A DYNAMIC EXCHANGE RATE MODEL WITH ENDOGENOUS AND HETEROGENOUS BELIEFS

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- We analyze an heterogenous agent model (HAM) in which there are heterogenous and endogenous beliefs on exchange rate, determined by the real markets.

- *‘A more attractive alternative would be to allow for time variation in both the fundamental value of the exchange rate as well as in the beliefs about it. In the agent based literature it is sometimes assumed the fundamental value of assets evolves as a random walk[..]. In all these models, however,* ***the fundamental is*** *a time varying variable but it* ***still exogenously determined****. That is,* ***it is not connected to the real part of the economy*** *in any way.’* (De Grauwe and Rovira, 2012 JEDC)

In finance, the Fundamental is the sum of the discounted future incomes generated by the asset;

However, in general, the Fundamental is a reference value or a focal point or a belief about the long-run value of a variable;

Therefore, it is better to talk about beliefs. The beliefs about the future have a subjective dimension: hardly agents reach the true fundamental value and it is really unlikely that agents have the same beliefs.

The fundamental/beliefs usually are:

- Fixed (Brock and Hommes, 1998; Naimzada and Ricchiuti,2009; De Grauwe and Rovira, 2012) vs. Time-varying (Westerhoff, 2003; Manzan and Westerhoff, 2005; De Grauwe and Grimaldi, 2006, Proaño, 2011)

In particular, among the fixed cases:

- In B-H (1998) and in DG-R (2012) there are opposite biases or pessimists/optimists;
- In N-R (2009) we assumed that the *true* fundamental value is unknown and agents have just different beliefs, that we (in a misleading way) called heterogenous fundamentals.

Recently different authors have started to endogenize the fundamental value:

- *Lengnick and Wohltmann (2010)*: in a NK framework, the beliefs are function of the output gap;
- *Proaño* (2011) analyzes a two country macroeconomic model for the determination nominal exchange rate, in which the fundamental value depends on the purchasing power parity;
- *Westerhoff (2012)*: linked a stock market with heterogeneous speculators with a Income-Expenditure Keynesian model for a closed economy;
- *Naimzada and Pireddu (2014)*: in line with W(2012), they assume agents make their decisions using a linear combination between an exogenous and an endogenous value of both national income and level of stock market.
- *Gori and Ricchiuti (2014)*: We overcome the exogenous determination of fundamentals, linking the foreign exchange market with goods market of two countries as in the Absorption model. However, this model missed the effects of interest rates on the exchange rate.

There are two countries (Europe and US), who freely trade without restrictions and who have both a flexible exchange rate regime.

$E(t)$ is the exchange rate between the two countries at time t, expressed as dollars per euros.

The actual exchange rate is determined by a market maker who looks at the excess of demand of euros in the system.

$$E(t+1) = E(t) + \omega g_1(ED(t)). \tag{1}$$

where g_1 is a regular function such that $g_1(0) = 0$ and $g_1'(.) > 0$.

$$ED(t) = \left[n_a\alpha_a \left(\hat{E}^a(t+1) - E(t) \right) + n_b\alpha_b \left(\hat{E}^b(t+1) - E(t) \right) \right] \tag{2}$$

where $\hat{E}^i(t+1)$ are the *endogenous* and *heterogenous* beliefs of two groups of agents that act as fundamentalists.

The demand/expenditure of European citizens in euros at time t is:

$$D_1(t) = A_1 + mpc_1 Y_1(t) - dr_1(t) + \frac{mpi_2 Y_2(t)}{E(t)} - mpi_1 Y_1(t)$$

and that of US citizens in dollars at time t is:

$$D_2(t) = A_2 + mpc_2 Y_2(t) - dr_2(t) + mpi_1 Y_1(t)E(t) - mpi_2 Y_2(t).$$

The European GDP at time $t+1$ is:

$$Y_1(t+1) = Y_1(t) + \gamma_1 g_2(D_1(t) - Y_1(t))$$

while the US GDP at time $t+1$ is:

$$Y_2(t+1) = Y_2(t) + \gamma_2 g_3(D_2(t) - Y_2(t))$$

where g_2 and g_3 are regular functions defined on $\mathbb{R}$ such that the first derivative is positive and $g_2(0) = 0$ and $g_3(0) = 0$.

We know that:

$$M^s = L + \Delta RU \tag{3}$$

M is the exogenous supply of money and RU are official reserves. On the other hand, the demand of money L is the function of :

$$L_i(t) = L_i(Y_i, r_i(t)) \tag{4}$$

Our assumption is that the interest rate depends on disequilibrium in the money market, that is:

$$r_1(t+1) = r_1(t) + \varphi_1 g_4[L_1(t) - M_1 - \Delta RU_1(t)] \tag{5}$$

$$r_2(t+1) = r_2(t) + \varphi_2 g_5[L_2(t) - M_2 - \Delta RU_2(t)] \tag{6}$$

Differently from Gori and Ricchiuti (2004), we assume that *'foreign exchange may be purchased and sold freely for all purposes* ***including*** *capital movements'*, therefore the BoP is:

$$CA + KA = \Delta RU \tag{7}$$

where CA is the Current Account and KA is the Capital/Financial Account. The two BoPs are strictly linked, note that $\Delta RU_1(t)E(t) = -\Delta RU_2(t)$, and we know that KA is a function of both, the spread of the interest rates and openness of the capital account, hence:

$$\Delta RU_2(t) = CA_2 + KA_2 = mpi_1 Y_1(t)E(t) - mpi_2 Y_2(t) + KA(k, r_2(t) - r_1(t)) \tag{8}$$

where k is an index of the openness of the capital account.

