Dynamics of Asset Prices in the CAPM under Autoregressive Forecasting and Noise*

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Abstract

The paper studies the impact of autoregressive forecasting rules in the CAPM with heterogeneous investors with CARA utilities which are characterized as so called fundamentalist or chartist rules.

Forecasts by chartists are made according to an adaptive mean reverting principle for the first two conditional moments of the asset price process. Fundamentalists' forecasts also use a mean reverting process, however, relative to a perceived so called *fundamental value* of asset price assumed to follow a stationary random walk.

We investigate the role of mean reversion on asset prices and returns showing the occurrence of a Neimark-Sacker bifurcation after a period doubling. For a large range of parameters these induce significant autocorrelations for long lags. These results provide evidence that adaptive expectations in many cases do not have a stabilizing influence on the asset price process.

The paper extends the analysis to study the impact of different random influences on aggregate asset supply, asset dividends, and on the fundamental asset price using numerical methods. For small enough noise, the period doubling and the Neimark-Sacker bifurcations are preserved inducing similar persistent autocorrelations. However, with larger perturbations and/or group switching, the bifurcations and autocorrelations disappear showing convergence to stationary solutions (random fixed points) without significant autocorrelations.

Keywords: Asset pricing; Stochastic Bifurcation; Autoregressive Expectations

JEL classification: C60, C61, D83, D84, E21, E32, G11, G12

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1 Introduction

– Attempting to explain asset prices and returns

– Can heterogeneity of behavior and beliefs of agents be a source of

Known Empirical Regularities

- fat tails
- skewness
- auto-correlation patterns

What explains the statistical difference of observed asset prices and of returns?

1. raw returns seem to have significant autocorrelation of the first lag, insignificant afterwards
2. absolute and squared returns show decaying autocorrelation patterns
3. $\Rightarrow$ almost no predictability –
4. but definitely some long memory

Possible structural sources/influences,
conjectured in one of the most popular work horse models to explain asset prices, are

1. non linearities induced by
   - expectations formation
   - heterogeneous investment strategies/behavior
   - learning/switching behavior based
2. noise in
   - perceived underlying fundamentals
   - dividend processes
   - market clearing mechanism
3. others ??

Literature:

– Brock & Hommes (1997)

– Lux (1995, 1997, 2005)

– Böhm & Chiarella (2005),

– Böhm & Wenzelburger (2005),

– Hillebrand & Wenzelburger (2006)

– Chiarella, Dieci & He (2007)

Here primarily:

– role of expectations based on past prices and returns: $\Longrightarrow$ autoregressive mechanism

– role of different sources of noise: $\Longrightarrow$ dividends, market, fundamentals

– role of non linearities: $\Longrightarrow$ switching/behavior

2 The Model

2.1 Investors

– one risky asset, one risk free asset with annual rate of return r

– there are $H \geq 1$ types of investors with $n_{h,t} \geq 0$ agents of type h at time t

– Let p_t be the ex dividend price per share of the risky asset at time t

– $\{y_t\}$ be the stochastic dividend process of the risky asset.

– If $W_{h,t}$ is the wealth at time t and

– $z_{h,t}$ is the number of shares of the risky asset purchased at t,

– $\Longrightarrow$ the wealth of investor h at $t+1$ is given by

$$W_{h,t+1} = RW_{h,t} + R_{t+1} z_{h,t}, \tag{2.1}$$

where $z_{h,t}$ is the number of shares of the risky asset purchased at t, and

$$R_{t+1} = p_{t+1} + y_{t+1} - Rp_t \tag{2.2}$$

Then it follows from (2.1) and (2.2) that

$$\begin{aligned} E_{h,t}(W_{t+1})(z_{h,t}) &= RW_t + E_{h,t}(R_{t+1}) z_{h,t}, \\ V_{h,t}(W_{t+1})(z_{h,t}) &= z_{h,t}^2 V_{h,t}(R_{t+1}). \end{aligned} \tag{2.3}$$

– each investor has a CARA (constant absolute risk aversion) utility function

$$u(W) = -e^{-a_h W} \qquad \text{with risk aversion coefficient} \quad a_h > 0$$

– maximizing expected utility of wealth,

– given expectations and taking the price p_t on the asset market parametrically,

$\Longrightarrow$ individual asset demand

$$z_{h,t} = \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})} = \frac{E_{h,t}(p_{t+1} + y_{t+1}) - Rp_t}{a_h V_{h,t}(p_{t+1} + y_{t+1})}, \tag{2.4}$$

– where $(p_{t+1} + y_{t+1})$ denotes the cum–dividend price of period $t + 1$ with dividend y_{t+1}.

$\Longrightarrow$ aggregate asset demand at time t is given by

$$\sum_h n_{h,t}\, z_{h,t} = \sum_h n_{h,t} \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})} \tag{2.5}$$

$$= \sum_h n_{h,t} \frac{E_{h,t}(p_{t+1} + y_{t+1})}{a_h V_{h,t}(p_{t+1} + y_{t+1})} - Rp_t \sum_h \frac{n_{h,t}}{a_h V_{h,t}(p_{t+1} + y_{t+1})}, \tag{2.6}$$

– which is a deterministic linear function of current prices p_t, given the characteristics $\left\{(n_{h,t}, a_h, E_{h,t}, V_{h,t})_1^H\right\}$ of traders: their distribution, risk aversion, and their subjective beliefs.

– Notice:

- the first sum is the aggregate of individual subjective mean-variance ratios $(E_{h,t}/a_h V_{h,t})$
- the second sum is the aggregate of individual variance adjusted risk tolerances $1/(a_h V_{h,t})$.

2.2 Equilibrium Asset Prices

– Let aggregate supply of the risky asset be given by the random variable $\tilde{z}_t = z^s + \tilde{\eta}_t$, where $z^s \in \mathbb{R}$ and $\tilde{\eta} \sim \mathcal{U}[-\tau, \tau], \tau \geq 0$.

– Then, the market clearing implies that there exists a unique asset price p clearing the market, i. e.

$$p_t = \frac{1}{R}\left(\sum_h \frac{n_{h,t}}{a_h V_{h,t}(p_{t+1}+y_{t+1})}\right)^{-1}\left(\sum_h n_{h,t}\frac{E_{h,t}(p_{t+1}+y_{t+1})}{a_h V_{h,t}(p_{t+1}+y_{t+1})} - (z^s + \tilde{\eta}_t)\right) \quad (2.7)$$

$$= \frac{1}{R}\left(\frac{SH_t - (z^s + \tilde{\eta}_t)}{AV_t}\right). \quad (2.8)$$

– where the characteristics of agents enter into the function through two sums only, i. e. the aggregate subjective mean–variance ratio

$$SH_t := \sum_h n_{h,t}\frac{E_{h,t}(p_{t+1}+y_{t+1})}{a_h V_{h,t}(p_{t+1}+y_{t+1})}$$

and the aggregate variance adjusted risk tolerance

$$AV_t := \sum_h \frac{n_{h,t}}{a_h V_{h,t}(p_{t+1} + y_{t+1})}$$

$\Longrightarrow$ the equilibrium asset price may be viewed as a family of random mappings $S : \mathbb{R}^{4H} \times \mathbb{R} \to \mathbb{R}$.

– If the characteristics of the agents do not change over time, i. e. if SH and AV are constant,

$\Longrightarrow$ asset prices are given by a simple random variable with additive noise which is a mirror image of random aggregate supply.

– These features are well known and they are essentially consequences of the CARA utility function (see for example Böhm & Chiarella (2005), Böhm (2002), Böhm & Wenzelburger (2005), Hillebrand & Wenzelburger (2006)).

2.3 Heterogeneous Beliefs and expectations formation

– Adopting the popular fundamentalist/chartist model (see for example Brock & Hommes 1997) by assuming that all investors can be grouped into either

- fundamentalists (type f) or
- chartists (type c)

– who differ in the way they form expectations.

2.3.1 Fundamentalists

– Fundamentalists believe that the future market price p_{t+1} is mean reverting to the subjectively perceived so called *fundamental value* p_t^*,

– which is assumed to follow a stationary random walk process[1]

$$p_t^* = p_{t-1}^* \exp\Big[-\frac{\sigma_p^2}{2} + \sigma_p \tilde{\epsilon}_{t-1}\Big], \qquad \tilde{\epsilon} \sim \mathcal{N}(0,1), \qquad \sigma_p \geq 0, \tag{2.9}$$

with constant conditional variance σ_1^2.

$\Longrightarrow$ The predictor of the fundamentalist for the two conditional moments $\psi^f : \mathbb{R}^2 \to \mathbb{R}^2$ are of the form

$$\begin{pmatrix} E_{f,t}(p_{t+1}) \\ V_{f,t}(p_{t+1}) \end{pmatrix} = \psi^f(p_t^\star, p_t) := \begin{pmatrix} p_t^* + g^f(p_t - p_t^*) \\ \sigma_1^2 \end{pmatrix}, \tag{2.10}$$

– where $0 \leq g^f \leq 1$ is the subjective speed of mean reversion and

– the true dividend process is assumed too be known and independent of prices with constant conditional expectations $\mathbb{E}(y_{t+1}) = \bar{y}$ and constant conditional variance $\mathbb{V}(y_{t+1}) = \sigma_y^2$.

– Then, the fundamentalist's demand, after using their forecasting technology, becomes

$$Z^f(p_t, p_t^\star) = \frac{p_t^* + g^f(p_t - p_t^*) + \bar{y} - Rp_t}{a_f(\sigma_1^2 + \sigma_y^2)} = \frac{(1-g^f)p_t^* + \bar{y} + (g^f - R)p_t}{a_f(\sigma_1^2 + \sigma_y^2)}, \tag{2.11}$$

– which is still a linear function in current prices p_t and in the fundamental price $p_t^\star$.

[1] This is essentially a discretization of a continuous time log-normal asset price process with zero-drift.

2.3.2 Chartists

– Chartists' beliefs are generated from the costless information contained in past prices.

– Their predictor combines two principles of statistical/autoregressive time series analysis.

1. they assume that the conditional mean m_t and variance v_t of asset prices can be estimated by a geometric decay processes with decay rate $0 \leq \delta \leq 1$:

$$m_t = (1-\delta)\sum_{k=0}^{\infty} \delta^k p_{t-k-1}, \qquad v_t = (1-\delta)\sum_{k=0}^{\infty} \delta^k [p_{t-k-1} - m_t]^2,$$

which induce the recursive formulae for the first two moments as

$$\begin{cases} m_t = \delta m_{t-1} + (1-\delta) p_{t-1}, \\ v_t = \delta v_{t-1} + \delta(1-\delta)(p_{t-1} - m_{t-1})^2. \end{cases} \tag{2.12}$$

2. they assume that the asset price is itself mean reverting with respect to their estimates, i. e.

– given these estimates, they choose the predictor $\psi^c : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ defined as[2]

$$\begin{pmatrix} E_{c,t}(p_{t+1}) \\ V_{c,t}(p_{t+1}) \end{pmatrix} = \psi^c(p_t, m_t, v_t) := \begin{pmatrix} p_t + g^c(p_t - m_t) \\ \sigma_1^2[1 + b v_t] \end{pmatrix} \qquad \begin{matrix} g^c \in \mathbb{R} \\ b \geq 0 \end{matrix} \tag{2.13}$$

– where $g^c \in \mathbb{R}$ is the subjective speed of reversion of the chartists.

[2] Strictly speaking, the composition of equations (2.12) and (2.13) should be called the predictor.

– Chartists are called *trend followers* when $g^c > 0$ and are called *contrarians* when $g^c < 0$.

– Both groups assume that the price process is mean reverting,

– but they chose different estimates for the mean price process.

– When $b = 0$, they agree on the same constant conditional variance for their forecast.

$\Longrightarrow$ in this case, the impact on the dynamics will result from a disagreement on first moments only.

– Combining equations (2.13) and (2.4) yields the expectations adjusted demand function of a chartist as

$$Z^c(p_t, m_t, v_t) := \frac{p_t + g^c(p_t - m_t) + \bar{y} - Rp_t}{a_c[\sigma_y^2 + \sigma_1^2(1 + bv_t)]} = \frac{\bar{y} - g^c m_t + (1 + g - R)p_t}{a_c[\sigma_y^2 + \sigma_1^2(1 + bv_t)]}, \tag{2.14}$$

– where it is again assumed that chartists know the true dividend process with $E(y_t) = \bar{y}, V(y_t) = \sigma_y^2$.

– **Note:**

- The demand is again linear in the current price p_t.
- with no random term

2.3.3 Temporary equilibrium

Define

$$a := \frac{a_c}{a_f}, \qquad A := a_f[\sigma_1^2 + \sigma_y^2], \qquad B := \frac{\sigma_1^2}{\sigma_1^2 + \sigma_y^2} b.$$

$\Longrightarrow$

– expectations adjusted individual demands (2.11), (2.14) can be written as

$$Z^f(p_t, p_t^\star) = \frac{1}{A}\Big((1-g^f)p_t^* + \bar{y} - (R-g^f)p_t\Big), \tag{2.15}$$

$$Z^c(p_t, m_t, v_t) = \frac{\bar{y} - g^c m_t - (R-1-g^c)p_t}{aA[1+Bv_t]}. \tag{2.16}$$

– Let $n_{f,t}$ and $n_{c,t}$ denote the number of fundamentalists and chartists respectively,

– and define $n_t := n_{f,t} - n_{c,t}$, implying $n_{f,t} = (1+n_t)/2$ and $n_{c,t} = (1-n_t)/2$.

– aggregate asset demand becomes

$$\begin{aligned} Z(p, m, v, n, p_t^*) &:= \frac{1+n_t}{2} Z^f(p_t, p^*) + \frac{1-n_t}{2} Z^c(p_t, m_t, v_t) \\ &= \frac{1+n_t}{2A}\Big(\bar{y} + (1-g^f)p_t^\star - (1+r-g^f)p_t\Big) + \frac{(1-n_t)}{2aA[1+Bv_t]}\Big(\bar{y} - g^c m_t - (r+g^c)p_t\Big), \end{aligned} \tag{2.17}$$

– which is linear in p_t.

Therefore, given the predictors $\psi = (\psi^f, \psi^c)$, and aggregate supply $\tilde{z}_t := z^s + \zeta_t$,

– there exists a unique equilibrium asset price p for all $(m, v, n, p^\star, \zeta)$ which is given by the solution of

$$Z(p, m, v, n, p^\star) = z^s + \zeta. \tag{2.18}$$

defining the price law $S_\psi : \mathbb{R}^5 \longrightarrow \mathbb{R}$

$$p = S_\psi(m, v, n, p^\star, \zeta) := \frac{S_1(m, v, n, p^\star, \zeta)}{S_2(v, n)}, \tag{2.19}$$

which can be factorized as the ratio of two functions

$$S_1(m, v, n, p^\star, \zeta) = (1+n)[p^\star(1-g^f) + \bar{y}] + (1-n)\frac{\bar{y} - g^c m_t}{a[1+Bv]} - 2A(z^s + \zeta),$$

$$S_2(v, n) = (1+n)(1+r-g^f) + (1-n)\frac{r - g^c}{a[1+Bv]}.$$

– When $b = 0 \Longrightarrow B = 0$, the denominator S_2 is a function of n alone.

– for $b = 0$ and n constant S_ψ becomes a linear delay equation of order two with additive noise.

$\Longrightarrow$ the parameter b determines

- whether actual prices are linear or non linear
- whether mean recursive deviation has an influence on dynamics
- an effect exclusively caused through recursive forecasting

2.4 The Dynamical system with constant group sizes

– Suppose that group sizes $n = n^f - n^c$ remain constant over time.

– combining the price law (2.19) with equations (2.12) of the chartist and with the fundamental price law (2.9) yields the system of equations

$$\begin{aligned} p_t &= S(m_t, v_t, n, p_t^\star, \zeta_{t-1}) && (2.20)\\ m_t &= \mathcal{M}(p_{t-1}, m_{t-1}) := \delta m_{t-1} + (1-\delta)p_{t-1}, && (2.21)\\ v_t &= \mathcal{V}((p_{t-1}, v_{t-1}, m_{t-1}) := \delta v_{t-1} + \delta(1-\delta)(p_{t-1} - m_{t-1})^2 && (2.22)\\ p_t^* &= p_{t-1}^* \exp\left[-\frac{\sigma_p^2}{2} + \sigma_p \tilde{\epsilon}_{t-1}\right], \qquad \tilde{\epsilon}_{t-1} \sim \mathcal{N}(0,1), \qquad \sigma_p \geq 0 && (2.23)\\ \zeta_{t-1} &\sim \mathcal{U}[-\tau, \tau], \qquad \tau \geq 0. && (2.24) \end{aligned}$$

– which defines a three dimensional non linear random dynamical system in the sense of Arnold (1998)

$$\begin{aligned} p_t &= \mathcal{S}(p_{t-1}, m_{t-1}, v_{t-1}, n, p_t^\star, \zeta_t) \\ &:= S_\psi(\mathcal{M}(p_{t-1}, m_{t-1}), \mathcal{V}(p_{t-1}, v_{t-1}, m_{t-1}), p_t^\star, \zeta_t) \\ m_t &= \mathcal{M}(m_{t-1}, p_{t-1}) := \delta m_{t-1} + (1-\delta)p_{t-1}, \\ v_t &= \mathcal{V}((p_{t-1}, v_{t-1}, m_{t-1}) := \delta v_{t-1} + \delta(1-\delta)(p_{t-1} - m_{t-1})^2 \end{aligned} \qquad (2.25)$$

– with generating stochastic process $\{\, \tilde{\epsilon}_t, \zeta_t\}_t$.

2.5 Profit performance and endogenous group sizes

– Consider group switching between fundamentalists and chartists on the basis of profit performance.

– Let $U_{f,t}$ and $U_{c,t}$ be the realized profit of the fundamentalists and chartists, respectively,

– defined by $U_{i,t} = R_t z_{i,t-1} - C_i,\ \ i = f, c$, where $C_i \geq 0$ measures the total cost.

– Let the updated fractions be formed on the basis of discrete choice probability

$$n_{i,t} = \frac{exp[\beta U_{i,t-1}]}{\sum_{i=f,c} exp[\beta U_{i,t-1}]} \tag{2.26}$$

– where $\beta \geq 0$ is the *intensity of choice* measuring how fast agents switch

– Let $n_{t-1} = n_{f,t} - n_{c,t}$.

$$\begin{aligned} n_t &= N(p_t, d_t, p_{t-1}, m_{t-1}, v_{t-1}, p^\star_{t-1}) \\ &:= \tanh \frac{\beta}{2} \left[(p_t + y_t - (1+r)p_{t-1}) \left(Z^f(p_{t-1}, p^\star_{t-1}) - Z^c(p_{t-1}, m_{t-1}, v_{t-1}) \right) - C \right], \end{aligned}$$

– where $C = C_f - C_c$.

2.6 The Dynamical system with group switching

– Then, the random dynamical system with switching consists of the set of equations

$$\begin{aligned} p_t &= \mathcal{S}(p_{t-1}, m_{t-1}, v_{t-1}, n_{t-1}, p_t^\star, \zeta_t) \\ m_t &= \mathcal{M}(p_{t-1}, m_{t-1}) := \delta m_{t-1} + (1-\delta)p_{t-1}, \\ v_t &= \mathcal{V}(p_{t-1}, m_{t-1}, v_{t-1}) := \delta v_{t-1} + \delta(1-\delta)(p_{t-1} - m_{t-1})^2 \\ n_t &= \mathcal{N}(p_{t-1}, m_{t-1}, v_{t-1}, n_{t-1}, p_{t-1}^\star, p_t^\star, \zeta_t, d_t) \end{aligned} \tag{2.27}$$

– with the generating processes given by

$$\begin{aligned} p_t^* &= p_{t-1}^* \exp\Big[-\frac{\sigma_p^2}{2} + \sigma_p \epsilon_{t-1} \Big], \\ \epsilon_t &\sim \mathcal{N}(0,1), \qquad \sigma_p \geq 0, \\ \zeta_t &\sim \mathcal{U}[-\tau, \tau], \qquad \tau \geq 0, \\ d_t &\sim \mathcal{U}[-\theta, \theta], \qquad \theta \geq 0, \end{aligned} \tag{2.28}$$

– and where

$$\begin{aligned} & \mathcal{S}(p_{t-1}, m_{t-1}, v_{t-1}, n_{t-1}, p_t^\star, \zeta_t) \\ := \quad & S_\psi(\mathcal{M}(p_{t-1}, m_{t-1}), \mathcal{V}((p_{t-1}, m_{t-1}, v_{t-1}), n_{t-1}, p_t^\star, \zeta_t)). \end{aligned} \tag{2.29}$$

and

$$\begin{aligned} & \mathcal{N}(p_{t-1}, m_{t-1}, v_{t-1}, n_{t-1}, p_{t-1}^\star, p_t^\star, \zeta_t, d_t) \\ := \quad & N(\mathcal{S}(p_{t-1}, m_{t-1}, v_{t-1}, n_{t-1}, p_t^\star, \zeta_t), d_t, p_{t-1}, m_{t-1}, v_{t-1}, p_{t-1}^\star). \end{aligned} \tag{2.30}$$

$\Longrightarrow$ one obtains

– a four dimensional system in (p, m, v, n)

– with a three dimensional random generating process $(\epsilon_t, \zeta_t, d_t)$, assumed to be i. i. d. processes.

– Figure 2.1 displays the interaction between the equilibrium price map S_ψ and

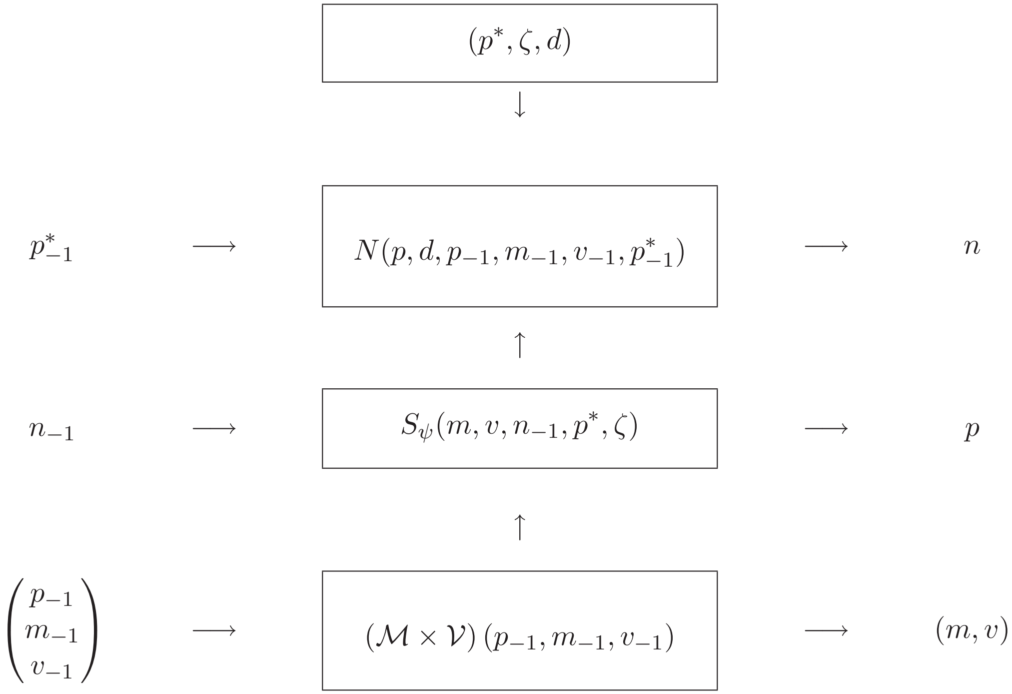


Figure 2.1: Flow Chart

– Observe that the realization of the dividend process d_t matters only for the switching,

– while the other two enter into the determination of the equilibrium price.

3 Cycles, Periodicities, and Bifurcations of random dynamical systems

– If deterministic maps $F : X \times \mathbb{R}^m \to X$
with interesting dynamic behavior are subjected to noise $\xi_t \in \mathbb{R}^m$, $\{\xi_t\}_t$ with small variation:

– what is relationship between the long run behavior of F for a given value of ξ and its perturbed version for small noise?

– how much complexity/periodicity derives from the deterministic function F,

– how much from the noise process $\{\xi_t\}$?

– what are the bifurcation scenarios?

– What is the relationship of the long run behavior of

- the deterministic mapping and
- the random system when there is only small noise?
- is there smoothing by noise?

3.1 Periodicity and Stationarity: Statistical dynamics

Definition 3.1

A continuous mapping $F : X \times \mathbb{R}^m \longrightarrow X$ implying a random difference equation

$$x_{n+1} = F(x_n, \xi_n), \qquad \{\xi_n\}_{n\in\mathbb{N}}$$

exhibits a periodicity of order $k \geq 1$ for a Markov operator $\bar{P}$ for densities, if there exists a finite list of densities $f_i \in D^1 \subset L^1$, $i = 1, \cdots, k$ with

$$supp f_i \cap supp f_j = \emptyset \qquad all \qquad i \neq j \tag{3.1}$$

such that for all $i = 1, \cdots, k$

$$\bar{P}^k f_i = f_i \tag{3.2}$$

$$\bar{P} f_i = f_{i+1} \qquad mod \quad k \tag{3.3}$$

$\Longrightarrow$ dynamic movement will occur between k disjoint sets

– with fixed deterministic sequence

– implying frequency k and

– a distribution with k modes[3]

[3] see Lasota & Mackey (1994)

3.2 Random fixed points

Definition 3.2

[4]*Consider a random family of continuous mappings* $F : X \times \mathbb{R}^m \longrightarrow X$ *with real noise process* $u_t = u \circ \vartheta^t$, $u : \Omega \longrightarrow \mathbb{R}^m$ *measurable, over the ergodic dynamical system* $(\Omega, \mathcal{F}, \mathbb{P}, (\vartheta^t))$.

For any x_0, *the iteration of the map* F *under the perturbation* ω *induces a measurable map* $\phi : \mathbb{Z} \times \Omega \times X \longrightarrow X$ *defined by*

$$\phi(t, \omega, x_0) := \begin{cases} (F(\vartheta^{t-1}\omega) \circ \ldots \circ F(\omega))x_0 & \textit{if} \quad t > 0 \\ x_0 & \textit{if} \quad t = 0 \end{cases} \tag{3.4}$$

such that $x_t = \phi(t, \omega, x_0)$ *is the state of the system at time* t.

A ***random fixed point*** *of* ϕ *is a random variable* $x_* : \Omega \longrightarrow X$ *on* $(\Omega, \mathcal{F}, \mathbb{P})$ *such that almost surely*

$$x_*(\vartheta\omega) = \phi(1, \omega, x_*(\omega)) = F(x_*(\omega), u(\omega)) \quad \textit{for all} \quad \omega \in \Omega', \tag{3.5}$$

where $\Omega' \subset \Omega$ *is a* ϑ*-invariant set of full measure,* $\mathbb{P}(\Omega') = 1$.

[4] (see p. 483 Arnold 1998), also Schmalfuß (1996, 1998))

Properties of random fixed points

– a random fixed point is a stationary solution of the stochastic difference equation.

$\Longrightarrow x_*(\vartheta^{t+1}\omega) = F(x_*(\vartheta^t\omega), u(\vartheta^t\omega))$ for all times t.

– Stationarity and ergodicity of ϑ implies that $\{x_*(\vartheta^t)\}_{t\in\mathbb{N}}$ is stationary and ergodic.

– The random fixed point x_* induces an invariant distribution $x_*\mathbb{P}$ on $\mathbb{R}^K$ defined by

$$(x_*\mathbb{P})(B) \;:=\; \mathbb{P}((x_*)^{-1}(B)) = \mathbb{P}\{\omega \in \Omega \,|\, x_*(\omega) \in B\} \tag{3.6}$$

The invariance of the measure $\mathbb{P}$ under the shift ϑ, i. e. $\mathbb{P} = \vartheta\mathbb{P} = \mathbb{P} \circ \vartheta^{-1}$ implies the invariance of $x_*\mathbb{P}$, since

$$((x_*\vartheta)\mathbb{P})(B) = \mathbb{P}(\vartheta^{-1} \circ (x_*)^{-1}(B)) = \mathbb{P}(x_*)^{-1}(B)) = (x_*\mathbb{P})(B) \tag{3.7}$$

– If, in addition, $\mathbb{E}||x_*|| < \infty$, then

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} 1_B(x_*(\vartheta^t\omega)) = x_*\mathbb{P}(B) := \mathbb{P}\{\omega \in \Omega | x_*(\omega) \in B\}$$

for every $B \in \mathcal{B}(X)$.

