MDEF2014

8th Workshop MDEF
Modelli Dinamici in Economia e Finanza
Dynamic Models in Economics and Finance

Dynamics of heterogeneous oligopolies with best response mechanisms

F. Cavalli, A. Naimzada, M. Pireddu

Department of Economics, Management and Statistics,
Università di Milano-Bicocca

Urbino, September 18-20, 2014

GOAL

Study dynamic models for cournotian oligopolies of generic size N, in which the firms are heterogeneous in terms of the behavioral rules, namely at least two firms adopt different adjustment mechanisms.

Each firm can adopt a behavioral rule, selected from a set of *two* different rules.
The way each rule is chosen gives rise to two possible frameworks:

- the rules are exogenously chosen and firms do not change their rule(**Fixed fractions**)
- the firms can change their rule accordingly to some criteria (**Evolutionary fractions**)

The rules we focus on are based on best response mechanisms and differentiate because of the rationality degree of agents.

Behavioral rules

Best Response mechanisms

We consider **best response** mechanisms with **different rationality degrees**

Rational (R) player

- full informational and computational capabilities
- complete knowledge of economic setting (demand and cost functions)
- endowed with *perfect foresight* of other players strategies
- able to optimally respond to the other players strategies

Best Response (BR) player

- complete knowledge of economic setting (demand and cost functions)
- NOT endowed with perfect foresight, *static expectation*
- able to optimally respond to the other players (expected) strategies

Local Monopolistic Approximation (LMA) player

- incomplete knowledge of economic setting (market price p_t, the produced quantity Q_t, local knowledge of the demand function in (p_t, Q_t))
- conjecture a demand function (local linear approximation), solve optimization

Homogeneous oligopolies

All firms adopt the *same decisional rule*.
Several works focus on stability thresholds with respect to oligopoly size

- Linear demand function: Palander (1939), Theocharis (1959), Canovas et al (2008).
- Isoelastic demand function: Puu (1991), Lampart (2012).
- LMA adjustment: Bischi et al.(2007) and Naimzada and Tramontana (2009).

Common behavior: **increasing** oligopoly **size** leads to **instability**. LMA is "more stable" than Best Response.

Heterogeneous oligopolies

Several couplings of different adjustment mechanisms for **duopolistic** markets: Agiza and Elsadany (2003,2004), Angelini et. al (2009), Tramontana(2010), C. and Naimzada (2014).
Droste et al. (2002) (linear demand function, no oligopoly size, only evolutionary fractions), Hommes et al. (2011) (linear demand function), Bischi et al. (2014)

Questions

Oligopoly size N
Oligopoly composition ω

Does increasing N always lead to instability?

How local stability is affected by ω?

Have the most rational behavioral rules always a stabilizing effect?

Framework

Economic setting

Isoelastic (inverse) demand function (Cobb-Douglas preferences)

$$p(Q) = \frac{1}{Q}$$

Constant marginal costs c_i:

$$C(q_i) = c_i q_i$$

Identical marginal costs for firms adopting the **same rule**

Model

- We compute the best response of each player, depending on his rationality degree
- We consider the 1D/2D discrete dynamical system obtained coupling the decisional rules of two different generic players.
- We study the models for continuous parameters (ω, N) and we focus on results for economically significant discrete values.

Behavioral rules

Game

Set a game in which the N players are divided into two sets F_i with

$$\#F_1 = \omega N, \qquad \#F_2 = (1-\omega)N$$

We remark that we are in **heterogeneous oligopolies**, so each rule is adopted by at least a firm $\omega = k/N$, with $k = 1, \dots, N-1$.

The rules have to be different.

We assume that

- F_1 players are the most rational (R/BR),
- F_2 players are the least rational (BR/LMA).
- players belonging to the same set are identical (for BR and LMA players, this means that they share the initial strategy). Hence, they have the same strategies.

Behavioral rules

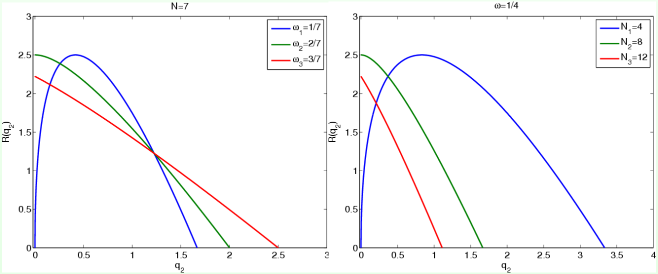
Generic R player

→ Compute the best response to the (correctly foreseen) strategies at time $t+1$ of remaining R players and F_2 players

→ The strategies of R players are identical: compute a (pseudo) best response to the (correctly foreseen) strategies at time $t+1$ of F_2 players

$$q_1^t = R_\omega(q_2^t) = \max\left\{ \frac{-2c_1\omega(1-\omega)N^2 q_2^t + (\omega N - 1) + \sqrt{\Delta(q_2^t,\omega)}}{2c_1\omega^2 N^2}, 0 \right\}$$

where $\Delta(q_2^t,\omega) = (\omega N - 1)^2 + 4c_1\omega(1-\omega)N^2 q_2^t$.



