FIRMS' CHOICES
IN IMPERFECT GENERAL EQUILIBRIUM

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Abstract. Imperfect general equilibrium was introduced and analysed in Nicola (1994), as a type of general equilibrium for a multisectoral many{person dynamic model, in which all price decisions are directly taken by individual ¯rms, period after period. To simplify, in that model the only input considered was labour. This paper extends the analysis by considering ¯rms whose inputs are labour and commodities produced by other ¯rms. It is proved that a solution exists for this ¯rm problem, both in the short run and in the long run, so that this generalized ¯rm problem is suited to be included into the imperfect general equilibrium model.

KEYWORDS: Firm's theory, Market signals, Dynamic (dis{)equilibrium, Short{ and long{run choices, Accumulation of capital.

1. Introduction

Walrasian general equilibrium, as presented by Arrow{Debreu (1954), has put a heavy strait{jacket on microeconomic theory, by insisting on the permanent equality, period after period, between demand and supply for every commodity. If this model is considered as representing a one period economy then surely no dynamics is present by de¯nition. But one can look at an Arrow{Debreu's economy also as a many-period economy, assuming that all interested agents perfectly know from the start the temporal evolution of all fundamentals, so that each one of them chooses in the ¯rst period, once and for all, a best intertemporal program, to be implemented period after period. In both cases, Walrasian general equilibrium does not seem to be the best starting point to study dynamic problems, mainly because it is easy to see that real world economies experiment a rich variety of time paths, going from quasi{steady states to chaotic{like motions.

One preliminary, albeit trivial, point to be put forward is that non Walrasian theoretical models, when they are built to operate under non equilibrium prices, are capable to generate truly dynamic solutions even when fundamentals are perfectly stationary. While it is plain that every dynamic behaviour, due to changing fundamentals, must produce an intrinsically exogenous dynamics, because by de¯nition fundamentals can change only in a completely exogenous way, it is equally plain that any path generated by the solution to a given model, when all fundamentals are stationary, is almost by de¯nition endogenous dynamics, whenever there is one. This consideration strengthen the previous observation that Walrasian general equilibrium is unable to cope with genuine dynamic situations.

The aim of this paper is to generalize what has been called elsewhere imperfect general equilibrium (Nicola, 1994), namely, the study of a temporary general (dis-)equilibrium model where all fundamentals are perfectly stationary and no Walrasian auctioneer is present to steer prices, in logical time, towards a Walrasian equilibrium, i.e. a state of the economy in which for every commodity demand does not exceed supply. In imperfect general equilibrium, the task to choose prices is given in all time periods to each seller, i.e. ¯rm,[1] whose external knowledge is limited to a subjectively estimated demand function for the commodity it sells. Every such function is statistically updated and improved, in calendar time, by the interested ¯rm, as it collects more and more data in real time.

When, from the start, all fundamentals are considered perfectly stationary, it seems that, according to Schumpeter (1911), there is no place for considering the \entrepreneur". Indeed, Schumpeter had a sharp distinction between

\... two types of individuals: mere managers and entrepreneurs."
(Schumpeter, 1961, p.83).

To Schumpeter, entrepreneurs have a place in an economy only when some innovations occur, namely, when new goods or new production processes are introduced into the economy. Of course, innovations are completely absent by de¯nition when fundamentals are assumed stationary. But, for instance according to Casson (1987, p.151), outside neoclassical theory, i.e. Walrasian economics, there are no \perfect information" and no \perfect markets", and this o®ers enough space to introduce entrepreneurs as agents of the story. Because in imperfect equilibrium there are neither perfect information nor perfect markets, some of the features attributed to the entrepreneur are present in the following model. In some sense, here we are midway between true entrepreneurs and mere managers, so the neutral term \¯rm" is adopted in the following pages. In the imperfect general equilibrium model to be considered, the entrepreneurial feature of the ¯rm's problem is the need to choose output prices in every time period while, because goods and technologies are perfectly stationary, the ¯rm's maximization problems remain typically manager's problems.

In general, it is surely a very interesting subject of study for economic historians, for instance Chandler (1992), to understand how modern ¯rms were born in the last century. But it seems almost fruitless and vain to try to introduce, into a formal economic model, mechanisms able to \generate" new ¯rms. Indeed, in mathematical reasoning all conclusions, i.e. theorems, are always hidden in the premises, i.e. assumptions; put di®erently, in a given time period a new ¯rm is born simply because from the start the chosen assumptions do imply the birth of the ¯rm in that period. Moreover, it is sure that a lot of non economic causes are implied in such a birth; entrepreneurship, according to Schumpeter (1911), is surely the most important quality needed to estabilish a new ¯rm, and it is doubtful that this quality, owned by a limited number of human beings, can be fully ascribed to, and described in, purely economic terms.

