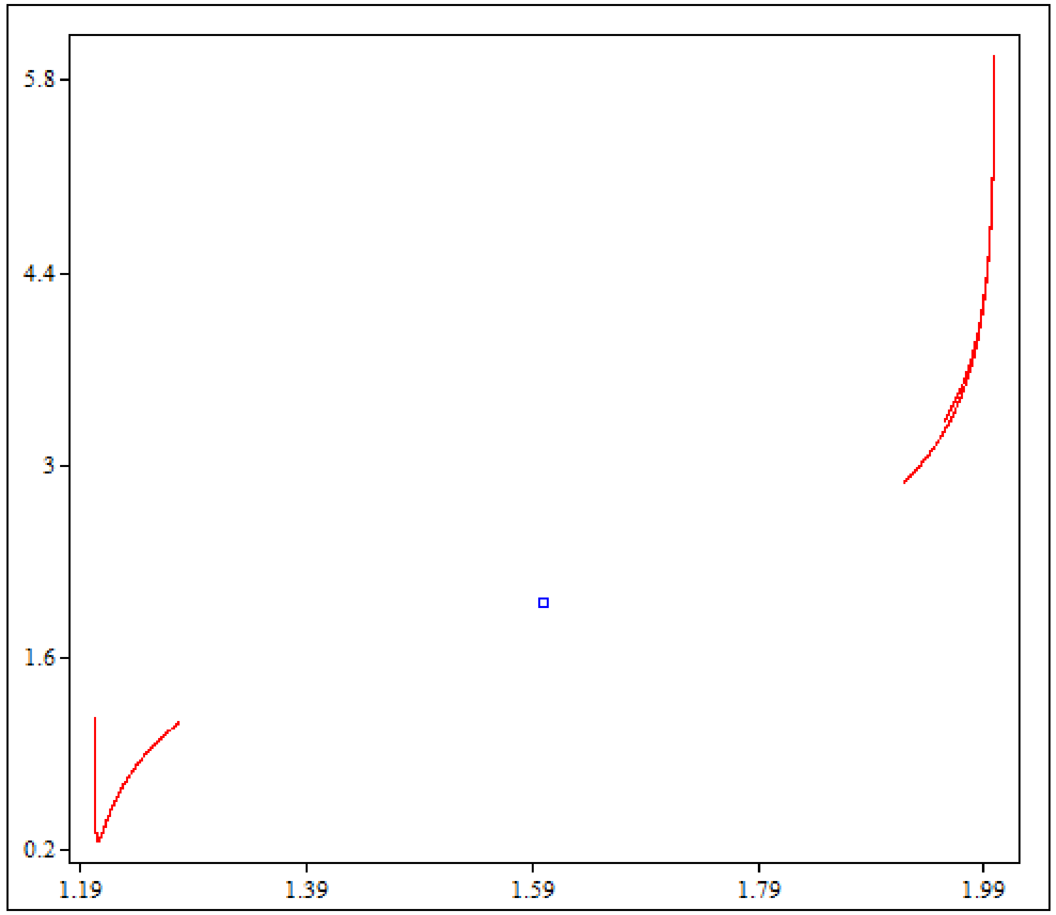

FIGURE 12: The (X, P)-phase portrait for $\beta = 1.6$.

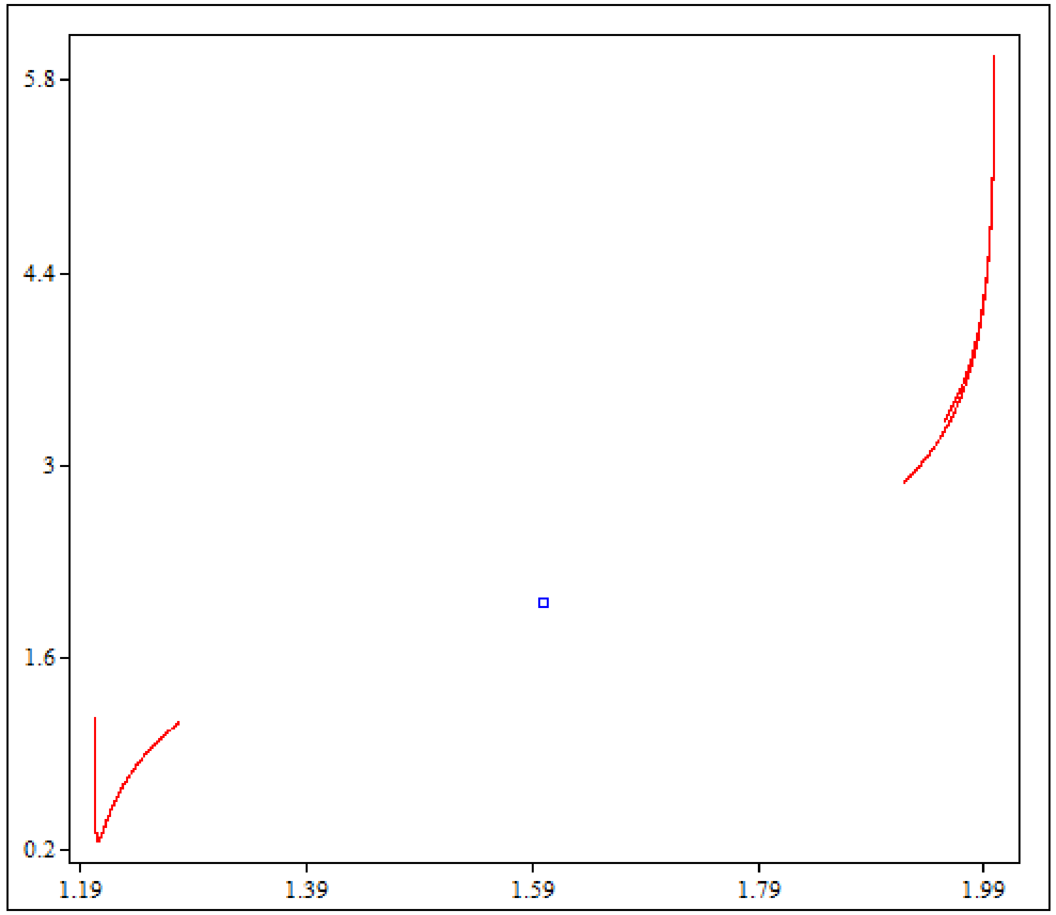

FIGURE 12: The (X, P)-phase portrait for $\beta = 1.6$.