As in Gori and Ricchiuti (2014), the group (a) believes that the long-run value of the exchange rate leads the Current Account of the BoP to zero:

$$\hat{E}^{a}(t+1) = \frac{mpi_2 Y_2(t)}{mpi_1 Y_1(t)}$$

On the other hand, the group b) chooses the long-run value of the exchange rate that satisfies the Uncovered Interest Parity (UIP):

$$(1 + r_1(t)) = (1 + r_2(t)) \frac{E(t)}{\hat{E}^{b}(t+1)}$$

where therefore, $r_i(t)$ is the interest rate determined in the money market of country i; therefore:

$$\hat{E}^{b}(t+1) = \frac{(r_2 - r_1)}{(1 + r_1)} E(t) + E(t)$$

Therefore the map is as follows:

$$
\begin{cases}
E(t+1) = E(t) + \omega g_1(n_a\alpha_a(\hat{E}^a(t+1) - E(t)) + n_b\alpha_b(\hat{E}^b(t+1) - E(t))) \\
Y_1(t+1) = Y_1(t) + \gamma_1 g_2(D_1(t) - Y_1(t)) \\
Y_2(t+1) = Y_2(t) + \gamma_2 g_3(D_2(t) - Y_2(t)) \\
r_1(t+1) = r_1(t) + \varphi_1 g_4(L_1(t) - M_1 - \Delta RU_1(t)) \\
r_2(t+1) = r_2(t) + \varphi_2 g_5(L_2(t) - M_2 - \Delta RU_2(t))
\end{cases}
\tag{9}
$$

We do not have an explicit expression of the equilibrium in which the excess demand for both groups is zero, that is $\hat{E}^a = \hat{E}^b = E^*$.

Such an equilibrium can be explicitly calculated with further restrictions on parameters, for example $A_1 = A_2$ and $mpc_1 = mpc_2$.

Therefore, we have studied the case of two perfectly symmetric countries (all parameters are equal), when the Exchange Rate is constant and equal to one, $E^* = 1$:

$$r_1 = r_2 = r^* = \frac{A\beta - M(1 - mpc)}{\beta d + \delta(1 - mpc)}$$

$$Y_1 = Y_2 = Y^* = \frac{A - dr^*}{1 - mpc}$$

This steady state is general and does not depend on g_i.

We calculate the stability conditions for the case in which g_i are linear and $KA(k, r_2(t) - r_1(t)) = k(r_2(t) - r_1(t))$.

The Jury stability conditions are:

1. 1-Tr+Det>0
$$\gamma \left[\varphi\delta \left(1 - mpc\right) + d\varphi\beta\right] > 0$$
2. 1+ Tr+Det>0
$$4 - 2\varphi\delta - 2\gamma(1 - mpc) + \gamma \left[\varphi\delta \left(1 - mpc\right) + d\varphi\beta\right] > 0$$
3. 1-det>0
$$\varphi\delta - \gamma d\varphi\beta + \gamma(1 - mpc)(1 - \varphi\delta) > 0$$

With perfect symmetry without $E^* = 1$, the system becomes five dimensional.

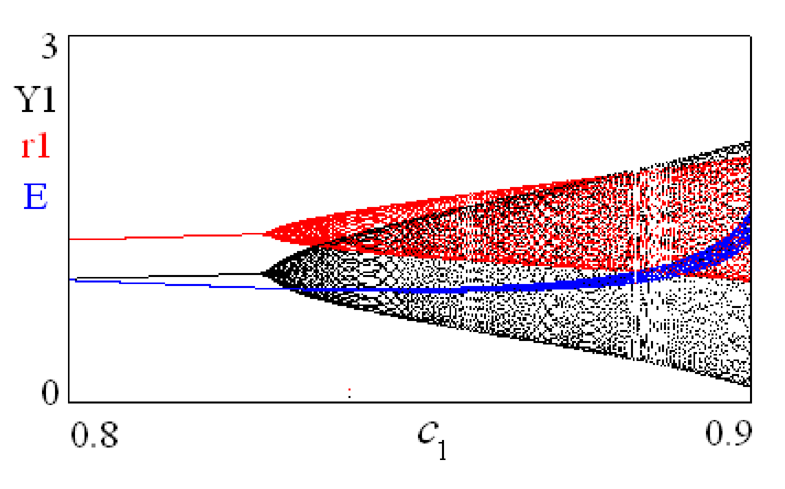
The previous equilibrium still exists but the stability conditions are not same.