The paper studies, in a reasonably general way, the dynamic behaviour of a stylized ¯rm, already in existence. The main point on which the paper is built is the observation that in all real economies there is no omniscient Walrasian auctioneer to steer prices towards a hypothetical general equilibrium with respect to all supplies and corresponding demands. In imperfect general equilibrium the very economic agents have the power to choose prices, period after period, in calendar time. Of

[1] Under the assumption of no possibility of collusion among ¯rms.

course, apart very speci¯c circumstances, it is unusual that consumers take an active part in determining current prices of the goods they buy; this task is usually reserved to ¯rms. But, in general, exchanges take place not only between ¯rms and consumers; a lot of transactions occur between ¯rms, in which case it is not at all certain that a seller chooses its ¯nal price independently from buyers.

How a price is arrived at, as the basis to a transaction between two parties, is a fertile territory for bargaining theory as presented, for instance, in the essays edited by Binmore and Dasgupta (1987), or in the monograph by Osborne and Rubinstein (1990). In particular, the last two authors, in Part 2 of their monograph, devoted to the study of models of decentralized trade, examine very deeply how numerous buyers and sellers can meet in pairs to reach an agreement in the market for a single commodity. While very interesting, their analysis completely ignores the connections among the numerous markets active in every real economy;[2] their models are a sort of Marshallian partial equilibrium models, since they do not present a full analysis of trades in a general equilibrium setting.

Another type of model, which does not seem far from imperfect general equilibrium, is exempli¯ed by the contributions by Dubey and his co{authors, summarized in Dubey (1994). In simple terms, Dubey assumes that every agent sends two types of signals, one concerning the quantities of various goods o®erred for sale, the other concerning the quantities of `¯at' money he/she is ready to spend for every commodity he/she likes to buy; no explanation is o®ered about the way every agent chooses his/her signals, and this is a major drawback of all these models. After all signals are di®used into the economy, the network of transactions is organized in a number of posts, one for every commodity, and each post is managed by somebody very similar to the auctioneer.

The imperfect general equilibrium strongly advocates the viewpoint that in real world economies there are no active posts to guide transactions, but sellers and buyers must generally meet in some informal ways to arrange transactions, which are always of this type: some quantity of a speci¯ed commodity is supplied against payment of a speci¯ed quantity of money.

As already noted, when there are inputs other than labour, it is not at all obvious that the price of a commodity, produced by a ¯rm and used as an input by another ¯rm, is chosen exclusively by the producing ¯rm, and submitted to the possible customer ¯rm, on the basis of the \take-it-or-leave-it" criterion. In principle, it is possible that a bargaining takes place between the two ¯rms, to arrive at an agreement on the price at which to transact a quantity of the speci¯ed commodity. But there are two main observations to be remembered. On one side, very likely the selling ¯rm has a greater knowledge of production costs than the buyer has; on the other side, in modern economies every commodity has many substitutes, more or less perfect, so that it is generally possible for any buyer to choose one among many substitute inputs, and this means that for each input it needs, a ¯rm has really the implicit bargaining power of choosing one among many potential suppliers. Thus, it is here postulated that in imperfect general equilibrium a net of posted{o®er markets is active, instead of, for instance, a double{auction market,[3] namely, in our model selling ¯rms choose prices and buying ¯rms choose sellers which, if necessary,

[2]Except in parr.8.4{8.7, pp.156{170, where a model containing a ¯nite number of divisible goods, and a continuum of agents, is considered under very restrictive assumptions.

[3]See Davis and Holt, 1993.

implement a non manipulable rationing scheme. As we shall see in what follows, it is postulated that, when rationed, a buyer always finds some suitable substitute for the commodity he/she is unable to buy as his/her first choice.

The monograph by Nicola (1994), to bypass the bargaining problem and to work in a reasonably simple but general framework, assumed that all firms employ one input only, namely one type of labour, whose wage rate is chosen, period after period, by a Public Authority (P. A.). Under stationary fundamentals, this assumption gives rise to a sort of "pure dynamics", entirely due to the endogenous changes in individual decisions. As previously said, in the present paper the analysis of the individual firm in imperfect general equilibrium is generalized, with respect to Nicola (1994), by considering firms whose inputs are labour and other firms' outputs. From a dynamic point of view, this generalization has an important consequence, because the accumulation of capital, namely, how productions are split, in every period, between consumption and investment, becomes an important part of the imperfect equilibrium story. Due to this fact, dynamics is no more "pure dynamics", because now it is due both to subjective factors, such as temporally changing estimated demand functions by firms, and to the accumulation of capital.

