Testing for Non-Linear Structure in

an Artificial Financial Market

by Shu-Heng Chen, Thomas Lux and Michele Marchesi

Abstract: We present a stochastic simulation model of a prototype financial market. Our market is populated by both noise traders and fundamentalist speculators. The dynamics covers switches in the prevailing mood among noise traders (optimistic or pessimistic) as well as switches of agents between the noise trader and fundamentalist group in response to observed differences in profits. The particular behavioral variant adopted by an agent also determines his decision to enter on the long or short side of the market. Short-run imbalances between demand and supply lead to price adjustments by a market maker or auctioneer in the usual Walrasian manner. Our interest in this paper is in exploring the behavior of the model when testing for the presence of chaos or non-linearity in the simulated data. As it turns out, attempts to determine the fractal dimension of the underlying process give unsatisfactory results in that we experience a lack of convergence of the estimate. Explicit tests for non-linearity and dependence (the BDS and Kaplan tests) also give very unstable results in that both acceptance and strong rejection of IIDness can be found in different realizations of our model. All in all, this behavior is very similar to experience collected with empirical data and our results may point towards an explanation of why robustness of inference in this area is low. However, when testing for dependence in second moments and estimating GARCH models, the results appear much more robust and the chosen GARCH specification closely resembles the typical outcome of empirical studies.

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1. Introduction

More than one decade ago, first applications of empirical methods from chaos theory raised the hope of detecting low-dimensional chaotic motion in financial data (cf., for example, Scheinkman and LeBaron, 1989; Frank and Stengos, 1989; or Medio and Gallo, 1992). However, these early positive results were questioned a few years later by other authors (for example, Ruelle, 1990; Gilmore, 1993). By now, a certain consensus seems to have emerged that the search for low-dimensional chaos has not been successful. However, experience also shows that the null hypotheses of either linearity or IIDness are often rejected with financial data. Furthermore, one also knows that much of the deviations from IIDness stems from the volatility dynamics and can be captured to some degree by GARCH time series models.

In this paper, we take the findings reported above as stylized facts of financial data and ask whether model-generated data from an 'artificial' market could reproduce these features. This continues the line of research of Lux and Marchesi (1999, 2000) who show that their artificial financial market generates time series of prices and returns sharing some even more elementary stylized facts of empirical data: both the presence of a unit root in the asset price dynamics as well as heteroscedasticity and leptokurtosis of returns can be found in simulations of the model. These results are even in good quantitative agreement with empirical findings: as with almost all real-life data, the tails of the distribution of returns (r_t) are characterized by power-law behavior, i.e. $F(|r_t| > x) \approx c \cdot x^{-\mu}$, with a 'tail index' μ in the range of about 2 to 4. Furthermore, both squared and absolute returns seem to exhibit long-term dependence, i.e. a slow (hyperbolic) decline of the autocorrelation function, with a realistic magnitude of the relevant statistics. At the same time, raw returns have only small degrees of autocorrelation which implies that the (artificial) market appears rather *efficient* on first sight since price increments are almost uncorrelated.

When testing for chaos and nonlinearity with simulated time series from the Lux/Marchesi framework in this paper, our results will turn out to conform to empirical behavior in even greater detail. The plan of the remainder is as follows: the next section reviews the basic building blocks of the model and explains what kind of mechanism leads to its interesting dynamics. Section 3 reports and evaluates the results of various statistical procedures. Section 4 provides concluding remarks.

2. The Artificial Financial Market

Among the various recent approaches towards dynamic behavioral modeling of financial markets (for example, Day and Huang, 1990; Kirman, 1991; Brock and LeBaron, 1996; or

Arthur *et al*., 1997) the characteristic feature of the model presented in Lux and Marchesi (1999, 2000) is its use of a mass-statistical approach which only considers a few key behavioral variants and formalizes agents' switching between these alternatives in a stochastic manner. Three groups of agents are considered in the model: first, the fixed number of traders in the market (N) is split up into the camps of noise traders and fundamentalists with $n_n(t)$ and $n_f(t)$ denoting the (time-varying) numbers of agents in both groups ($n_n + n_f = N$). Second, the noise trader group itself consists of optimistic and pessimistic individuals whose numbers are given by $n_+(t)$ and $n_-(t)$ with $n_+(t) + n_-(t) = n_n$. Given this classification of behavioral variants, the dynamics is encapsulated in six *transition probabilities* for changes between groups.

First, the probabilities of switches of agents from the pessimistic to the optimistic subgroup and *vice versa* during a small time increment Δt are denoted by $\pi_{+-}\Delta t$ and $\pi_{-+}\Delta t$ where π_{+-} and π_{-+} are concretized as follows:

$$(1) \quad \pi_{+-} = v_1 \frac{n_n}{N} \exp(U_1) \ , \ \pi_{-+} = v_1 \frac{n_n}{N} \exp(-U_1) \, , \quad U_1 = \alpha_1 x + \frac{\alpha_2}{v_1} \frac{dp/dt}{p} \, .$$

Here, the basic influences on the noise traders' formation of opinion are the majority opinion of their fellow traders, $x = \frac{n_+ - n_-}{n_c}$, and the actual price trend, $\frac{dp/dt}{p}$. The first component may be seen as a short-hand reflecting herd behavior or the attempt to trace out underlying information from the behavior of others. The second component may be interpreted as being representative of trend following practices. Parameters v_1, α_1, and α_2 are measures of the frequency of revaluation of opinion and the importance of majority opinion and trend, respectively. The transition probabilities are multiplied by the actual fraction of chartists (that means, potential transitions are restricted to such a fraction) because we will also allow interaction with fundamentalist traders in the next step.