– In other words, the empirical law of an orbit is well defined and it is equal to the distribution $x_*\mathbb{P}$ of x_*.

Random fixed points of order k and Periodicities

Definition 3.3

A ***random fixed point of order*** $k \geq 1$ *is a random variable* $x_* : \Omega \longrightarrow X$ *on* $(\Omega, \mathcal{F}, \mathbb{P})$ *such that for* $j = 0, \ldots, k-1$ *almost surely*

$$x_*(\vartheta^{k+j}\omega) = \phi(k, \omega, (x_*(\vartheta^j\omega)) \quad \textit{for all} \quad \omega \in \Omega', \tag{3.8}$$

where $\Omega' \subset \Omega$ *is a* ϑ*-invariant set of full measure,* $\mathbb{P}(\Omega') = 1$*.*

Properties

– a random fixed point of order k induces k distinct stationary solutions

– does existence of k-th order fixed point imply k-th order periodicity of k densities (of Markov kernels) with disjoint supports?

– answer unclear in general, true for smooth maps under small noise

– Does existence of k-th order periodicity of k densities (of Markov kernels) with disjoint support imply existence of k-th order fixed point?

– answer most likely yes for smooth maps; example by Keener (1980)

3.3 Bifurcations of random dynamical systems

Definition 3.4

*A random fixed point x_* is called **asymptotically stable** with respect to a norm $\|\cdot\|$, if there exists a random neighborhood $U(\omega) \subset X$, $\omega \in \Omega$ such that $\mathbb{P}$- a.s.*

$$\lim_{t\to\infty} ||\phi(t,\omega,x_0) - x_*(\vartheta^t\omega)|| = 0 \quad \textit{for all } x_0(\omega) \in U(\omega).$$

Bifurcations

– Consider a parameterized random family of continuous mappings $F : \mathbb{R}^k \times X \times \mathbb{R}^m \longrightarrow X$

– with real noise process $u_t = u \circ \vartheta^t$, $u : \Omega \longrightarrow \mathbb{R}^m$ measurable,

– bifurcation parameter $\mu \in \mathbb{R}^k$

$\Longrightarrow$ for every $\mu \in \mathbb{R}^k$ its associated random flow is defined by

$$\phi_\mu(t,\omega,x_0) := \begin{cases} (F_\mu(\vartheta^{t-1}\omega) \circ \ldots \circ F_\mu(\omega))x_0 & \text{if} \quad t > 0 \\ x_0 & \text{if} \quad t = 0 \end{cases} \tag{3.9}$$

such that $x_t = \phi_\mu(t,\omega,x_0)$ is the state of the system at time t.

– if F is smooth, standard methods of bifurcation theory can be applied to $F(\mu,\cdot,\lambda)$

– intuitive that the type of fixed points and bifurcations of $F(\mu,\cdot,\xi)$ for given μ and fixed λ are preserved

– when i. i. d. perturbations occur with $\xi \sim [\lambda-\epsilon, \lambda+\epsilon]$ are preserved for small ϵ.

4 Dynamics with constant group size

without noise

– consider first the dynamical system when there is no noise, i.e. with $\sigma_p = \tau = 0$.

$\Longrightarrow$ (2.25) becomes a 3-dimensional deterministic system, for which we obtain the following stability result.

Proposition 4.1
Let $(\bar{p}, \bar{m}, \bar{v})$ denote the steady state of the deterministic system (2.25) with $\bar{p} = p^{*5}$. Then

a). *the steady state $(\bar{p}, \bar{m}, \bar{v})$ of the deterministic system (2.25) is uniquely given by*

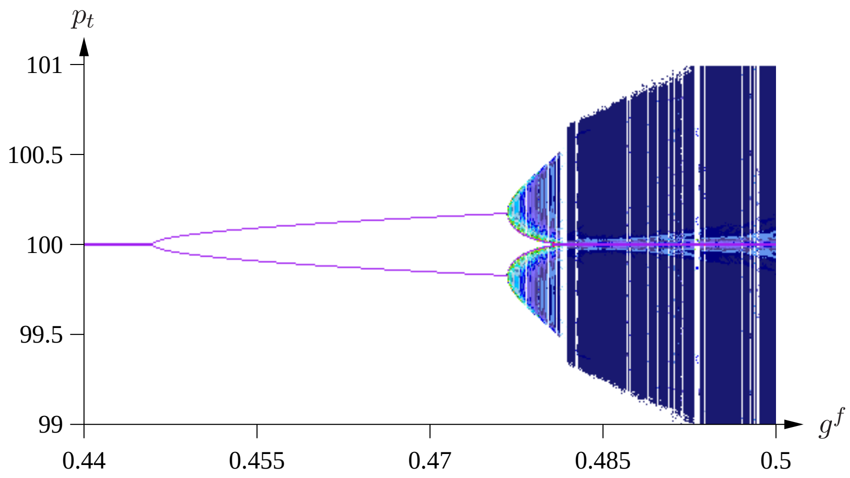
$$\bar{p} = \bar{m} = \frac{1}{R-1}\left[\bar{y} - \frac{2Az^s}{(1+1/a)+(1-1/a)n}\right], \qquad \bar{v} = 0; \tag{4.1}$$

b). *the steady state is globally asymptotically stable if $n = 1$;*

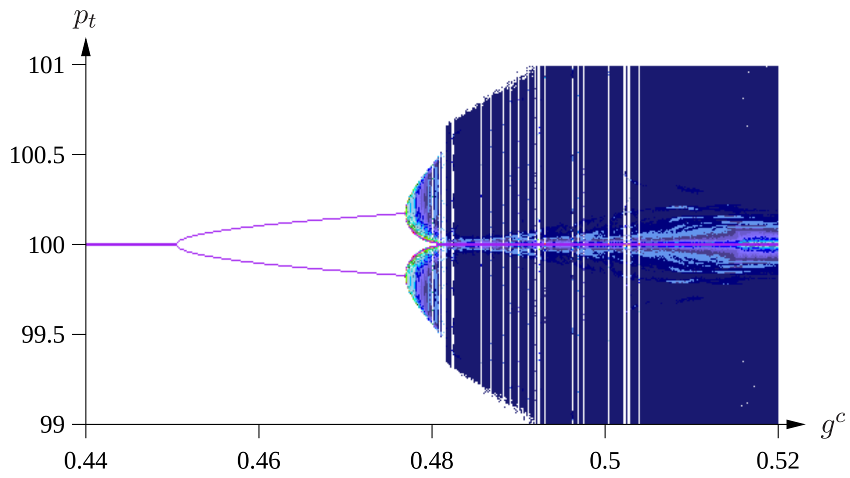
c). *for $n \in [-1, 1)$, the steady state is locally asymptotically stable (LAS) if*

$$g^c < g_{-1} := \frac{1+\delta}{2}\bar{g}, \qquad \bar{g} := r + a(1 + r - g^f)\frac{1+n}{1-n}.$$

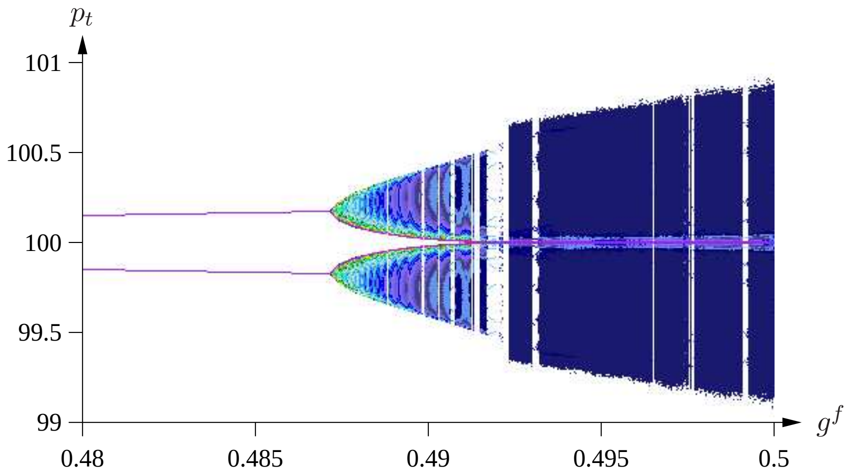
d). *a flip bifurcation occurs at $g^c = g_{-1}$ followed by a Neimark-Sacker bifurcation of the second iterate.*



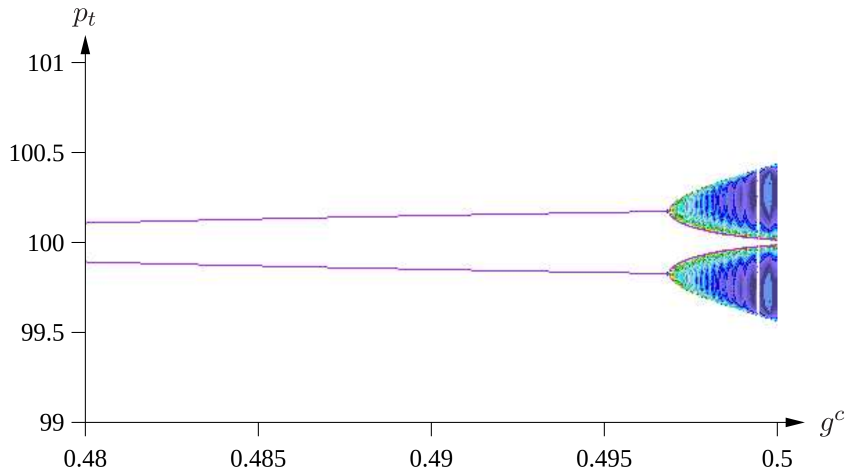
(a) $\sigma_p = \tau = 0$



(b) $\sigma_p = \tau = 0$

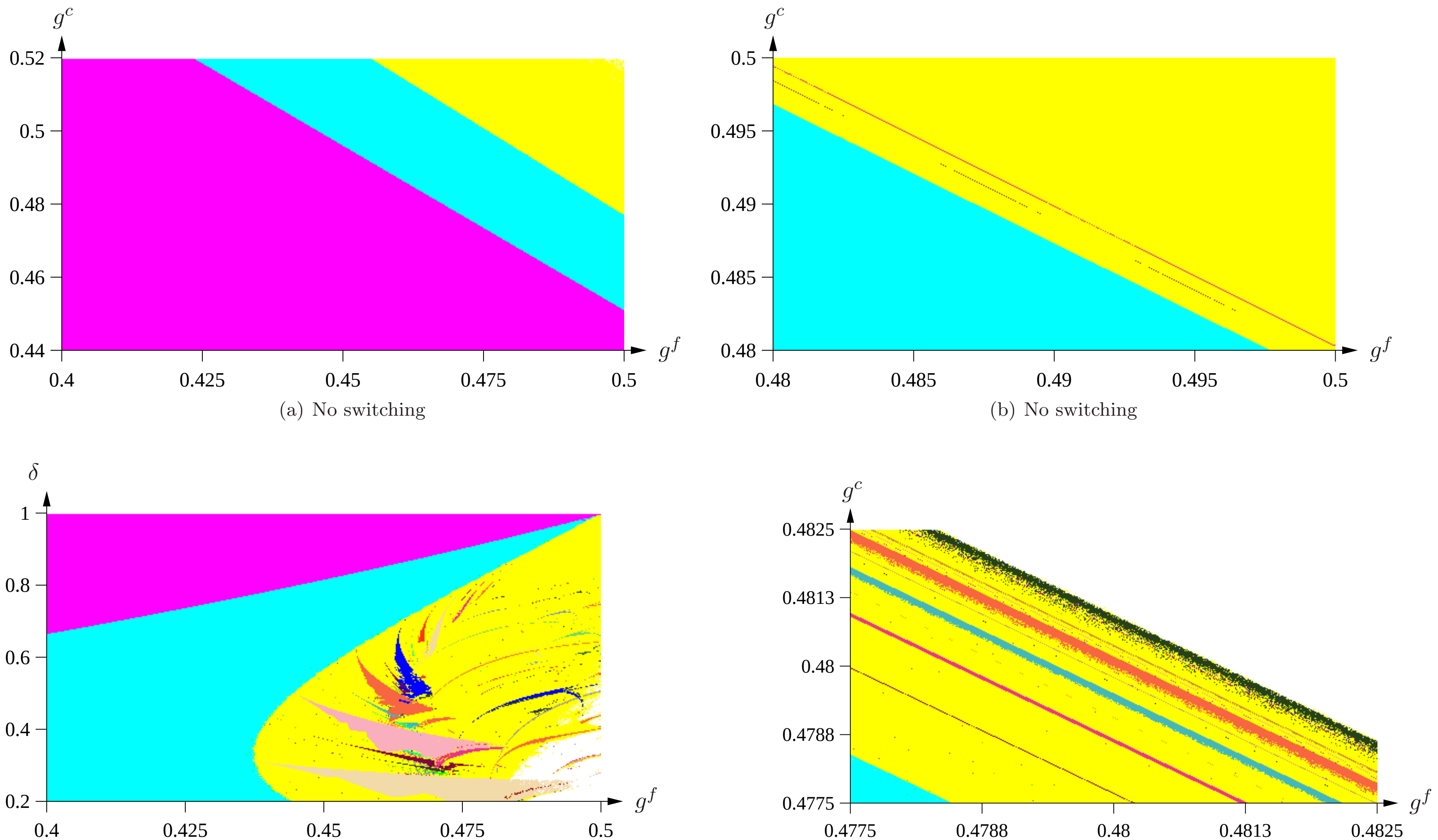


(c) $\sigma_p = \tau = 0$



(d) $\sigma_p = \tau = 0$

Figure 4.1: Role of parameters g^c and g^f; $a^c = a^f = 0.05$, $g^c = g^f = 0.5, b = 2, \delta = 0.8$



(a) No switching

(b) No switching

(c) No switching

(d) No switching

Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8
Order 9	Order 10	Order 11	Order 12	Order 13	Order 14	Order 15	Order 16
Order 17	Order 18	Order 19	Order 20	Order 21	Order 22	Order 23	Order 24
Order 25	Order 26	Order 27	Order 28	Order 29	Order 30	oor	aperiodic

(e) Color code

Figure 4.2: No noise; Role of parameters g^c and g^f;

Features

– diagrams show a Neimark-Sacker originating after a period doubling

– bifurcations show respective periodic windows corresponding to periodicity tongues

– since the Neimark-Sacker originates from a two-cycle,
it is unclear which periodicity they will have

– However,
the convergence of the periodic solutions is VERY SLOW, convergence requires at least ten thousand iterations!!

– they are difficult to be identified in the periodicity diagram (4.2) of $g^c - g^f$.

– They show better in the $g^f - \delta$ diagram.

with noise

Proposition 4.2
Let $(\bar{p}, \bar{m}, \bar{v})$ denote the unique steady state of the deterministic system (2.25) with $\bar{p} = p^*$, and let $\tau > 0$.

a). *there exists a corresponding unique random fixed point $(p_\star, m_\star, v_\star)$*

b). *$(p_\star, m_\star, v_\star)$ is globally asymptotically stable if $n = 1$, ;*

c). *for $n \in [-1, 1)$, $(p_\star, m_\star, v_\star)$ is locally asymptotically stable (LAS) if*

$$g^c < g_{-1} := \frac{1+\delta}{2}\bar{g}, \qquad \bar{g} := r + a(1 + r - g^f)\frac{1+n}{1-n}.$$

d). *for τ small, the system undergoes a period doubling bifurcation for parameter values at $g^c = g_{-1}$, inducing a random fixed point of order two*

e). *followed by a "random Neimark-Sacker" bifurcation for changes in g_f and g_c, i. e.*

f). *whose stationary distribution has a projection of the form of two disjoint "random rings" in $\mathbb{R}^2$*

g). *as τ increases, fixed points of order two as well as the ring structure disappear*

h). *no evidence was found for stable random fixed points of order higher than two.*

Remarks, comments, and conjectures

– diagrams show the appearance of the "random Neimark-Sacker" originating after a period doubling

– regions of periodicity tongues could not be identified, they may not exist

– since the Neimark-Sacker originates from a two-cycle,
it is unclear which periodicity they will have

– However, there must be stationary solutions with higher order periodicities

– these results may not hold with noise of the fundamental price or with switching

– However, numerically the same features could be portrayed under all forms of noise

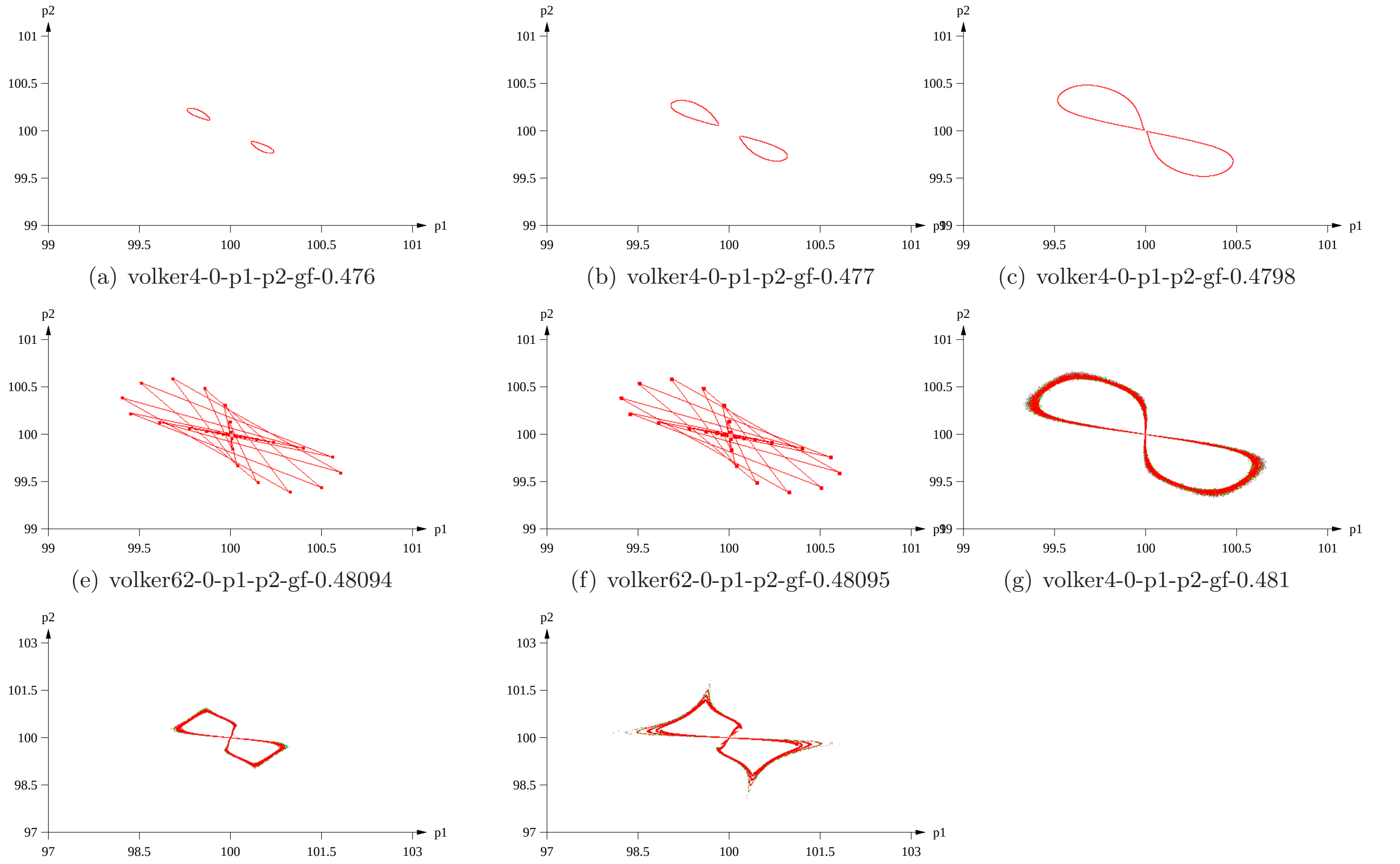
– MORE search/research needed !!!

5 Numerical Results: constant group sizes

feste Parameter		variable Parameter			
Parameter	Standardwert	Parameter	Standardwert	Min-Wert	Max-Wert
beta	0.01	sigps	0.0	0.0	10^{-5}
delta	0.8	af	0.05	0.01	0.1
sigysquared	0.2	ac	0.05	0.01	0.1
b	2	gc	0.5	0.4	0.6
C	0.0	gf	0.5	0.4	0.6
ybar	0.02	tau	0.0	0.0	0.01
zs	0.0	theta	0.0	0.0	2
r	0.05	swich1	0	0	1
K	250	swichf	0	0	1
p0	100	swichm	0	0	1
n0	0				

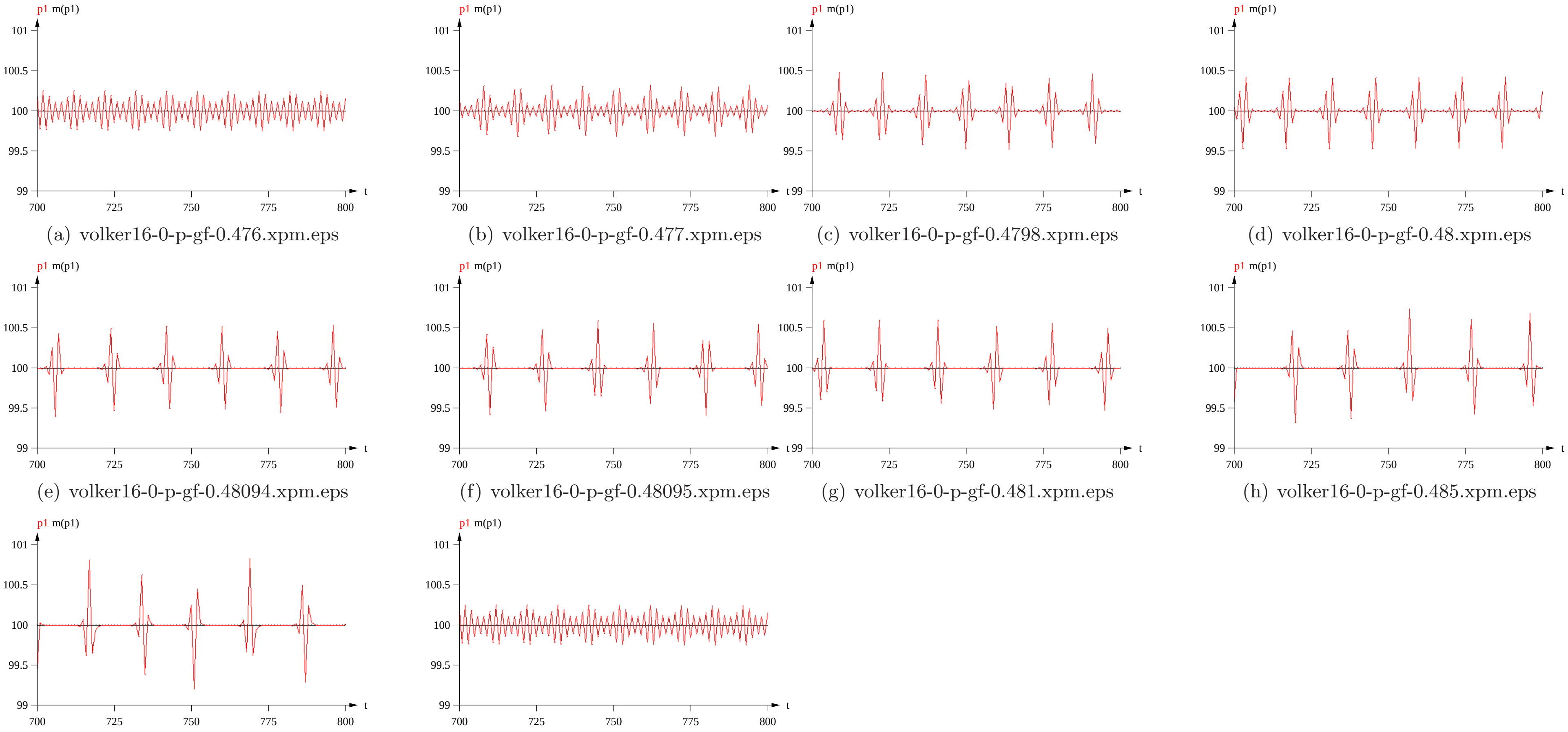
5.1 Variation of g^f - no switch - no noise

The standard parameter set	
Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigps	0.0
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.0
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	0
swichf	0
swichm	0



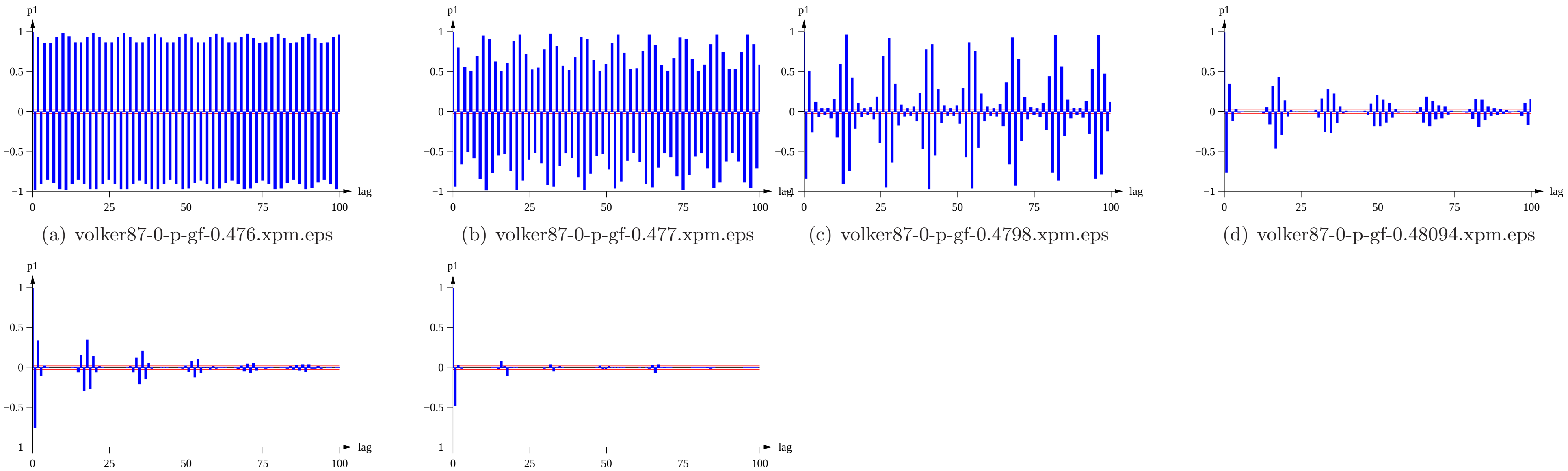
(a) volker4-0-p1-p2-gf-0.476
(b) volker4-0-p1-p2-gf-0.477
(c) volker4-0-p1-p2-gf-0.4798
(d) volker62-0-p1-p2-gf-0.48
(e) volker62-0-p1-p2-gf-0.48094
(f) volker62-0-p1-p2-gf-0.48095
(g) volker4-0-p1-p2-gf-0.481
(h) volker4-0-p1-p2-gf-0.485
(i) volker4-0-p1-p2-gf-0.49
(j) volker4-0-p1-p2

Figure 5.1: **Role of subjective speed of reversion g_f:** Attractors



(a) volker16-0-p-gf-0.476.xpm.eps
(b) volker16-0-p-gf-0.477.xpm.eps
(c) volker16-0-p-gf-0.4798.xpm.eps
(d) volker16-0-p-gf-0.48.xpm.eps
(e) volker16-0-p-gf-0.48094.xpm.eps
(f) volker16-0-p-gf-0.48095.xpm.eps
(g) volker16-0-p-gf-0.481.xpm.eps
(h) volker16-0-p-gf-0.485.xpm.eps
(i) volker16-0-p-gf-0.49.xpm.eps
(j) volker16-0-p.xpm.eps