2. Input Subsets and Cost Minimization

In the imperfect general equilibrium model (Nicola, 1994), under stationary fundamentals, there are n goods, each one 'a priori' produced by a different firm, so there are n distinct firms, indexed $i = 1, 2, \ldots, n$. At the start of a generic time period all firms choose simultaneously their selling prices, $p_1, p_2, \ldots, p_n$, while the P.A. chooses the wage rate w.[4] Later we shall see how prices are chosen; in this par. the analysis is limited to consider a specific firm i, after all current prices have already been chosen and communicated to all agents in the economy. Commodity i is produced by firm i only, by means of labour, q_i, whose price is w, and material inputs, z_{ji} ($j = 1, 2, \ldots, n$), with prices p_j, according to a given stationary production function $g_i : \Re_+ \times \Re^n_+ \to \Re_+$, where

$$y_i = g_i(q_i, z_{1i}, \ldots, z_{ni}) = g_i(q_i, z^i) \tag{1}$$

is the maximum output of commodity i, obtained by the specified amounts of all inputs.

Assumption 1 Function $g_i : \Re^{1+n}_+ \to \Re_+$ is continuous and for every $z \geq 0$ one has $g_i(0, z) = 0$.

Economically, labour is always needed to get a positive output.

As a simple observation of what happens in the real world, and as a parallel to consumer's analysis in Nicola (1994), it is evident, as remembered in the Introduction, that generally every input has many, more or less perfect, substitutes, with the exception of labour, here assumed of one type only. Thus, the set $N = \{1, 2, \ldots, n\}$ of all possible non labour inputs can be partitioned into a family of r disjoint subsets, N_{ki} ($k = 1, 2, \ldots, r$), $1 \leq r < n$, so that, for every (q_i, z^i), one has

$$g_i(q_i, z^i) \equiv \tilde{g}_i(q_i, \tilde{z}_{1i}, \ldots, \tilde{z}_{ri}); \tag{2}$$

[4] According to Nicola (1994, par.6.4).

with

(2a) $$\bar{z}_{ki} = \sum_{h \in N_{ki}} \theta_{hi} z_{hi} \qquad (k = 1, 2, \ldots, r);$$

where $\theta_{hi} > 0$ are given technical coefficients, whose two by two ratios measure the degree of substitutability inside each subset of inputs. For instance, if $N_{1i} = \{1, 2\}$ and $\theta_{1i} = 1 = \theta_{2i}$ then one has $\bar{z}_{1i} = z_{1i} + z_{2i}$, meaning that inputs '1' and '2' are perfect substitutes. The new variables $\bar{z}_{ki}$ economically define aggregate inputs. Whatever the specification of g_i, from (2a) it is possible to formulate some interesting considerations about the optimal technology, (q_i, z^i), as a function of all prices, and of the output, with respect to elements N_{ki}s of the previous partition.

Let (w, p) denote current period wage rate and input prices, already chosen, respectively, by P.A. and by selling firms; all prices become public knowledge. Because any optimal technology implies the minimization of total cost, let us consider the customary problem: for every $y_i > 0$, solve

(3) $$\min_{(q_i, z^i)} \left\{ wq_i + \sum_{j=1}^{n} p_j z_{ji} \mid \bar{g}_i(q_i, \bar{z}_{1i}, \ldots, \bar{z}_{ri}) \geq y_i \right\}:$$

Of course, all variables $\bar{z}_{ki}$s are continuous functions of z_{hi}s; thus, by Assumption 1, $\bar{g}_i$ too is continuous, and problem (3) has a solution, because the objective function is continuous and lower bounded, while the constraint defines a closed set. Thus we have

Proposition 1 Under Assumption 1 problem (3) has a solution.

Assuming there is one solution only to this problem,[5] we can write

(4) $$z_{ji} = \psi_{ji}(p, w, y_i) \qquad (j = 1, 2, \ldots, n);$$

to denote firm i's input demands as functions of all input prices and of the output.

Let us now remember that if g_i is C^1 then $\bar{g}_i$ too is C^1; it is useful to introduce

Assumption 2 Production function g_i is at least C^1, and its first derivative, $\partial g_i = (\partial g_i/\partial q_i, \partial g_i/\partial z_{1i}, \ldots, \partial g_i/\partial z_{ni})$ is positive in $\Re^{1+n}_{++}$.