Switching from the noise trader to the fundamentalist group and vice versa is formalized in a similar manner. The notational convention in the transition probabilities below is again that the first index denotes the subgroup to which a trader moves who had changed his mind and the second index gives the subgroup to which he formerly belonged (hence, as an example, $\pi_{+f}\,\Delta t$ is the probability for a fundamentalist to switch to the optimistic noise traders' group within a small time interval Δt):

$$\pi_{+f} = v_2 \frac{n_+}{N} \exp(U_{2,1}),\; \pi_{f+} = v_2 \frac{n_f}{N} \exp(-U_{2,1})$$

(2)

$$\pi_{-f} = v_2 \frac{n_f}{N} \exp(U_{2,2}),\; \pi_{f-} = v_2 \frac{n_f}{N} \exp(-U_{2,2}).$$

The forcing terms $U_{2,1}$ and $U_{2,2}$ for these transitions depend on the difference between the (momentary) profits earned by noise traders and fundamentalists:

$$U_{2,1} = \alpha_3 \{ \underbrace{\frac{r + \frac{1}{v_2}\frac{dp}{dt}}{p} - R}_{\substack{\text{profit of noise} \\ \text{traders from } n_+ \text{ group}}} - \underbrace{s \cdot \left| \frac{p_f - p}{p} \right|}_{\text{fundamentalists' profit}} \}$$

(3)

$$U_{2,2} = \alpha_3 \{ \underbrace{R - \frac{r + \frac{1}{v_2}\frac{dp}{dt}}{p}}_{\substack{\text{profits of noise} \\ \text{traders from } n_- \text{ group}}} - \underbrace{s \cdot \left| \frac{p_f - p}{p} \right|}_{\text{fundamentalists' profit}} \}.$$

Profits enjoyed by noise traders from the optimistic group (who are buyers and, thus, increase the fraction of the asset in their portfolio) are composed of nominal dividends (r) and capital gains due to the price change (dp/dt). Dividing by the actual market price gives the revenue per unit of the asset. Excess returns are computed by subtracting the average real risk-adjusted return (R) available from other investments.[1] Fundamentalists, on the other hand, consider the deviation between price and fundamental value p_f (irrespective of its sign) as the source of arbitrage opportunities. As the gains from arbitrage occur only in the future (and depend on the uncertain time for reversal to the fundamental value) the latter are discounted by a factor $s < 1$. Furthermore, neglecting the dividend term in fundamentalists' profits is justified by assuming that they correctly perceive the (long-term) real returns to be equal to the average return of alternative investments (i.e. $r/p_f = R$) so that the only source of excess profits in their view is arbitrage when prices are 'wrong' ($p \neq p_f$).

As concerns the second U-function, $U_{2,2}$, we consider profits from the viewpoint of pessimistic noise traders who in order to avoid losses will rush out of the market and sell the asset under consideration. Their fall-back position by acquiring other assets is given by the

[1] Usually, one would think of R as a risk-free rate. However, as our model lacks risk aversion on the part of speculators, we have no basis for computation of a risk premium. An extension of the model to the case of risk-averse traders is on our agenda. In such a framework, one would hope to be able to account for a positive, time-varying risk premium as well as for leverage effects (dependence of volatility on the sign of returns) which so far the model is unable to produce.

average return R which they compare with nominal dividends plus price change (which, when negative, amounts to a capital *loss*) of the asset they sell. This explains why the first two items in the forcing term are interchanged when proceeding from $U_{2,1}$ to $U_{2,2}$.

Lastly, the dynamics of the asset's price results from the market operations of our agents and the ensuing price adjustment by a market maker who reacts on imbalances between demand and supply. With optimistic (pessimistic) noise traders entering on the demand (supply) side of the market, excess demand within this group depends on the number of individuals in both groups. Assuming a constant average trading volume per transaction, t_n, this amounts to: $ED_n = (n_+ - n_-)\, t_n$. Excess demand of fundamentalists, on the other hand, typically obeys a law of the type: $ED_f = n_f \cdot \gamma \frac{p_f - p}{p}$, γ being a parameter for the strength of reaction on differences between p and p_f. In order to conform with the general structure of our framework, we also formalize the price adjustment process in terms of (Poisson) transition probabilities. As a stochastic version of the standard Walrasian adjustment we use the following probabilities for the price to increase (decrease) by a small percentage $\Delta p = \pm\, 0.001$ p during a time increment Δt:[2]

(4) $\pi_{\uparrow p} = \max[0, ß(ED+\mu)]$, $\pi_{\downarrow p} = -\min[ß(ED+\mu), 0]$, $ED = ED_f + ED_n$,

where ß is a parameter for the price adjustment speed and μ is a small random component which is added to the speculators' excess demand.

Note that using Poisson transition probabilities for all dynamic processes, we have formulated a continuous-time model with *asynchronous* changes of behavior. In our simulations, updating of p and the number of individuals in the various subgroups is performed with sufficiently small time increments (ranging in a flexible manner between Δt = 0.002 and Δt = 0.01) in order to achieve a close approximation to the underlying continuous process. For the statistical analyses, the realized sample paths are extracted at integer time steps.