Figure 5.2: **Role of subjective speed of reversion g_f:** time series



(a) volker87-0-p-gf-0.476.xpm.eps

(b) volker87-0-p-gf-0.477.xpm.eps

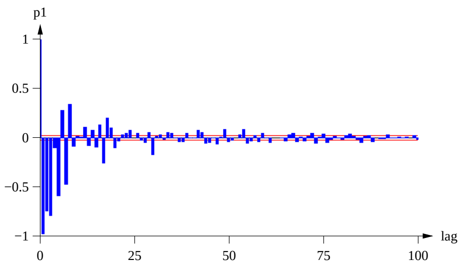
(c) volker87-0-p-gf-0.4798.xpm.eps

(d) volker87-0-p-gf-0.48094.xpm.eps

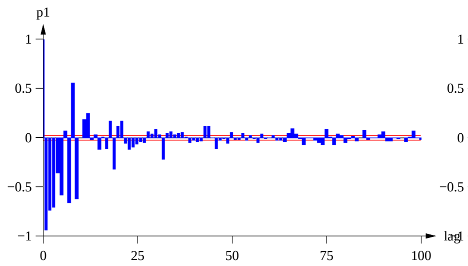
(e) volker87-0-p-gf-0.481.xpm.eps

(f) volker87-0-p.xpm.eps

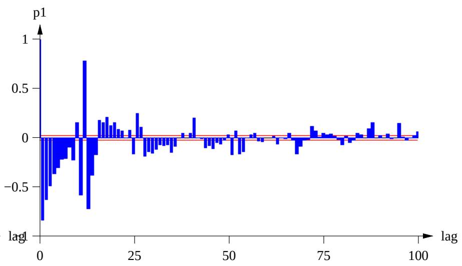
Figure 5.3: **Role of subjective speed of reversion g_f:** ACF



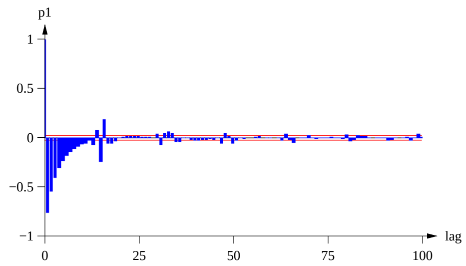
(a) volker88-0-p-gf-0.476.xpm.eps



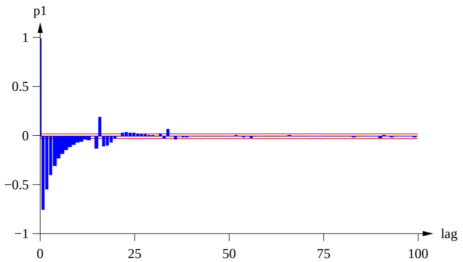
(b) volker88-0-p-gf-0.477.xpm.eps



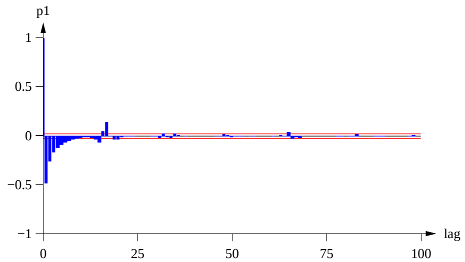
(c) volker88-0-p-gf-0.4798.xpm.eps



(d) volker88-0-p-gf-0.48094.xpm.eps

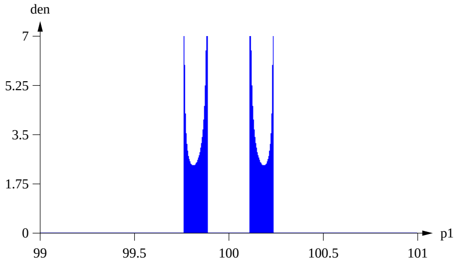


(e) volker88-0-p-gf-0.481.xpm.eps

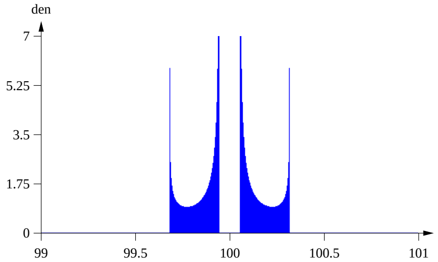


(f) volker88-0-p.xpm.eps

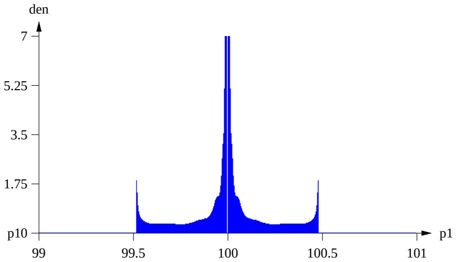
Figure 5.4: **Role of subjective speed of reversion g_f:** PACF



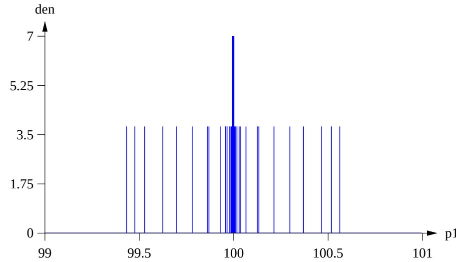
(a) volker41-0-p-gf-0.476.xpm.eps



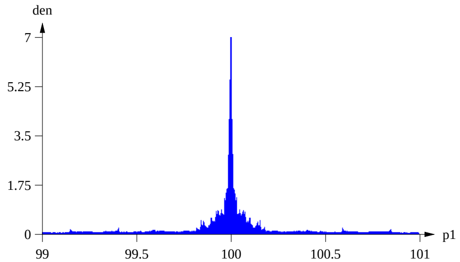
(b) volker41-0-p-gf-0.477.xpm.eps



(c) volker41-0-p-gf-0.4798.xpm.eps



(d) volker41-0-p-gf-0.48094.xpm.eps



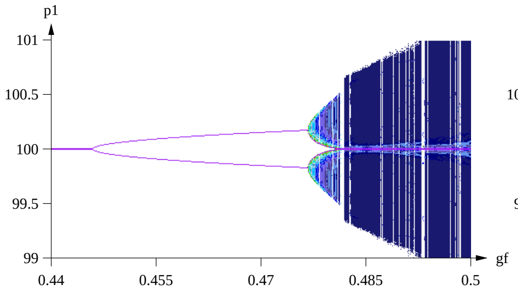
(e) volker41-0-p.xpm.eps

Figure 5.5: **Role of subjective speed of reversion g_f:** Densities

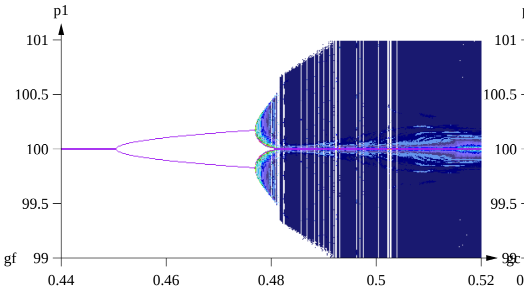
statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48094$	$g^f = 0.5$
mean	100	100	100	100	100
variance	0.0302131	0.0313924	0.0343456	0.0344019	0.0777563
standard deviation	0.173819	0.177179	0.185326	0.185477	0.278848
skewness	9.82925e-06	5.703e-05	-3.44669e-05	-0.000160688	-0.00301204
kurtosis	-1.74968	-0.920925	1.64379	3.58844	8.40534
quantile (0.95)	100.292	100.296	100.299	100.262	100.202

5.2 Variation of g^f - no switch, but noise($\tau = 10^{-3}$)

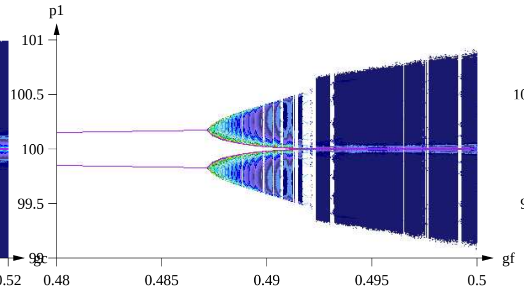
The standard parameter set	
Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigps	0.0
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.001
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	0
swichf	0
swichm	0



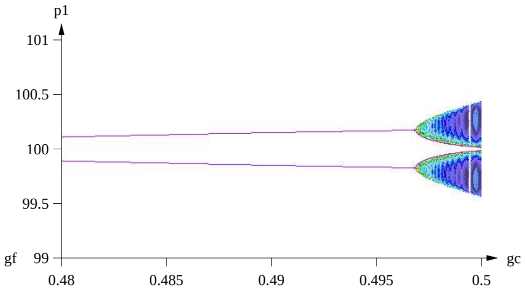
(a) $\sigma_p = \tau = 0$



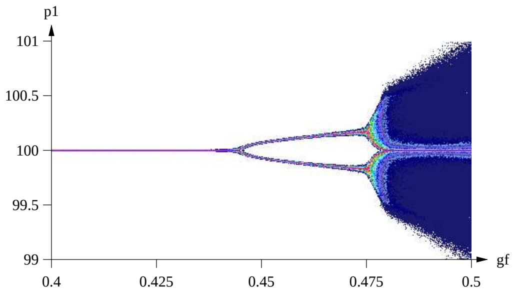
(b) $\sigma_p = \tau = 0$



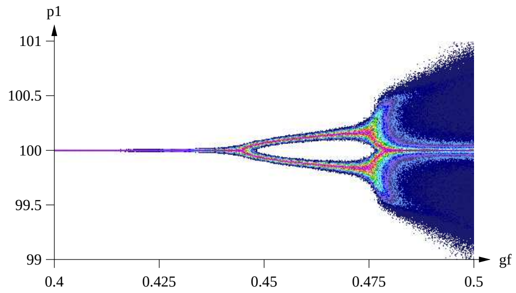
(c) $\sigma_p = \tau = 0$



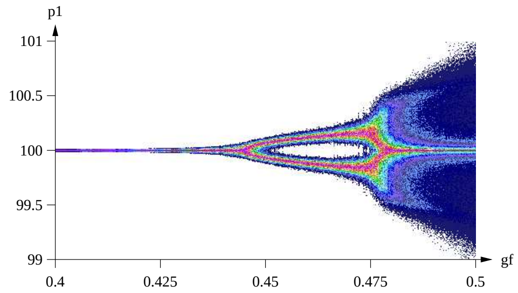
(d) $\sigma_p = \tau = 0$



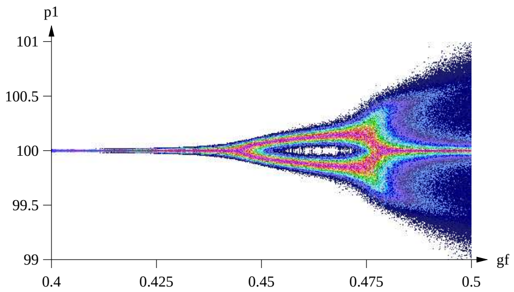
(e) volker31-0-p-tau-0.001-gf-0.4-0.5



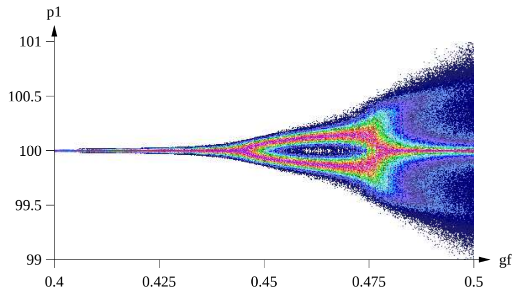
(f) volker31-0-p-tau-0.003-gf-0.4-0.5



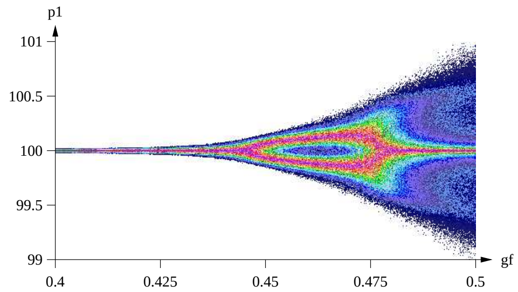
(g) volker31-0-p-tau-0.005-gf-0.4-0.5



(h) volker31-0-p-tau-0.007-gf-0.4-0.5

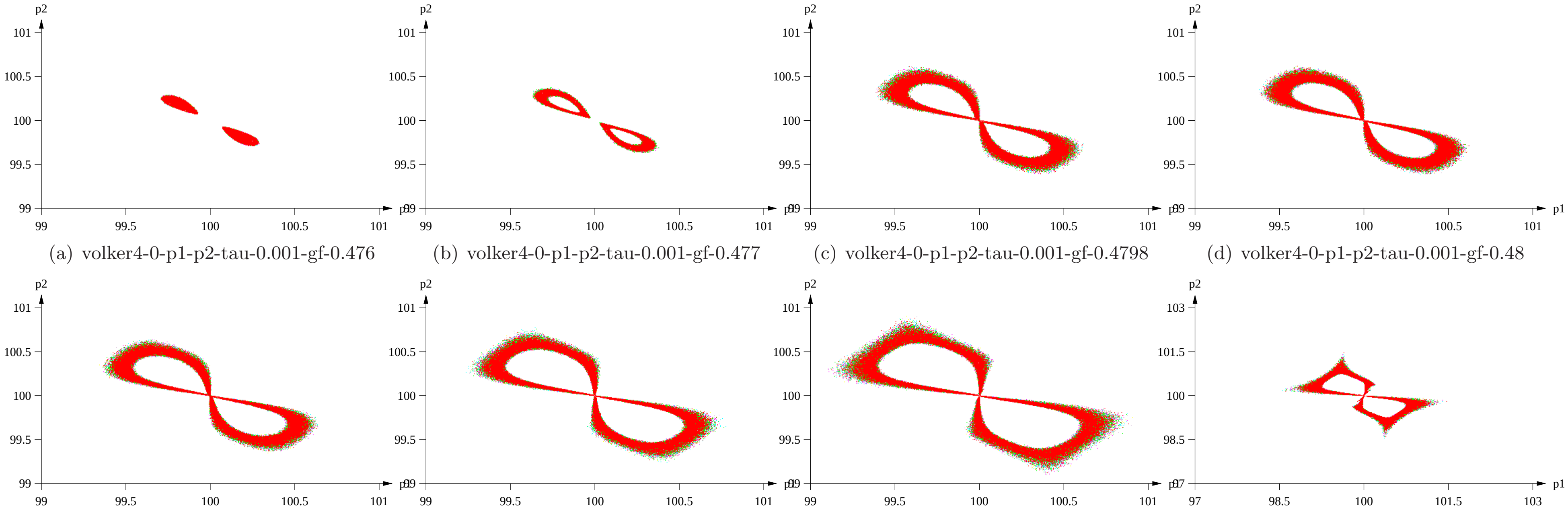


(i) volker31-0-p-tau-0.008-gf-0.4-0.5



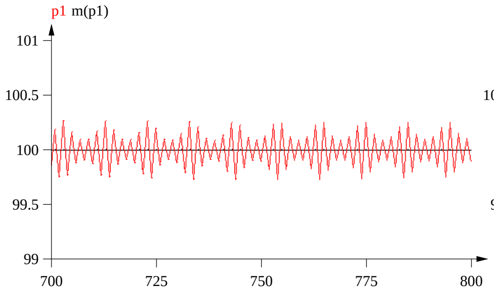
(j) volker31-0-p-tau-0.01-gf-0.4-0.5

Figure 5.6: **The role of noisy supply:** Bifurcations

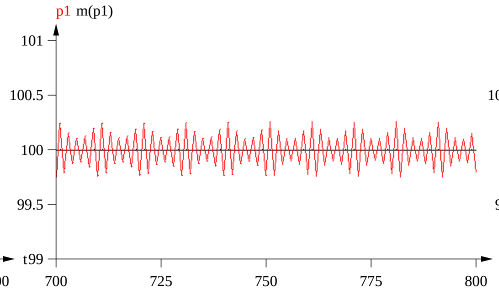


(a) volker4-0-p1-p2-tau-0.001-gf-0.476
(b) volker4-0-p1-p2-tau-0.001-gf-0.477
(c) volker4-0-p1-p2-tau-0.001-gf-0.4798
(d) volker4-0-p1-p2-tau-0.001-gf-0.48
(e) volker4-0-p1-p2-tau-0.001-gf-0.481
(f) volker4-0-p1-p2-tau-0.001-gf-0.485
(g) volker4-0-p1-p2-tau-0.001-gf-0.49
(h) volker4-0-p1-p2-tau-0.001

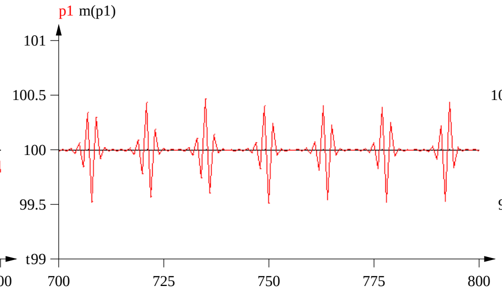
Figure 5.7: **The role of noisy supply:** Attractorplots



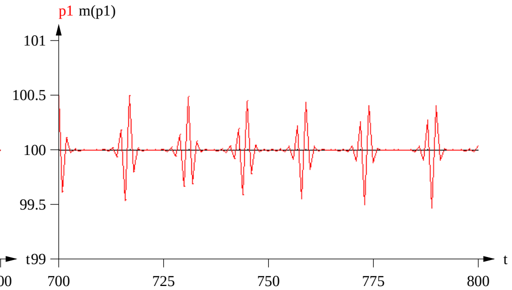
(a) volker16-0-p-tau-0.001-gf-0.476



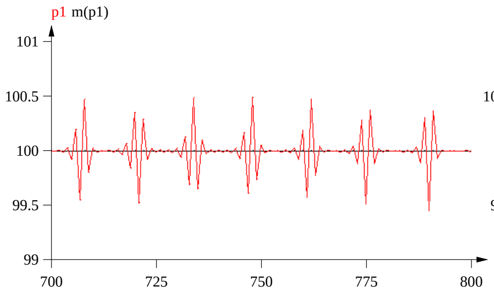
(b) volker16-0-p-tau-0.001-gf-0.477



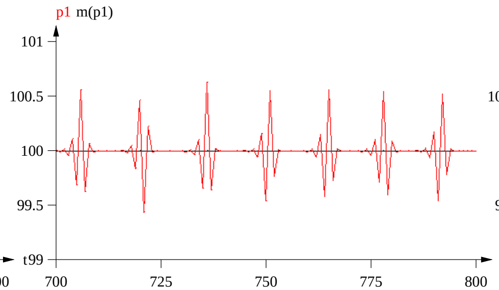
(c) volker16-0-p-tau-0.001-gf-0.4798



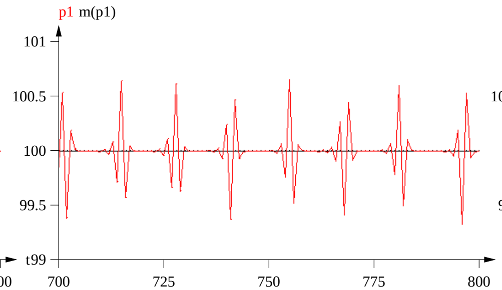
(d) volker16-0-p-tau-0.001-gf-0.48



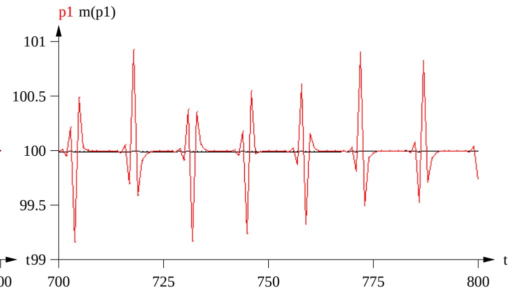
(e) volker16-0-p-tau-0.001-gf-0.481



(f) volker16-0-p-tau-0.001-gf-0.485

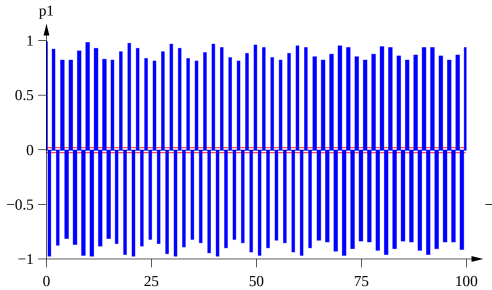


(g) volker16-0-p-tau-0.001-gf-0.49

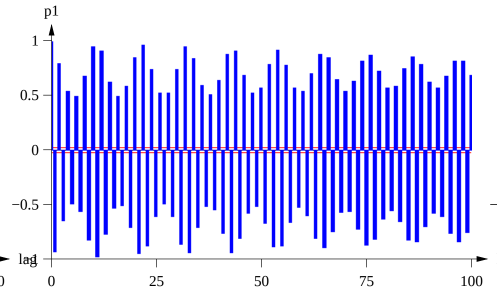


(h) volker16-0-p-tau-0.001

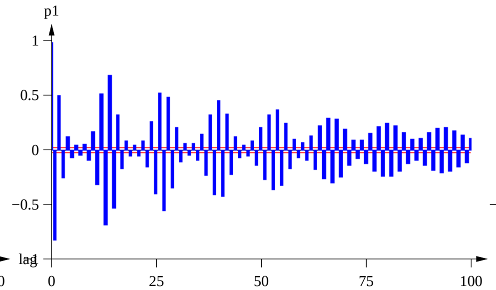
Figure 5.8: **The role of noisy supply:** time series



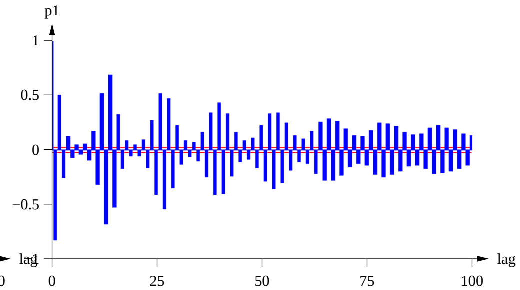
(a) volker87-0-p-tau-0.001-gf-0.476



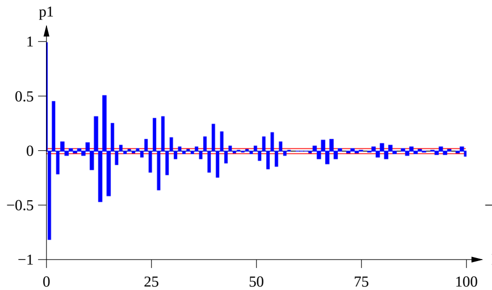
(b) volker87-0-p-tau-0.001-gf-0.477



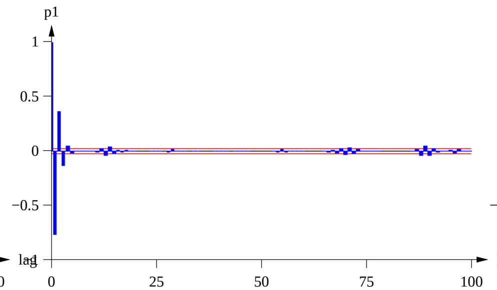
(c) volker87-0-p-tau-0.001-gf-0.4798



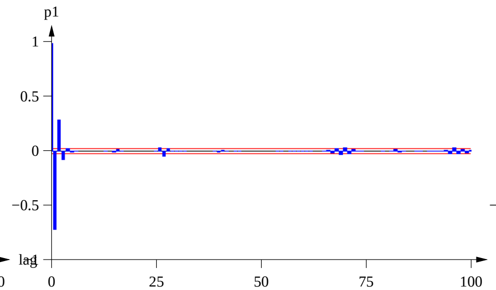
(d) volker87-0-p-tau-0.001-gf-0.48



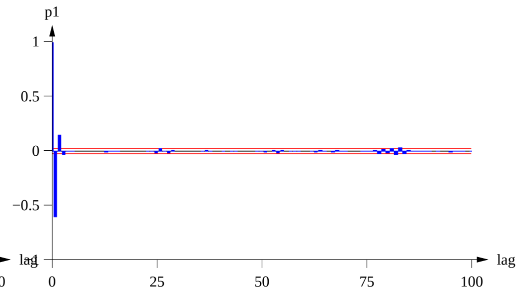
(e) volker87-0-p-tau-0.001-gf-0.481



(f) volker87-0-p-tau-0.001-gf-0.485

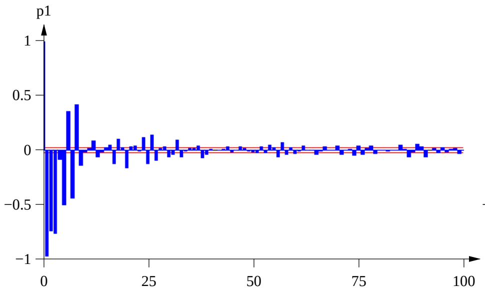


(g) volker87-0-p-tau-0.001-gf-0.49

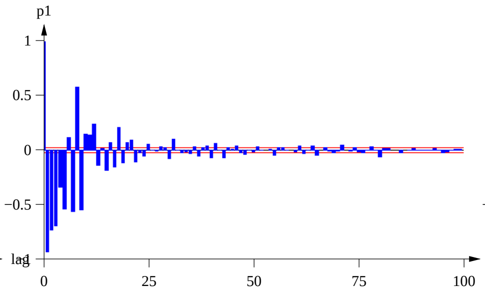


(h) volker87-0-p-tau-0.001

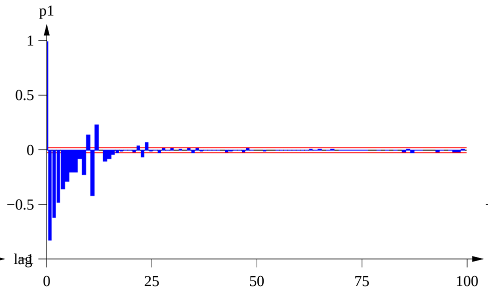
Figure 5.9: **The role of noisy supply:** ACF