Namely, all marginal productivities are positive in the interior of the domain of g_i.

Under the previous Assumptions 1, 2, it is easy to characterize a solution to problem (3), whose Lagrangian is:

$$L_i(q_i, z_{1i}, \ldots, z_{ni}, \lambda_i) = wq_i + \sum_{j=1}^{n} p_j z_{ji} + \lambda_i[y_i - g_i(q_i, z_{1i}, \ldots, z_{ni})]:$$

Remembering Assumption 1 on the labour input, the customary first order condition, which is also sufficient when the production function is quasi concave, for a constrained minimum is:

(5) $$\begin{cases} \dfrac{\partial L_i}{\partial q_i} = w - \lambda_i \dfrac{\partial g_i}{\partial q_i} = 0; \\ \dfrac{\partial L_i}{\partial z_{ji}} = p_j - \lambda_i \dfrac{\partial g_i}{\partial z_{ji}} \geq 0; \qquad z_{ji}\dfrac{\partial L_i}{\partial z_{ji}} = 0 \qquad (j = 1, 2, \ldots, n); \\ \dfrac{\partial L_i}{\partial \lambda_i} = y_i - g_i(q_i, z^i) = 0: \end{cases}$$

[5] Outside a set of Lebesgue's measure zero!

Considering the aggregate inputs, $\bar{z}_{ki}$s, and the aggregate production function, $\bar{g}_i$, we can write:

$$\frac{\partial g_i}{\partial z_{ji}} = \frac{\partial \bar{g}_i}{\partial \bar{z}_{ki}} \frac{\partial \bar{z}_{ki}}{\partial z_{ji}} = \frac{\partial \bar{g}_i}{\partial \bar{z}_{ki}} \theta_{ji} \qquad (j \in N_{ki}).$$

By means of these formulae, it is possible to write (5) as follows:

$$\text{(6)} \qquad \begin{cases} w - \lambda_i \dfrac{\partial \bar{g}_i}{\partial q_i} = 0; \\ p_j - \lambda_i \dfrac{\partial \bar{g}_i}{\partial \bar{z}_{ki}} \theta_{ji} \geq 0; \quad z_{ji} \dfrac{\partial L_i}{\partial \bar{z}_{ki}} = 0 \qquad (j = 1, 2, \ldots, n); \\ y_i = g_i(q_i; z^i). \end{cases}$$

Let us consider one of the subsets N_{ki}s, and two indices $\alpha, \beta \in N_{ki}$. Previous relations (6) give the following result:

$$\text{(7)} \qquad \begin{cases} \dfrac{\theta_{\alpha i}}{p_\alpha} < \dfrac{\theta_{\beta i}}{p_\beta} \quad \Rightarrow \quad z_{\alpha i} = 0; \ z_{\beta i} > 0 \\ \dfrac{\theta_{\alpha i}}{p_\alpha} > \dfrac{\theta_{\beta i}}{p_\beta} \quad \Rightarrow \quad z_{\alpha i} > 0; \ z_{\beta i} = 0; \end{cases}$$

namely, in any subset N_{ki} of substitute inputs the choice is on the input, or one of those inputs, for which the ratio θ_{ji}/p_j is maximum. This ratio is a measure of the "return to the dollar" for input j.

In practical terms, one can take into consideration a situation where, for instance, a car maker contacts many "subcontractors" from which to buy spare parts;[6] the θ_{ji}s are quality indexes, used by the car maker to weigh prices in order to choose the economically best input in the set N_{ki}. This means that generally a firm can choose among many suppliers for each type of its inputs, in search for the most favorable contracts. This is a multilateral bargaining between the firm and its potential suppliers in every subset N_{ki}. It is important to note that it is reasonable to assume that the firm total expenditure on every N_{ki}, once total cost has been minimized, becomes a constant

$$\gamma_{ki} = p_j z_{ji} \qquad (j \in N_{ki}; \ k = 1, \ldots, r);$$

indeed, if the firm cannot buy the chosen input, z_{ji}, then in the present period its "second best" choice is to demand the next to optimal input, $z_{j'i}$, in the amount $z_{j'i} = \gamma_{ki}/p_{j'}$, and so on, remembering that in the production function g_i all marginal productivities are positive, by Assumption 2. In practical terms, we see that while the firm does its best to minimize costs, at last it contents itself to obtain a "satisficing" result, according to the principle introduced long ago by Simon (1956), and elaborated by his followers. Of course, for the theory of the firm, as here conceived, it is very important that the assumption on the existence of many close substitutes with respect to every input type be true. Indeed, in the real world it is very unlikely that an input type is supplied only by a very limited number of firms, let alone by one firm; this could happen, from time to time, when new goods or new production processes are introduced into the economy, but not in our "scenario", where all fundamentals are assumed perfectly stationary.