$\Rightarrow$ coexistence between the fixed point and a chaotic attractor in two pieces.

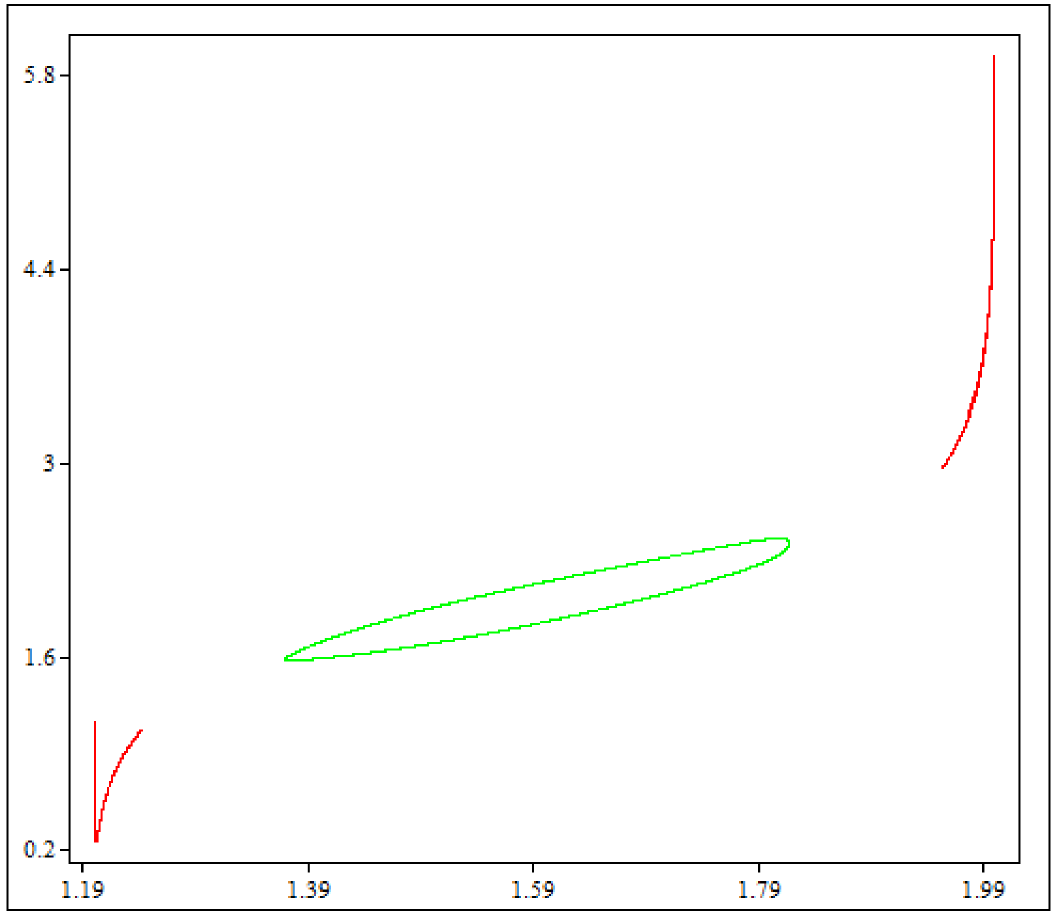

FIGURE 13: The (X, P)-phase portrait for $\beta = 1.9$.