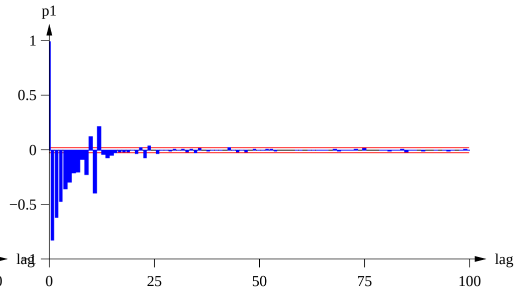
(a) volker88-0-p-tau-0.001-gf-0.476



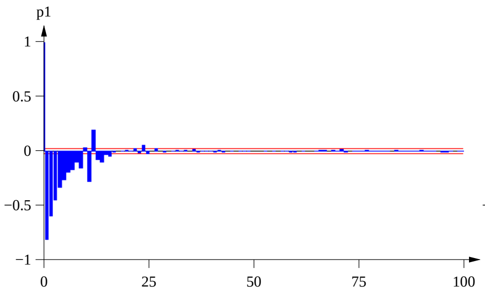
(b) volker88-0-p-tau-0.001-gf-0.477



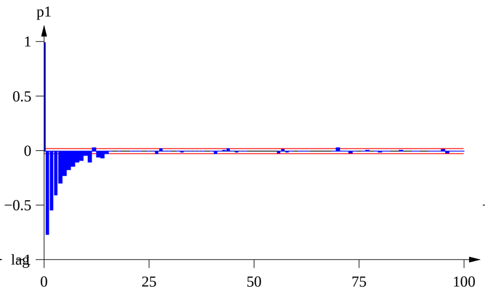
(c) volker88-0-p-tau-0.001-gf-0.4798



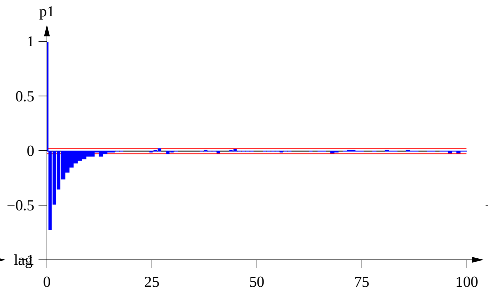
(d) volker88-0-p-tau-0.001-gf-0.48



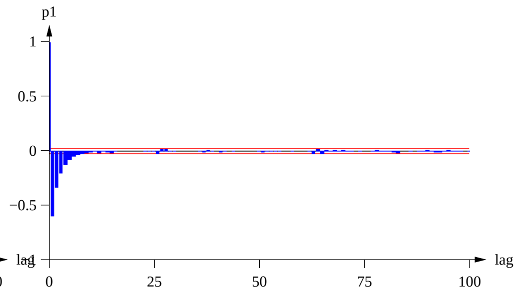
(e) volker88-0-p-tau-0.001-gf-0.481



(f) volker88-0-p-tau-0.001-gf-0.485

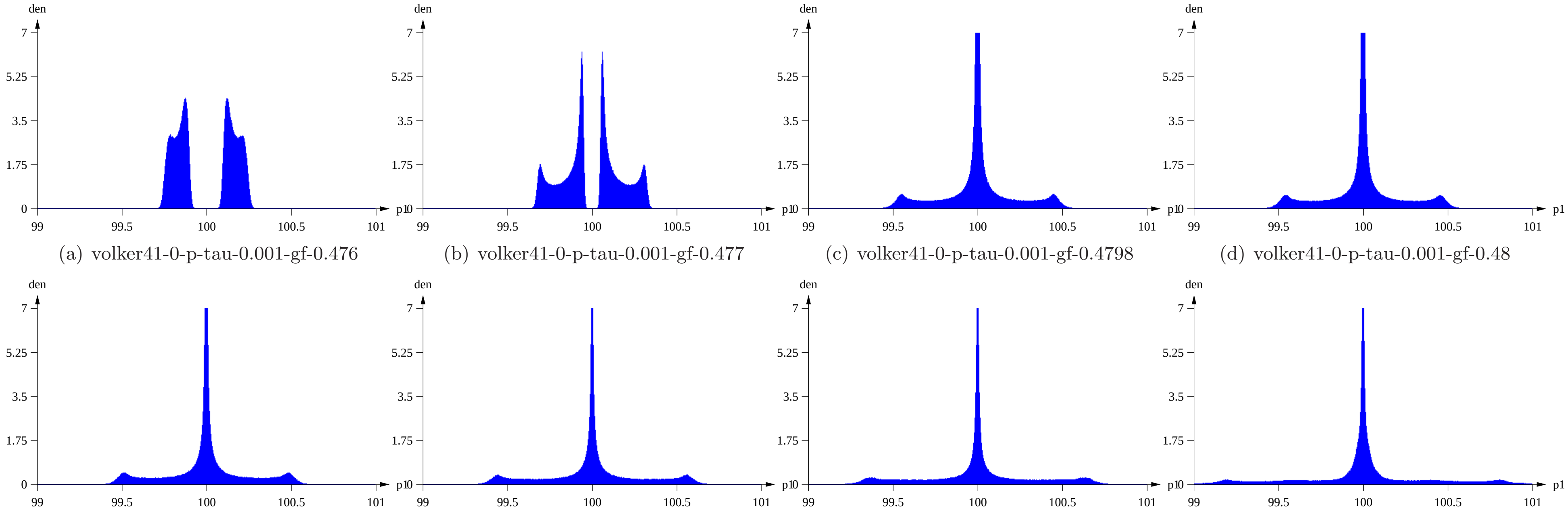


(g) volker88-0-p-tau-0.001-gf-0.49



(h) volker88-0-p-tau-0.001

Figure 5.10: **The role of noisy supply:** PACF



(a) volker41-0-p-tau-0.001-gf-0.476
(b) volker41-0-p-tau-0.001-gf-0.477
(c) volker41-0-p-tau-0.001-gf-0.4798
(d) volker41-0-p-tau-0.001-gf-0.48
(e) volker41-0-p-tau-0.001-gf-0.481
(f) volker41-0-p-tau-0.001-gf-0.485
(g) volker41-0-p-tau-0.001-gf-0.49
(h) volker41-0-p-tau-0.001

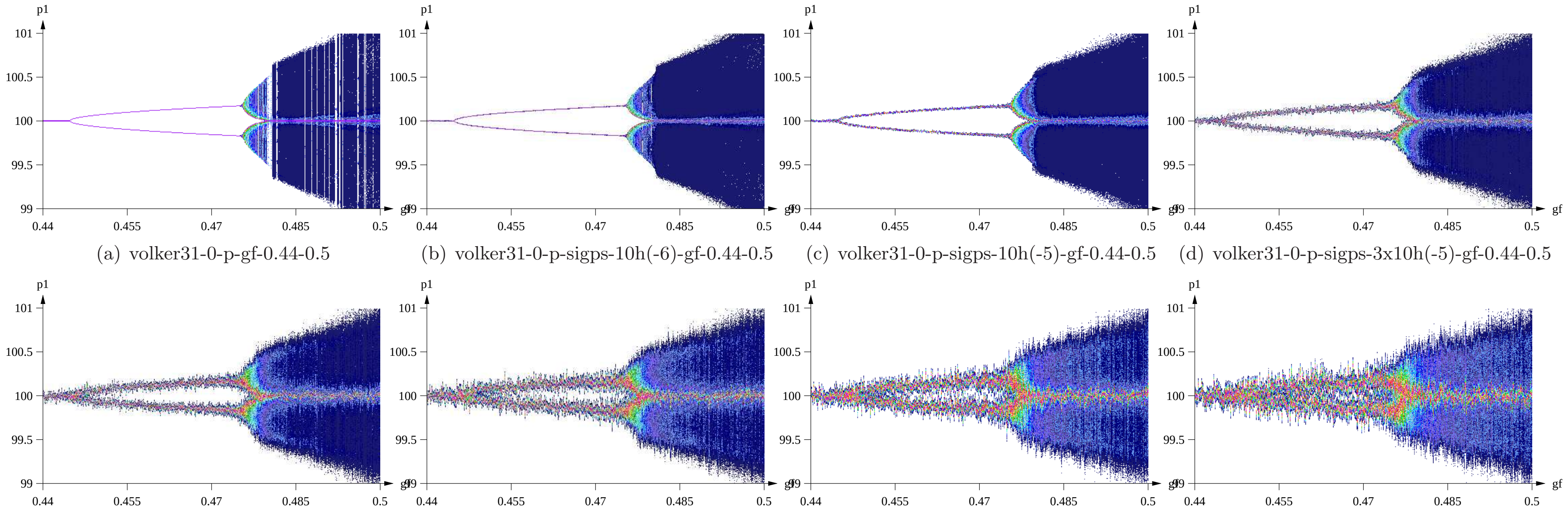
Figure 5.11: **The role of noisy supply:** Densities

statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	100	100	100	100
variance	0.0302148	0.0313883	0.0341746	0.0343815
standard deviation	0.173824	0.177167	0.184864	0.185422
skewness	2.76039e-05	-1.31805e-05	3.37117e-05	-0.00103497
kurtosis	-1.76156	-0.868867	1.7026	1.85163
quantile (0.95)	100.292	100.296	100.298	100.297

statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100	100	100	100
variance	0.0356348	0.0418547	0.0506633	0.0718015
standard deviation	0.188772	0.204584	0.225085	0.267958
skewness	-0.00050188	0.00297984	0.00261488	-0.00478176
kurtosis	2.30212	2.87366	3.29419	4.40906
quantile (0.95)	100.298	100.313	100.331	100.328

5.3 Variation of g^f - no switch, but noise ($\sigma_p = 10^{-5}$)

The standard parameter set	
Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.0
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	0
swichf	0
swichm	0



(a) volker31-0-p-gf-0.44-0.5

(b) volker31-0-p-sigps-10h(-6)-gf-0.44-0.5

(c) volker31-0-p-sigps-10h(-5)-gf-0.44-0.5

(d) volker31-0-p-sigps-3x10h(-5)-gf-0.44-0.5

(e) volker31-0-p-sigps-5x10h(-5)-gf-0.44-0.5

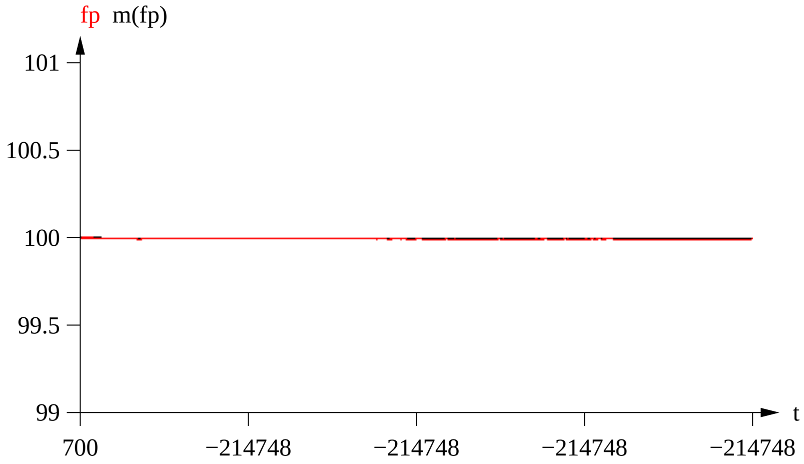
(f) volker31-0-p-sigps-8x10h(-5)-gf-0.44-0.5

(g) volker31-0-p-sigps-10h(-4)-gf-0.44-0.5

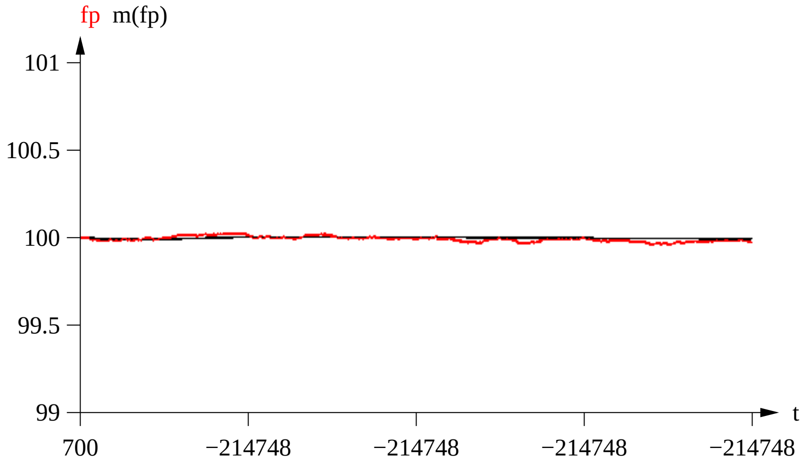
(h) volker31-0-p-sigps-13x10h(-4)-gf-0.44-0.5

Figure 5.12: **Role of noise in fundamental:** Bifurcations

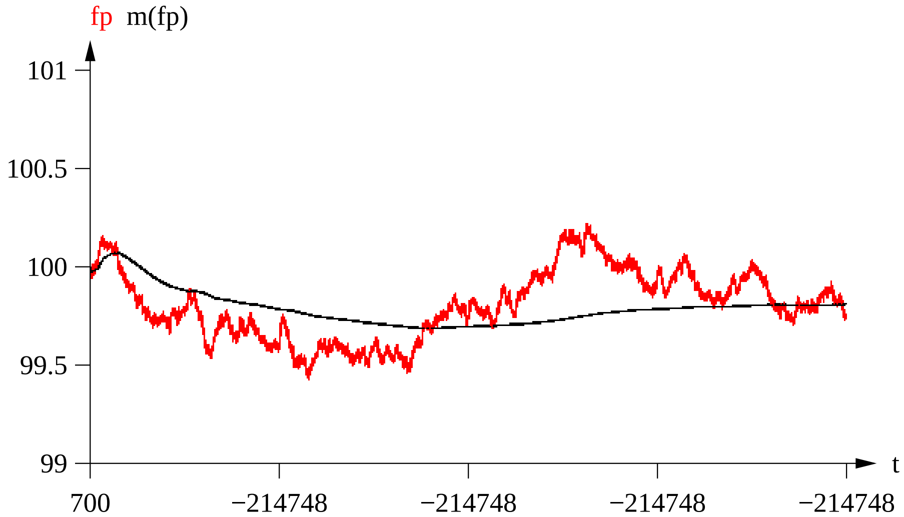
Role of noise on the fundamental price



(a) $\sigma_p = 10^{-6}$;volker16-1-fp-sigps-10h(-6)-gf-0.476

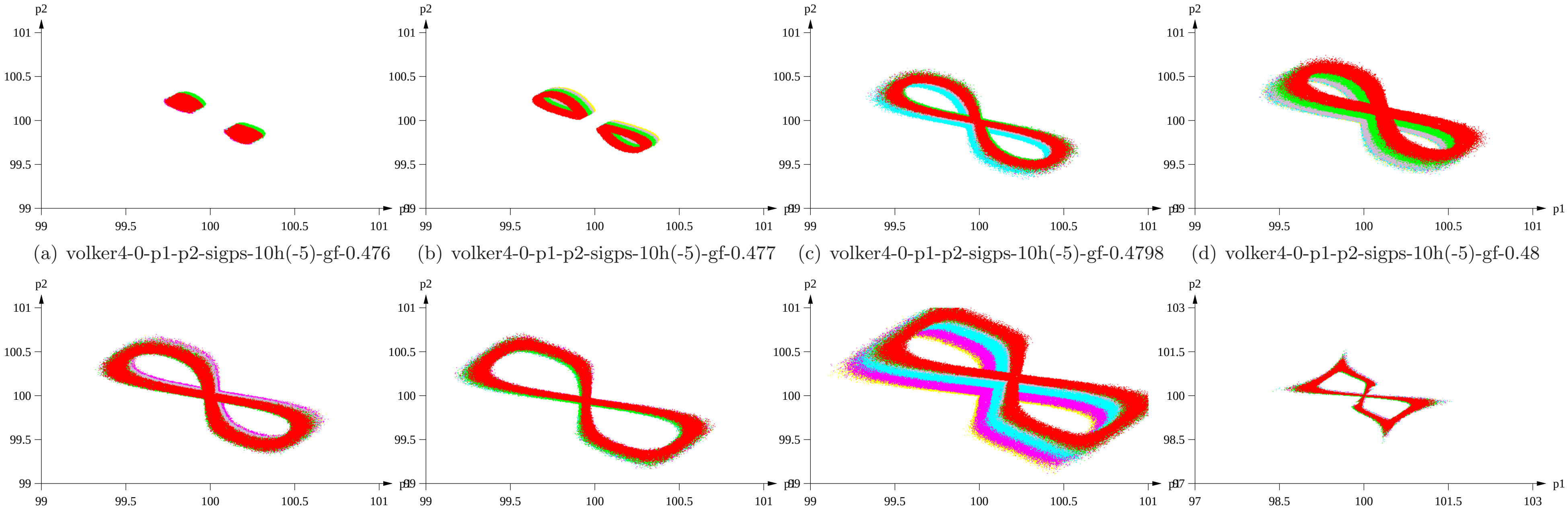


(b) $\sigma_p = 10^{-5}$; volker16-1-fp-sigps-10h(-5)-gf-0.476



(c) $\sigma_p = 10^{-4}$; volker16-1-fp-sigps-10h(-4)-gf-0.476

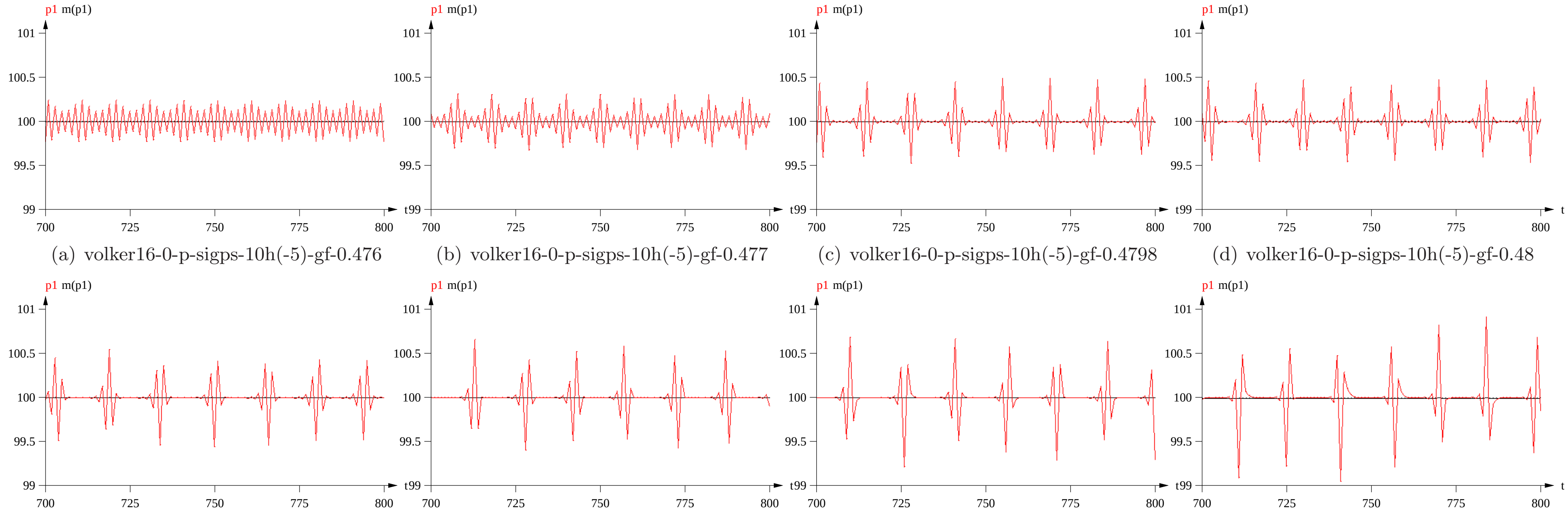
Figure 5.13: **Role of noise on the fundamental price:** time series of the fundamental price



(a) volker4-0-p1-p2-sigps-10h(-5)-gf-0.476 (b) volker4-0-p1-p2-sigps-10h(-5)-gf-0.477 (c) volker4-0-p1-p2-sigps-10h(-5)-gf-0.4798 (d) volker4-0-p1-p2-sigps-10h(-5)-gf-0.48

(e) volker4-0-p1-p2-sigps-10h(-5)-gf-0.481 (f) volker4-0-p1-p2-sigps-10h(-5)-gf-0.485 (g) volker4-0-p1-p2-sigps-10h(-5)-gf-0.49 (h) volker4-0-p1-p2-sigps-10h(-5)

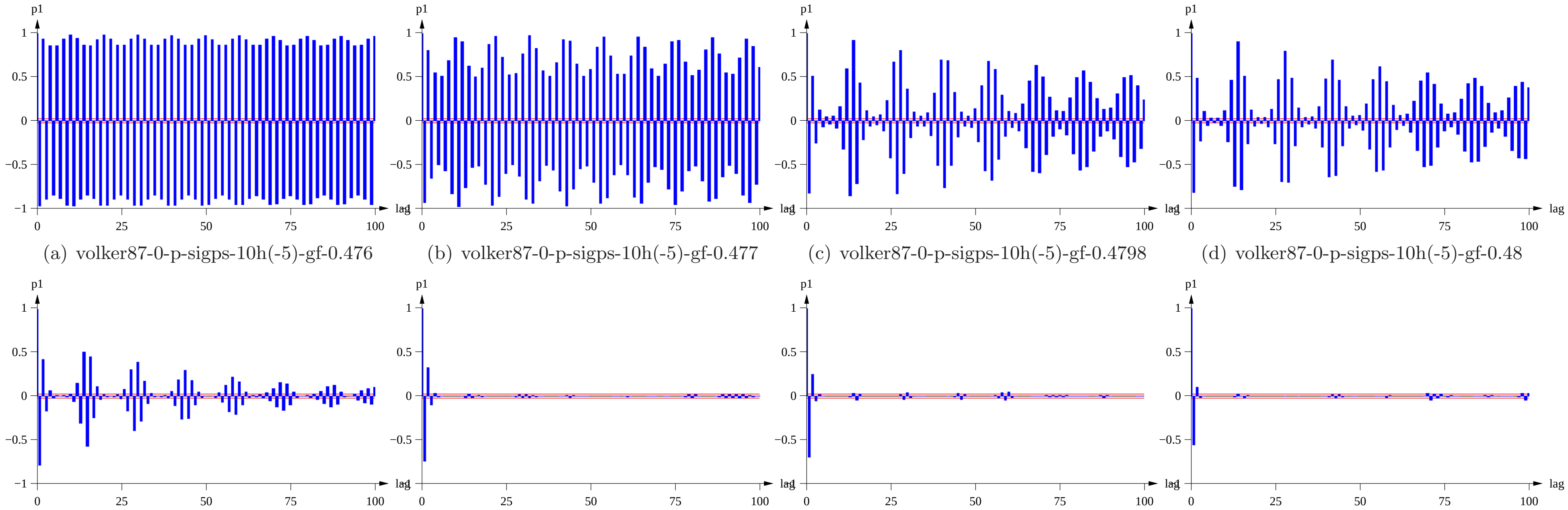
Figure 5.14: **Role of noise in fundamental:** Attractors



(a) volker16-0-p-sigps-10h(-5)-gf-0.476 (b) volker16-0-p-sigps-10h(-5)-gf-0.477 (c) volker16-0-p-sigps-10h(-5)-gf-0.4798 (d) volker16-0-p-sigps-10h(-5)-gf-0.48

(e) volker16-0-p-sigps-10h(-5)-gf-0.481.xpm.eps (f) volker16-0-p-sigps-10h(-5)-gf-0.485.xpm.eps (g) volker16-0-p-sigps-10h(-5)-gf-0.49.xpm.eps (h) volker16-0-p-sigps-10h(-5).xpm.eps

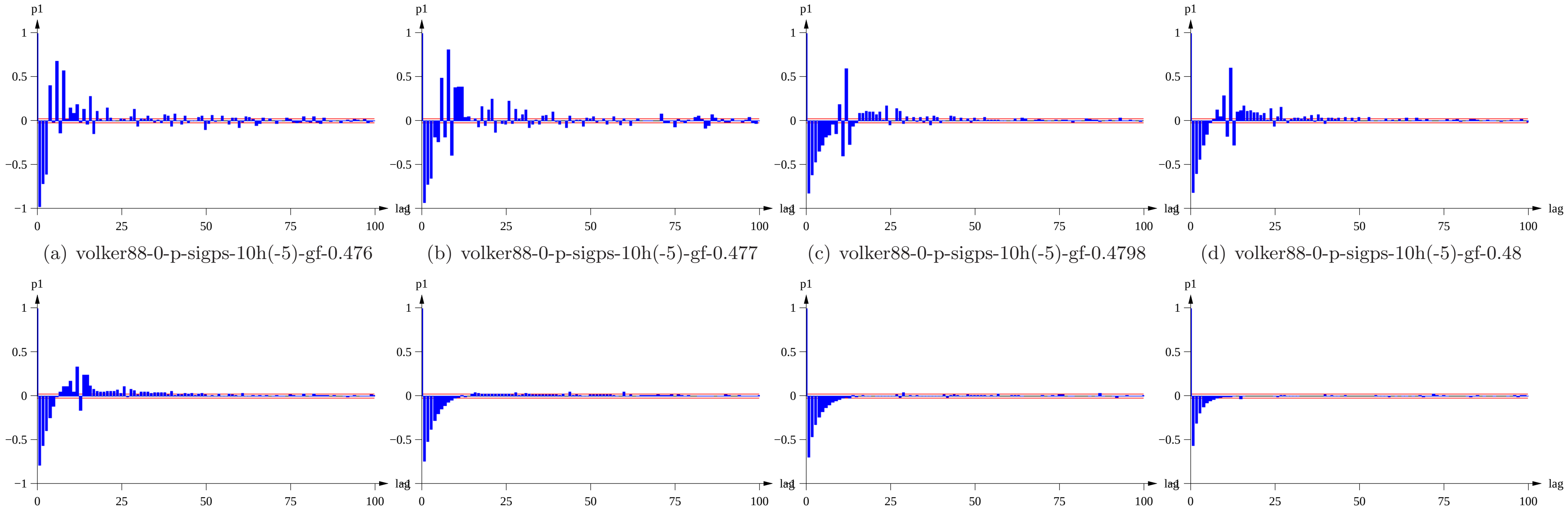
Figure 5.15: **Role of noise in fundamental:** time series



(a) volker87-0-p-sigps-10h(-5)-gf-0.476 (b) volker87-0-p-sigps-10h(-5)-gf-0.477 (c) volker87-0-p-sigps-10h(-5)-gf-0.4798 (d) volker87-0-p-sigps-10h(-5)-gf-0.48

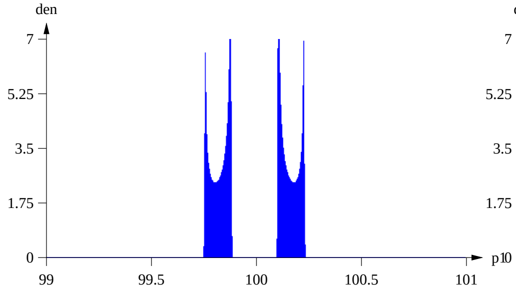
(e) volker87-0-p-sigps-10h(-5)-gf-0.481 (f) volker87-0-p-sigps-10h(-5)-gf-0.485 (g) volker87-0-p-sigps-10h(-5)-gf-0.49 (h) volker87-0-p-sigps-10h(-5)

Figure 5.16: **Role of noise in fundamental:** ACF

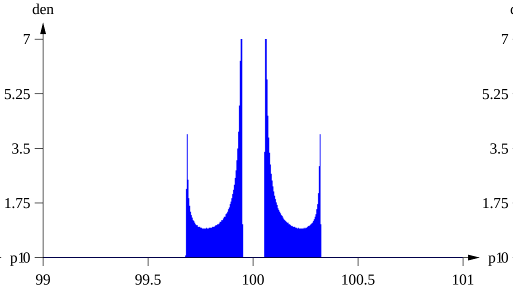


(a) volker88-0-p-sigps-10h(-5)-gf-0.476
(b) volker88-0-p-sigps-10h(-5)-gf-0.477
(c) volker88-0-p-sigps-10h(-5)-gf-0.4798
(d) volker88-0-p-sigps-10h(-5)-gf-0.48
(e) volker88-0-p-sigps-10h(-5)-gf-0.481
(f) volker88-0-p-sigps-10h(-5)-gf-0.485
(g) volker88-0-p-sigps-10h(-5)-gf-0.49
(h) volker88-0-p-sigps-10h(-5)

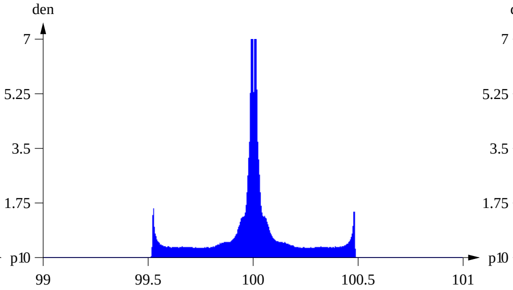
Figure 5.17: **Role of noise in fundamental:** PACF



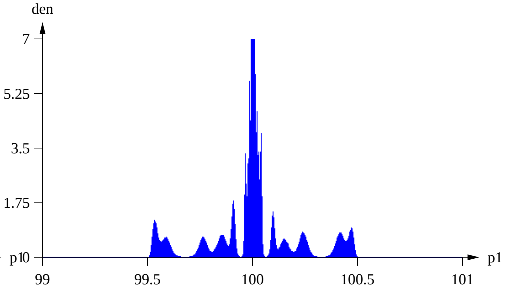
(a) volker41-0-p-sigps-10h(-5)-gf-0.476.xpm.eps



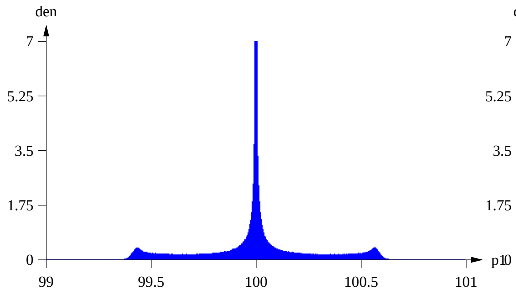
(b) volker41-0-p-sigps-10h(-5)-gf-0.477.xpm.eps



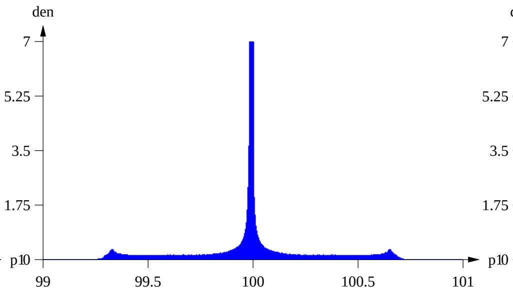
(c) volker41-0-p-sigps-10h(-5)-gf-0.4798.xpm.eps