Behavioral rules

Generic LMA player:

Approximated best response depends on own LMA player strategy q_t^t and on aggregated strategy Q^t

$$q_2^{t+1} = L_\omega(q_2^t, Q^t) = \max\left\{\frac{1}{2}q_2^t + \frac{1}{2}\left(1 - c_2 Q^t\right)Q^t, 0\right\}.$$

Generic BR player

Classical best response to the others' expected aggregated strategy Q_{-i}^t (static expectations) $i = 1, 2$

$$q_i^{t+1} = B_\omega(Q_{-i}^t) = \max\left\{\sqrt{\frac{Q_{-i}^t}{c_i}} - Q_{-i}^t 0\right\},$$

First model: Rational vs. LMA

One dimensional model

$$q_2^{t+1} = \max\left\{\frac{1}{2}q_2^t + \frac{1}{2}\left(1 - c_2 Q^t\right)Q^t, 0\right\},$$

where $Q^t = \omega N R_\omega(q_2^t) + (1-\omega) N q_2^t$.

We focus on identical marginal costs $c = c_1 = c_2$

Proposition

The Nash equilibrium is the only positive steady state.

Proposition

If $N \leq 4$, the Nash equilibrium is stable for all $\omega \in [0, 1]$. For $N \geq 5$, stability requires

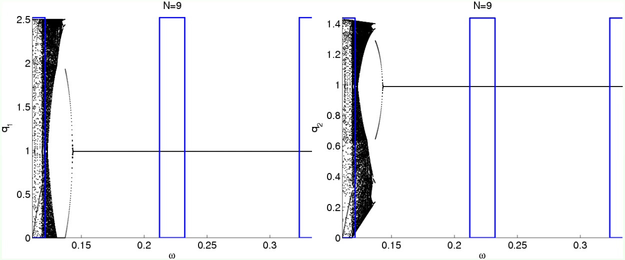
$$1 - \frac{3}{4(N-2)} < \omega \leq 1.$$

Corollary: discrete values

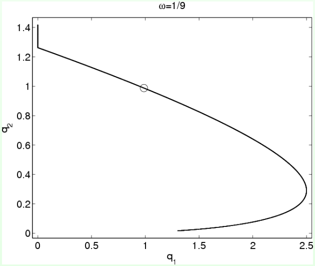
- *Equilibrium is stable provided that oligopoly has a sufficient number of R players ($f(N) \leq \omega$).*
- *For $N = 2, 3, 4$ all the compositions are stable (actually those homogeneous).*
- *For $N = 5, 6, 7$ all compositions are stable (in this case only those heterogeneous).*
- *For $N \geq 8$ only compositions with $\omega > 1/4$ are stable.*
- *For a fixed composition, increasing N can be either neutral or destabilizing.*
- *Adding R players leads to stability, adding LMA players leads to instability.*

Rational vs. LMA

Bifurcation diagrams ($c = 0.1$)



Attractor



Second model: Best Response vs. LMA

Two dimensional system with inertial mechanism (inertia α_i)

$$\begin{cases} q_1^{t+1} = q_1^t + \alpha_{BR}\left(\sqrt{\dfrac{Q_{-1}^t}{c_1}} - q_1^t\right) - Q_{-1}^t, \\ q_2^{t+1} = q_2^t + \alpha_{LMA}\left(\dfrac{1}{2}q_2^t + \dfrac{1}{2}\left(1 - c_2 Q^t\right)Q^t - q_2^t\right) \end{cases}$$

where $Q_{-1}^t = (\omega N - 1)q_1^t + (1-\omega)Nq_2^t$ and $Q^t = \omega N q_1^t + (1-\omega)Nq_2^t$

We focus on identical marginal costs $c = c_1 = c_2$.
Inertia has to be considered, otherwise only for small oligopolies ($N < 5$) equilibrium can be stable.

Proposition

The Nash equilibrium is the only positive steady state.

