The effects of R&D investments in International Environmental Agreements with Asymmetric Countries

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MDEF 2014 - Urbino, 18-20 September 2014

International Environmental Agreements - IEA

- Over the last two decades, the interest in international environmental problems such as climate change, ozone depletion, marine pollution has grown immensely and it has drived an increased sense of interdependence among countries.
- Cooperation results in IEA such as:
  - Helsinky Protocol (1985);
  - Oslo Protocol (1994);
  - Montreal Protocol (1987);
  - Kyoto Protocol on the reduction of greenhouse gases causing global warming (1997).
  - Copenhagen (2009)

IEA

In these IEAs, the number of signatories varies considerably.

- Why IEAs are ratified only by a fraction of the countries?
- Which strategies can increase the number of signatories?
- Will the agreement be stable?
- What does mean stable?

- Agreements must be profitable (there must be gains to all signatory countries).
- Agreements must be self-enforcing (in the absence of any international authority, there must be incentives for countries to join and to remain in an agreement).

Both Cooperative and Non-Cooperative game theory have been used to study coalition formation.

- In the Cooperative Game framework, using core concepts and implementing transfers to solve the heterogeneity of the countries, Chander and Tulkens (1995) reach the conclusion that the grand coalition is stable.
- In the Non-Cooperative Game framework the concepts of Internal and External stability have been applied to obtain the size of a coalition. The idea is to check for which size of a coalition an individual country is indifferent between remaining in the coalition or leaving it.

Following Rubio and Casino (2005), in a continuous time setting and, in a time-discrete setting de Zeeuw (2008), in this talk we study a two-stage game.

- In the first stage (the membership game) each country decides noncooperatively wheter or not to join an IEA.
- In the second stage (the abatement differential game) signatories and nonsignatories determine their abatement levels in a dynamic continuous time setting.

- Let assume to have n countries that we suppose to divide in two kinds: n_h is the number of Developed countries and n_l is the number of Developing ones. These countries decide to abate emissions in order to reduce the environmental pollution;
- Let us assume that there are m_l out of n_l developing countries and m_h out of n_h developed countries that jointly form a non trivial coalition; obviously $m_h + m_l \geq 2$.
- We use the notation $[m_h, m_l]$ to represent the coalition formed by m_h developed countries and m_l developing countries. Moreover we denote by $m = m_h + m_l$. All the others players form only singleton coalition.

We denote by:

- $a_h(t)$ for $h = 1 \cdots n_h$ and $a_l(t)$, for $l = 1 \cdots n_l$, the abatement quantities provided by each developed and developing country, respectively.
- The dynamic of accumulated emissions is given by the following differential equation:

$$\dot{s}(t) = L - \sum_{h=1}^{n_h} a_h(t) - \sum_{l=1}^{n_l} a_l(t) - ks(t); \quad s(0) = s_0; \quad (1)$$

where $s_0 \in \mathbb{R}$ the initial level of accumulated emissions, L stands for a constant source of pollutant and k a positive rate of natural decay.

The cost functions for each developed and developing country are, respectively:

$$C_h = \frac{1}{2}a_h^2(t) + \frac{1}{2}p_h s(t) + c; \quad C_l = \frac{1}{2}a_l^2(t) + \frac{1}{2}p_l s(t) + c$$

• They are the sum of three terms: the abatement costs that are given by the quadratic form, the damage costs due to remaining pollution that are linear, the R&D investment in green capital that has to be developed at a fixed cost c.

• The parameter $p > 0$ is as a measure of the environmental awareness of the country; i.e. it denotes the weight of the damage costs of remaining emissions as compared to the abatement costs. The developed countries have a higher priority to environmental conservation than developing ones. So we can state that $p_l \leq p_h$, where we denote by p_l and p_h the environmental awareness of developing and developed.

- In the differential game proposed, Feedback Nash equilibria are calculated in order to determine both the optimal paths of the abatement levels and the stock of pollutant.
- We use a Non-Cooperative game framework solved in a backward order. As result of the first stage, m cooperators and $n - m$ defectors exist.
- We assume that when countries cooperate they coordinate their R&D activities and share the R&D investments to avoid duplication of green activities. Otherwise, outsiders have to provide on their own R&D costs of green capital.