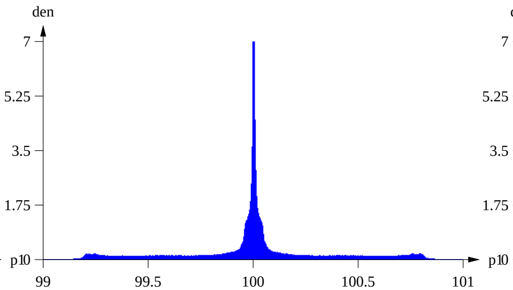
(d) volker41-0-p-sigps-10h(-5)-gf-0.48.xpm.eps



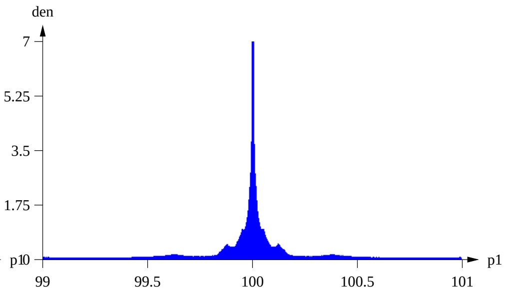
(e) volker41-0-p-sigps-10h(-5)-gf-0.481.xpm.eps



(f) volker41-0-p-sigps-10h(-5)-gf-0.485.xpm.eps



(g) volker41-0-p-sigps-10h(-5)-gf-0.49.xpm.eps



(h) volker41-0-p-sigps-10h(-5).xpm.eps

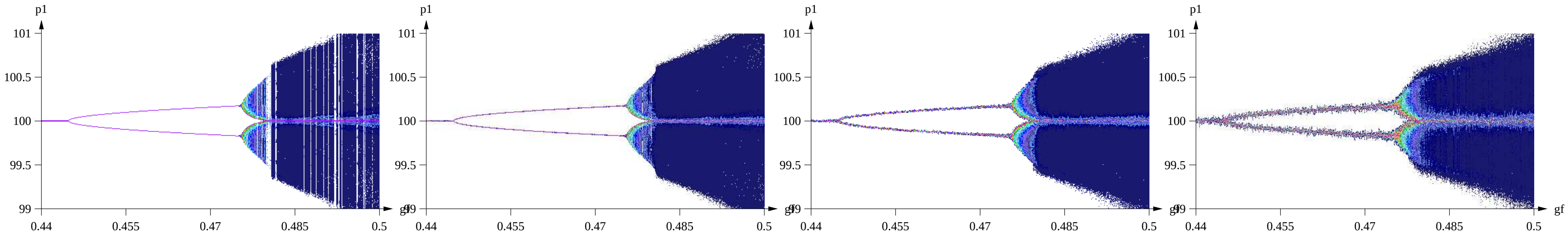
Figure 5.18: **Role of noise in fundamental:** density plots

statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	99.9948	100.003	100.01	100.004
variance	0.0302228	0.0313967	0.0343747	0.0344986
standard deviation	0.173847	0.177191	0.185404	0.185738
skewness	-1.13053e-05	1.96533e-06	0.000331643	-0.00286535
kurtosis	-1.71056	-0.873329	1.6333	1.86846
quantile (0.95)	100.287	100.299	100.309	100.302

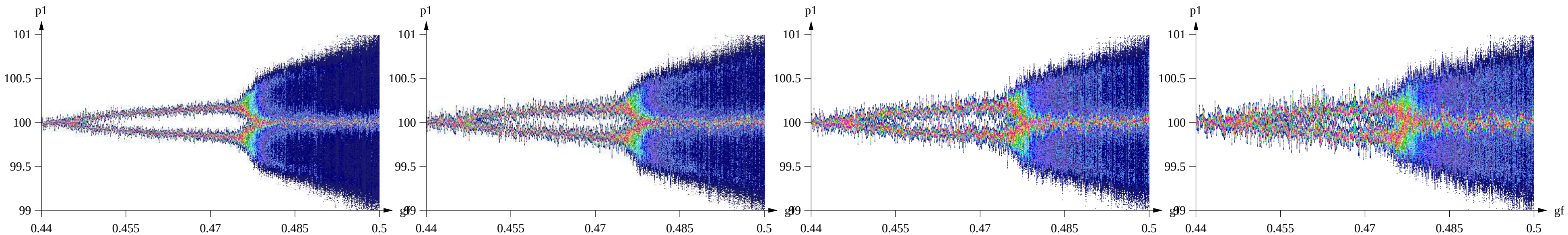
statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100.002	99.9929	100.004	100.003
variance	0.0339969	0.0403881	0.0503853	0.0782341
standard deviation	0.184383	0.200968	0.224467	0.279704
skewness	0.000345495	-0.00233278	0.00471148	0.00241466
kurtosis	3.94155	4.90437	5.48951	7.58664
quantile (0.95)	100.248	100.219	100.234	100.223

5.4 Variation of g^f - no switch, but noise ($\sigma_p = 10^{-6}$)

The standard parameter set	
Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.0
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	0
swichf	0
swichm	0

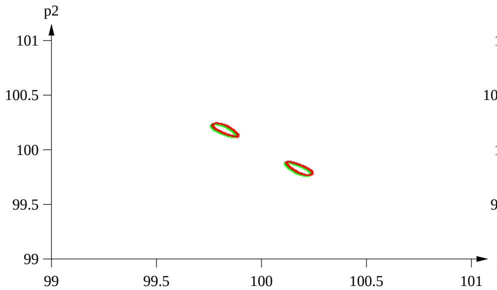


(a) volker31-0-p-gf-0.44-0.5 (b) volker31-0-p-sigps-10h(-6)-gf-0.44-0.5 (c) volker31-0-p-sigps-10h(-5)-gf-0.44-0.5 (d) volker31-0-p-sigps-3x10h(-5)-gf-0.44-0.5

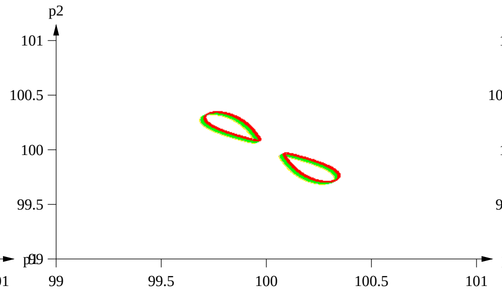


(e) volker31-0-p-sigps-5x10h(-5)-gf-0.44-0.5 (f) volker31-0-p-sigps-8x10h(-5)-gf-0.44-0.5 (g) volker31-0-p-sigps-10h(-4)-gf-0.44-0.5 (h) volker31-0-p-sigps-13x10h(-4)-gf-0.44-0.5

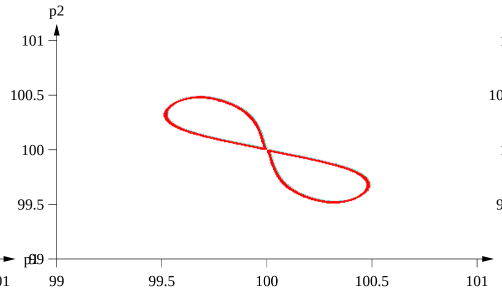
Figure 5.19: **Role of noise in fundamental:** Bifurcations



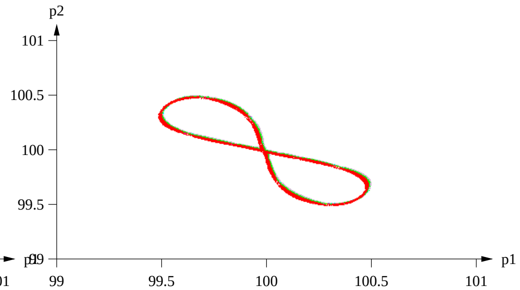
(a) volker4-0-p1-p2-sigps-10h(-6)-gf-0.476



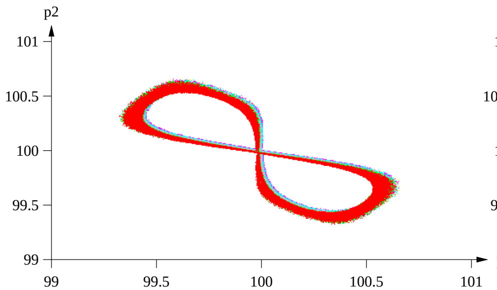
(b) volker4-0-p1-p2-sigps-10h(-6)-gf-0.477



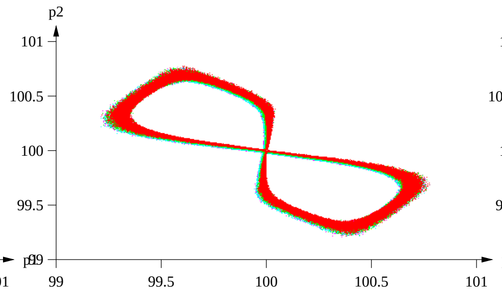
(c) volker4-0-p1-p2-sigps-10h(-6)-gf-0.4798



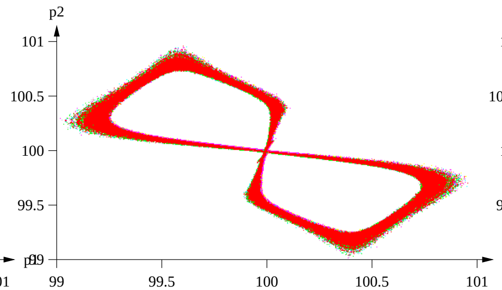
(d) volker4-0-p1-p2-sigps-10h(-6)-gf-0.48



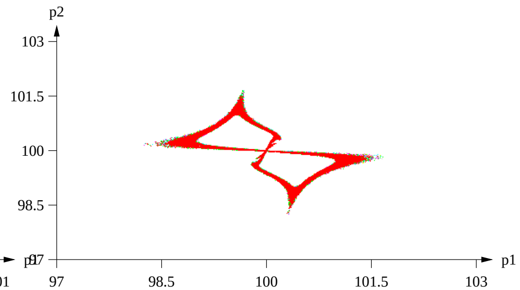
(e) volker4-0-p1-p2-sigps-10h(-6)-gf-0.481



(f) volker4-0-p1-p2-sigps-10h(-6)-gf-0.485

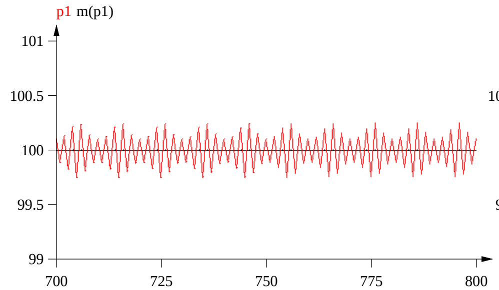


(g) volker4-0-p1-p2-sigps-10h(-6)-gf-0.49

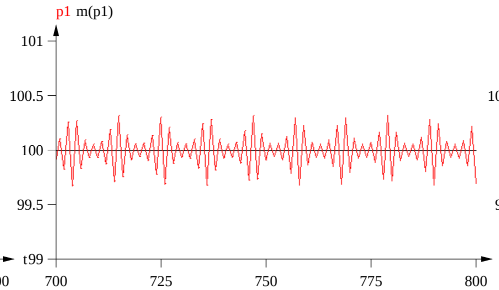


(h) volker4-0-p1-p2-sigps-10h(-6)

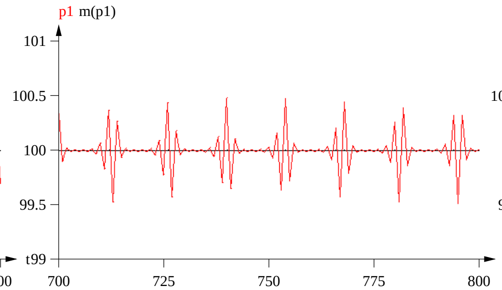
Figure 5.20: **Role of noise in fundamental:** Attractors



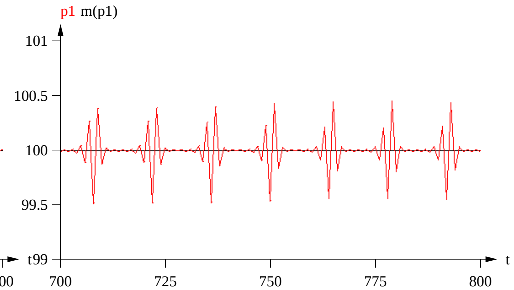
(a) volker16-0-p-sigps-10h(-6)-gf-0.476



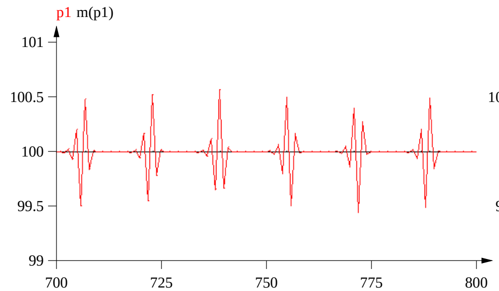
(b) volker16-0-p-sigps-10h(-6)-gf-0.477



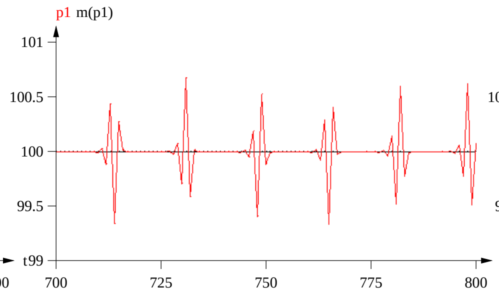
(c) volker16-0-p-sigps-10h(-6)-gf-0.4798



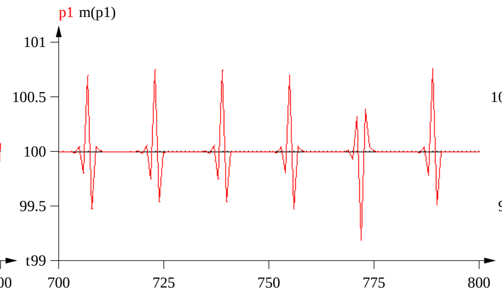
(d) volker16-0-p-sigps-10h(-6)-gf-0.48



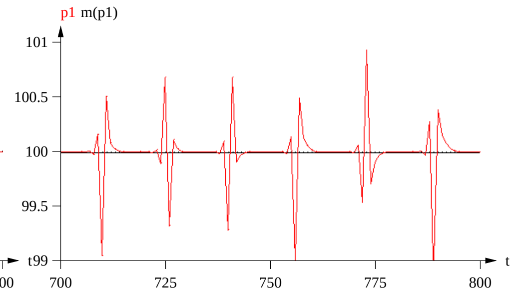
(e) volker16-0-p-sigps-10h(-6)-gf-0.481.xpm.eps



(f) volker16-0-p-sigps-10h(-6)-gf-0.485.xpm.eps

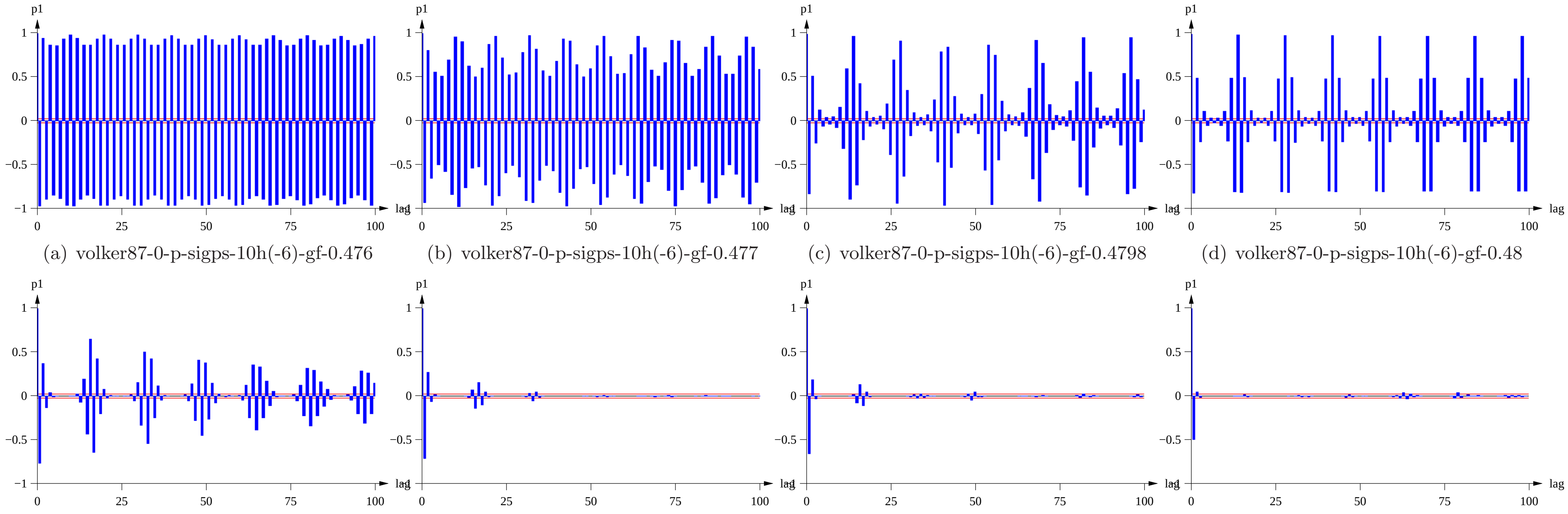


(g) volker16-0-p-sigps-10h(-6)-gf-0.49.xpm.eps



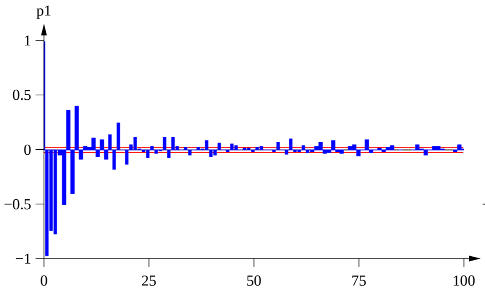
(h) volker16-0-p-sigps-10h(-6).xpm.eps

Figure 5.21: **Role of noise in fundamental:** time series

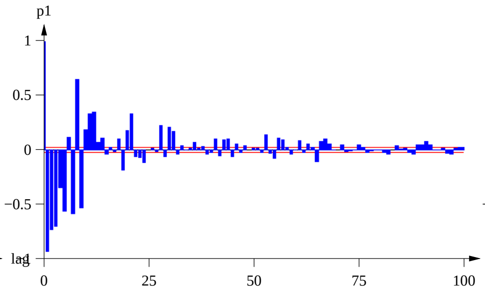


(a) volker87-0-p-sigps-10h(-6)-gf-0.476
(b) volker87-0-p-sigps-10h(-6)-gf-0.477
(c) volker87-0-p-sigps-10h(-6)-gf-0.4798
(d) volker87-0-p-sigps-10h(-6)-gf-0.48
(e) volker87-0-p-sigps-10h(-6)-gf-0.481
(f) volker87-0-p-sigps-10h(-6)-gf-0.485
(g) volker87-0-p-sigps-10h(-6)-gf-0.49
(h) volker87-0-p-sigps-10h(-6)

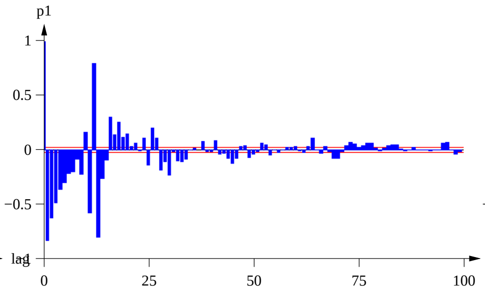
Figure 5.22: **Role of noise in fundamental:** ACF



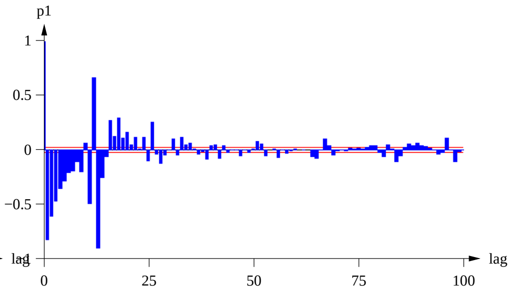
(a) volker88-0-p-sigps-10h(-6)-gf-0.476



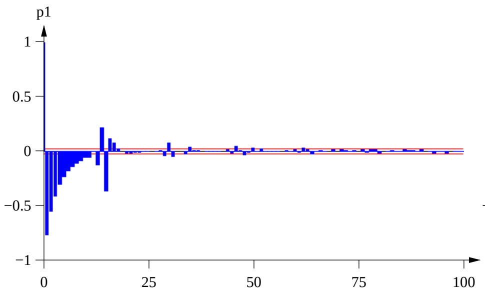
(b) volker88-0-p-sigps-10h(-6)-gf-0.477



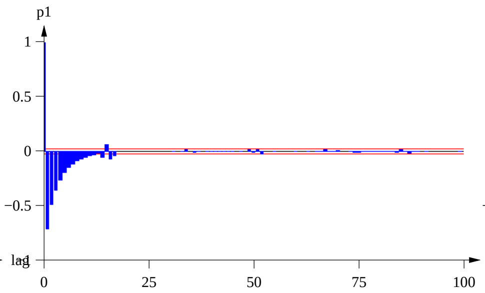
(c) volker88-0-p-sigps-10h(-6)-gf-0.4798



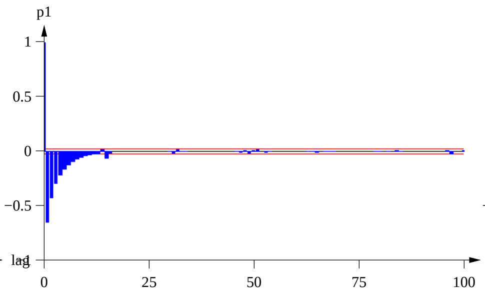
(d) volker88-0-p-sigps-10h(-6)-gf-0.48



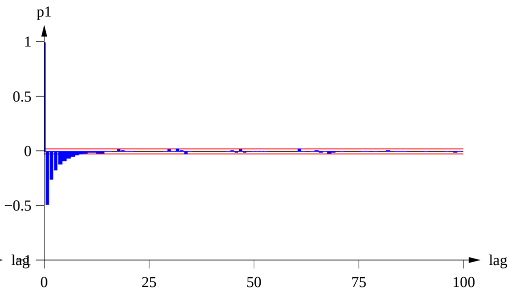
(e) volker88-0-p-sigps-10h(-6)-gf-0.481



(f) volker88-0-p-sigps-10h(-6)-gf-0.485

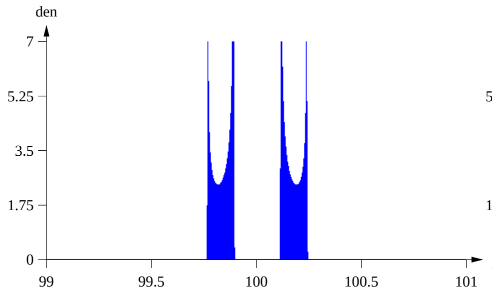


(g) volker88-0-p-sigps-10h(-6)-gf-0.49

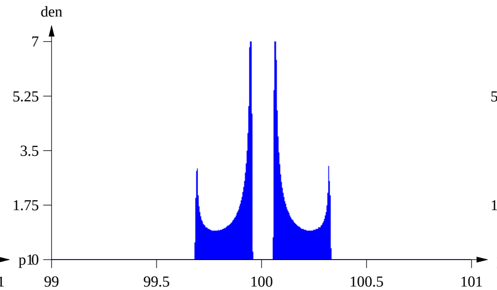


(h) volker88-0-p-sigps-10h(-6)

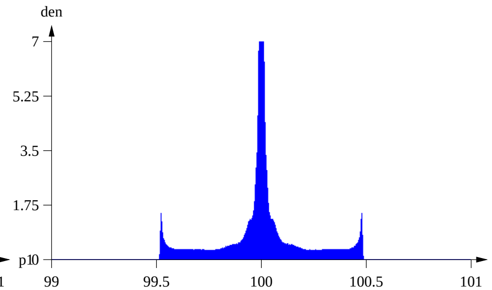
Figure 5.23: **Role of noise in fundamental:** PACF



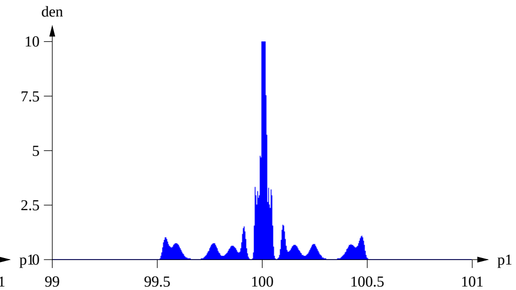
(a) volker41-0-p-sigps-10h(-6)-gf-0.476.xpm.eps



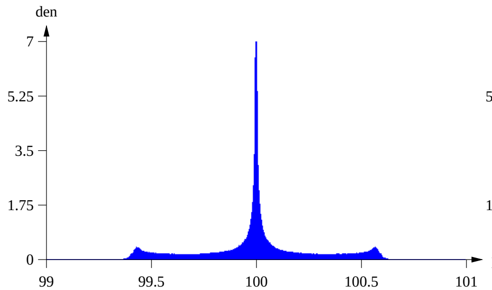
(b) volker41-0-p-sigps-10h(-6)-gf-0.477.xpm.eps



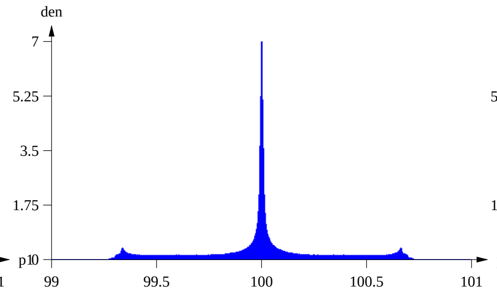
(c) volker41-0-p-sigps-10h(-6)-gf-0.4798.xpm.eps



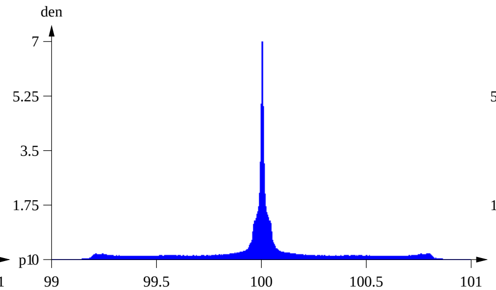
(d) volker41-0-p-sigps-10h(-6)-gf-0.48.xpm.eps



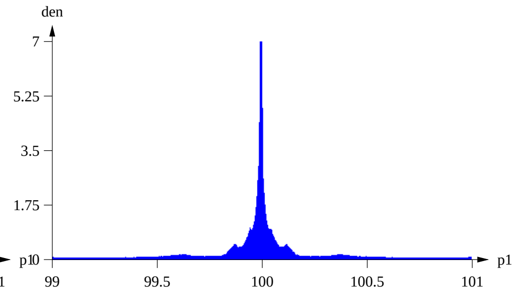
(e) volker41-0-p-sigps-10h(-6)-gf-0.481.xpm.eps