As a benchmark for the analysis of the resulting price dynamics, we introduce an exogenous *news arrival process*. Our assumption here is that the log of the fundamental value follows a Wiener process and, hence,

(5) $\ln(p_{f,t}) = \ln(p_{f,t-\Delta t}) + \varepsilon_t \Delta t$

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[2] The increment Δp has been chosen as small as possible in order to avoid artificial lumpiness of price changes with concentration of the distribution of returns at a few values only.

with increments ε_t identically and independently distributed according to a Normal distribution with mean zero and (time-invariant) variance σ_ε^2. This specification ensures that neither fat tails nor volatility clustering nor any kind of non-linear dependence are brought about by the exogenous news arrival process.[3] Hence, emergence of these characteristics in market prices would *not* be driven by similar characteristics of the news, but would rather have to be attributed to the trading process itself. In fact, the major finding from our earlier work on this artificial market (Lux and Marchesi, 1999, 2000) is that the trading process itself generates realistic dynamics of asset returns, i.e., market interactions of agents *magnify and transform* exogenous noise (news) into fat tailed returns with clustered volatility (cf. Fig. 2 for an example of the resulting dynamics of returns).

Lux and Marchesi (2000) provide a theoretical analysis of the mean-value dynamics of the model that allows to gain some insights into the origin of this dynamic behavior. They show that the above system is characterized by a continuum of equilibria[4] with a market price which (on average) equals the fundamental value, balanced disposition among noise traders, and indeterminate fraction of agents within the noise trader and fundamentalist group. The reason or this indeterminacy can be understood by taking into account that neither group has any advantage in a situation where no arbitrage opportunities exist ($p = p_f$) and no deviations from the equilibrium price are expected (which amounts to $dp/dt = 0$). This implies, that switches of individual agents between groups become random in the neighborhood of an equilibrium, so that the system moves in an erratic manner along its continuum of equilibria. The relevant equilibrium 'selected' in any period, then, depends on the whole history of the process.[5]

Another theoretical result is that *stability* of an equilibrium depends on the fraction of noise traders among agents. A critical value for the fraction of chartists can be computed that separates the region of stable and unstable equilibria. When the configuration comes close to this critical point, volatility increases due to destabilizing reactions of the now larger chartist group. However, these destabilizing forces are kept in check by a tendency of agents to switch

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[3] Another stylized fact, unit root behavior, is, in fact, shared by both the fundamental value and market prices. It seems to us, however, that this is not an entirely trivial consequence of our assumptions as the price dynamics follows a different, much more complicated process than fundamentals. In fact, as shown in Lux and Marchesi (2000), one can get non-rejection of unit roots from the price process even with stationary fundamentals.

[4] More precisely, these are the equilibria of a first-order approximation to the time development of mean values of the relevant variables (cf. Lux, 1995; 1997; 1998). In the original stochastic system, these equilibria are stationary in the sense that the systematic factors in the transition probabilities vanish (all U-functions in eq. 1 to 3 are identical zero as is excess demand) and the dynamics is confined to stochastic fluctuations.

[5] In out-of-equilibrium episodes, however, one finds more systematic motions between groups. There one also finds changing majorities of optimistic or pessimistic noise traders. Note also that with a price close to the fundamental value, balanced disposition of noise traders implies that their excess demand is close to a random process with mean zero. Hence, as long as no bubbles built up our noise traders do essentially behave like the 'liquidity traders' of standard market microstructure models.

back to a fundamentalist behavior in the presence of large deviations between price and fundamental value. The resulting decline of the number of noise traders, then, leads to reduction of the amplitude of the fluctuations again. As a result, destabilization is only a temporal phenomenon which, nevertheless, occurs repeatedly in the course of the market's development. Note that for the fraction of noise traders this stabilizing mechanism amounts to some kind of re-injection (this is similar to, but more complicated than a random walk with a reflecting boundary).

As this temporary destabilization does not lead to lasting deviations from fundamental valuation, the resulting picture still seems to be consistent with *efficiency* of the price formation process, but the market can also be characterized by a certain *fragility* with a tendency towards 'unnecessarily' large fluctuations and alternation between tranquil and turbulent periods. This behavior resembles a phenomenon called on-off intermittency in natural science (cf. Heagy *et al*. 1994).[6]

3. Testing for Non-Linear Structure in the Simulated Data

We now proceed with the statistical analyses of our model-generated data. The parameter values used for simulations are the same as in Lux and Marchesi (1999):

$N = 500$, $v_1 = 2$, $v_2 = 0.6$, $ß = 4$, $t_n = 0.001$, $\gamma = 0.01$, $\alpha_1 = 0.6$, $\alpha_2 = 1.5$, $\alpha_3 = 1$,
$R = 0.0004$ ($r = R\ p_f$), $s = 0.75$,

The random variables μ and ε are assumed to follow Normal distributions with mean zero and standard deviations $\sigma_\varepsilon = 0.005$ and $\sigma_\mu = 0.05$.

'Fine-tuning' of the model's parameters was not necessary in order to get 'realistic' statistical attributes. What we did in choosing the parameters was some scale adjustment in order to get an interval of price changes which fits with empirical observations in industrialized economies (with absolute returns over unit time intervals not exceeding 0.2 to 0.3). With changes of the parameters, we are, of course, able to evoke fluctuations which are either wilder or more moderate, but these nevertheless share the same statistical characteristics as the data shown in this paper. Now turn to the details of the statistical analysis.

Fractal (or correlation) dimension: First, we follow many of the early empirical chaos papers in attempting to estimate the so-called fractal dimension of our data. The fractal

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6 Another economic example of its occurrence is given in Youssefmir and Huberman (1997).

dimension (denoted by D_c in the following) is a measure to determine the degree of complexity of a time series which, for data from a chaotic attractor, would assume some non-integer value $D_c > 1$. With a reasonably low estimate, say < 4, the hope would emerge of understanding the underlying dynamics, while a higher dimension estimate (if its existence could be assured at all) would imply that the dynamics is close to truly random generating mechanisms.