However, if ω (the reaction coefficient of the mkt maker) is small enough the previous conditions could be a good landmark.

In the simulations, we have assumed:

$g_i = arctan(.)$
$KA(k, r_2(t) - r_1(t)) = k(r_2(t) - r_1(t))$
$\gamma_1 = \gamma_2 = 3.5$
$\varphi_3 = \varphi_4 = 0.4$
$\omega = 0.55$
$A_1 = A_2 = 0.8$
$mpc_1 = mpc_2 = 0.8$
$mpi_1 = mpi_2 = 0.03$
$\beta = 1.2$
$\delta = 0.8$
$d = 0.45$
$k = 0.2$
$M_1 = M_2 = 0.15$
$n_a = 0.5$
$\alpha_a = \alpha_b = 0.25$

Increasing mpc_1



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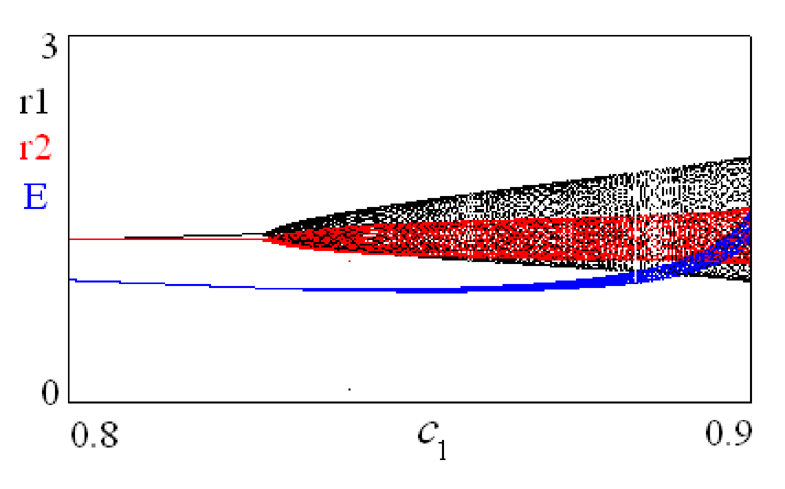

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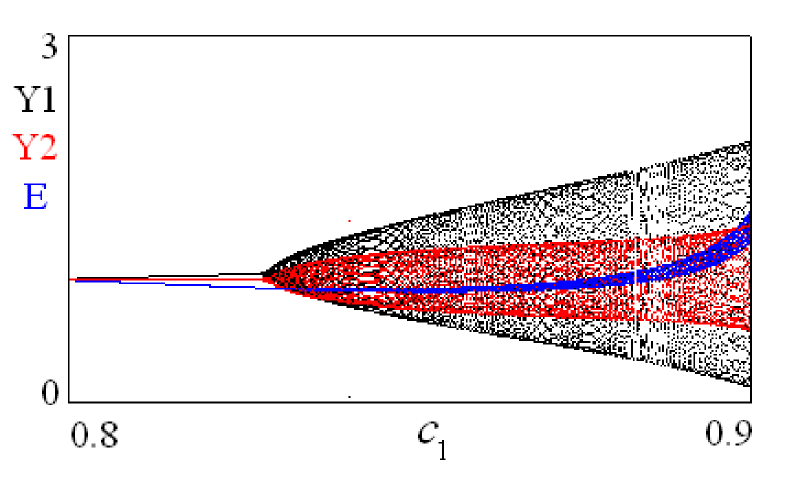

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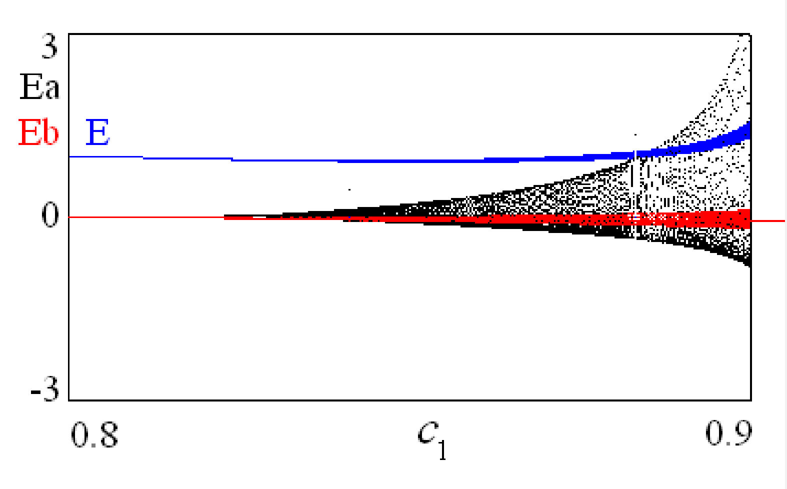

- We overcome the exogenous determination of fundamentals/beliefs, linking the foreign exchange market with both goods and money markets of two countries;
- It is a preliminary work, but we could say that complex dynamics can arise also when the reaction parameters are small, mainly due to the insurgence of asimmetry/heterogeneity among countries;