- In the second stage, taken as given the abatement levels of outsiders, cooperators commit to a level of abatement that minimizes the discounted value of aggregate costs of m countries:

$$C_{\alpha,i}=\min_{a_{\alpha,i}}\left[\sum_{z=1}^{m_h}\int_0^{+\infty} e^{-\delta t}\left(\frac{1}{2}a_{h,z}(t)^2+\frac{1}{2}p_h\, s(t)+\frac{c}{m_h+m_l}\right)dt\right.$$
$$\left.+\sum_{z=1}^{m_l}\int_0^{+\infty} e^{-\delta t}\left(\frac{1}{2}a_{l,z}(t)^2+\frac{1}{2}p_l\, s(t)+\frac{c}{m_h+m_l}\right)dt\right]$$

for $\alpha = h$ for developed country and $\alpha = l$ for a developing one.

- In the second stage, taken as given the abatement levels of cooperators, defectors commit to a level of abatement that minimizes the discounted value of their costs:

$$C_{\alpha,j} = \min_{a_{\alpha,j}} \int_0^{+\infty} e^{-\delta t} \left(\frac{1}{2} a_{\alpha,j}(t)^2 + \frac{1}{2} p_\alpha \, s(t) + c \right) dt$$

  for $\alpha = h$ for developed country and $\alpha = l$ for a developing one.
- In both cases, countries face the same dynamic:

$$\dot{s}(t) = L - \sum_{i=1}^{m_h} a_{h,i} - \sum_{i=1}^{m_l} a_{l,i} - \sum_{j=m_h+1}^{n_h} a_{h,j} - \sum_{j=m_l+1}^{n_l} a_{l,j} - ks(t)$$

- The Hamilton-Jacobi-Bellman equation for cooperators is:

$$\delta V_{\alpha,i} = \max_{\{a_{\alpha,i}\}} \Bigg[-\sum_{z=1}^{m_h} \left(\frac{1}{2} a_{h,z}^2 + \frac{1}{2} p_h\, s + \frac{c}{m_h + m_l} \right) - \sum_{z=1}^{m_l} \left(\frac{1}{2} a_{l,z}^2 + \frac{1}{2} p_l\, s + \frac{c}{m_h + m_l} \right) + \\ + V'_{\alpha,i} \left(L - \sum_{i=1}^{m_h} a_{h,i} - \sum_{i=1}^{m_l} a_{l,i} - \sum_{j=m_h+1}^{n_h} a_{h,j} - \sum_{j=m_l+1}^{n_l} a_{l,j} - ks \right) \Bigg] \tag{2}$$

with $\alpha = h$ for a developed country and $\alpha = l$ for a developing one.

- The Hamilton-Jacobi-Bellman equation for defectors is:

$$\delta V_{\alpha,j} = \max_{\{a_{\alpha,j}\}} \left[- \left(\frac{1}{2} a_{\alpha,j}^2 + \frac{1}{2} p_\alpha \, s + c \right) + \right. \\ \left. + V'_{\alpha,j} \left(L - \sum_{i=1}^{m_h} a_{h,i} - \sum_{i=1}^{m_l} a_{l,i} - \sum_{j=m_h+1}^{n_h} a_{h,j} - \sum_{j=m_l+1}^{n_l} a_{l,j} - ks \right) \right] \tag{3}$$

with $\alpha = h, l$ for a developed and developing country. Moreover, $V_{\alpha,i}(s)$ and $V_{\alpha,j}(s)$ denote the optimal control value functions associated with the optimization problem, $V'_{\alpha,i}$ and $V'_{\alpha,j}$ are the first derivatives with respect to the state variable s.

The optimal value of the control variables must satisfy the first order conditions that are:

$$-a_{\alpha,i} - V'_{\alpha,i} = 0; \qquad -a_{\alpha,j} - V'_{\alpha,j} = 0; \qquad \text{for} \quad \alpha = h, l \quad (4)$$

Substituting these levels in the Hamiton-Jacobi-Bellman equations, we obtain the non linear differential equations. After some manipulations, the solutions give us the optimal abatement levels:

Proposition

The Feedback Nash equilibrium abatements are given by:

$$a_{h,i} = a_{l,i} = \frac{m_h p_h + m_l p_l}{2(\delta + k)} \quad (5)$$

for developed and developing cooperators;

$$a_{h,j} = \frac{p_h}{2(\delta + k)} \quad (6)$$

for developed defectors;

$$a_{l,j} = \frac{p_l}{2(\delta + k)} \quad (7)$$

for developing defectors.