Of the various definitions of the fractal dimension, the *correlation dimension* is usually adopted in empirical work. To arrive at an estimate of D_c for a given time series $\{y_t\}$, t = 1,2,..., n, one first computes the so-called correlation function:

$$(6) \quad C_{\varepsilon,m} = \sum_{1 \le s} \sum_{\le t \le n} \frac{\mathbf{I}_{\varepsilon}(y_t^m, y_s^m)}{\binom{n}{2}}$$

with $y_t^m = (y_t, y_{t+\tau}, ..., y_{t+(\tau-1)m})$ an 'm-history' constructed from the underlying uni-variate data set and $\mathbf{I}_\varepsilon(\ )$ an indicator function: $\mathbf{I}_\varepsilon(y_t^m, y_s^m) = 1$ if $\left\| y_t^m - y_s^m \right\| < \varepsilon$ and 0 otherwise. The correlation function, thus, measures the relative frequency with which different points are within radius ε of each other. Here, m is called the embedding dimension, and the lag τ used in constructing the m-histories is chosen in a way to avoid too high a correlation between the elements of an m-tuple. It is usually recommended to set this lag equal to the first zero-crossing of the autocorrelation function and we follow this practice here. For chaotic attractors, $C_{\varepsilon,m}$ should behave like $C_{\varepsilon,m} \approx \text{constant} \cdot \varepsilon^{D_c}$. As stochastic processes exhibit increasing estimates of D_c with increasing 'embedding' dimensions, one looks for the development of the estimate when using m-histories y_t^m with increasing m. If the estimate D_c exhibits convergence to some almost constant value, this value is used as an estimate of the 'true' correlation dimension of the process under investigation.

In Fig. 1, we show the application of this procedure to a data set of 40,000 observations of returns over unit time steps simulated from our model. The upper part illustrates the behavior of the correlation integral with increasing embedding dimension m. It can readily be seen that at least for embeddings up to m = 12, the slope of the fitted linear curves increases with increasing m. Plotting the slopes as the estimates of D_c in the bottom part confirms that one cannot speak of convergence of the estimate which seems to increase monotonously and, finally, at m = 12, reaches a value of 8.81. Looking up early papers such as Scheinkman and LeBaron (1989) or Frank and Stengos (1989), this pattern appears quite familiar. For comparison, our plot also shows the behavior of the increments of the fundamental value (which are assumed to follow a Normal distribution). Here we see higher estimates of D_c coming close to the 45° line throughout and increasing up to a high 11.06 at m = 12. Thus, it appears that, although we are unable to establish convergence of the correlation dimension estimate, the price dynamics from the model appears less complex (less random) than the pseudo-random numbers underlying the dynamics of the fundamental value.

To check the significance of this apparently different behavior formally, we applied the ‘shuffle test’ (Scheinkman and LeBaron, 1989) and ‘surrogate data test’ (Theiler *et al*., 1992). In the former, the results for the original series are compared with estimates obtained for randomly reshuffled time series, whereas, in the latter, they are compared to that of synthetic data with similar distributional characteristics and (linear) autocorrelation. Table 1 shows the outcome of these tests for embedding dimensions m = 3, 6, 9, and 12. It can be seen that the dimension estimates from 20 sets of surrogate data are uniformly larger than those of the original data set. Reshuffling yields lower estimates than the surrogate data technique, but for the higher embeddings (m = 9 and 12) also leads to a rejection of the underlying null hypothesis (that the original data does not behave differently from the randomized ones) at the 95 percent level. Again, similar results are familiar from the empirical chaos literature, where randomly reshuffled series usually lead to higher estimates of D_c despite lack of convergence of the dimension estimate. The overall result is that like most of the empirical time series that have been analyzed in this way our computer-generated data show traces of hidden structure which, however, appears to be of a more complicated nature than time series from some low-dimensional deterministic dynamics.

Fig. 1 and Table 1 go about here

BDS and Kaplan test: Having been unable to establish a low-dimensional correlation dimension for our simulated data, we are turning to more modest goals. In the following we are interested in whether explicit tests for nonlinearity and IIDness would indicate at all that there is more in our data than a purely stochastic motion or short-term linear dependence. From the wealth of available procedures, we choose the BDS and Kaplan test (Brock *et al*., 1996; Kaplan, 1994).

The reason for this choice is that these two tests turned out to be the best performing ones in a recent competition among nonlinearity tests (cf. Barnett *et al*., 1998). Our application can also be viewed as adding another type of process to their competition. In order to facilitate comparison, we use the same sample size as Barnett *et al.* did in their ‘large’ samples, i.e. 2,000 observations, and consider 20 subsamples from a longer simulation run in order to perform a small Monte Carlo experiment.[7] The whole data series of 40,000 entries is shown in Fig. 2, it is the same series that was used in the above attempt at estimating the correlation

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[7] Of course, both the size of this experiment as well as the number of testing procedures applied to the data are restricted by computation time.

dimension.

Fig. 2 and Table 2 go about here

The idea of the *BDS test* is to look for significant deviations of the behavior of the correlation integral (6) from that expected under IIDness of the data. In particular, if the data under consideration are identically and independently distributed, then it can be shown that $\lim_{n\to\infty} C_{\varepsilon,m} = (\lim_{n\to\infty} C_{\varepsilon,1})^m$ almost surely for all ε > 0 and m = 2,3,... The pertinent test statistics is (Brock *et al.*, 1996):

(7) $V_{\varepsilon,m} = \sqrt{n}(C_{\varepsilon,m} - C^{m}_{\varepsilon,1})/\sigma_{\varepsilon,m}$,

which has a limiting standard Normal distribution under the null hypothesis of IID. With the estimate of the standard deviation $\sigma_{\varepsilon,m}$ given in Brock *et al.* application of the above is straightforward.