Best Response vs. LMA

Results

Proposition

For $N > 2$, let us define

$$\tilde{\omega} = -\frac{(4N - 4 - N\alpha_{BR})(\alpha_{LMA} - N\alpha_{LMA} + 4)}{4(N-2)(N\alpha_{LMA} - \alpha_{LMA} - N\alpha_{BR})}.$$

Then, setting $\hat{\alpha}_{BR} = 4/N$ and $\hat{\alpha}_{LMA} = 4/(N-1)$, we have

- *E^* is stable $\forall \omega \in (0,1) \Leftrightarrow$*

$$\begin{cases} N < 5, \\ \alpha_i \in (0,1], \end{cases} \quad \text{or} \quad \begin{cases} N \geq 5, \\ \alpha_{BR} \in (0, \hat{\alpha}_{BR}], \alpha_{LMA} \in (0, \hat{\alpha}_{LMA}] \\ (\alpha_{BR}, \alpha_{LMA}) \neq (\hat{\alpha}_{BR}, \hat{\alpha}_{LMA}). \end{cases}$$

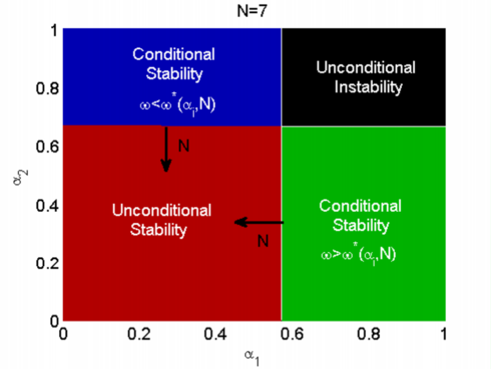
- *E^* is unstable $\forall \omega \in (0,1) \Leftrightarrow N \geq 5$ and $\alpha_{BR} \in [\hat{\alpha}_{BR}, 1], \alpha_{LMA}[\hat{\alpha}_{LMA}, 1]$.*
- *E^* is conditionally stable on ω for*

$$\omega \in (0, \tilde{\omega}) \Leftrightarrow \begin{cases} N \geq 5, \\ \alpha_{BR} \in (\hat{\alpha}_{BR}, 1], \\ \alpha_{LMA} \in (0, \hat{\alpha}_{LMA}), \end{cases} \qquad \omega \in (\tilde{\omega}, 1) \Leftrightarrow \begin{cases} N \geq 5, \\ \alpha_{BR} \in (0, \hat{\alpha}_{BR}), \\ \alpha_{LMA} \in (\hat{\alpha}_{LMA}, 1]. \end{cases}$$

Best Response vs. LMA

Results

Looking at stability bounds, several situations are possible:

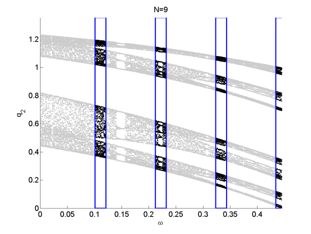


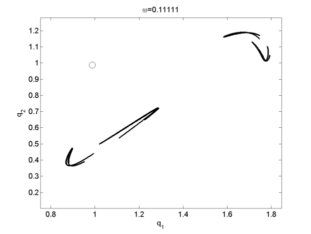
We have both scenarios of LMA stabilizing players and BR stabilizing players.
The constraint on α_{BR} is more severe than that on α_{LMA}.
If $\alpha_{BR} = \alpha_{LMA}$, adding LMA players improves stability

Best Response vs. LMA

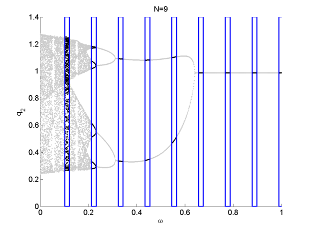
Simulations

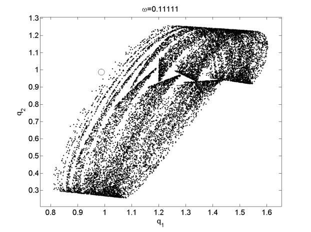
Unconditionally unstable scenario ($\alpha_{BR} = 0.556, \alpha_{LMA} = 0.65, c = 0.1$)



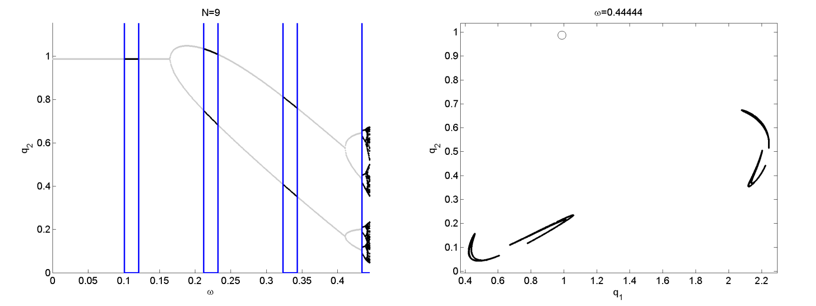


BR Stabilizing scenario ($\alpha_{BR} = 0.3344, \alpha_{LMA} = 0.7, c = 0.1$)