Proposition

The optimal path for the state variable $s(t)$ is:

$$s(t) = s_0 e^{-kt} + \frac{A}{k}(1 - e^{-kt})$$

where

$$A = L - \frac{(m_l + m_h)(m_l p_l + m_h p_h)}{2(\delta + k)} - \frac{(n_h - m_h)p_h}{2(\delta + k)} - \frac{(n_l - m_l)p_l}{2(\delta + k)}$$

is the remaining pollution after the abatement.

• We will select coalitions $[m_h, m_l]$ that guarantee the positivity of parameter A so that the pollution stock $s(t)$ is positive.

Finally, the individual costs borne by each country are:

$$C_{h,i}[m_h, m_l] = \frac{p_h s_0}{2(\delta + k)} + \frac{(m_l p_l + m_h p_h)^2}{8\delta(\delta + k)^2} + \frac{p_h A}{2\delta(\delta + k)} + \frac{c}{\delta(m_h + m_l)};$$

$$C_{l,i}[m_h, m_l] = \frac{p_l s_0}{2(\delta + k)} + \frac{(m_l p_l + m_h p_h)^2}{8\delta(\delta + k)^2} + \frac{p_l A}{2\delta(\delta + k)} + \frac{c}{\delta(m_h + m_l)};$$

$$C_{h,j}[m_h, m_l] = \frac{p_h s_0}{2(\delta + k)} + \frac{p_h^2}{8\delta(\delta + k)^2} + \frac{p_h A}{2\delta(\delta + k)} + \frac{c}{\delta};$$

$$C_{l,j}[m_h, m_l] = \frac{p_l s_0}{2(\delta + k)} + \frac{p_l^2}{8\delta(\delta + k)^2} + \frac{p_l A}{2\delta(\delta + k)} + \frac{c}{\delta};$$

- One of the earliest definitions of a self-enforcing agreement used in the literature of IEAs was the stability concept proposed by D'Aspremont et al. (1983).
- A coalition of size m is internally stable if no member has an incentive to leave the coalition because the costs for an outsider in respect to a coalition of size $m-1$ are larger than the costs for a member of an m-sized coalition.
- External stability means that no country has an incentive to join a coalition of size m because the cost for a member of a coalition of size $m+1$ is higher than for an outsider to a coalition of size m .
- A coalition of size m is stable if it is both *internally* and externally stable.

In our case, because we have two kinds of countries, the internal and external stability become:

- $C_{h,i}[mh, ml] \leq C_{h,j}[mh-1, ml]; \quad C_{h,i}[mh+1, ml] \geq C_{h,j}[mh, ml]$ for a developed country;
- $C_{l,i}[mh, ml] \leq C_{l,j}[mh, ml-1]; \quad C_{l,i}[mh, ml+1] \geq C_{l,j}[mh, ml]$ for a developing country.

To check the stability conditions, we use the auxiliary functions:

$$\begin{aligned}
\Phi_1 &= C_{h,j}[m_h-1, m_l] - C_{h,i}[m_h, m_l];\\
\Phi_2 &= C_{h,i}[m_h+1, m_l] - C_{h,j}[m_h, m_l];\\
\Phi_3 &= C_{l,j}[m_h, m_l-1] - C_{l,i}[m_h, m_l];\\
\Phi_4 &= C_{l,i}[m_h, m_l+1] - C_{l,j}[m_h, m_l];
\end{aligned}$$

We can assert that a coalition of size $[m_h, m_l]$ is stable if functions $\Phi_z, \quad z = 1, \cdots, 4$ are simultaneously positive.

• We fix the values of enviromental awareness p_h and p_l and we consider as variables the R&D costs c and the number of cooperators m_h and m_l.

• We assume that:
$n = 10,\ k = 0.01,\ \delta = 0.04,\ L = 500,\ s_0 = 100$.