The outcomes of a sequence of BDS tests applied to twenty data windows are given in Table 2 and are visualized in Fig. 2. In constructing the m-tuples, we tried embedding dimensions ranging from 2 to 5 and, since linear dependence had been removed by ARMA filtering,[8] we set the lag length τ equal to 1. We classify the results as 'acceptance' ('rejection') of IIDness, if none (all) of the test statistics over m = 2 to 5 are significant at the 95 percent level. Mixed results are classified as 'ambiguous'.

Kaplan's test (Kaplan, 1994) is a test based upon continuity in phase space of deterministic dynamics. Continuity implies that nearby points on a trajectory from a deterministic process should also be nearby in phase space, while, with data from a purely stochastic dynamics, nearby points (in time) may be further apart in phase space. More formally, this amounts to testing whether for pairs of data points which are within some small distance $d_{ij} = \|y_i - y_j\| < r$, the average of the differences of their iterations $\varepsilon_{ij} = \|y_{i+1} - y_{j+1}\|$ is found to be smaller than some threshold value. The significance of this test statistics is judged by comparison with surrogate data. As the computational burden of this test allows only limited experimentation, we resorted to the conservative choice m = 2 for the length of the vectors y_i and again set τ = 1. Furthermore, we performed 20 replications with surrogate data and

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8 The optimal ARMA(p,q) model was estimated by the PSC (predictive stochastic complexity) algorithm, cf. Chen and Tan (1999). The finding of small positive orders of the ARMA parameters is mainly due to a negative spike at the first lag which appears somewhat too large when compared to empirical numbers. The reason for this negative short-run autocorrelation is probably a too swiftly change of noise traders from optimistic to pessimistic mood and *vice versa* during turbulent episodes. Further attempts at fine-tuning of parameters may eliminate this feature.

adopted two variants of this test: in the first, we computed the test statistic K as the average ε_{ij} from the 500 smallest distances d_{ij}, while in the second variant, we performed a linear regression on these smallest pairs $(d_{ij}, \varepsilon_{ij})$ and considered the intercept at $d_{ij} = 0$. In both cases, the resulting test statistics K is compared to the minimum K from 20 time series of surrogate data. With the latter greater (smaller) than the actual one, we accept (reject) linearity of the data and report ‘ambiguous’ results, if both cases have divergent outcomes.

Detailed results are given in an unpublished Appendix which is available upon request. As can be seen from Table 2, results from both tests are similar within most subperiods, but are in no way uniform across subsamples. Interestingly, comparing test results with the visual appearance of the relevant parts of the time series, there seems to be a general tendency towards rejection in periods with larger fluctuations, while in periods with moderate volatility both the BDS and Kaplan test do not reject IID or linearity.[9] It is interesting to compare this behavior of our model with de Lima’s recent findings for U.S. stock market data: Considering daily S&P 500 data during the eighties, he was unable to reject IIDness in all subsamples *prior* to the crash in 1987. However, once he extended the sample to include this event and the following episodes, the outcome was overwhelming rejection (de Lima, 1998). Apparently, the results from our model are quite similar to de Lima’s findings with the BDS and Kaplan statistics becoming significant in periods of high volatility only.[10] However, note that, in our model, both the rejection and non-rejection periods are generated from one and the same simulation run without any change of the underlying mechanism. This serves to question the suggested interpretation in de Lima’s paper that non-stationarity rather than dependence may be the source of rejection of IIDness in the eighties. In fact, here it is shown that one can conceive processes that look practically random for extended time spans (and, the theoretical arguments outlined in sec. 2, in fact, suggest, that the dynamics *is* close to random near the equilibrium), but encapsulate non-linear forces which only show up in the dynamics and in the test statistics in certain subperiods. It is worth emphasizing that what the process does is exactly what the tests indicate: switching between tranquil phases in which the dynamics is practically indistinguishable from purely random motion and more turbulent phases where

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[9] The four panels of 10,000 time steps each in Fig. 2 have been scaled according to the maximum fluctuations occurring within each window. In the second and third panel, no excessively large fluctuations appear. However, in the first and fourth panel we have some very large fluctuations which may artificially give the impression of a lower average volatility over the remaining parts of the window. This is, however, an optical illusion only as the usual bandwidth of fluctuations which are observed over the total of the second and third panel is dominated here by a few outliers.

[10] Performing recursive BDS tests as in de Lima (1999) we often get exactly the same picture with the statistic ‘jumping’ right across the critical values when large fluctuations set in.

some structural elements can be detected.[11]

GARCH estimation: As it is well known that most of the non-linearity in financial data seems to be contained in their second moments, we proceed by carrying out a sequence of tests and parameter estimates on volatility dynamics. Our first step is to test for the presence of GARCH effects by applying the Ljung-Box and Lagrange multiplier tests to squared entries of our data. In both cases the number of lags considered is 12. In order to conserve space, we confine ourselves to a short summary here instead of giving all the details (which are available upon request): both tests gave uniform results for each of our 20 subsamples with usually overwhelming rejection of the null hypothesis in the majority (17) of cases. As can be seen in Table 2, those periods without rejection of independence of squared returns (nos. 5, 15, and 20) are also periods without rejection of IIDness with any of the variants of the BDS test or the Kaplan test. On the other hand, with 17 out of 20 rejections at the 95 percent level, the presence of GARCH effects seems to be more robust than the rejection of IIDness from the non-linearity tests.