LMA Stabilizing scenario ($\alpha_{BR} = 0.86, \alpha_{LMA} = 0.39, c = 0.1$)



Third model: Rational vs. Best Response

One dimensional model

$$q_2^{t+1} = \max\left\{ \sqrt{\frac{Q_{-2}^t}{c_2}} - Q_{-2}^t, 0 \right\},$$

where $Q_{-2}^t = \omega N R_\omega(q_2^t) + (1-\omega)(N-1)q_2^t$

We consider **different marginal costs**, we focus on $c_1 \geq c_2$

Proposition

The only positive steady state is the Nash Equilibrium

$$q_1^* = \frac{(c_1 + N(1-\omega)(c_2 - c_1))(N-1)}{N^2(c_2(1-\omega)+c_1\omega)^2},\ q_2^* = \frac{(c_1 N\omega - c_2(N\omega - 1))(N-1)}{N^2(c_2(1-\omega)+c_1\omega)^2}.$$

Rational vs. Best Response

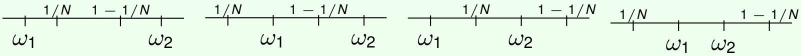
Proposition

Let

$$\omega_{1,2} = \frac{c_2\left(3c_1N - 2c_1 - c_2N - 2c_2 \pm \sqrt{2\widetilde{\Delta}}\right)}{2c_1c_2N + c_1^2N - 3c_2^2N},$$

where $\widetilde{\Delta} = c_2^2N^2 + 2c_1^2N^2 + c_1^2 + c_2^2 - 2c_1^2N - 2c_1c_2N^2 + 2c_1c_2 - 2c_2^2N$. *Then equilibrium is stable provided that* $\omega \in (\omega_1, \omega_2)$.

With respect to the R player fraction, four possible scenarios arise



Neutrally stable | Stabilizing | Destabilizing | Mixed

REMARK : LINEAR DEMAND FUNCTION ONLY GIVES RISE TO STABILIZING SCENARIO

Rational vs. Best Response revisited

Negativity issue: when system loses stability, interesting dynamics give rise to negative trajectories.

Improved model

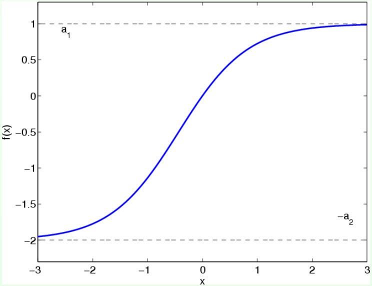
One dimensional model

$$q_2^{t+1} = q_2^t + f(\gamma(BR(Q_{-2}^t) - q_2^t))$$

where f is an increasing, sign preserving, bounded function and γ is the reaction speed of the BR agents.

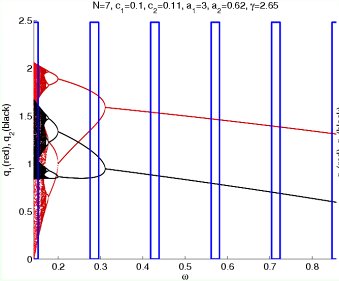
Example: sigmoid function

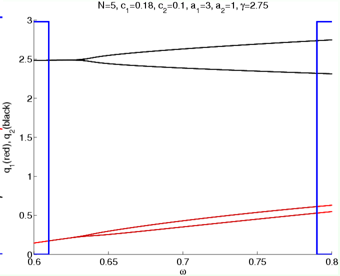
$$f(x) = a_2 \left(\frac{a_1 + a_2}{a_2 + a_1 \exp(-x)} - 1 \right)$$

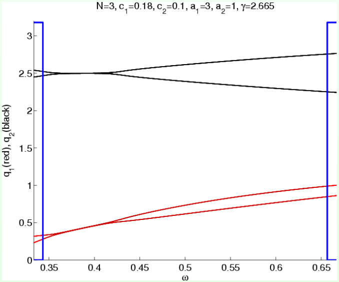


Rational vs. Best Response revisited

Simulations







Answers

Oligopoly size N
Oligopoly composition ω

Does increasing N always lead to instability?

No (Suitable R vs. LMA compositions are stable for all N)

How local stability is affected by ω?

Both stabilizing and destabilizing (Example of BR vs. LMA)

Have the most rational behavioral rules always a stabilizing effect?

It seems that different possible scenarios occur, improve investigation (Example of R vs. BR)

Thanks!