• We consider two cases:
a) in the first the environmental awareness are very close, i.e. $p_h = 1$ and $p_l = 0.80$ (Tables 1 and 2);
b) in the second the environmental awareness are faraway, i.e. $p_h = 1$ and $p_l = 0.10$ (Table 3).

We can observe that Φ_z are linear functions with respect to c for which it is possible to determine the unique solutions that $\Phi_z(c) = 0$. They are:

1) $\Phi_1(c) = 0 \iff c_1 = \frac{((m_h p_h + m_l p_l)^2 - 2 p_h m_l p_l + 3 p_h^2 - 4 p_h^2 m_h - 2 p_h^2 m_l)(m_h + m_l)}{8(\delta + k)^2 (m_h - 1 + m_l)}$;

2) $\Phi_2(c) = 0 \iff c_2 = \frac{((m_h p_h + m_l p_l)^2 - 2 p_h^2 (m_h + m_l))(m_h + m_l + 1)}{8(m_h + m_l)(\delta + k)^2}$;

3) $\Phi_3(c) = 0 \iff c_3 = \frac{((m_h p_h + m_l p_l)^2 - 2 p_l m_h p_h + 3 p_l^2 - 4 p_l^2 m_l - 2 p_l^2 m_h)(m_h + m_l)}{8(\delta + k)^2 (m_h - 1 + m_l)}$;

4) $\Phi_4(c) = 0 \iff c_4 = \frac{((m_h p_h + m_l p_l)^2 - 2 p_l^2 (m_h + m_l))(m_h + m_l + 1)}{8(m_h + m_l)(\delta + k)^2}$

• We observe that $c_4 > c_2$. Moreover, $\Phi_1(c)$ and $\Phi_3(c)$ are increasing functions, instead Φ_2 and Φ_4 are decreasing ones.

Proposition

If the constraint $A > 0$ is satisfied, a coalition of size $[m_h, m_l]$ is stable if

- $\max\{0, c_3\} \leq c \leq c_2$ *when* $c_1 < c_3 < c_2$*;*
- $\max\{0, c_1\} \leq c \leq c_2$ *when* $c_3 < c_1 < c_2$

otherwise, the coalition $[m_h, m_l]$ is not stable.

Proposition

If the constraint $A > 0$ is satisfied, the grand coalition $[n_h, n_l]$ is stable if

- $c > 0$ *when* $c_1 < 0; c_3 < 0$*;*
- $c \geq \max\{c_1, c_3\}$*, in the other cases.*