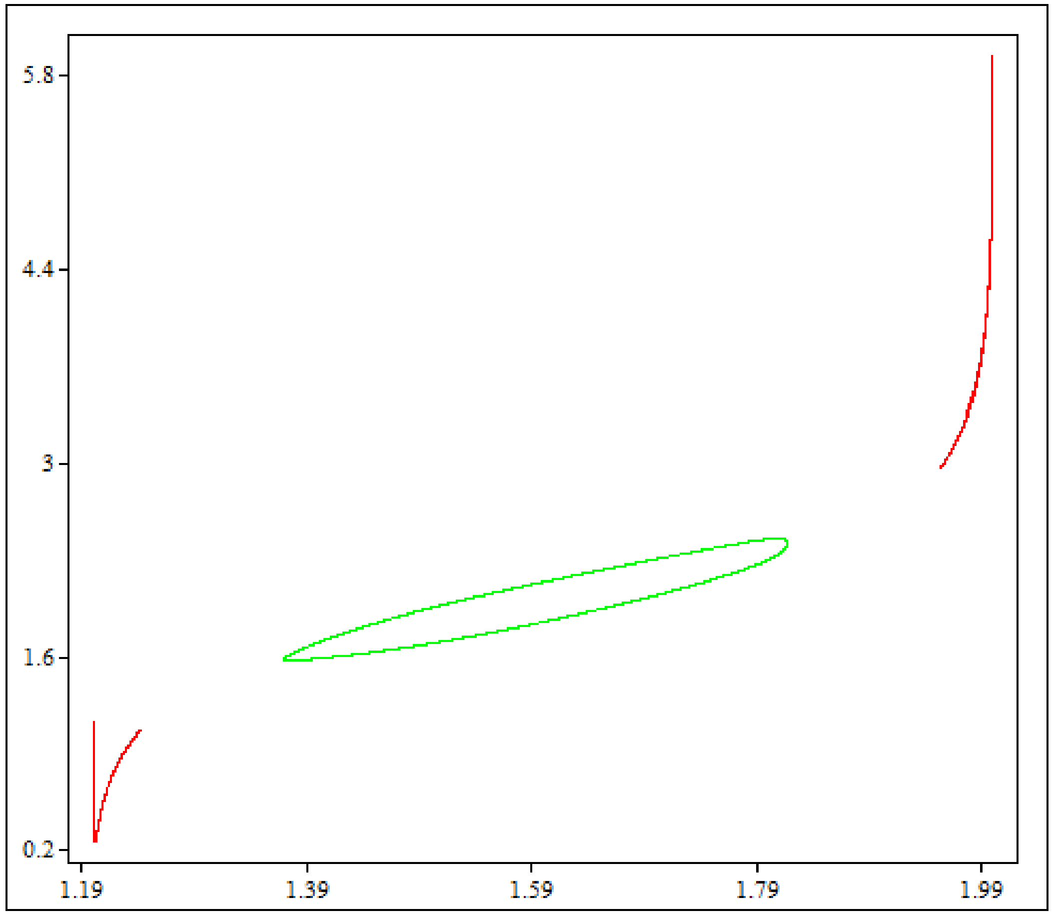

FIGURE 13: The (X, P)-phase portrait for $\beta = 1.9$.

$\Rightarrow$ **coexistence between an invariant curve and a chaotic attractor in two pieces.**

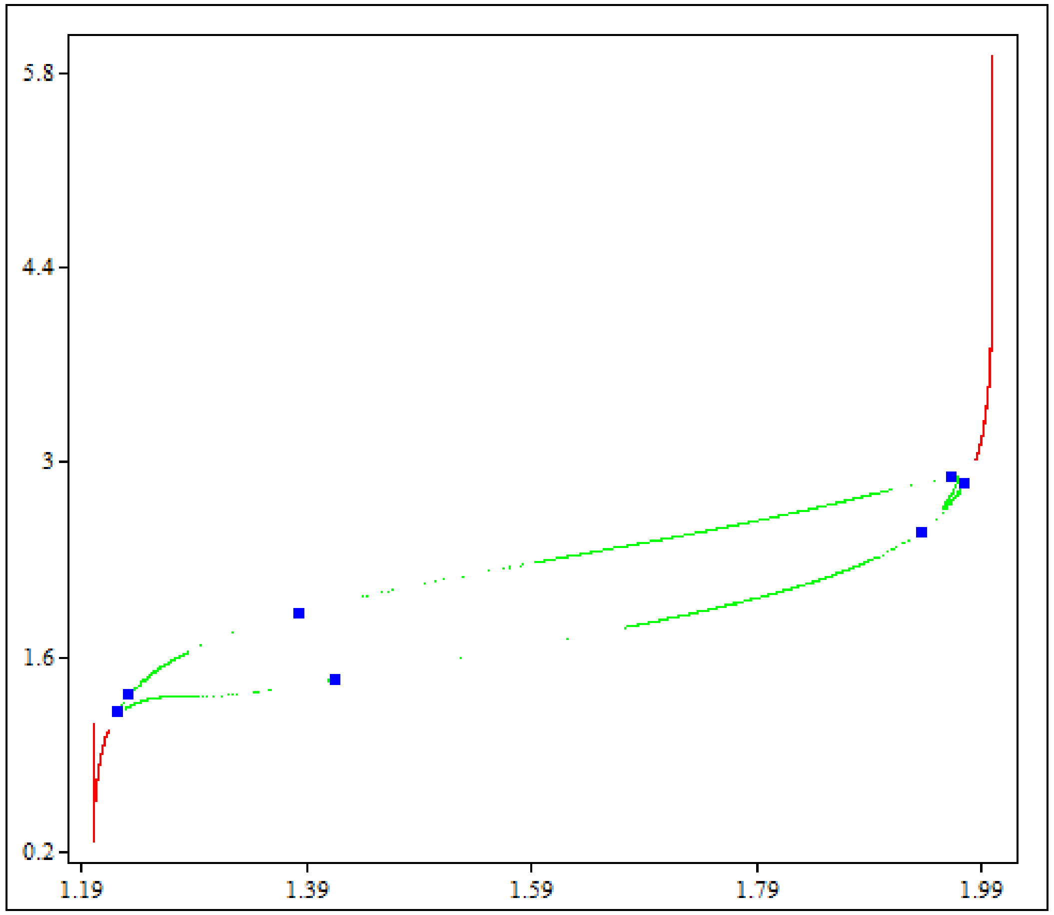

FIGURE 14: The (X, P)-phase portrait for $\beta = 2.604$.