In the second stage of GARCH modeling we attempted to specify the appropriate model from the GARCH(p,q) family and estimate its parameters. This amounts to the following specification of the returns generating process:

$$(8) \qquad r_t = \mu + h_t^{1/2} \cdot \varepsilon_t, \qquad h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} ,$$

with ε_t IID normal innovations and the restrictions $\alpha_0 > 0$, α_i, $ß_i \geq 0$ and $\sum_i \alpha_i + \sum_i \beta_i < 1$.

For the selection of the optimal number of lags in the variance equation we adopted the BIC criterion. Overall results as shown in Table 3 point to the parsimonious GARCH (1,1) specification as the optimal model for 16 out of the 17 time series under consideration, while for the remaining sample GARCH (1,2) has been chosen. Table 3 gives the detailed parameter estimates which are relatively uniform across samples: browsing through the rows we find for all (1, 1) specifications a small influence of the most recent innovation ($\alpha_1 < 0.1$ throughout) coming along with strong persistence of the variance coefficient ($\beta_1 > 0.9$). A glance at the relevant literature shows that such parameter estimates are rather common when considering returns from share markets of foreign exchange rates at daily frequencies (cf. de Vries, 1994; Pagan, 1996). It is also interesting to observe that the sum of the coefficients $\alpha_1 + \beta_1$ (+ β_2) is close to one in all cases, i.e. the process is close to an Integrated GARCH process. Again, the

[11] As noted in the introduction, the tails of the distribution of returns drop off with a power-law index around 3. This assures existence of second moments which can also be confirmed by the convergence of recursive variances. The process should, thus, be covariance stationary, so that the variation in the results of the BDS and Kaplan tests should not be due to moment condition failures.

whole chain of results found for the GARCH framework is astonishingly similar to what one usually extracts from real-life data.

Table 3 goes about here

4. Conclusions

The aim of this paper was to investigate the time series behavior of simulated data from a simple model of a financial market with interacting agents. Extending earlier work on the unconditional distributional properties and scaling laws of our model, we were interested in the dependence structure in our data and the outcome of various tests for non-linearity. We found mixed results with the omnibus tests by Brock *et al.* and Kaplan. Hence, without knowledge of the generating mechanism a researcher would probably not find it easy to classify our data and would perhaps even find it doubtful that all the samples have been generated from one and the same underlying mechanism. One of the contributions of this paper is to point exactly to this possibility of obtaining seemingly divergent results from an extended simulation of our artificial market. This behavior may account for the appearance of non-stationarity of stock market indices during the eighties (cf. de Lima, 1998). More generally, such mechanisms may provide a possible explanation for the lack of robustness of the results of non-linearity tests both over time periods and between tests (cf. Barnett *et al.*, 1996). On the other hand, the last part of our experiments showed that the finding of GARCH effects appears to be much more robust and, in most subsamples, yields realistic parameter estimates. Even the numerical estimates fall into a very narrow and realistic range for those 16 samples where GARCH (1,1) appears appropriate.

It is worth emphasizing that, in our model, all these interesting qualitative features arise endogenously from the trading process and the interactions of our agents. With the assumption of IID Normal innovations of the fundamental value, none of these characteristics can be attributed to exogenous influences. Taking together our earlier results on the unconditional distributions (Lux and Marchesi, 1999, 2000) and the present findings, it seems that a large part of the stylized facts of financial data can be explained by relatively simple models of interacting agents.

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Table 1: Estimates of Correlation Dimension

embedding dimension	raw data	shuffled data		surrogate data	
		min	max	min	max
3	2.21	2.20	2.28	2.65	2.71
6	4.51	4.40	4.55	5.26	5.42
9	6.54	6.66	6.89	7.86	8.15
12	8.81	8.84	9.19	9.69	11.28

Table 2: Overall results of Nonlinearity Tests

subsample	**ARMA**	**BDS**	**Kaplan**	**GARCH**
1	(0, 0)	**reject**	**reject**	(1, 1)
2	(1, 0)	accept	*ambiguous*	(1, 1)
3	(0, 0)	accept	*ambiguous*	(1, 1)
4	(1, 0)	accept	*ambiguous*	(1, 1)
5	(1, 0)	accept	accept	no GARCH
6	(1, 0)	accept	accept	(1, 1)
7	(1, 0)	accept	accept	(1, 1)
8	(1, 0)	accept	accept	(1, 1)
9	(2, 2)	*ambiguous*	*ambiguous*	(1, 1)
10	(1, 0)	accept	accept	(1, 1)
11	(0, 2)	*ambiguous*	*ambiguous*	(1, 1)
12	(0, 0)	**reject**	*ambiguous*	(1, 1)
13	(0, 2)	*ambiguous*	*ambiguous*	(1, 1)
14	(1, 0)	**reject**	**reject**	(1, 2)
15	(1, 0)	accept	accept	no GARCH
16	(0, 2)	*ambiguous*	*ambiguous*	(1, 1)
17	(2, 1)	**reject**	**reject**	(1, 1)
18	(0, 2)	accept	*ambiguous*	(1, 1)
19	(2, 2)	*ambiguous*	*ambiguous*	(1, 1)
20	(1, 0)	accept	accept	no GARCH