(f) volker41-0-p-sigps-10h(-6)-gf-0.485.xpm.eps



(g) volker41-0-p-sigps-10h(-6)-gf-0.49.xpm.eps



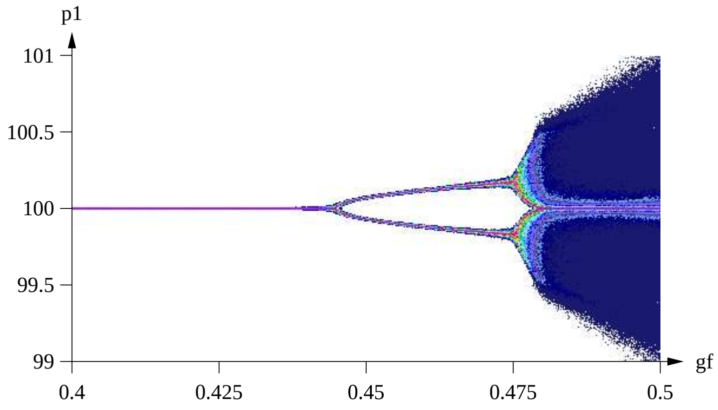
(h) volker41-0-p-sigps-10h(-6).xpm.eps

Figure 5.24: **Role of noise in fundamental:** density plots

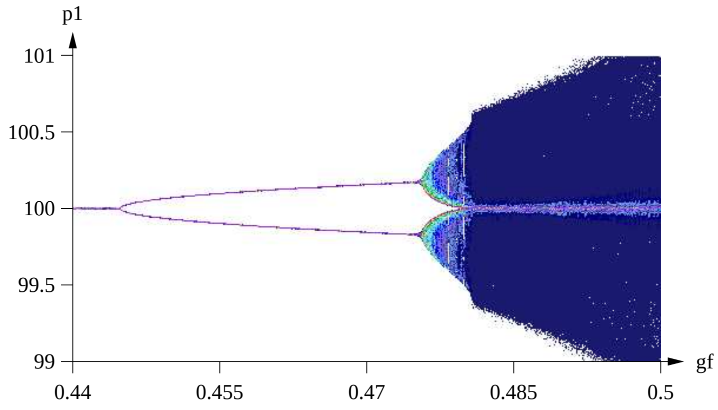
statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	100.006	100.007	100.002	100.008
variance	0.030223	0.0314084	0.0343527	0.0344973
standard deviation	0.173848	0.177224	0.185345	0.185735
skewness	-1.76458e-05	2.79356e-05	7.50891e-06	0.0169171
kurtosis	-1.70047	-0.892341	1.5852	1.85561
quantile (0.95)	100.297	100.303	100.302	100.307

statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100.001	100.002	100.005	99.9953
variance	0.0339963	0.040352	0.0503822	0.0782122
standard deviation	0.184381	0.200878	0.22446	0.279665
skewness	1.86987e-05	0.00113865	0.00546533	0.00520293
kurtosis	3.96716	4.91226	5.50793	7.58878
quantile (0.95)	100.246	100.227	100.234	100.215

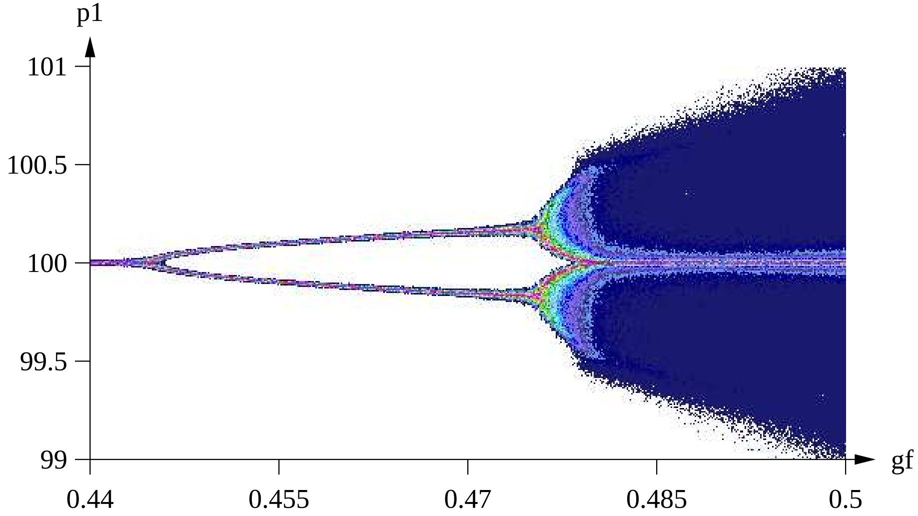
5.5 Variation of g^f - no switch, but noise ($\sigma_p = 10^{-6}$ and $\tau = 10^{-3}$)



(a) volker31-0-p-sigps-tau-10h(-3)-gf-0.44-0.5

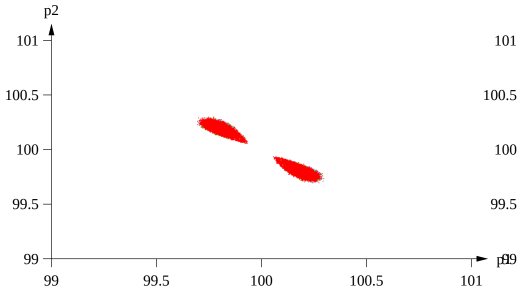


(b) volker31-0-p-sigps-10h(-6)-gf-0.44-0.5

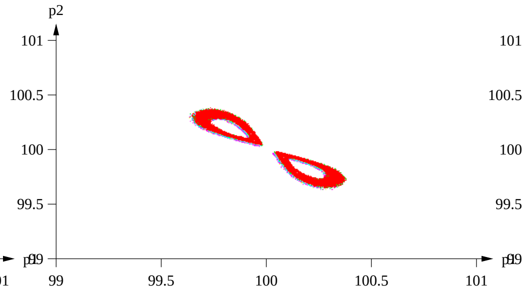


(c) volker31-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.44-0.5

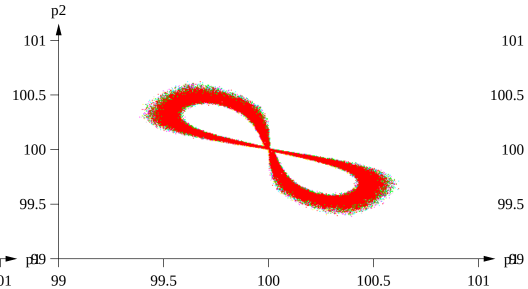
Figure 5.25: **Role of noise in fundamental and supply:** Bifurcations



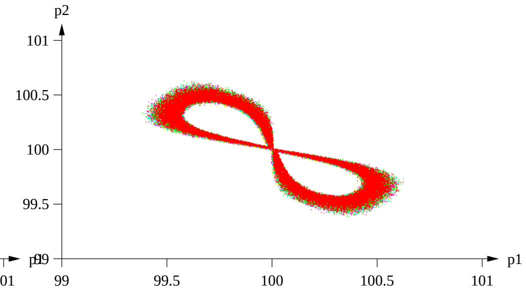
(a) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)-gf-0.476



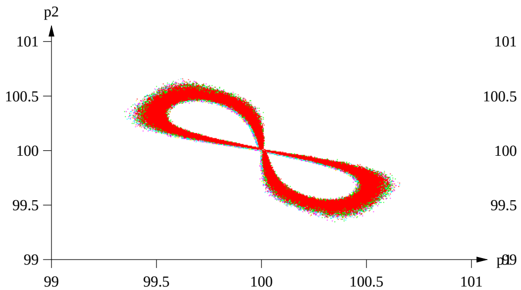
(b) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)-gf-0.477



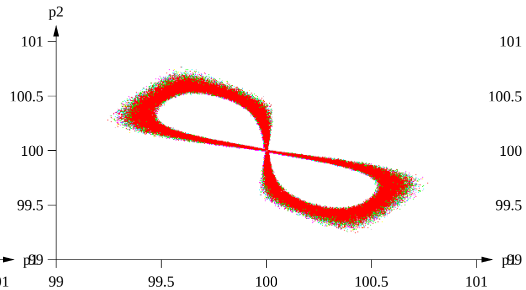
(c) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)-gf-0.4798



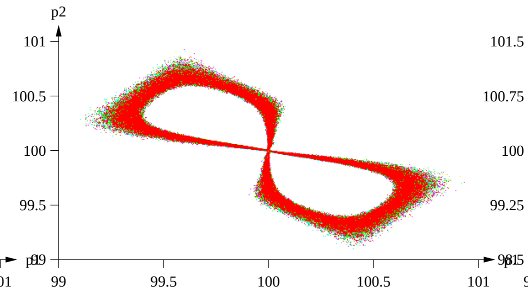
(d) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)-gf-0.48



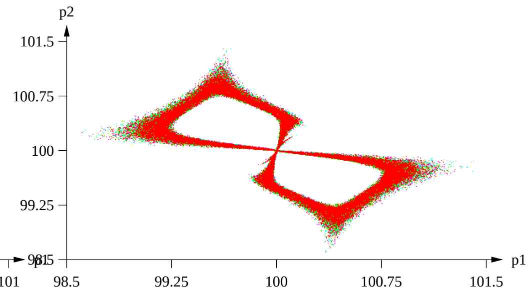
(e) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)-gf-0.481



(f) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)-gf-0.485

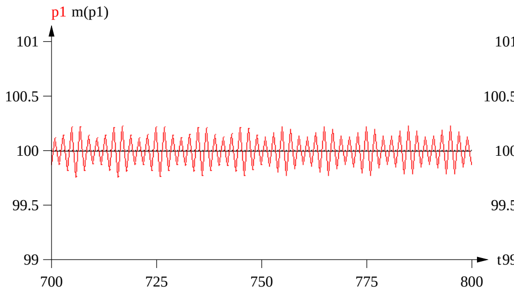


(g) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)-gf-0.49

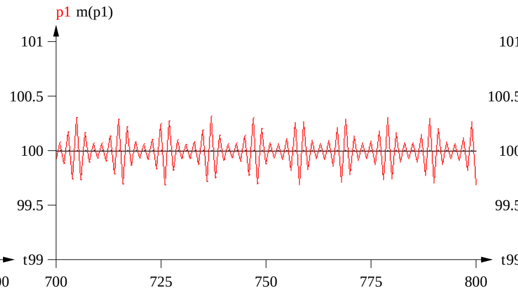


(h) volker4-0-p1-p2-sigps-10h(-6)-tau-10h(-3)

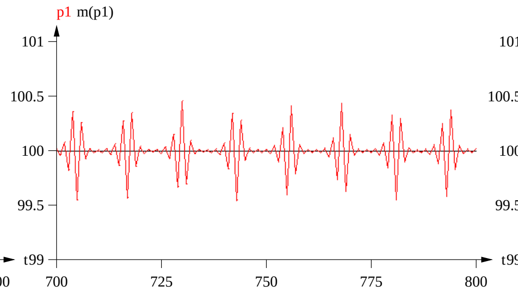
Figure 5.26: **Role of noise in fundamental and supply:** Attractors



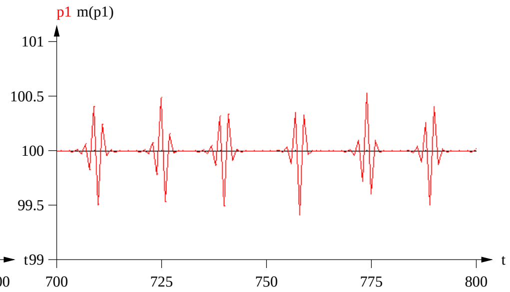
(a) volker16-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.476



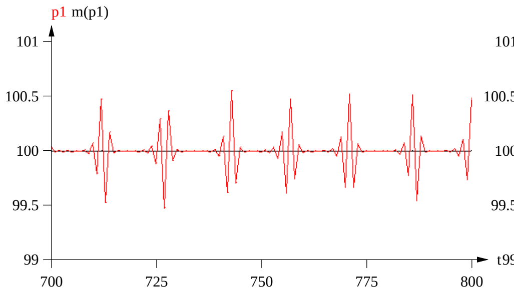
(b) volker16-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.477



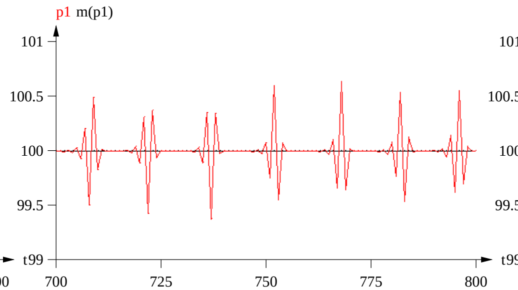
(c) volker16-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.4798



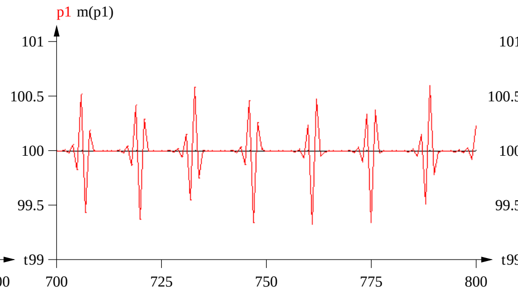
(d) volker16-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.48



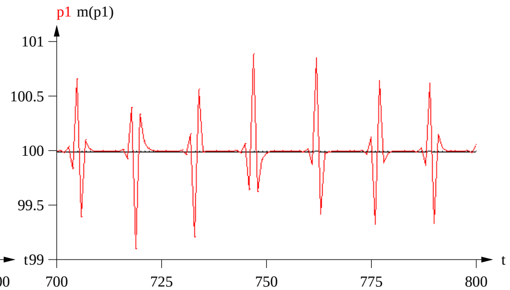
(e) volker16-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.481



(f) volker16-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.485

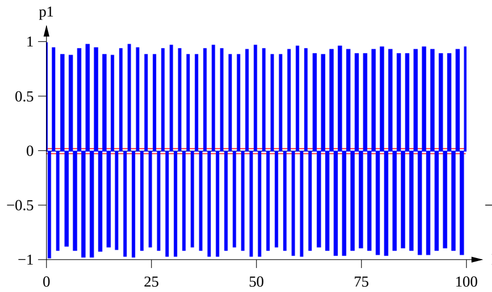


(g) volker16-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.49

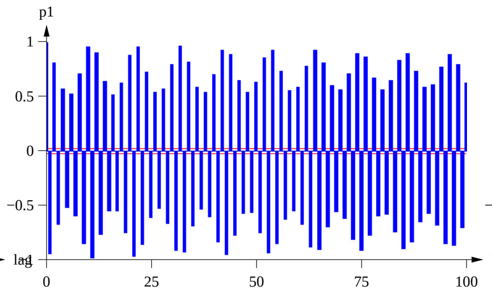


(h) volker16-0-p-sigps-10h(-6)-tau-10h(-3)

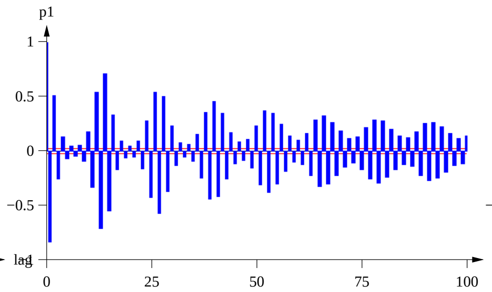
Figure 5.27: **Role of noise in fundamental and supply:** time series



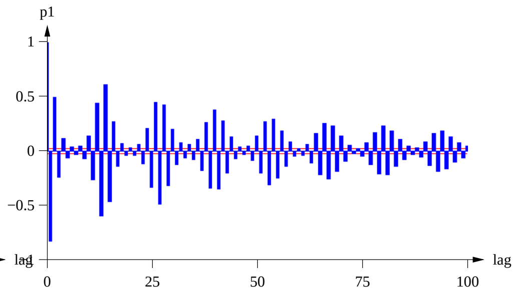
(a) volker87-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.476



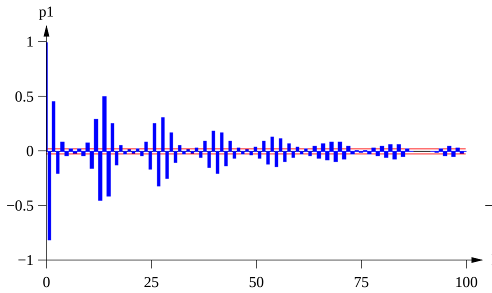
(b) volker87-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.477



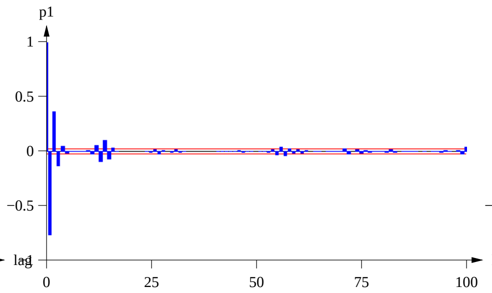
(c) volker87-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.4798



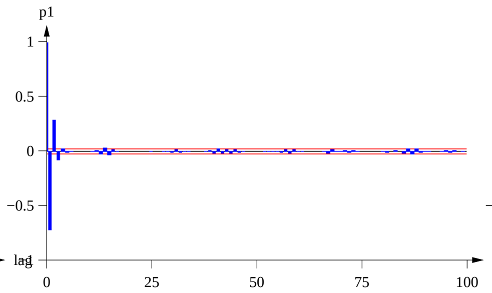
(d) volker87-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.48



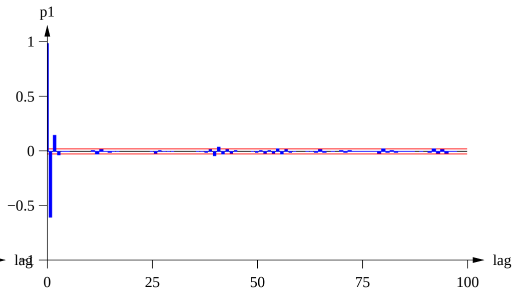
(e) volker87-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.481



(f) volker87-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.485

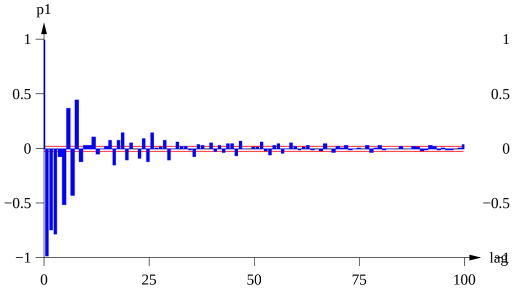


(g) volker87-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.49

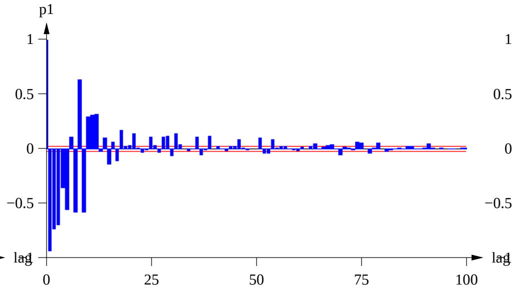


(h) volker87-0-p-sigps-10h(-6)-tau-10h(-3)

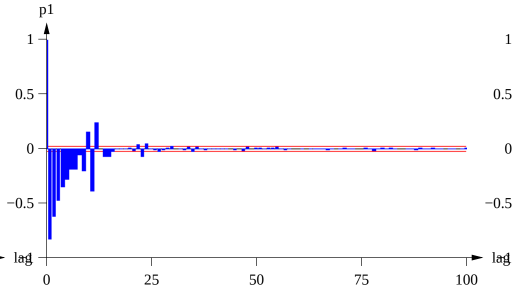
Figure 5.28: **Role of noise in fundamental and supply:** ACF



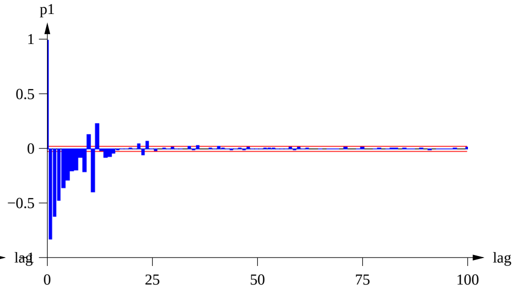
(a) volker88-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.476



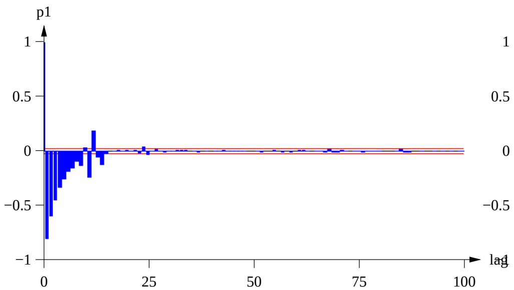
(b) volker88-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.477



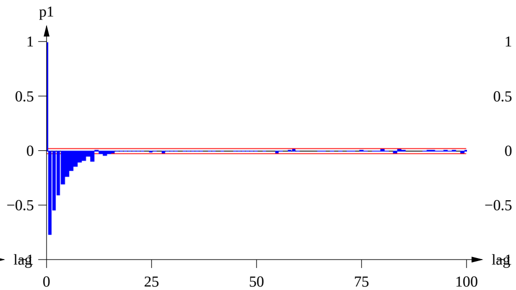
(c) volker88-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.4798



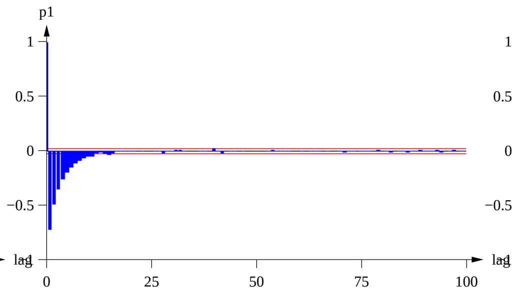
(d) volker88-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.48



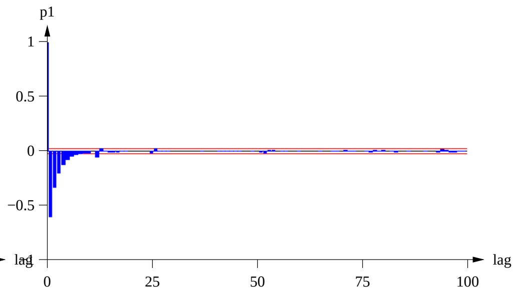
(e) volker88-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.481



(f) volker88-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.485

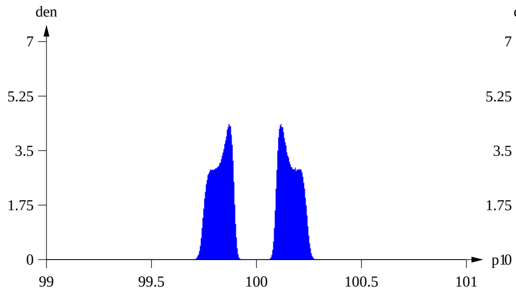


(g) volker88-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.49

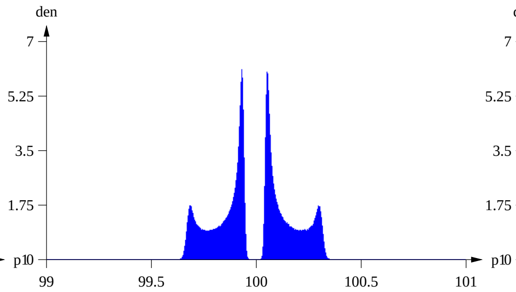


(h) volker88-0-p-sigps-10h(-6)-tau-10h(-3)

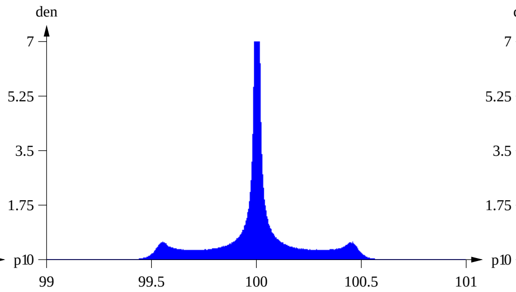
Figure 5.29: **Role of noise in fundamental and supply:** PACF



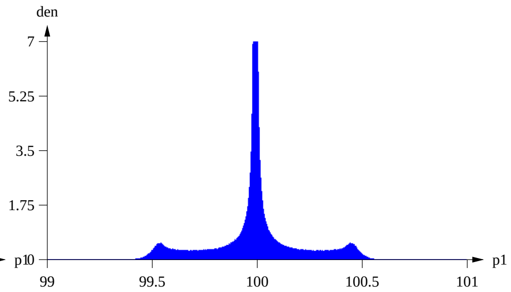
(a) volker41-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.476



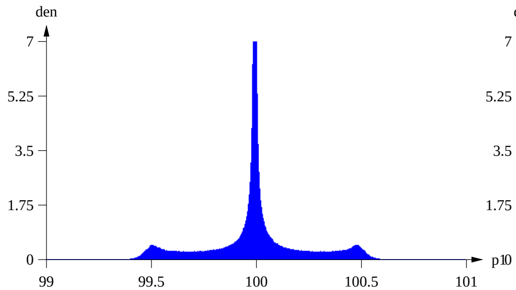
(b) volker41-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.477



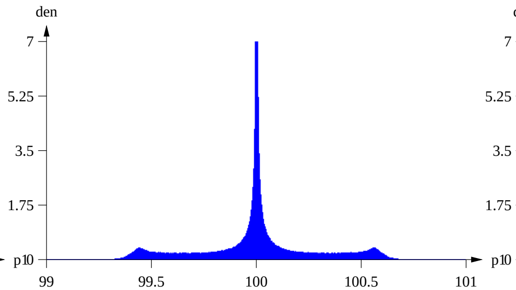
(c) volker41-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.4798



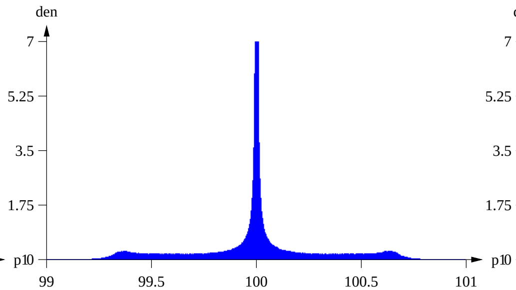
(d) volker41-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.48



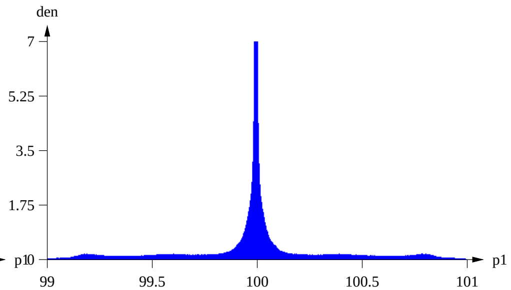
(e) volker41-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.481



(f) volker41-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.485



(g) volker41-0-p-sigps-10h(-6)-tau-10h(-3)-gf-0.49



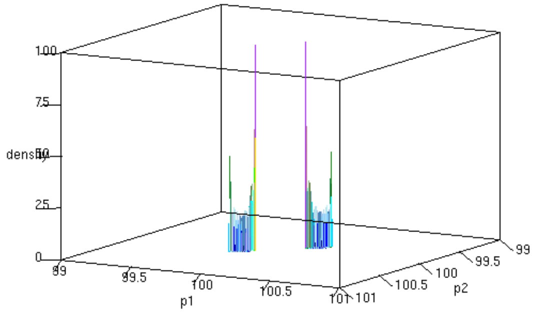
(h) volker41-0-p-sigps-10h(-6)-tau-10h(-3)

Figure 5.30: **Role of noise in fundamental and supply:** density plots

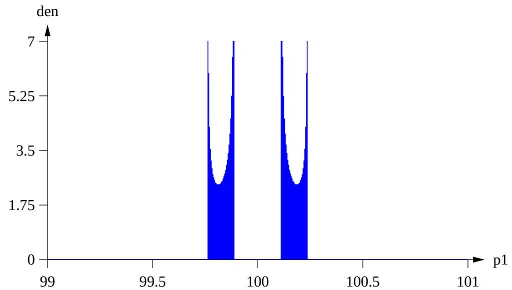
statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	99.9969	99.9938	100.004	99.9946
variance	0.0302177	0.0313949	0.0341815	0.0343855
standard deviation	0.173832	0.177186	0.184882	0.185433
skewness	2.48405e-05	2.16842e-05	0.000660813	0.001197
kurtosis	-1.75231	-0.872905	1.71163	1.86696
quantile (0.95)	100.289	100.29	100.302	100.292

statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	99.9959	100.002	100.002	99.997
variance	0.0356429	0.0418639	0.0506767	0.071847
standard deviation	0.188793	0.204607	0.225115	0.268043
skewness	-0.000594823	-0.000276075	0.000860999	-0.000830048
kurtosis	2.2859	2.8645	3.2846	4.40804
quantile (0.95)	100.294	100.315	100.332	100.326