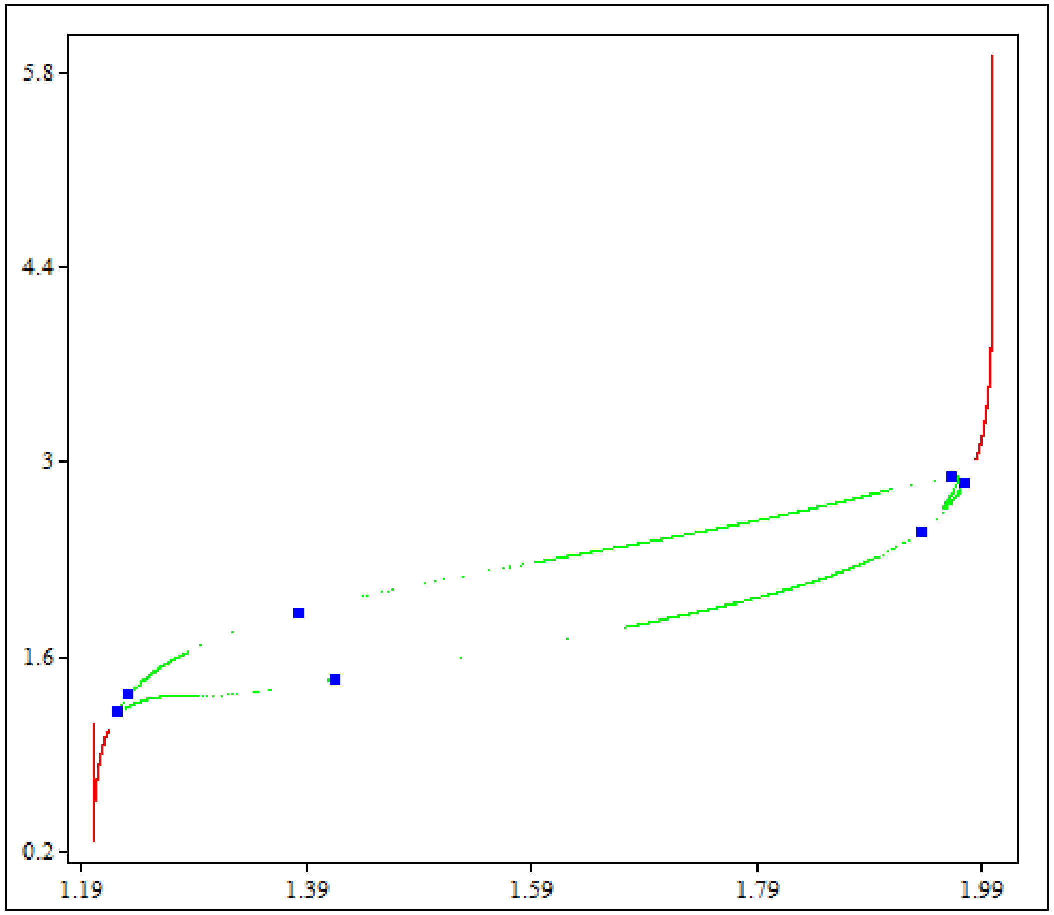

FIGURE 14: The (X, P)-phase portrait for $\beta = 2.604$.

$\Rightarrow$ **coexistence among a period-7 cycle and two chaotic attractors.**

5. Possible extensions

Endogenous switching mechanism

Endogenous switching mechanism

$$
\begin{cases}
X(t+1) = F - \Delta\left(\frac{1}{1+e^{-\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\
Y(t+1) = F + \Delta\left(\frac{1}{1+e^{\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\
P(t+1) = P(t) + \gamma a_2\left(\frac{a_1+a_2}{a_1 e^{-(\omega(t)\sigma_X(X(t)-P(t))+(1-\omega(t))\sigma_Y(Y(t)-P(t)))}+a_2} - 1\right) \\
\omega(t+1) = \frac{1}{1+e^{-\mu(\pi_X(t+1)-\pi_Y(t+1))}}
\end{cases}
$$

Endogenous switching mechanism

$$\begin{cases} X(t+1) = F - \Delta\left(\frac{1}{1+e^{-\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\ Y(t+1) = F + \Delta\left(\frac{1}{1+e^{\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\ P(t+1) = P(t) + \gamma a_2\left(\frac{a_1+a_2}{a_1 e^{-(\omega(t)\sigma_X(X(t)-P(t))+(1-\omega(t))\sigma_Y(Y(t)-P(t)))}+a_2} - 1\right) \\ \omega(t+1) = \frac{1}{1+e^{-\mu(\pi_X(t+1)-\pi_Y(t+1))}} \end{cases}$$

Logit mechanism by Brock and Hommes (1997) (or a different mechanism based on squared errors between fundamentals and price, like in Naimzada and Ricchiuti 2008, 2009).

Endogenous switching mechanism

$$
\begin{cases}
X(t+1) = F - \Delta\left(\frac{1}{1+e^{-\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\
Y(t+1) = F + \Delta\left(\frac{1}{1+e^{\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\
P(t+1) = P(t) + \gamma a_2\left(\frac{a_1+a_2}{a_1 e^{-(\omega(t)\sigma_X(X(t)-P(t))+(1-\omega(t))\sigma_Y(Y(t)-P(t)))}+a_2} - 1\right) \\
\omega(t+1) = \frac{1}{1+e^{-\mu(\pi_X(t+1)-\pi_Y(t+1))}}
\end{cases}
$$

Logit mechanism by Brock and Hommes (1997) (or a different mechanism based on squared errors between fundamentals and price, like in Naimzada and Ricchiuti 2008, 2009).