[6] For instance, with the help of so called "Yellow Pages".

3. Short Run Decisions

Up to now, what we have done is mainly intended to justify the fact that some search activity[7] must be undertaken by the firm we are considering, with respect to inputs it chooses to buy at the start of every time period. This is an important point, given that in imperfect equilibrium no Walrasian auctioneer is present to determine clearing market prices, period after period. According to the previous discussion, it is reasonable to assume that the firm takes as given, and unchangable in the short run, the prices charged by all other firms.

The type of economy here considered is one in which at the start of period t the P. A., according to the rules considered in Nicola (1994, Ch.3), chooses a positive wage rate, $w^*(t)$, which becomes at once public knowledge, while firm i must choose, simultaneously with, and independently of, all other firms, its current price, at which to sell its disposable stock. Here we consider the case when all goods are durable, in the sense that, once produced, a commodity can be sold in any future time period, so allowing for stock accumulation at zero cost.

Let $\bar{y}_i(t-1)$ be firm i's output at the end of period $t-1$. If the firm started period $t-1$ with a stock $\bar{s}_i(t-1)$ and sold the amount $\bar{e}_i(t-1)$, then in the current period t sales by this firm are bounded by the stock

$$\bar{s}_i(t) = \bar{s}_i(t-1) - \bar{e}_i(t-1) + \bar{y}_i(t-1).$$

In the short run, firm i must choose, as every other firm must, the price $p_i(t)$ at which to sell currently its disposable stock, $\bar{s}_i(t)$. Of course, the firm is well aware that its output, generally demanded by households and by other firms, faces many potential substitutes, both in consumption and in production. We assume there is an objective demand function, at least partially unknown to the firm and to be discovered experimentally by it, expressing the market demand for firm i output as a function of all prices, $w(t), p(t)$. Essentially, the firm solves a statistical fitting problem consisting in updating, period after period, an expected demand function according to the data collected by the firm; generally, least squares on past data will prove very useful. Here we need only take as given this statistical process. Let $\eta_{it} : \Re^{1+n}_{++} \to \Re_+$ be the expected demand function in period t, namely $(w(t), p(t)) \mapsto \eta_{it}[w(t), p(t)]$ is the quantity of commodity i that firm i's expects to sell at present. The $n+1$ prices have different meanings to firm i; $w(t)$ is chosen by the P. A., $p_i(t)$ is chosen by the firm, $p_h(t)$ $(h \neq i)$ are chosen by other firms. Thus, firm i not only must choose η_{it}, but it must also guess the most likely prices to be quoted at present by all other firms whose prices enter firm's i demand function. All this is due to the fact that prices are chosen simultaneously by all firms, so firm i cannot wait, to choose $p_i(t)$, that all other firms have chosen their respective prices. Hence, the firm must choose a more or less sophisticated set of extrapolating price functions, statistically chosen too, to estimate the actual prices charged by other firms, and very likely obtained by means of extrapolations on past time sequences of such prices.

Formally, let us write $p^e_{-i}(t) = (p^e_1(t), \ldots, p^e_{i-1}(t), p^e_{i+1}(t), \ldots, p^e_n(t))$ to mean the vector of all prices, different from the i-th price, expected in period t by firm i; ψ_{it} to mean the functions used to calculate such expected prices; and $p^*(t-1), p^*(t-2), \ldots$ to mean the sequence of effective past prices. Then firm i guess is

$$p^e_{-i}(t) = \psi_{it}[p^*(t-1), p^*(t-2), \ldots]. \tag{8}$$

[7] At no cost!

A standard assumption about these functions is:

Assumption 3 Functions ψ_{it} are continuous, and for every positive λ one has

$$\psi_{it}(p, \lambda^{-1}p, \lambda^{-2}p, \ldots) = \lambda p_{-i}.$$

Namely, steady prices in past periods are expected by the firm to persist in the future.