Costs	$n_h = 5$ $n_l = 5$	$n_h = 6$ $n_l = 4$	$n_h = 4$ $n_l = 6$	$n_h = 7$ $n_l = 3$	$n_h = 3$ $n_l = 7$	$n_h = 8$ $n_l = 2$	$n_h = 2$ $n_l = 8$	$n_h = 9$ $n_l = 1$	$n_h = 1$ $n_l = 9$
$c = 0$	[2, 0] [3, 0]	[2, 0] [3, 0]	[2, 0] [3, 0]	[2, 0] [3, 0]	[2, 0] [3, 0]	[2, 0] [3, 0]	[2, 0]	[2, 0]	
$c = 100$	[3, 0]	[3, 0]	[3, 0]	[3, 0]	[3, 0]	[3, 0]	[2, 0]	[3, 0]	[1, 2]
$c = 110$	[3, 0] [2, 1]	[3, 0] [2, 1]	[3, 0] [2, 1]	[3, 0] [2, 1]	[3, 0] [2, 1]	[3, 0] [2, 1]	[2, 1]	[3, 0] [2, 1]	[1, 2]
$c = 130$	[3, 0] [0, 4]	[3, 0] [0, 4]	[3, 0] [0, 4]	[3, 0]	[3, 0] [0, 4]	[3, 0]	[2, 1] [0, 4]	[3, 0]	[1, 2] [0, 4]
$c = 210$	[4, 0] [1, 3]	[4, 0] [1, 4] [1, 3]	[4, 0] [1, 3]	[4, 0] [1, 3]	[3, 0] [1, 3]	[4, 0]	[2, 1] [1, 3]	[4, 0]	[1, 3]
$c = 300$	[4, 0] [2, 2]	[4, 0] [2, 2] [1, 4]	[4, 0] [2, 2]	[4, 0] [2, 2]	[3, 0] [2, 2]	[4, 0] [2, 2]	[2, 2]	[4, 0]	[1, 3]
$c = 350$	[4, 0] [3, 1] [0, 5]	[4, 0] [3, 1] [1, 4]	[4, 0] [3, 1] [0, 5]	[4, 0] [3, 1]	[3, 1] [0, 5]	[4, 0] [3, 1]	[2, 2] [0, 5]	[4, 0] [3, 1]	[1, 3] [0, 5]
$c = 400$	[4, 0] [3, 1] [1, 4]	[4, 0] [3, 1] [1, 4]	[4, 0] [3, 1] [1, 4]	[4, 0] [3, 1]	[3, 1] [1, 4]	[4, 0] [3, 1]	[2, 2]	[4, 0] [3, 1]	[1, 3]
$c = 450$	[4, 0] [1, 4]	[4, 0] [1, 4]	[4, 0] [1, 4]	[4, 0]	[3, 1] [1, 4]	[4, 0]	[2, 2] [1, 4]	[4, 0]	[1, 4]
$c = 500$	[5, 0] [4, 0] [2, 3]	[5, 0] [4, 0] [2, 3] [2, 4]	[4, 0] [2, 3]	[5, 0] [4, 0] [2, 3]	[3, 1] [2, 3]	[5, 0] [4, 0]	[2, 3]	[5, 0] [4, 0]	[1, 4]
$c = 600$	[5, 0] [3, 2]	[5, 0] [3, 2] [2, 4]	[4, 0] [3, 2] [0, 6]	[5, 0] [3, 2]	[3, 2] [0, 6]	[5, 0] [3, 2]	[2, 3] [0, 6]	[5, 0]	[1, 4] [0, 6]
$c = 700$	[5, 0] [4, 1] [1, 5]	[5, 0] [4, 1] [2, 4]	[4, 1] [1, 5]	[4, 1] [5, 0]	[3, 2] [1, 5]	[5, 0] [4, 1]	[2, 3] [1, 5]	[5, 0] [4, 1]	[1, 5]

Table 1: Coalition Stability assuming $n = 10$; $L = 500$; $\delta = 0.04$; $k = 0.01$; $s_0 = 100; p_h = 1$ and $p_l = 0.80$

Costs	$n_h = 5$ $n_l = 5$	$n_h = 6$ $n_l = 4$	$n_h = 4$ $n_l = 6$	$n_h = 7$ $n_l = 3$	$n_h = 3$ $n_l = 7$	$n_h = 8$ $n_l = 2$	$n_h = 2$ $n_l = 8$	$n_h = 9$ $n_l = 1$	$n_h = 1$ $n_l = 9$
$c = 800$	$[5,0]$ $[2,4]$	$[5,0]$ $[2,4]$	$[4,1]$ $[3,3]$ $[2,4]$	$[5,0]$	$[3,2]$ $[2,4]$	$[5,0]$	$[2,4]$	$[5,0]$	$[1,5]$
$c = 900$	$[5,0]$ $[3,3]$	$[6,0]$ $[5,0]$ $[3,3]$ $[3,4]$	$[4,1]$ $[3,3]$	$[6,0]$ $[5,0]$ $[3,3]$	$[3,3]$ $[0,7]$	$[6,0]$ $[5,0]$	$[2,4]$ $[0,7]$	$[6,0]$ $[5,0]$	$[1,5]$ $[0,7]$
$c = 1\,000$	$[5,0]$ $[4,2]$ $[3,3]$	$[6,0]$ $[4,2]$ $[3,3]$ $[3,4]$	$[4,2]$ $[3,3]$	$[6,0]$ $[4,2]$ $[3,3]$	$[3,3]$	$[6,0]$ $[4,2]$	$[2,4]$	$[6,0]$	$[1,5]$
$c = 1\,100$	$[5,0]$ $[4,2]$	$[6,0]$ $[3,4]$	$[4,2]$ $[1,6]$	$[6,0]$ $[4,2]$	$[3,3]$ $[1,6]$	$[6,0]$ $[4,2]$	$[2,4]$ $[1,6]$	$[6,0]$	$[1,6]$
$c = 1\,200$	$[5,1]$ $[2,5]$	$[6,0]$ $[5,1]$ $[3,4]$	$[4,2]$ $[2,5]$	$[6,0]$ $[5,1]$	$[3,3]$ $[2,5]$	$[6,0]$ $[5,1]$	$[2,5]$	$[6,0]$ $[5,1]$	$[1,6]$
$c = 1\,300$	$[5,1]$ $[3,4]$	$[6,0]$ $[3,4]$	$[4,2]$ $[3,4]$	$[6,0]$	$[3,4]$	$[6,0]$	$[2,5]$	$[6,0]$	$[1,6]$
$c = 1\,400$	$[5,1]$ $[4,3]$	$[6,0]$ $[4,3]$	$[4,3]$	$[6,0]$ $[4,3]$	$[3,4]$	$[6,0]$	$[2,5]$	$[6,0]$	$[1,6]$
$c = 1\,600$	$[5,2]$	$[6,0]$ $[5,2]$	$[4,3]$	$[5,2]$	$[3,4]$	$[5,2]$			
$c = 1\,700$		$[6,1]$	$[4,3]$						
$c = 1\,900$		$[6,1]$							
$c = 2\,200$									