Table 3: Details of GARCH Estimation Results

subsample	μ	α_0	α_1	β_1	β_2
1	$3.82*10^{-5}$ (0.241)	$9.77*10^{-7}$ (3.476)	0.07466 (8.522)	0.91276 (93.067)	-
2	$-2.43*10^{-4}$ (-1.734)	$4.78*10^{-7}$ (1.741)	0.02932 (3.740)	0.95979 (79.004)	-
3	$-7.30*10^{-5}$ (-0.530)	$7.35*10^{-7}$ (2.277)	0.03332 (4.367)	0.94903 (70.968)	-
4	$2.12*10^{-4}$ (1.498)	$1.02*10^{-6}$ (2.536)	0.03124 (4.591)	0.94429 (65.950)	-
5	no evidence of GARCH				
6	$1.27*10^{-4}$ (0.961)	$1.30*10^{-6}$ (1.410)	0.02185 (2.231)	0.94130 (28.590)	-
7	$-6.74*10^{-5}$ (-0.508)	$6.67*10^{-7}$ (1.814)	0.02316 (3.359)	0.95898 (62.986)	-
8	$-1.02*10^{-4}$ (-0.765)	$4.84*10^{-7}$ (1.721)	0.02691 (4.034)	0.96051 (79.321)	-
9	$1.01*10^{-4}$ (0.616)	$6.65*10^{-7}$ (2.384)	0.04534 (5.234)	0.94431 (87.341)	-
10	$-1.63*10^{-5}$ (-0.119)	$3.48*10^{-7}$ (1.302)	0.01563 (2.551)	0.97553 (84.008)	-

11	$-1.36*10^{-5}$ (-0.089)	$1.11*10^{-6}$ (2.898)	0.04598 (6.718)	0.93318 (76.466)	-
12	$-8.94*10^{-5}$ (-0.583)	$4.72*10^{-7}$ (2.726)	0.04255 (7.429)	0.95004 (139.042)	-
13	$4.00*10^{-5}$ (0.262)	$3.13*10^{-7}$ (2.020)	0.02975 (4.860)	0.96464 (129.026)	-
14	$2.40*10^{-4}$ (1.627)	$4.83*10^{-7}$ (2.438)	0.11720 (4.519)	-0.08093 (-2.890)	0.95432 (101.419)
15	no evidence of GARCH				
16	$3.54*10^{-4}$ (2.224)	$5.48*10^{-7}$ (2.374)	0.03987 (5.308)	0.95111 (99.659)	-
17	$-1.45*10^{-4}$ (-0.848)	$4.59*10^{-7}$ (3.084)	0.076136 (10.356)	0.92184 (159.800)	-
18	$3.90*10^{-5}$ (0.284)	$4.63*10^{-7}$ (2.466)	0.03296 (4.892)	0.95634 (103.815)	-
19	$-1.05*10^{-4}$ (-0.701)	$4.01*10^{-7}$ (2.313)	0.03428 (4.695)	0.95814 (108.365)	-
20	no evidence of GARCH				