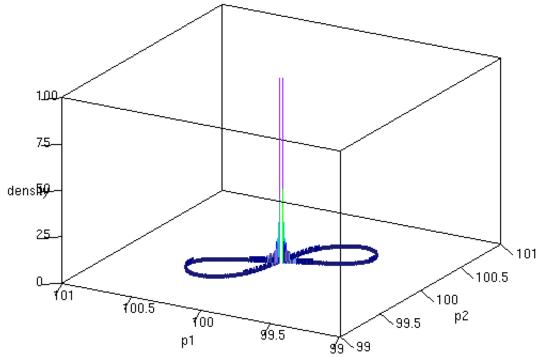
5.6 3D Densities on Attractors



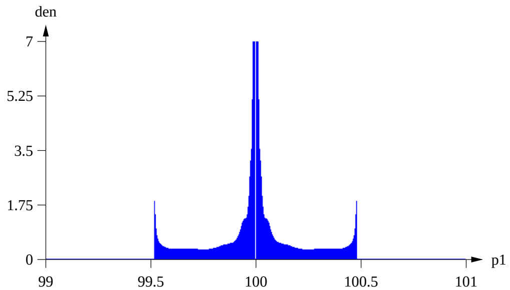
(a) $gf = 0.476$



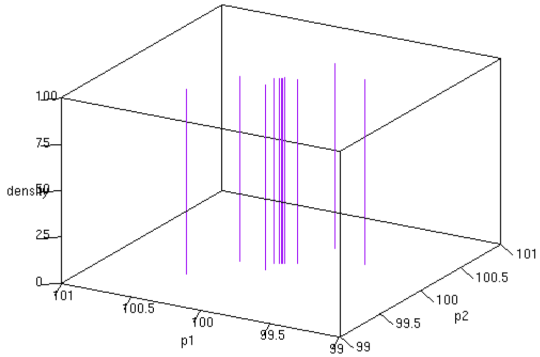
(b) volker41-0-p-gf-0.476.xpm.eps



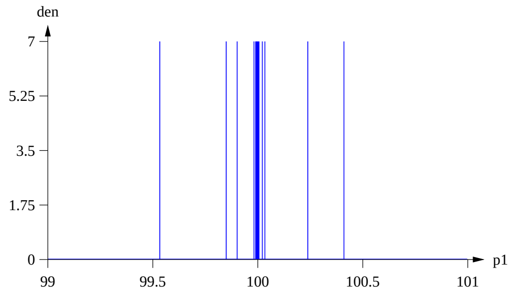
(c) $gf = 0.4798$



(d) volker41-0-p-gf-0.4798.xpm.eps

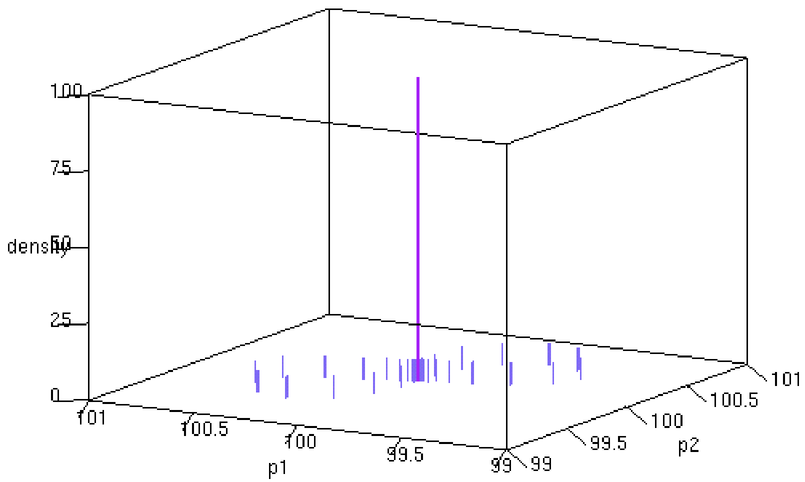


(e) $gf = 0.48$

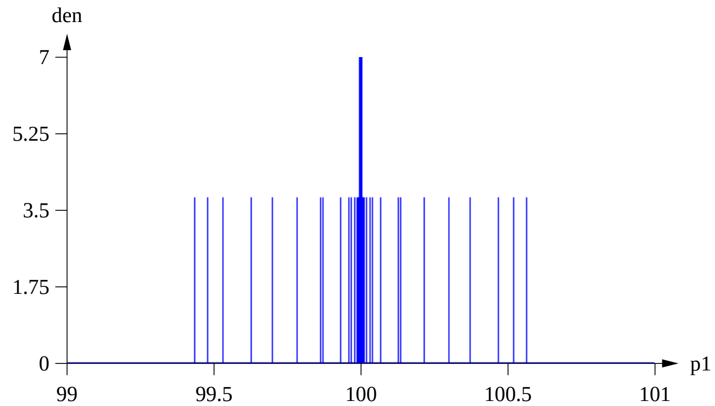


(f) volker41-0-p-gf-0.48.xpm.eps

Figure 5.31: variation of gf - no noise

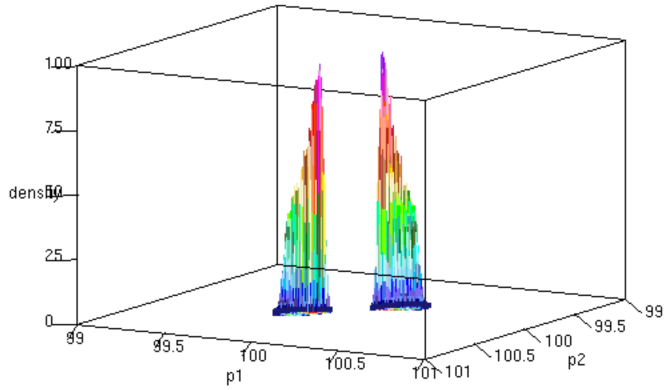


(a) $gf = 0.48094$

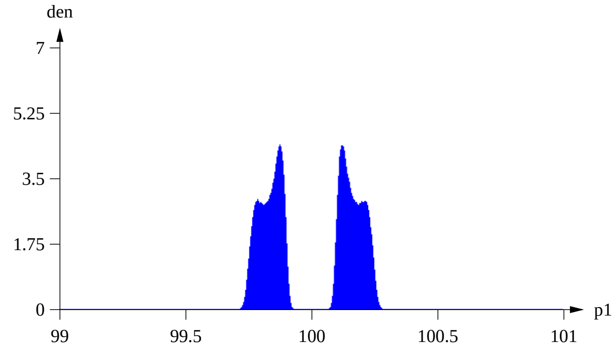


(b) volker41-0-p-gf-0.48094.xpm.eps

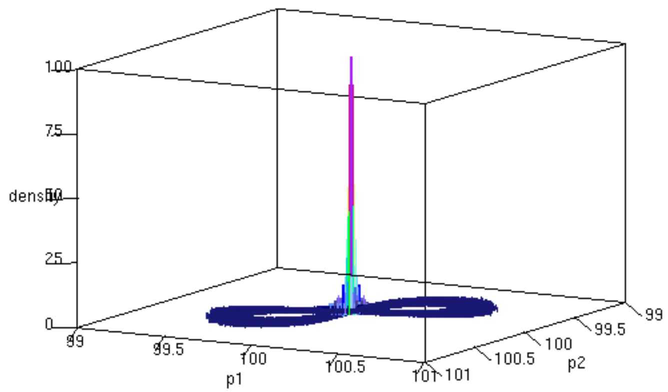
Figure 5.32: variation of gf - no noise



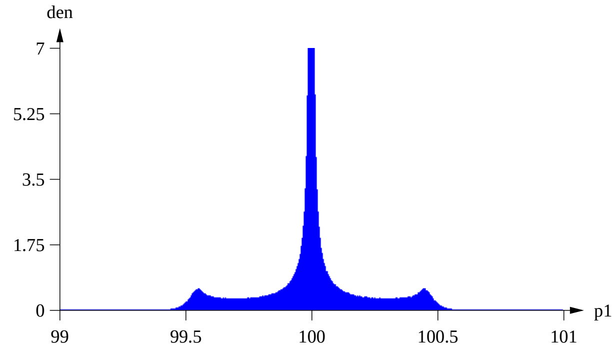
(a) $gf = 0.476$



(b) volker41-0-p-tau-10h(-3)-gf-0.476

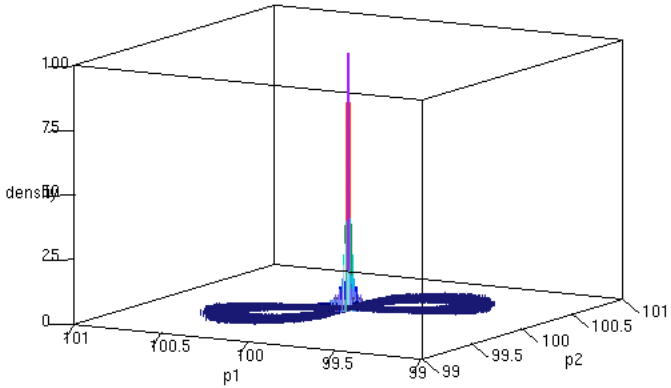


(c) $gf = 0.4798$

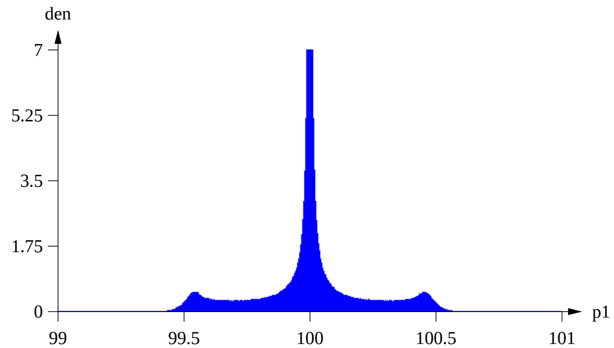


(d) volker41-0-p-tau-10h(-3)-gf-0.4798

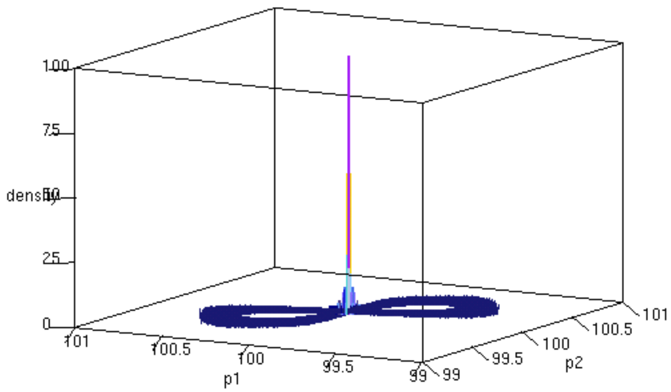
Figure 5.33: variation of gf - $\tau = 10^{-5}$



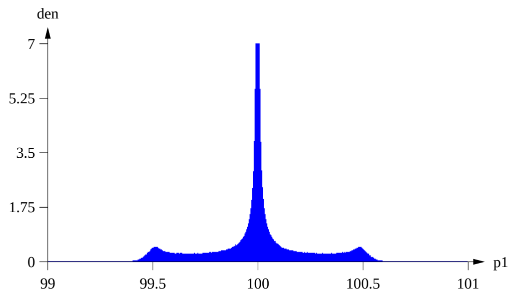
(a) $gf = 0.48$



(b) volker41-0-p-tau-10h(-3)-gf-0.48

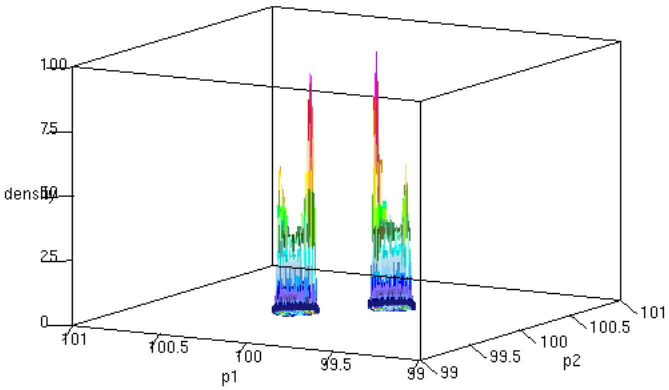


(c) $gf = 0.48$

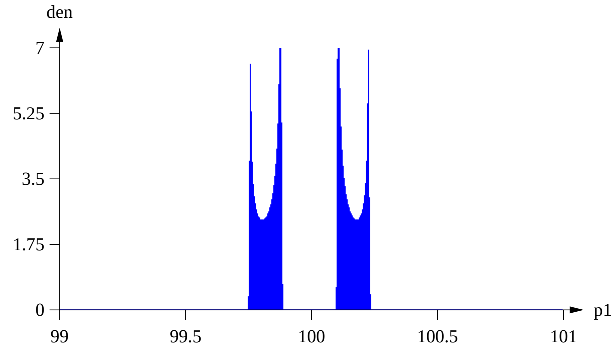


(d) volker41-0-p-tau-10h(-3)-gf-0.481

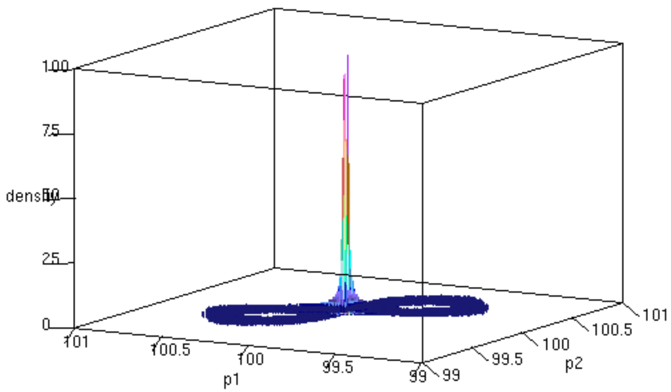
Figure 5.34: variation of gf - $\tau = 10^{-3}$



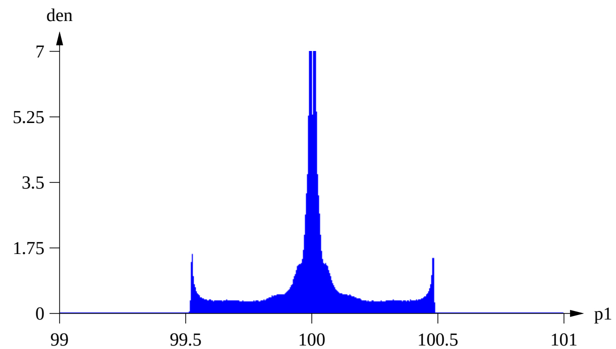
(a) $gf = 0.476$



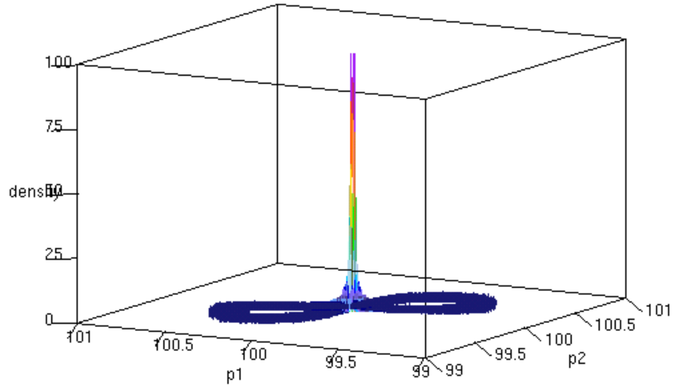
(b) volker41-0-p-sigps-10h(-5)-gf-0.476.xpm.eps



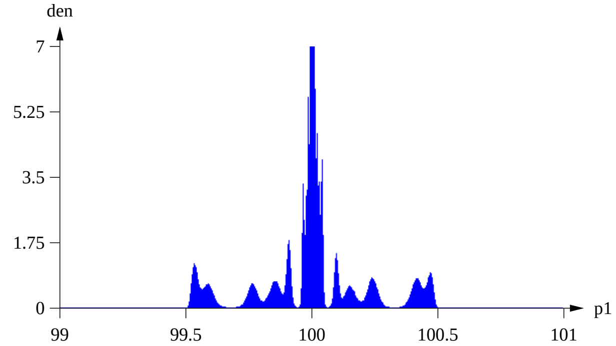
(c) $gf = 0.4798$



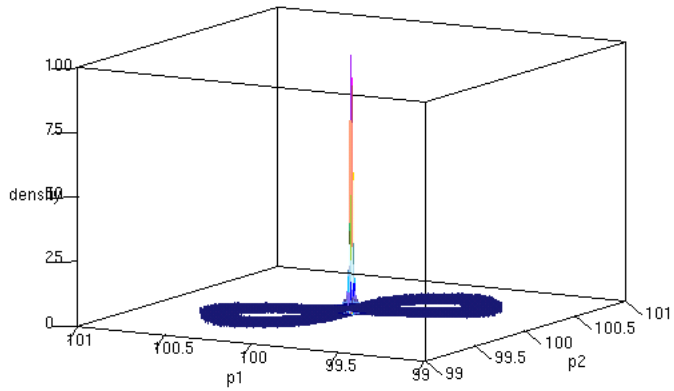
(d) volker41-0-p-sigps-10h(-5)-gf-0.4798.xpm.eps



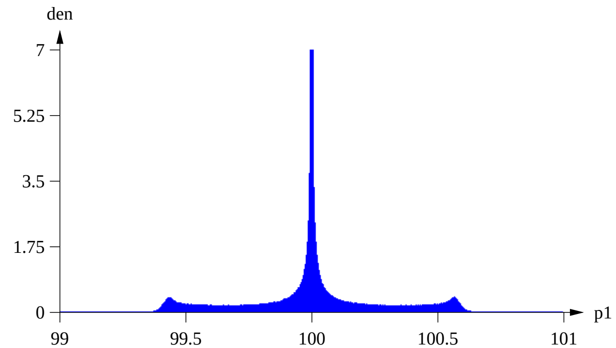
(e) $gf = 0.48$



(f) volker41-0-p-sigps-10h(-5)-gf-0.48.xpm.eps

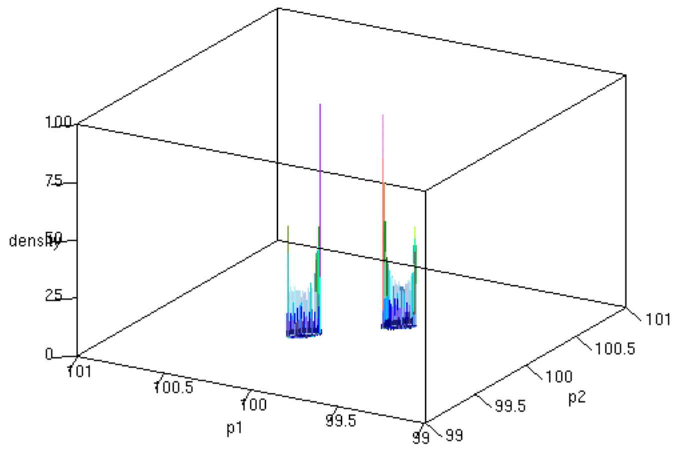


(g) $gf = 0.481$

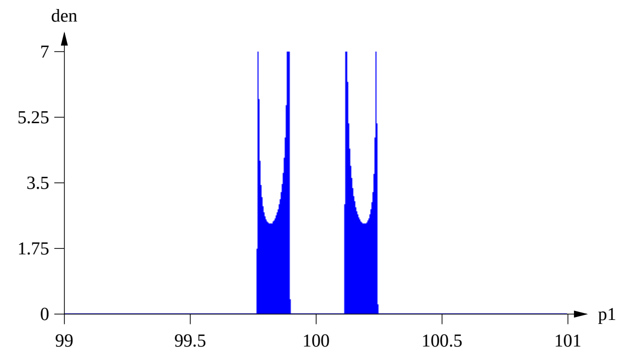


(h) volker41-0-p-sigps-10h(-5)-gf-0.481.xpm.eps

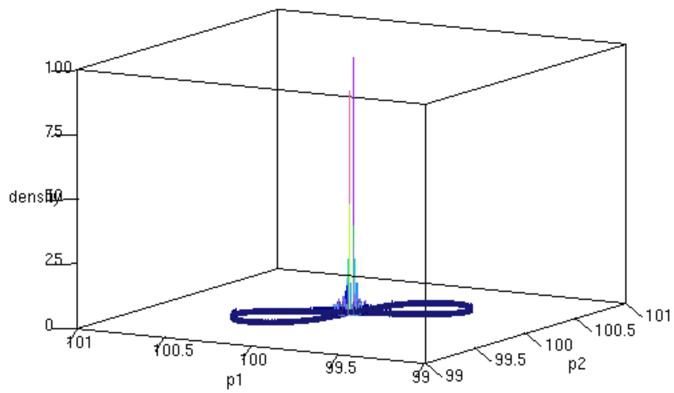
Figure 5.35: variation of gf - $\sigma_p = 10^{-5}$



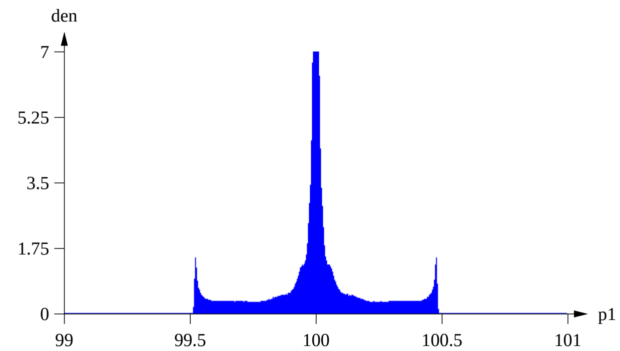
(a) $gf = 0.476$



(b) volker41-0-p-sigps-10h(-6)-gf-0.476.xpm.eps

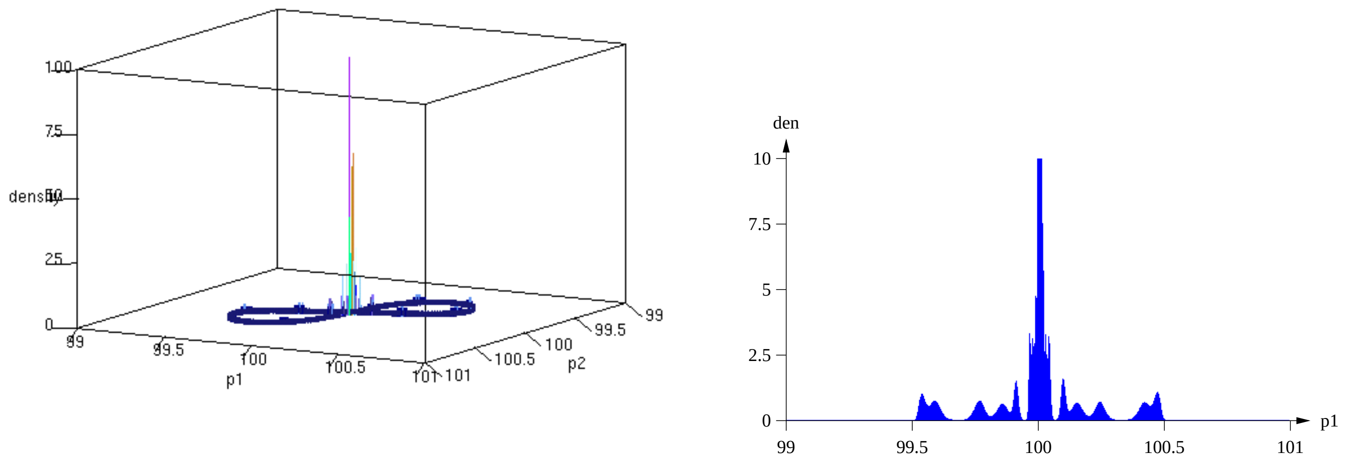


(c) $gf = 0.4798$

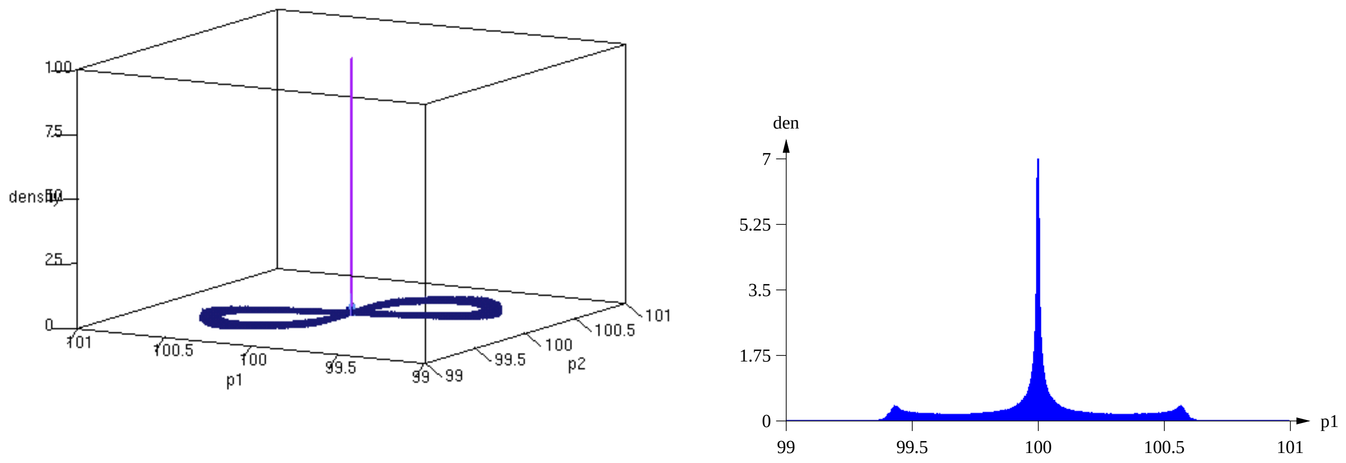


(d) volker41-0-p-sigps-10h(-6)-gf-0.4798.xpm.eps

Figure 5.36: variation of gf - $\sigma_p = 10^{-6}$



(a) $gf = 0.48$ (b) volker41-0-p-sigps-10h(-6)-gf-0.48.xpm.eps



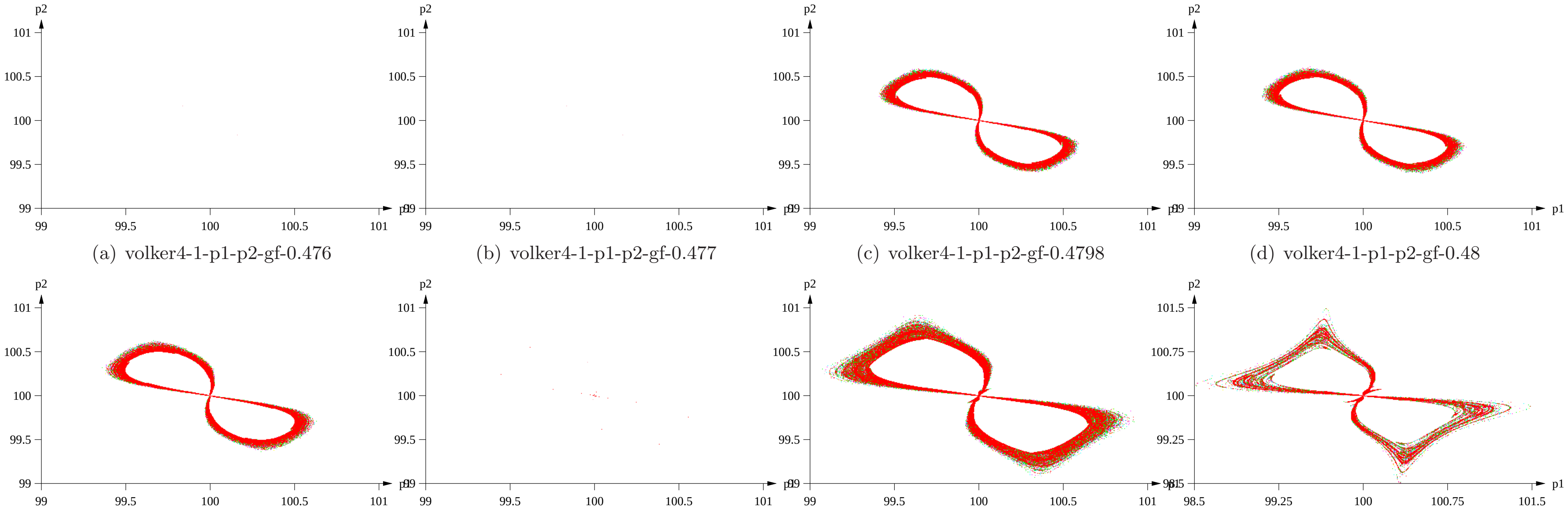
(c) $gf = 0.481$ (d) volker41-0-p-sigps-10h(-6)-gf-0.481.xpm.eps