Introducing these expectation functions into the expected demand function of firm i, and remembering that $w^*(t)$ is the wage rate chosen and announced by the P. A. for the current period, it is possible to define the expected revenue, $\rho_i^e(t)$, of firm i from its period t sales:

$$\rho_i^e(t) = p_i(t)\zeta_{it}[w^*(t), p_i(t), p_{-i}^e(t)]. \tag{9}$$

Let us introduce

Assumption 4 Function $p_i\zeta_{it}$ is continuous and uniformly bounded in $\Re_+^{1+n}$, and for every w, p_{-i} one has

$$\lim_{p_i \to 0^+}[p_i\zeta_{it}(w, p_i, p_{-i})] = 0.$$

This statement means economically that the firm is perfectly aware that when its price goes to zero its expected revenue too tends to zero, even if its demand becomes unbounded.

As far as period t is concerned, it seems sensible to assume that the firm aims at maximizing its present revenue, given that it can sell at most the quantity $\bar{s}_i(t)$. So the firm chooses $p_i(t)$ to maximize its revenue, given by (9), under the expectation functions (8), and constraint

$$\zeta_{it}[w^*(t), p_i(t), \psi_{it}(p^*(t-1), p^*(t-2), \ldots)] \le \bar{s}_i(t). \tag{10}$$

Given the continuity conditions stated by Assumptions 3, 4, and the boundedness expressed by Assumption 4, an immediate application of the standard Weierstrass' extremum theorem proves

Proposition 2 Under Assumptions 3, 4 firm i's short run problem has at least one solution.

When there is more than one solution, very likely the firm chooses the one corresponding to the higest price, $p_i^*(t)$, because for "normal" demand functions this implies the minimum amount sold, thus, the maximum amount of stock kept at no cost for future sales. $p_i^*(t)$ is the price definitely chosen by the firm, at which it expects to sell the quantity $c_i(t) = \zeta_{it}[w^*(t), p_i^*(t), p_{-i}^e(t)]$. As we shall see soon, because generally the whole price vector $p^*(t) = [p_1^*(t), \ldots, p_n^*(t)]$ does not correspond to a Walrasian equilibrium while, moreover, very likely $p_{-i}^e(t) \neq p_{-i}^*(t)$ is true, almost certainly firm's effective sales will differ from $c_i(t)$.

4. Long Run Decisions

One of the main difficulties in defining imperfect equilibrium, when firms use inputs which are goods produced by other firms, is the fact that bilateral exchanges, when values exchanged are different, must be balanced by means of money, here considered to be 'fiat' money. On one side, firm i gets money by selling its output; on the other side it spends money to buy both labour and non labour inputs.[8] Because production takes time, we have assumed that while inputs enter production at the start of every time period, output comes out, ready for sale, only at the end of the same period. Thus, it seems plausible to assume that in the economy money tokens circulate at most once per period. This implies that money from sales in one period can be spent only in the next period. Moreover, it seems natural enough to consider that firm i's production plans are constrained by its money disposability at the start of the period, $\bar{m}_i(t)$, and are independent of current sales, which are expected sales when the firm plans its actual production decisions. So, let us start by considering how firm i arrives at owning the amount of money $\bar{m}_i(t)$; to this aim, let us go back to period $t-1$. Apart from taxes, which are considered in the imperfect general equilibrium model but are here inessential, if $\bar{e}_i(t-1); \bar{q}_i(t-1); \bar{z}^i(t-1)$ stand for, respectively, output sold, labour input hired, non labour inputs bought in period $t-1$, we have the obvious equality

$$(11)\quad \bar{m}_i(t) = \bar{m}_i(t-1) - w^*(t-1)\bar{q}_i(t-1) - p^*(t-1)\cdot\bar{z}^i(t-1) + p^*_i(t-1)\bar{e}_i(t-1);$$

implying that actually the amounts of all inputs firm i can buy must satisfy relation

$$w^*(t)q_i(t) + p^*(t)\cdot z^i(t) \le \bar{m}_i(t);$$

together with the feasibility constraint given by the stationary production function.

With respect to long run choices, the firm must also guess future wage rates; let us write

$$(12)\qquad w^e_i(t+t') = \psi^0_{i;t+t'}[w^*(t-1); w^*(t-2); \ldots] \qquad (t' = 1; 2; \ldots):$$

A standard assumption about these functions is:

Assumption 5 Functions $\psi^0_{i;t+t'}$ are continuous and for every positive λ one has

$$\psi^0_{i;t+t'}(w; \lambda^{-1}w; \lambda^{-2}w; \ldots) = \lambda w \qquad (t' = 1; 2; \ldots):$$