$\beta = 0 \;\Rightarrow\; X(t+1) = F - \frac{\Delta}{2}, \quad Y(t+1) = F + \frac{\Delta}{2}.$

Endogenous switching mechanism

$$\begin{cases} X(t+1) = F - \Delta\left(\frac{1}{1+e^{-\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\ Y(t+1) = F + \Delta\left(\frac{1}{1+e^{\beta(\pi_X(t+1)-\pi_Y(t+1))}}\right) \\ P(t+1) = P(t) + \gamma a_2\left(\frac{a_1+a_2}{a_1 e^{-(\omega(t)\sigma_X(X(t)-P(t))+(1-\omega(t))\sigma_Y(Y(t)-P(t)))}+a_2} - 1\right) \\ \omega(t+1) = \frac{1}{1+e^{-\mu(\pi_X(t+1)-\pi_Y(t+1))}} \end{cases}$$

Logit mechanism by Brock and Hommes (1997) (or a different mechanism based on squared errors between fundamentals and price, like in Naimzada and Ricchiuti 2008, 2009).

$$\beta = 0 \;\Rightarrow\; X(t+1) = F - \frac{\Delta}{2}, \quad Y(t+1) = F + \frac{\Delta}{2}.$$

Hence, with $\beta = 0$ we are in the framework by De Grauwe and Rovira Kaltwasser (2012) with bias $a = \frac{\Delta}{2}$, except for our nonlinear price adjustment mechanism.

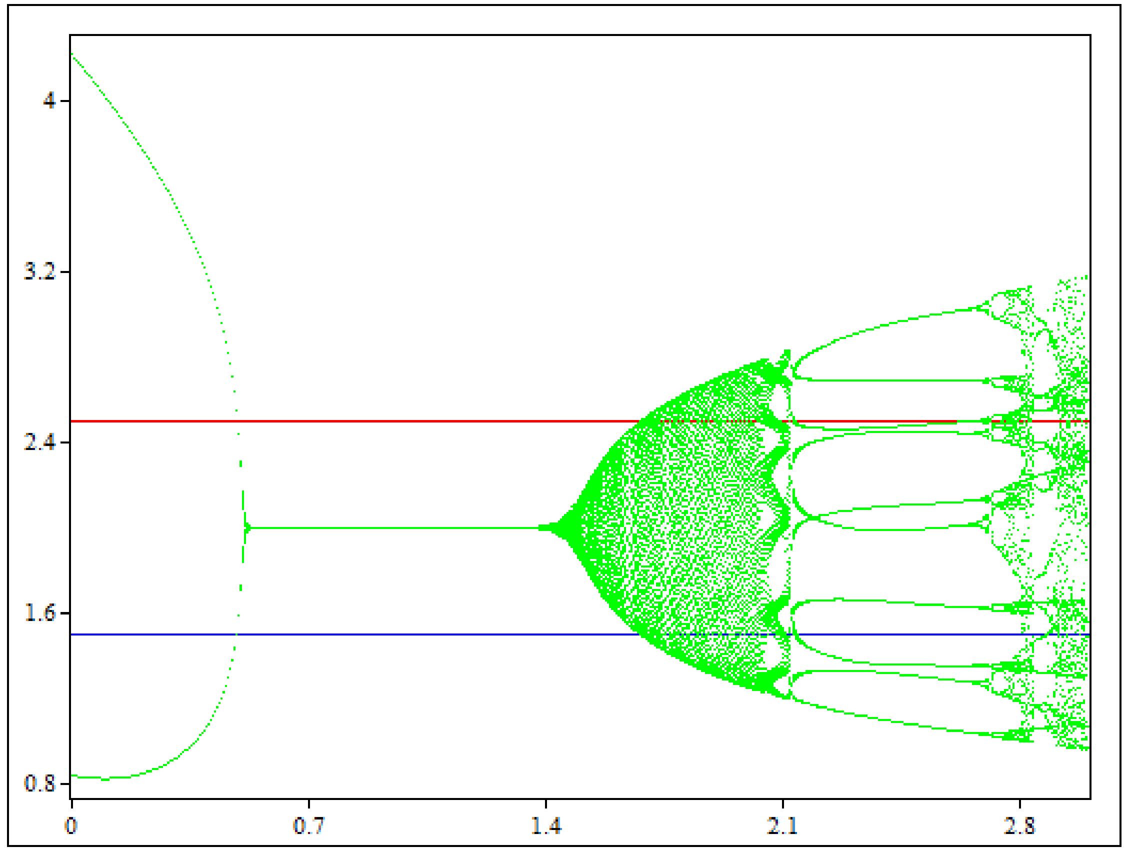

FIGURE 15: The bifurcation diagram with respect to $\mu \in [0, 3]$ for X in blue, Y in red and P in green, for $\gamma = 4$, $F = 2$, $\beta = 0$, $a_1 = 2$, $a_2 = 1$, $\Delta = 1$, and the initial conditions $X(0) = 1.6$, $Y(0) = 2.8$ and $P(0) = 3$.