Table 2: Coalition Stability assuming $n = 10$; $L = 500$; $\delta = 0.04$; $k = 0.01$; $s_0 = 100$; $p_h = 1$ and $p_l = 0.80$

Costs	$n_h = 5$ $n_l = 5$	$n_h = 6$ $n_l = 4$	$n_h = 4$ $n_l = 6$	$n_h = 7$ $n_l = 3$	$n_h = 3$ $n_l = 7$	$n_h = 8$ $n_l = 2$	$n_h = 2$ $n_l = 8$	$n_h = 9$ $n_l = 1$	$n_h = 1$ $n_l = 9$
$c = 0$	[3,0] [2,0]	[3,0] [2,0]	[3,0] [2,0]	[3,0] [2,0]	[3,0] [2,0]	[3,0] [2,0]	[2,0]	[3,0] [2,0]	
$c = 90$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	[1,2]
$c = 100$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	[1,3]
$c = 110$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	[1,4]
$c = 120$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	[1,5]
$c = 130$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	[1,6]
$c = 140$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	[1,7]
$c = 160$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	[1,8]
$c = 180$	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]	[3,0]		[3,0]	**[1,9]**
$c = 295$	[4,0]	[4,0]	[4,0]	[4,0]	[3,0]	[4,0]	[2,2]	[4,0]	**[1,9]**
$c = 300$	[4,0]	[4,0]	[4,0]	[4,0]	[3,0]	[4,0]	[2,3]	[4,0]	**[1,9]**
$c = 320$	[4,0]	[4,0]	[4,0]	[4,0]	[3,0]	[4,0]	[2,4]	[4,0]	**[1,9]**
$c = 330$	[4,0]	[4,0]	[4,0]	[4,0]	[3,0]	[4,0]	[2,5]	[4,0]	**[1,9]**
$c = 320$	[4,0]	[4,0]	[4,0]	[4,0]	[3,0]	[4,0]	[2,6]	[4,0]	**[1,9]**
$c = 380$	[4,0]	[4,0]	[4,0]	[4,0]	[3,0]	[4,0]	[2,7]	[4,0]	**[1,9]**
$c = 400$	[4,0]	[4,0]	[4,0]	[4,0]	[3,0]	[4,0]	**[2,8]**	[4,0]	**[1,9]**
$c = 600$	[5,0]	[5,0]	[4,0]	[5,0]	[3,2]	[5,0]	**[2,8]**	[5,0]	**[1,9]**
$c = 610$	[5,0]	[5,0]	[4,0]	[5,0]	[3,3]	[5,0]	**[2,8]**	[5,0]	**[1,9]**
$c = 630$	[5,0]	[5,0]	[4,0]	[5,0]	[3,4]	[5,0]	**[2,8]**	[5,0]	**[1,9]**
$c = 670$	[5,0]	[5,0]	[4,0]	[5,0]	[3,5]	[5,0]	**[2,8]**	[5,0]	**[1,9]**
$c = 700$	[5,0]	[5,0]	[4,0]	[5,0]	[3,6]	[5,0]	**[2,8]**	[5,0]	**[1,9]**
$c = 800$	[5,0]	[5,0]	[4,0]	[5,0]	**[3,7]**	[5,0]	**[2,8]**	[5,0]	**[1,9]**
$c = 1\,000$	[5,0]	[6,0]	[4,1]	[6,0]	**[3,7]**	[6,0]	**[2,8]**	[6,0]	**[1,9]**
$c = 1\,010$	[5,0]	[6,0]	[4,2]	[6,0]	**[3,7]**	[6,0]	**[2,8]**	[6,0]	**[1,9]**
$c = 1\,030$	[5,0]	[6,0]	[4,3]	[6,0]	**[3,7]**	[6,0]	**[2,8]**	[6,0]	**[1,9]**
$c = 1\,050$	[5,0]	[6,0]	[4,4]	[6,0]	**[3,7]**	[6,0]	**[2,8]**	[6,0]	**[1,9]**
$c = 1\,100$	[5,0]	[6,0]	[4,5]	[6,0]	**[3,7]**	[6,0]	**[2,8]**	[6,0]	**[1,9]**
$c = 1\,200$	[5,0]	[6,0]	**[4,6]**	[6,0]	**[3,7]**	[6,0]	**[2,8]**	[6,0]	**[1,9]**
$c = 1\,500$	[5,1]	[6,0]	**[4,6]**	[7,0]	**[3,7]**		**[2,8]**		**[1,9]**
$c = 1\,520$	[5,2]	[6,0]	**[4,6]**	[7,0]	**[3,7]**		**[2,8]**		**[1,9]**
$c = 1\,550$	[5,3]	[6,0]	**[4,6]**	[7,0]	**[3,7]**		**[2,8]**		**[1,9]**
$c = 1\,600$	[5,4]	[6,0]	**[4,6]**	[7,0]	**[3,7]**		**[2,8]**		**[1,9]**
$c = 2\,100$		[6,1]	**[4,6]**	[7,0]	**[3,7]**		**[2,8]**		**[1,9]**
$c = 2\,150$		[6,2]	**[4,6]**	[7,0]	**[3,7]**		**[2,8]**		**[1,9]**
$c = 2\,800$			**[4,6]**		**[3,7]**		**[2,8]**		**[1,9]**