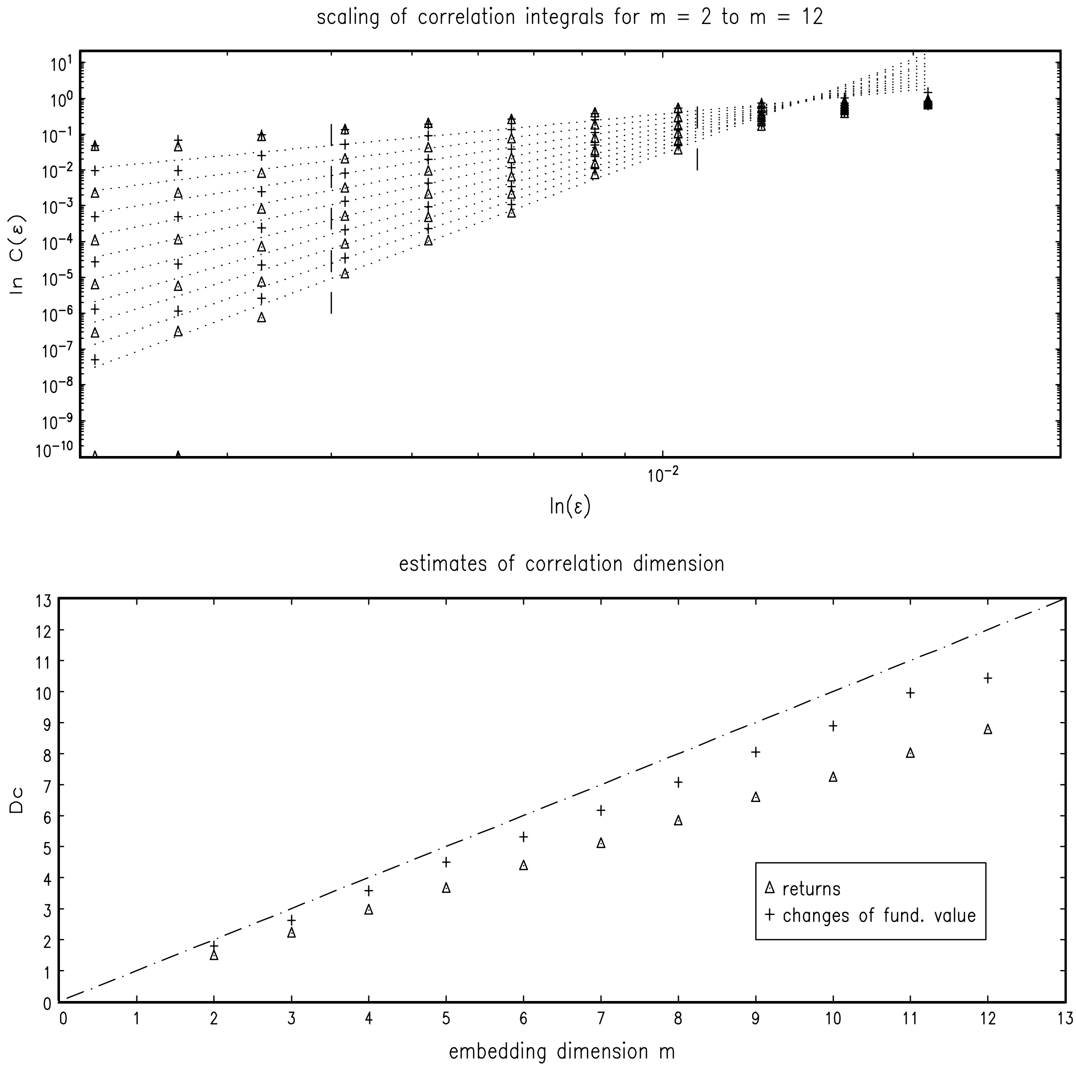

Fig. 1: *Estimation of the correlation dimension D_c for a sample of 40,000 observations*. Top: loglog plot of the scaling of the correlation integral with embedding dimension ranging from m = 2 to m = 12. The eleven curves proceed counter-clockwise from lower to higher numbers of m, the broken vertical lines demarcating the scaling region. As can be seen, the slope keeps increasing, so there is no saturation of the correlation dimension as can also be inferred from the bottom plot of D_c *versus* m. The bottom plot also shows that the randomly generated changes of the fundamental value are characterized by higher estimates of D_c at all embedding dimensions.

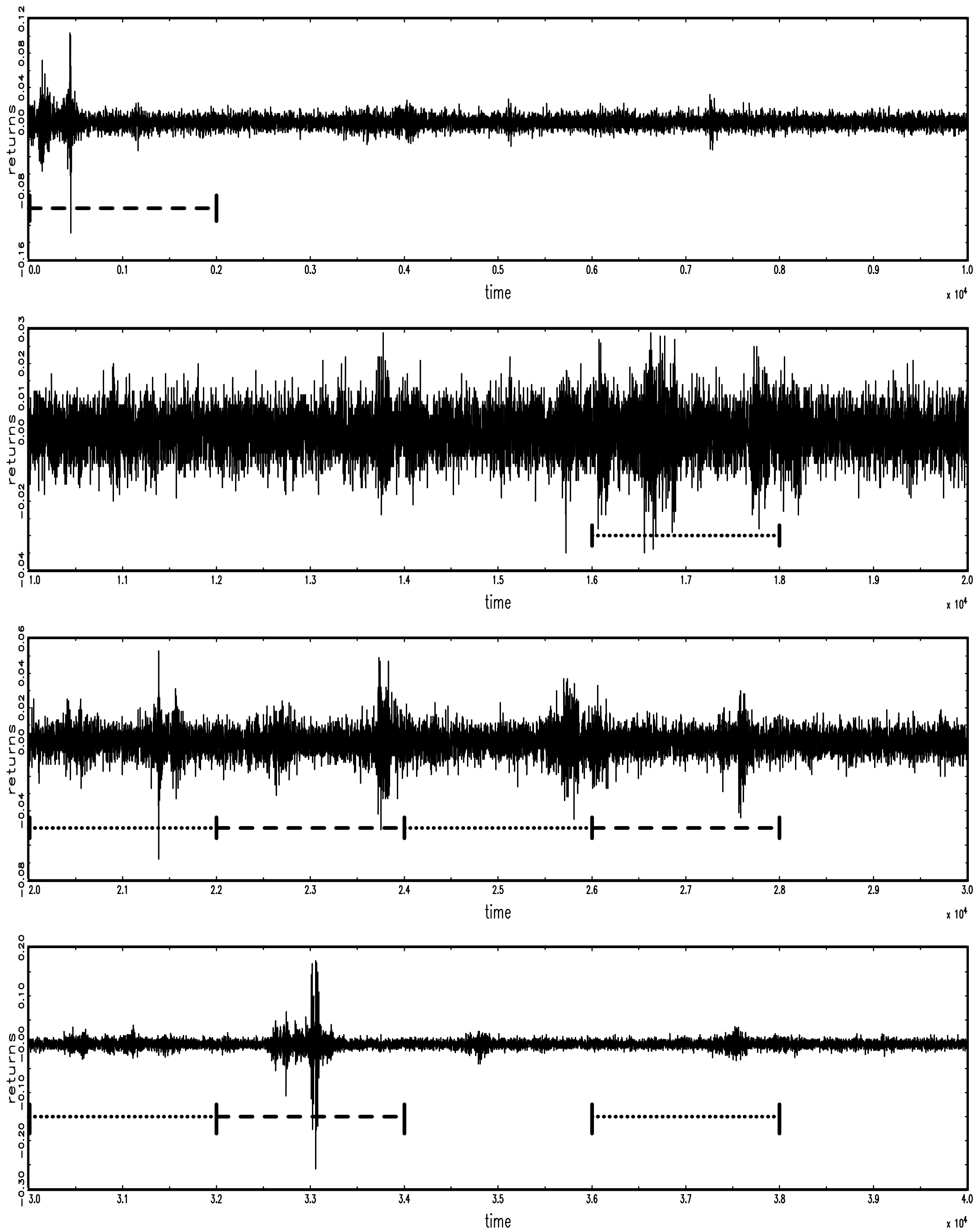

Fig. 2: Simulation run over 40,000 time steps. The parameter values underlying this simulation are: N = 500, v_1 = 2, v_2 = 0.6, ß = 4, t_n = 0.001, γ = 0.01, α_1 = 0.6, α_2 = 1.5, α_3 = 1, R = 0.0004 (r = R p_f), s = 0.75, and σ_ε = 0.005. The broken and dotted lines indicate those subperiods with clear rejection from the BDS test (----) and ambiguous results (......), respectively, cf. Table 1.