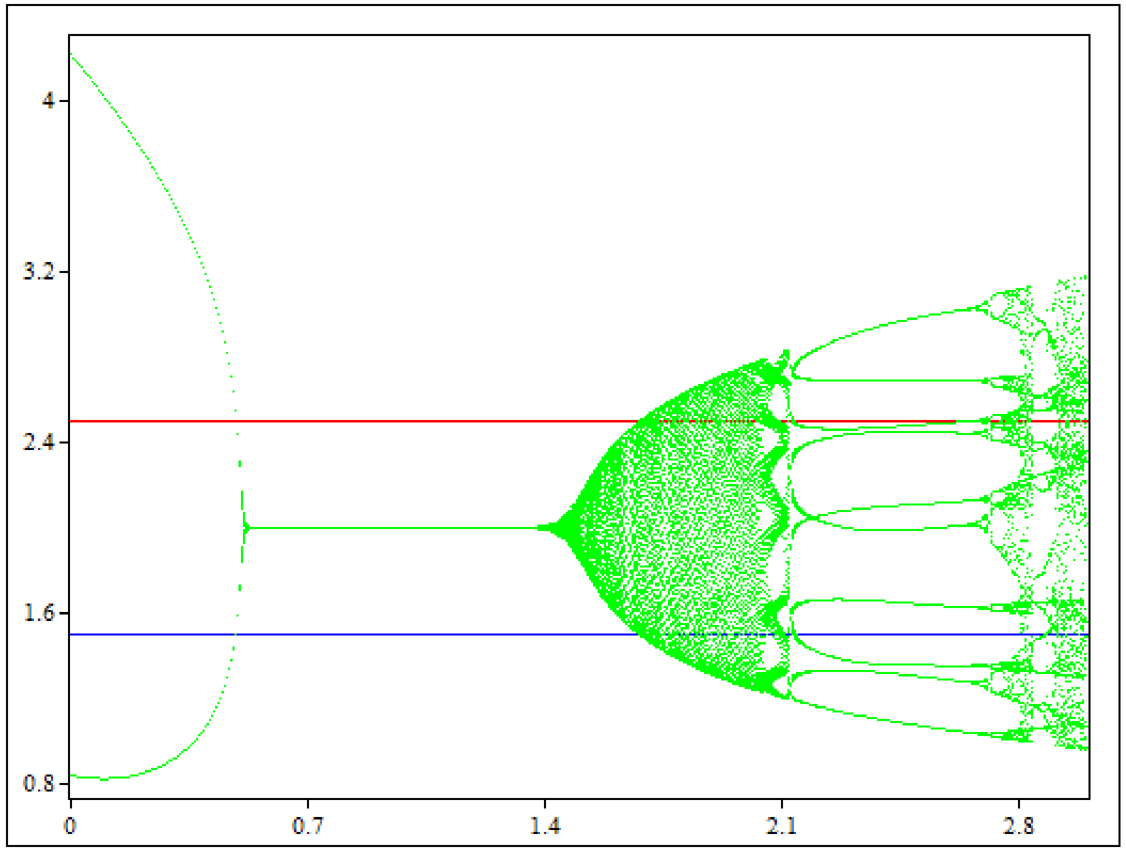

FIGURE 15: The bifurcation diagram with respect to $\mu \in [0, 3]$ for X in blue, Y in red and P in green, for $\gamma = 4$, $F = 2$, $\beta = 0$, $a_1 = 2$, $a_2 = 1$, $\Delta = 1$, and the initial conditions $X(0) = 1.6$, $Y(0) = 2.8$ and $P(0) = 3$.

$$\beta = 0 \;\Rightarrow\; X \equiv F - \frac{\Delta}{2} = 1.5, \quad Y \equiv F + \frac{\Delta}{2} = 2.5.$$

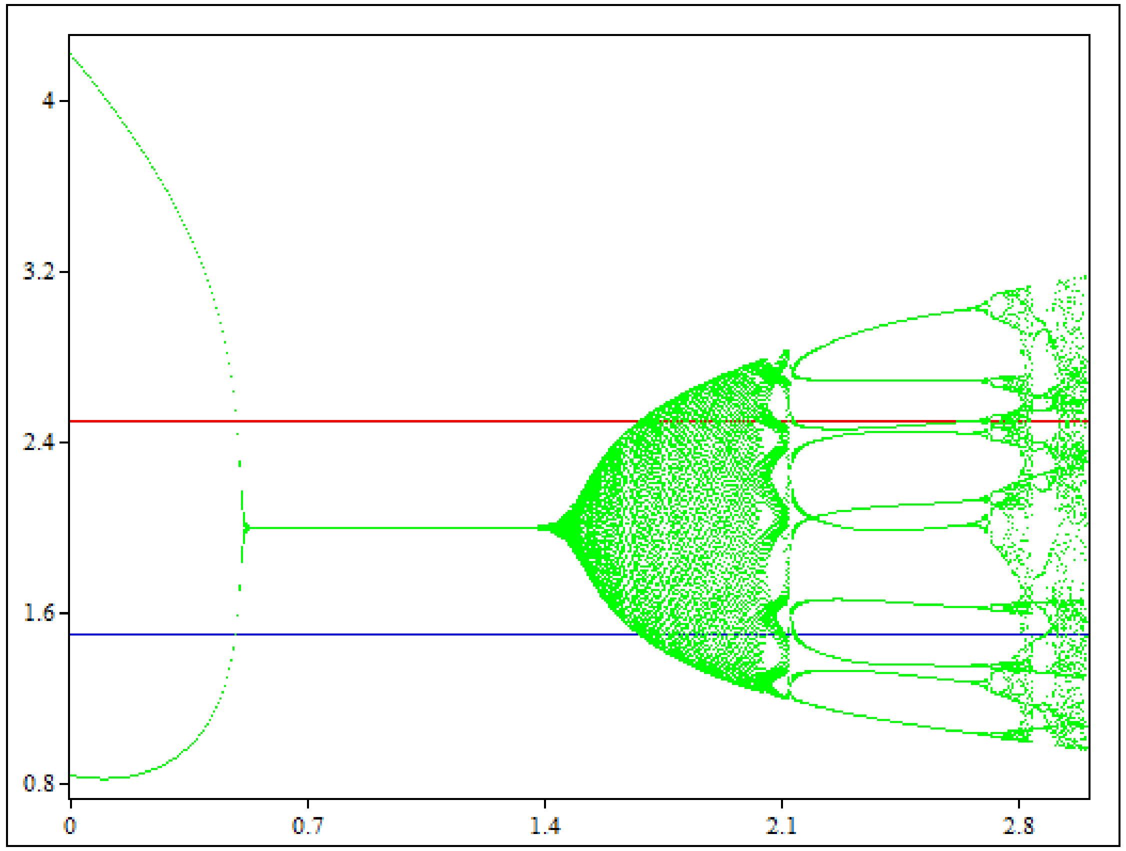

FIGURE 15: The bifurcation diagram with respect to $\mu \in [0, 3]$ for X in blue, Y in red and P in green, for $\gamma = 4$, $F = 2$, $\beta = 0$, $a_1 = 2$, $a_2 = 1$, $\Delta = 1$, and the initial conditions $X(0) = 1.6$, $Y(0) = 2.8$ and $P(0) = 3$.

$$\beta = 0 \;\Rightarrow\; X \equiv F - \frac{\Delta}{2} = 1.5, \quad Y \equiv F + \frac{\Delta}{2} = 2.5.$$

For the price, a flip bifurcation occurs for $\mu \simeq 0.55$ and a Hopf bifurcation occurs for $\mu \simeq 1.4$.