When the commodity is durable, as we have assumed, and there are variable returns to scale, firm i can discover that it is profitable to produce not only for short run sales, but also with an eye to long run sales. Namely, it can be part of an optimal intertemporal program to accumulate stocks in certain periods and decumulate them in others, depending on the time paths followed by expected prices. Thus, it is necessary to consider firm i's multiperiod objective function, arrived at by introducing expected profit for period t, $\pi^e_i(t)$, defined by formula

$$(13)\qquad \pi^e_i(t) = p^e_i(t+1)\zeta_{it}[w^e_i(t+1); p^e_i(t+1); p^e_{-i}(t+1)] - w^*(t)q_i(t);$$

[8] In the full imperfect general equilibrium model, profits are also distributed by means of money.

remembering that the output of one period can be sold only in period $t+1$, at a price which possibly differs from $p^*_i(t)$. Because, generally speaking, a firm has no natural end, we assume that the economic life of firm i is unbounded, and that its aim is to maximize long run expected profit, π^e_{it}, defined as

$$\pi^e_{it} = \sum_{t'=0}^{\infty} \beta_{i;t+t'}\pi^e_i(t+t') = \sum_{t'=0}^{\infty} \beta_{i;t+t'}\{p^e_i(t+1+t')\cdot \gamma_{it}[w^e_i(t+1+t'); p^e_i(t+1+t'); p^e_{-i}(t+1+t')] - w^e_i(t+t')q_i(t+t')\}; \tag{14}$$

where $\beta_i(t) = \{\beta_{i;t+t'}\}_{t'=0}^{\infty}$ is a sequence of positive subjective discount factors, $0 < \beta_{i;t+t'} < 1$, chosen by the firm. In writing formula (14), it has been assumed that money bears no interest and that the estimated demand function, γ_{it}, is, as seen from period t, the best estimate with respect to past data.

Function (14) is defined on an infinite dimensional linear space. For our purposes it is enough to take as the ambient space Hilbert's space l^2 (see, for instance, Nicola, 1994, p.62). In this space, the prototype of a compact, and convex, set is the so called Hilbert's cube, H, which can be defined as

$$H = \{x \mid 0 \le x_i \le 1/i;\ i = 1, 2, \ldots\}.$$

To guarantee the continuity of π^e_{it} in the topology of l^2, and the compactness of the appropriate domain, it is useful to introduce the following

Assumption 6 There is $\mu > 0$ so that all inputs are chosen by firm i to satisfy $q_i(t) = \{q_i(t+t')\}_{t'=1}^{\infty} \in \mu H$, $z_{hi}(t) = \{z_{hi}(t+t')\}_{t'=1}^{\infty} \in \mu H$ $(h = 1, 2, \ldots, n)$. The production function, g_i, is so that for given $(q; z)$ the output $g_i(q; z)$ is finite.

Denoting by l^2_+ the set of all nonnegative sequences in l^2, let us introduce also

Assumption 7 Functions ψ_{it} and ψ^0_{it} define price sequences, $p^e_i(t) = \{p^e_i(t+t')\}_{t'=1}^{\infty}$ and $w^e_i(t) = \{w^e_i(t+t')\}_{t'=1}^{\infty}$, belonging to l^2_+.

Assumption 8 There is a positive number, γ_0, so that we have $\gamma_{it}(0; \cdot; \cdot) \le \gamma_0$.

Under the previous assumptions, which together economically mean that the firm is aware that every maximum problem must be put in a proper (compact) space, it is possible to prove (Nicola, 1994, p.63) that one has $\pi_i(t) = \{\pi^e_i(t+t')\}_{t'=0}^{\infty} \in l^2$; so we can write π^e_{it} as a bilinear functional in $l^2_+ \times l^2$, namely, $\pi^e_{it} = \langle \beta_i(t); \pi_i(t)\rangle$. Let us now introduce the following technical

Assumption 9 There is a positive α so that

$$\alpha = \sup\{|\langle \beta_i(t); \pi_i(t)\rangle| : \|\beta_i(t)\| = 1 = \|\pi_i(t)\|\} < +\infty.$$

Namely, π^e_{it} is a bounded bilinear functional. The economic plausibility of this assumption can be justified as follows: in order that one has $\beta_i(t) \in l^2_+$ it is enough that the firm gives appropriate decreasing weights to future and future profits, while $\pi_i(t) \in l^2$ is due to the fact that we live in a finite universe, according to modern physicists. All bounded bilinear functionals are continuous (Halmos, 1957, pp.31–33), so π^e_{it} is continuous in $l^2_+ \times l^2$.