Table 3: Coalition Stability assuming $n = 10$; $L = 500$; $\delta = 0.04$; $k = 0.01$; $s_0 = 100$; $p_h = 1$ and $p_l = 0.10$

- We observe that the grand coalition $[n_h, n_l]$ is stable if it is verified the internal stability for developed and developing countries:

$$C_{h,i}[n_h, n_l] \leq C_{h,j}[n_h - 1, n_l]; \quad C_{l,i}[n_h, n_l] \leq C_{l,j}[n_h, n_l - 1];$$

- Moreover, the following condition:

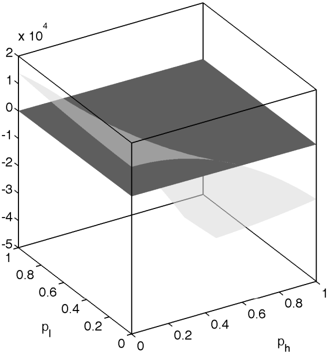
$$C_{h,j}[n_h - 1, n_l] - C_{h,i}[n_h, n_l] > C_{l,j}[n_h, n_l - 1] - C_{l,i}[n_h, n_l]$$

is satisfied if and only if $p_h < -\dfrac{p_l(4n_l + 2n_h - 3)}{4n_h + 2n_l - 3} \vee p_h > p_l$.

- As $-\frac{p_l(4n_l+2n_h-3)}{4n_h+2n_l-3} < 0$ and $p_h > p_l$, then the occurrence of the internal stability for developing countries implies that the internal stability of developed ones holds too.