$\beta \neq 0 \Rightarrow$ **we generalize the framework in De Grauwe and Rovira Kaltwasser (2012) (except for the price equation).**

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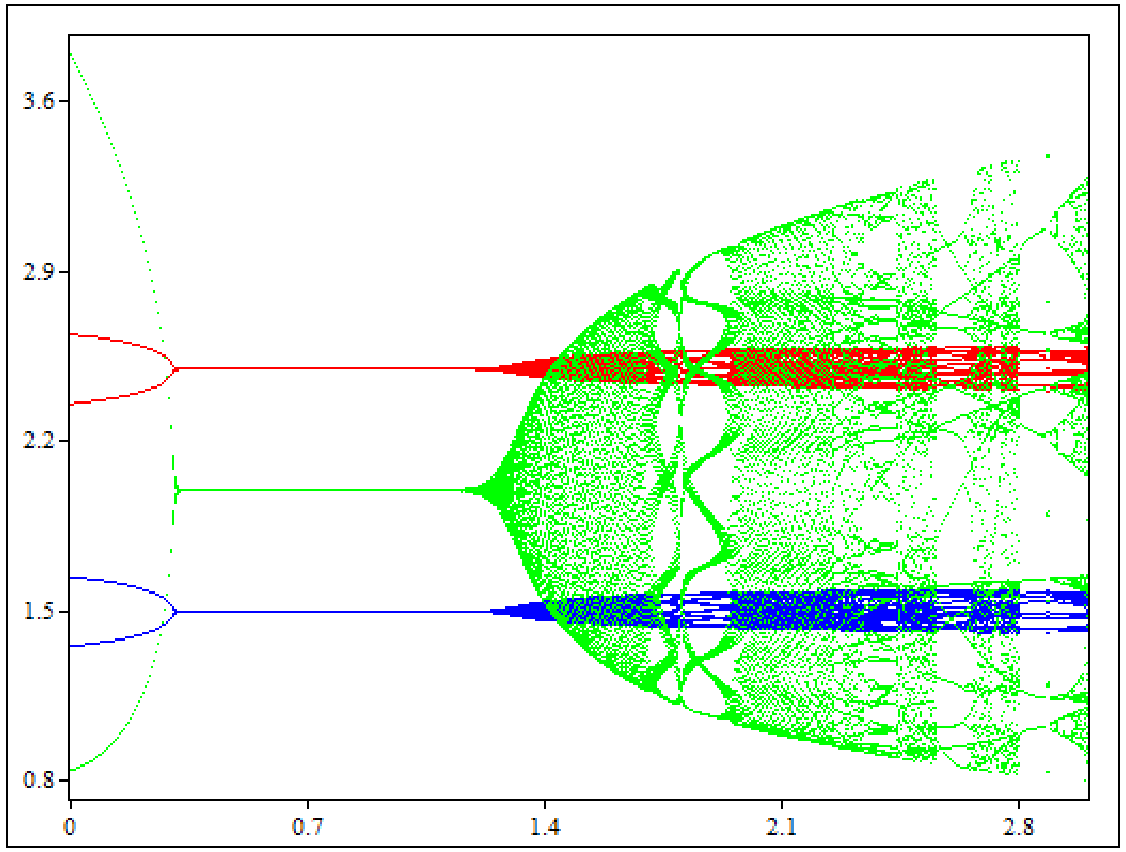

FIGURE 16: The bifurcation diagram with respect to $\mu \in [0, 3]$ for X in blue, Y in red and P in green, for $\gamma = 4$, $F = 2$, $\beta = 0.2$, $a_1 = 2$, $a_2 = 1$, $\Delta = 1$, and the initial conditions $X(0) = 1.6$, $Y(0) = 2.8$ and $P(0) = 3$.