Expected profit (14) is maximized under constraints:

(15i) $$y_i(t - 1 + t') = g_i[q_i(t - 1 + t'), z^i(t - 1 + t')];$$

(15ii) $$s_i(t + t') = \bar{s}_i(t - 1 + t') + y_i(t - 1 + t') - \bar{e}_i(t - 1 + t');$$

(15iii) $$\zeta_{it}[w^*(t), p_i^*(t), p^e_{-i}(t)] \le \bar{s}_i(t);$$

(15iv) $$\zeta_{it}[w_i^e(t + t'), p_i^e(t + t'), p^e_{-i}(t + t')] \le s_i(t + t');$$

(15v) $$w^*(t)q_i(t) + p^{ei}(t) \cdot z^i(t) \le m_i^*(t);$$

for $t' = 1, 2, \ldots$. With respect to money, there is one constraint only, namely (15v), concerning the current period, because only actual inputs are effectively bought and paid in money.

Let us now apply Weierstrass' extremum theorem, holding in every normed linear space, thus holding in l^2; because constraints (15) define a compact set, given the previous assumptions, we have

Proposition 3 *Under Assumptions 3–9 firm's long run problem has a solution.*

Let us remember that ζ_{it} is just a best estimate of the true demand function for the given firm, which estimate is improved period after period, as more and more data are collected by the firm. This helps to explain why the firm implements only the present part of its intertemporal program. Specifically, let us denote by $\hat{q}_i(t), \hat{z}^i(t), \hat{y}_i(t)$, together with $p_i^*(t)$, firm i's present optimal decision. To this choice one must associate the corresponding effective values, depending on the inputs supplied by the economy to firm i. Because actual prices, $w^*(t), p^*(t)$, in general are non Walrasian prices, there is at least one market in excess demand; thus, let us assume there is a random rationing, directly implemented by the firms producing the various outputs, and by the P. A. with respect to labour (see Nicola, 1994). Let $q_i^*(t), z^{*i}(t)$ be the inputs offered by the economy to firm i; then the corresponding amounts bought are

$$\bar{q}_i(t) = \min\{\hat{q}_i(t), q_i^*(t)\}, \qquad \bar{z}^i(t) = \min\{\hat{z}^i(t), z^{*i}(t)\},$$

where the second min operator is taken componentwise. Accordingly, the output of firm i is $\bar{y}_i(t) = g_i[\bar{q}_i(t), \bar{z}^i(t)]$.

With respect to current sales, firm i can sell up to the amount corresponding to its present stock; hence present effective sales of the firm are given by

$$\bar{e}_i(t) \le \bar{s}_i(t)\}.$$

Given all this, firm i will start period $t + 1$ with money endowment

$$\bar{m}_i(t + 1) = \bar{m}_i(t) + p_i^*(t)\bar{e}_i(t) - w^*(t)\bar{q}_i(t) - p^*(t) \cdot \bar{z}^i(t);$$

while its stock will be

$$\bar{s}_i(t + 1) = \bar{s}_i(t) + \bar{y}_i(t) - \bar{e}_i(t).$$

The whole story now repeats itself with the same production function g_i, but with new stocks $\bar{m}_i(t + 1), \bar{s}_i(t + 1)$, and with statistically updated expected functions $\zeta_{i,t+1}$, $\psi^0_{i,t+1}$, and $\psi_{i,t+1}$.

5. Concluding Remarks

As it happens in the real world economies, ¯rm i revises its optimal program period after period, and always implements only the part concerning the running period. Thus, imperfect general equilibrium is a type of temporary non Walrasian general equilibrium. At the present level of generality, it is almost impossible to obtain any general and safe conclusion about the qualitative properties of the solution trajectories; every other thing apart, they depend also on the way in which agents are randomly paired when doing exchanges, so that under the same conditions the results can change when the pairings change.

It is by means of computer simulations that one can verify the rich variety[9] of possible trajectories, which richness of course does not stem from fundamentals, here assumed perfectly stationary, but by the fact that ¯rms operate in an economy where there is no Walrasian auctioneer, and so prices are in general non Walrasian prices, with no clear tendency to converge to some Walrasian{type solution.

This conclusion, which may appear a bit discomforting, can be compared to the reassuring conclusion, obtained in the analysis of oligopolistic models `a la' Cournot or `a la' Bertrand, that starting from an economy containing a limited number of ¯rms, and increasing this number in an appropriate way, in the limit very often a Walrasian competitive solution is obtained.[10] On these and similar results, Allen and Polemarchakis (1994) present a concise and very good survey.

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[9] See, for instance, Nicola, 1994, Ch.8.

[10] But real economies do not seem to possess Walrasian equilibria.

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