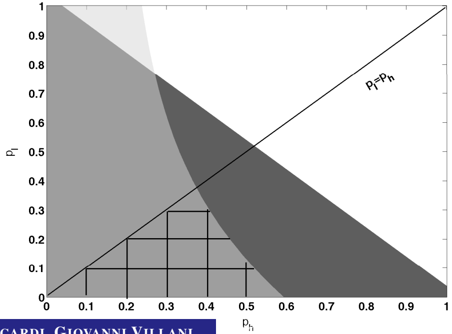
• We propose a numerical simulation in which $n_h = 5, n_l = 5$ and the R&D costs $c = 500$. The other parameters are $L = 500; \delta = 0.04; k = 0.01; s_0 = 100$. The variables are p_h and p_l.

• We are interested to the (p_h, p_l) values that guarantee the internal stability of developing and that satisfied the constraint A.



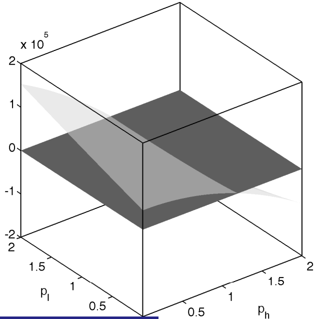
- The white surface represents the quantity $C_{l,j}[n_h, n_l - 1] - C_{l,i}[n_h, n_l]$, while the black surface denotes the constraint $A = 500 - 500p_h - 500p_l$.

This figure shows a projection of 3D graphic on the (p_h, p_l) plane, in which the constraint $A \geq 0$ and the internal stability condition $C_{l,j}[5,4] - C_{l,i}[5,5] > 0$ are both verified.
The grand coalition is stable for the (p_h, p_l) values which are in the squared region.



• We propose another numerical simulation increasing the R&D investment $c = 2\,500$.

• Working in the same manner of previous case, the squared region represents the p_h and p_l values for which the grand coalition is stable.



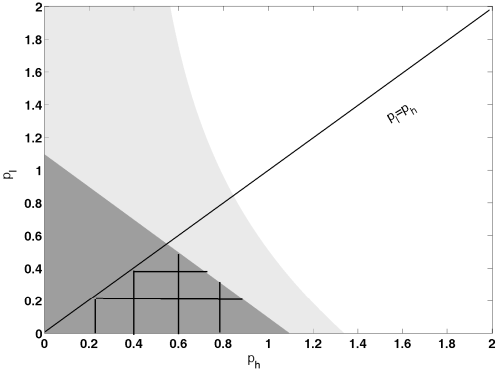


Figura: Projection 2D

- Comparing the two results, when the R&D costs are low, then p_h and p_l values that guarantee the stability of the grand coalition are identified by the internal stability condition $C_{l,j}[5,4] - C_{l,i}[5,5] > 0$ and by the constraint $p_l < p_h$.
- Otherwise, when the costs c are high, then the region that characterizes the stability of the grand coalition is determined by the constraints $A = 500 - 500p_h - 500p_l > 0$ and $p_l < p_h$.

• In the plane (p_h, p_l), the intersection between the lines $A = 0$ and $p_l = 0$ is:

$$p_h^* = \frac{2L(\delta + k)}{(n_h + n_l)n_h}$$

• Instead, the intersection between the curve $C_{l,j}[n_h, n_l - 1] - C_{l,i}[n_h, n_l] = 0$ and the line $p_l = 0$ is:

$$p_h' = \frac{2(\delta + k)\sqrt{2(n_h + n_l)c(n_h - 1 + n_l)}}{(n_h + n_l)n_h}$$

Proposition

When the R&D investment cost $c < \frac{L^2}{2(n_h+n_l)(n_h+n_l-1)}$*, then it results that* $p'_h < p^*_h$*, and so the stability of the grand coalition is determined by the internal stability condition represented by the curve* $C_{l,j}[n_h, n_l - 1] - C_{l,i}[n_h, n_l] = 0$*.*
Instead, when $c > \frac{L^2}{2(n_h+n_l)(n_h+n_l-1)}$*, then we have that* $p'_h > p^*_h$*, and the stability of the grand coalition is determined by the constraint* $A = 0$

- When the R&D investment c increases then the number of cooperators raises;
- If the environmental awareness of two type of countries is faraway, when the R&D investment c increases, the number of developed countries in the stable agreements goes up too and, only when all the developed countries are in the coalition, also the developing countries sign the agreement.
- We have analyzed conditions that guarantee the stability of the Grand Coalition.