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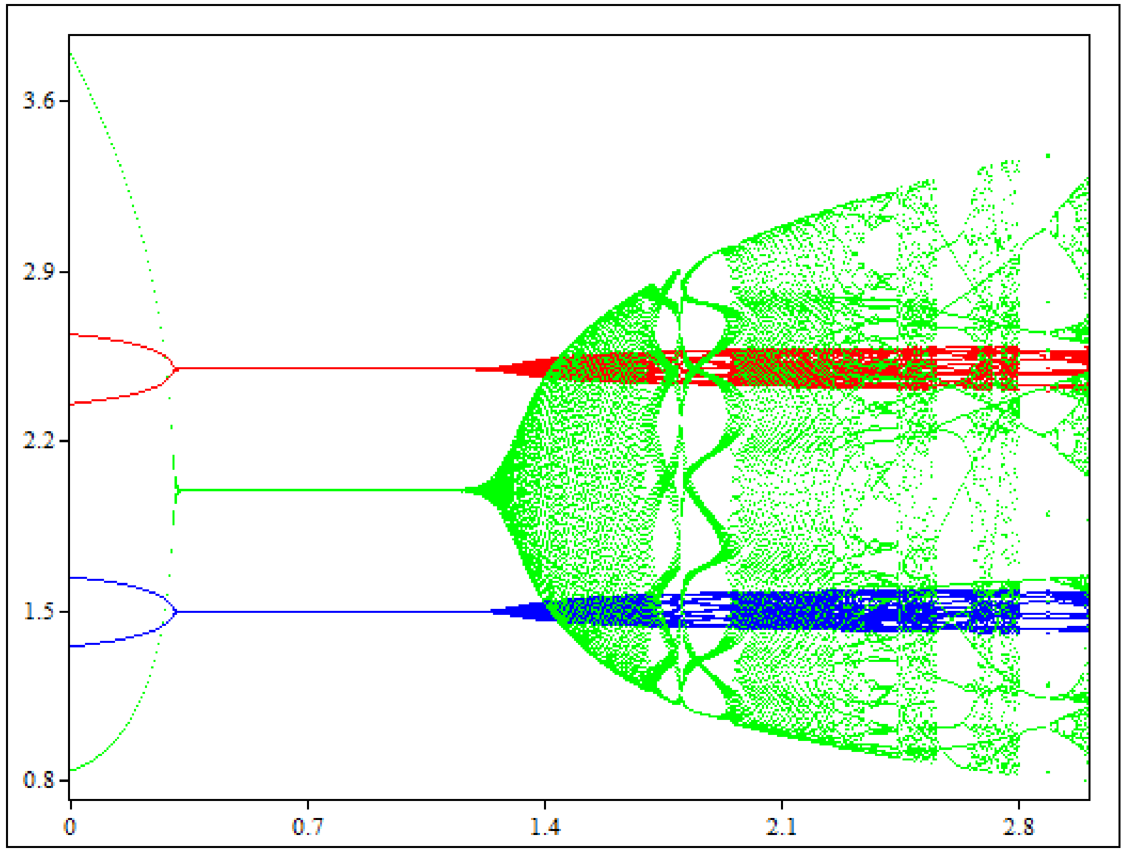

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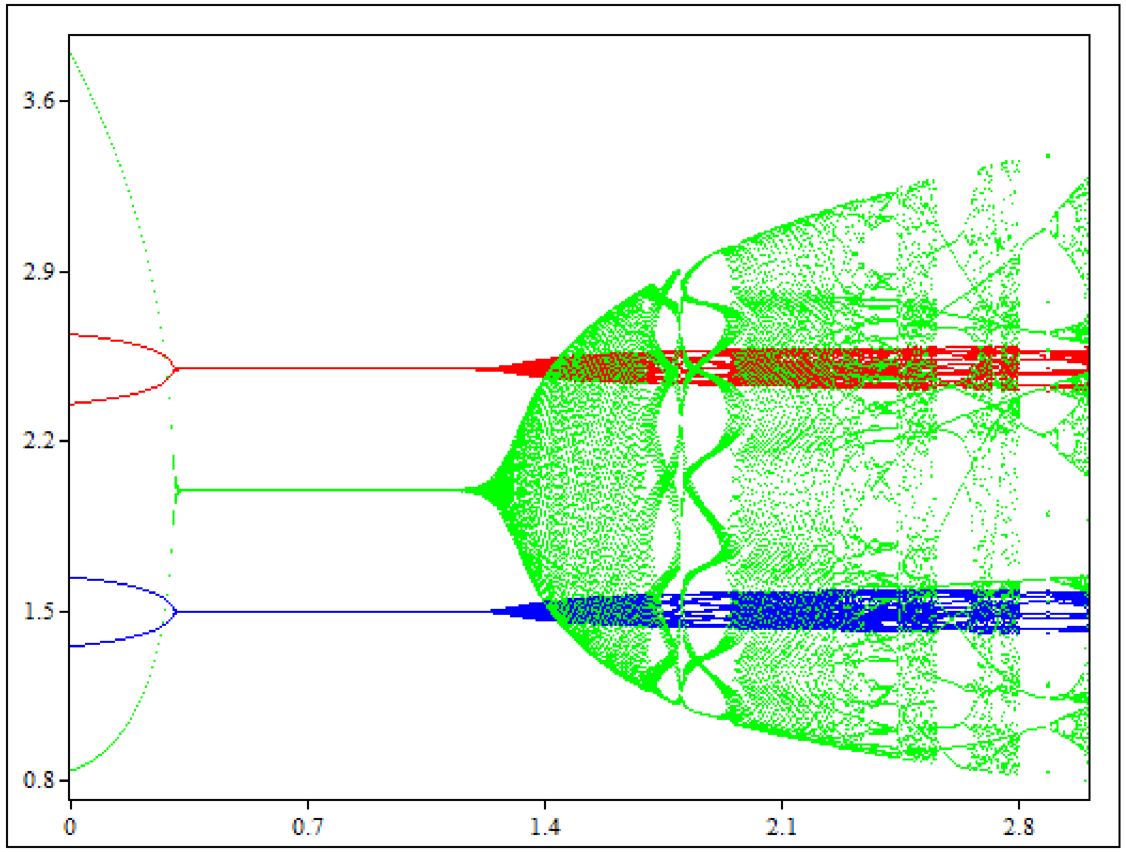

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A flip bifurcation occurs for $\mu \simeq 0.3$ **and a Hopf bifurcation occurs for** $\mu \simeq 1.25$.

Relaxing the symmetry condition

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Another extension: consider (possibly) different degrees of optimism and pessimism for agents, i.e., $\Delta_X \neq \Delta_Y \in [0, F]$.

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$\rightarrow$ **does the stability region become larger?**

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