Figure 5.37: variation of gf - $\sigma_p = 10^{-6}$

6 Numerical Results with group switching

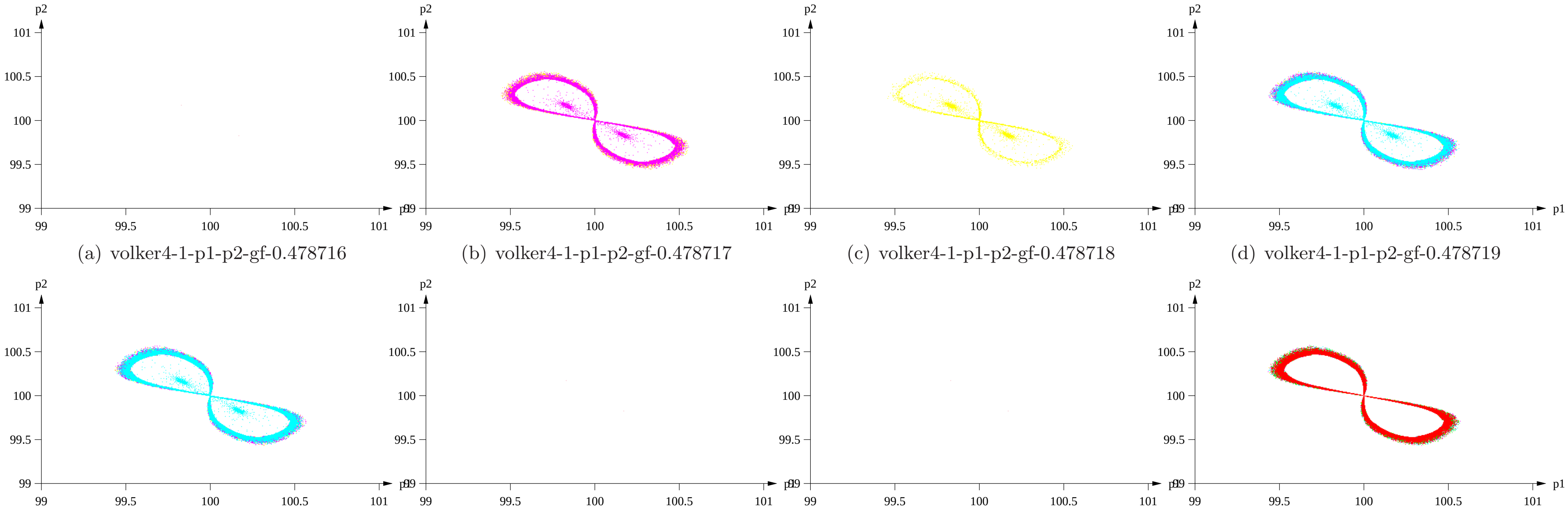
6.1 Variation of g^f - with switch - no noise ($\beta = 0.01$)

Standard Parameter Set	
Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.0
sigps	0.0
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	1
swichf	0
swichm	0



(a) volker4-1-p1-p2-gf-0.476
(b) volker4-1-p1-p2-gf-0.477
(c) volker4-1-p1-p2-gf-0.4798
(d) volker4-1-p1-p2-gf-0.48
(e) volker4-1-p1-p2-gf-0.481
(f) volker4-1-p1-p2-gf-0.485
(g) volker4-1-p1-p2-gf-0.49
(h) volker4-1-p1-p2

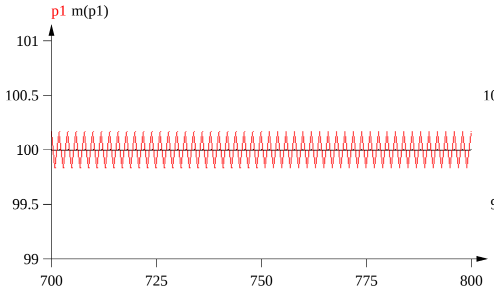
Figure 6.1: **switch - no noise with** $\beta = 0.01$ **:** Attractors



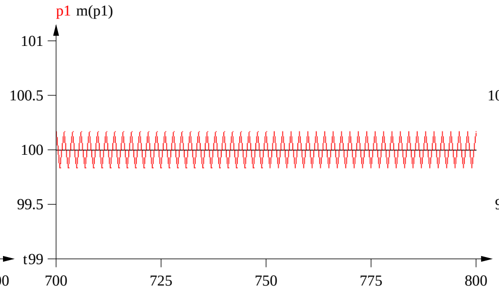
(a) volker4-1-p1-p2-gf-0.478716 (b) volker4-1-p1-p2-gf-0.478717 (c) volker4-1-p1-p2-gf-0.478718 (d) volker4-1-p1-p2-gf-0.478719

(e) volker4-1-p1-p2-gf-0.47872 (f) volker4-1-p1-p2-gf-0.478721 (g) volker4-1-p1-p2-gf-0.478722 (h) volker4-1-p1-p2-gf-0.478723

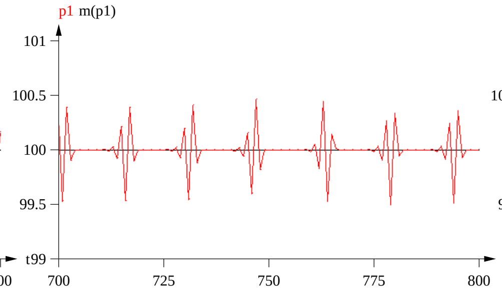
Figure 6.2: **switch - no noise with** $\beta = 0.01$ **:** Attractors



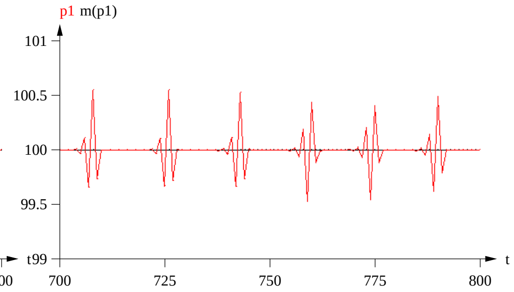
(a) volker16-1-p-gf-0.476.xpm.eps



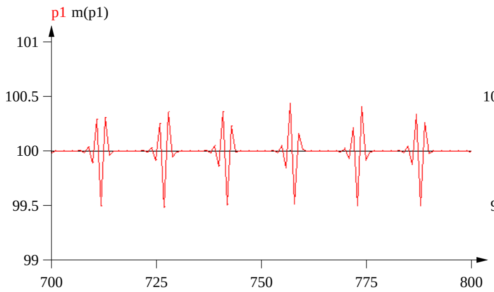
(b) volker16-1-p-gf-0.477.xpm.eps



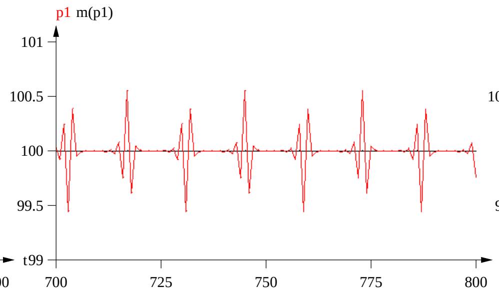
(c) volker16-1-p-gf-0.4798.xpm.eps



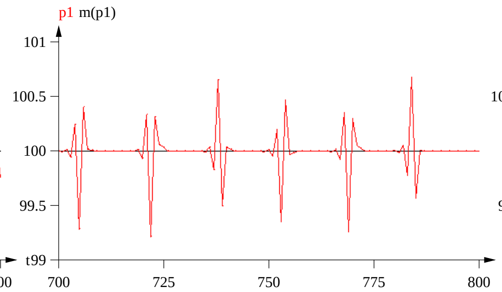
(d) volker16-1-p-gf-0.48.xpm.eps



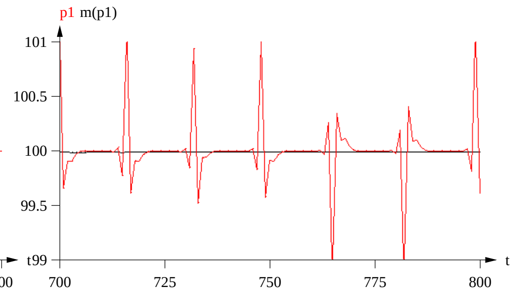
(e) volker16-1-p-gf-0.481.xpm.eps



(f) volker16-1-p-gf-0.485.xpm.eps

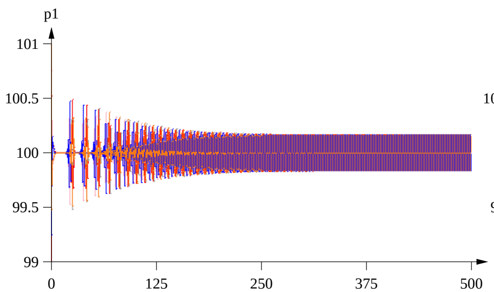


(g) volker16-1-p-gf-0.49.xpm.eps

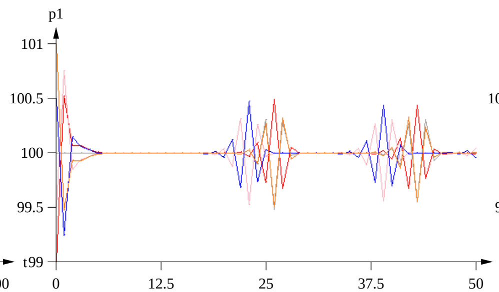


(h) volker16-1-p.xpm.eps

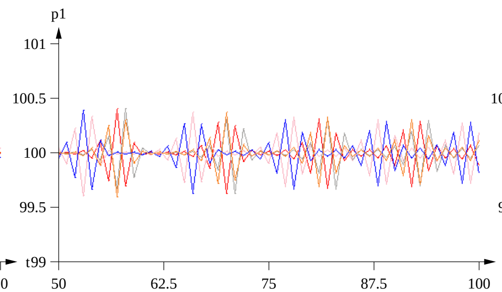
Figure 6.3: **switch - no noise with** $\beta = 0.01$ **:** time series



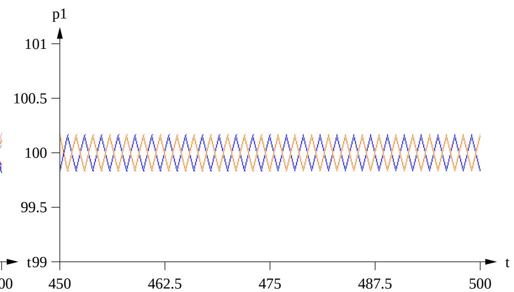
(a) volker17-1-p-gf-0.476-t-0-500.xpm.eps



(b) volker17-1-p-gf-0.476-t-0-50.xpm.eps

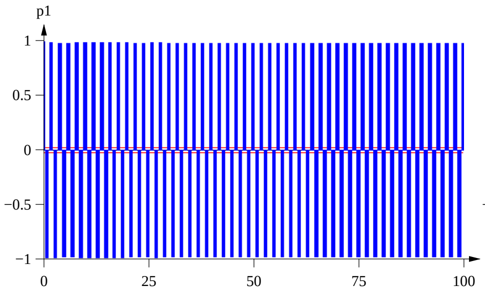


(c) volker17-1-p-gf-0.476-t-50-100.xpm.eps

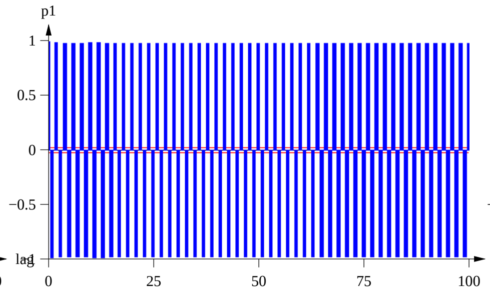


(d) volker17-1-p-gf-0.476-t-450-500.xpm.eps

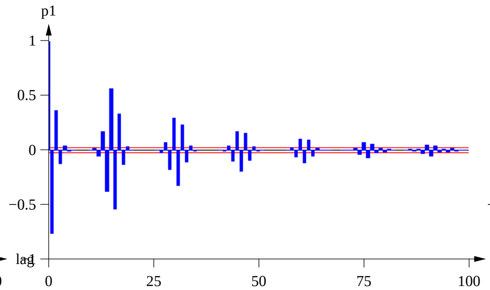
Figure 6.4: **switch - no noise with** $\beta = 0.01$ **:** multiple time series



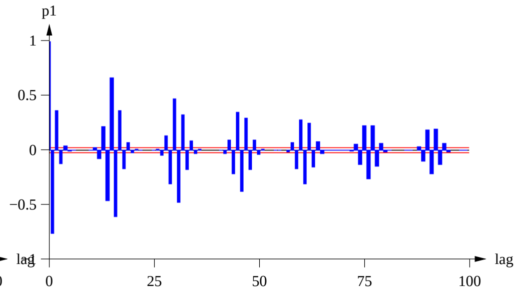
(a) volker87-1-p-gf-0.476.xpm.eps



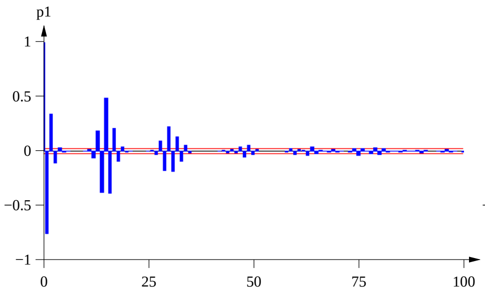
(b) volker87-1-p-gf-0.477.xpm.eps



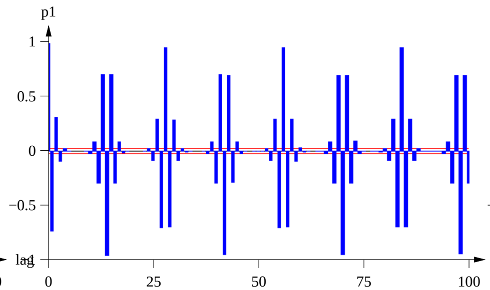
(c) volker87-1-p-gf-0.4798.xpm.eps



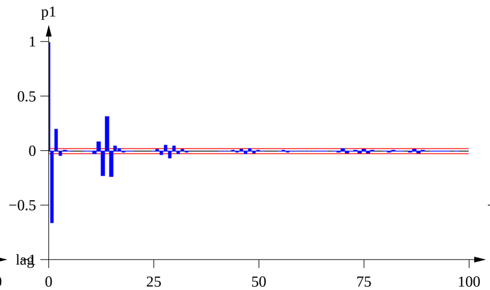
(d) volker87-1-p-gf-0.48.xpm.eps



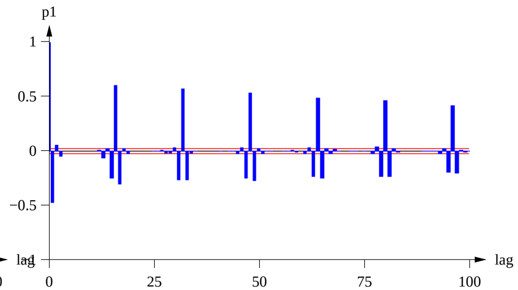
(e) volker87-1-p-gf-0.481.xpm.eps



(f) volker87-1-p-gf-0.485.xpm.eps

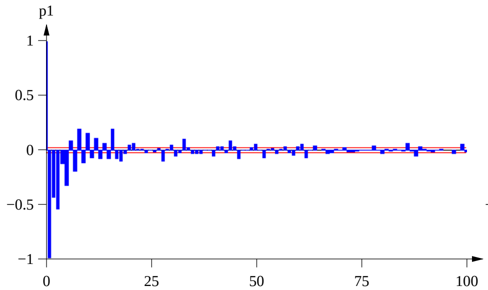


(g) volker87-1-p-gf-0.49.xpm.eps

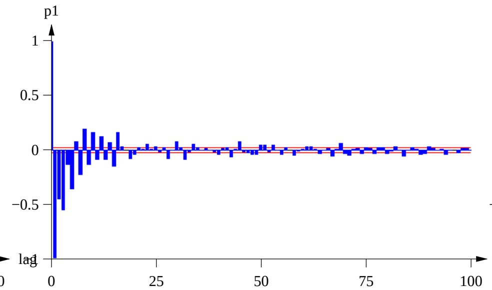


(h) volker87-1-p.xpm.eps

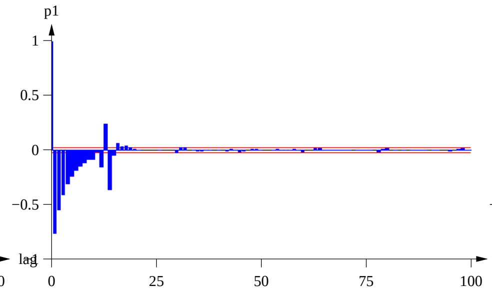
Figure 6.5: **switch - no noise with** $\beta = 0.01$ **:** ACF



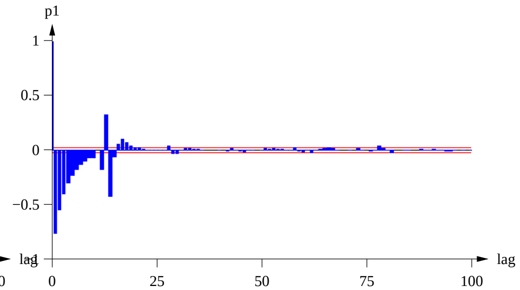
(a) volker88-1-p-gf-0.476.xpm.eps



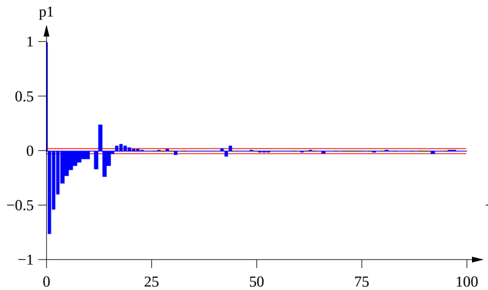
(b) volker88-1-p-gf-0.477.xpm.eps



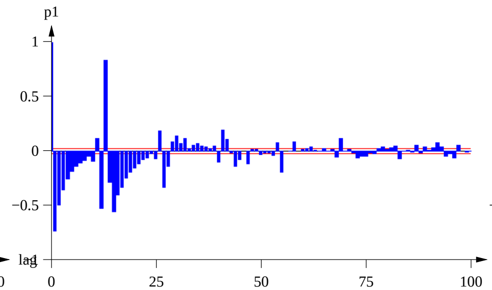
(c) volker88-1-p-gf-0.4798.xpm.eps



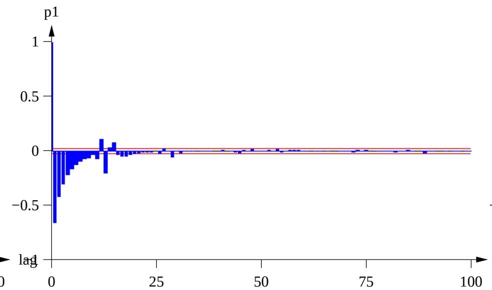
(d) volker88-1-p-gf-0.48.xpm.eps



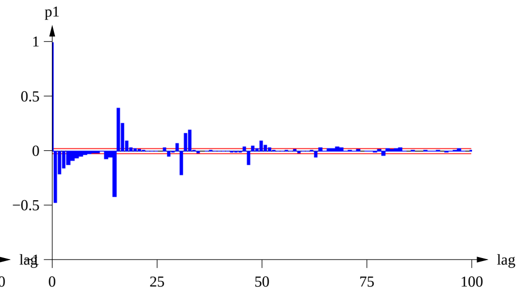
(e) volker88-1-p-gf-0.481.xpm.eps



(f) volker88-1-p-gf-0.485.xpm.eps

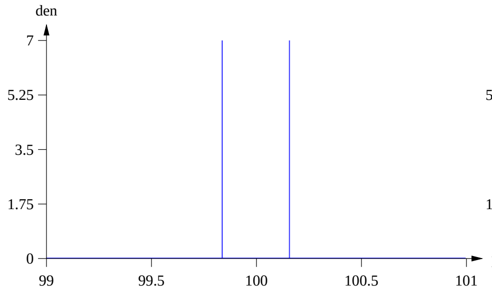


(g) volker88-1-p-gf-0.49.xpm.eps

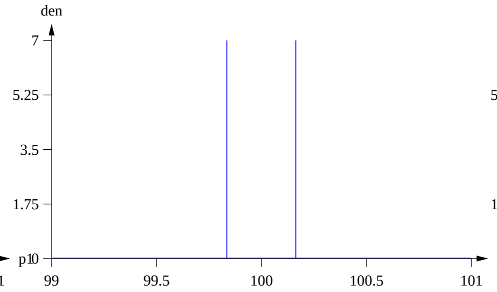


(h) volker88-1-p.xpm.eps

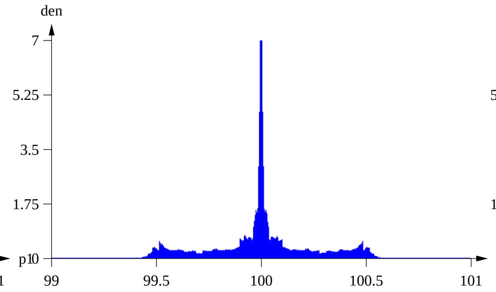
Figure 6.6: **switch - no noise with** $\beta = 0.01$ **:** PACF



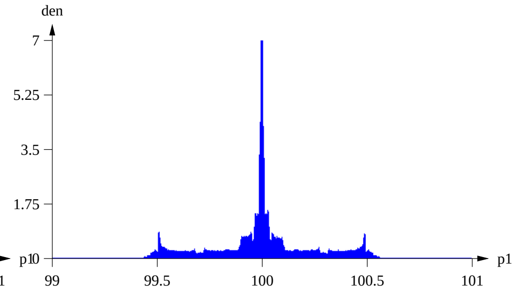
(a) volker41-1-p-gf-0.476.xpm.eps



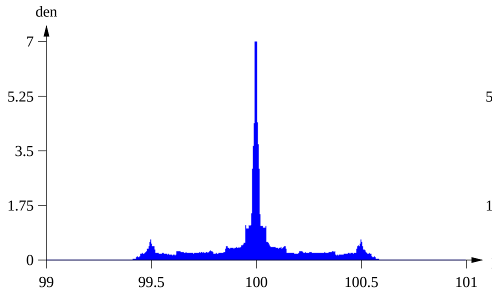
(b) volker41-1-p-gf-0.477.xpm.eps



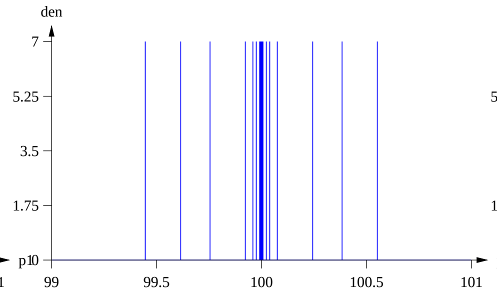
(c) volker41-1-p-gf-0.4798.xpm.eps



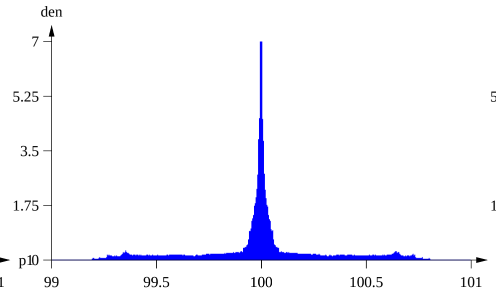
(d) volker41-1-p-gf-0.48.xpm.eps



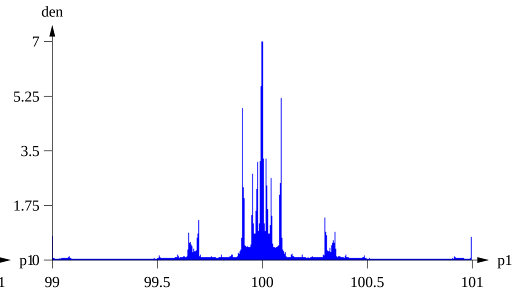
(e) volker41-1-p-gf-0.481.xpm.eps



(f) volker41-1-p-gf-0.485.xpm.eps



(g) volker41-1-p-gf-0.49.xpm.eps



(h) volker41-1-p.xpm.eps

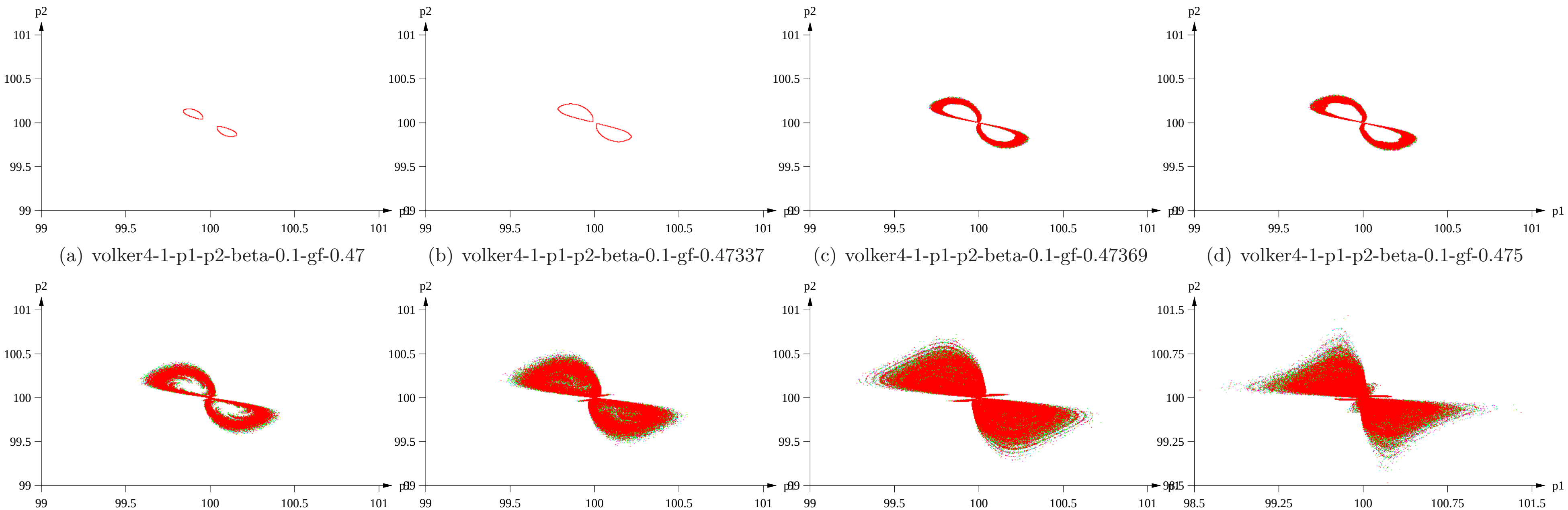
Figure 6.7: **switch - no noise with** $\beta = 0.01$ **:** Densities

statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	100	100	100	100
variance	0.026446	0.027347	0.0283736	0.0287874
standard deviation	0.162622	0.165369	0.168445	0.169669
skewness	1.02088e-05	7.33859e-06	-0.000467131	-0.00760042
kurtosis	-1.98896	-2.00847	3.47773	3.3415
quantile (0.95)	100.273	100.278	100.242	100.247

statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100	100	100	99.9999
variance	0.0302197	0.037511	0.0462322	0.0770309
standard deviation	0.173838	0.193678	0.215017	0.277544
skewness	0.00125277	9.56144e-06	0.000750769	-0.0232838
kurtosis	3.55228	3.1011	4.5675	7.90634
quantile (0.95)	100.247	100.29	100.256	100.211

6.2 Variation of g^f - with switch - no noise ($\beta = 0.1$)

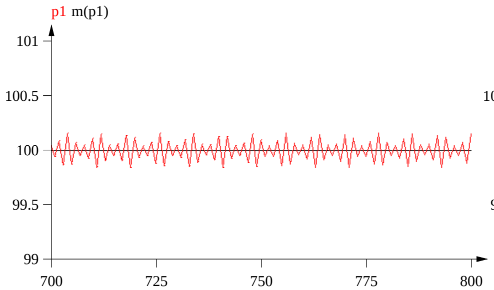
Standard Parameter Set	
Parameter	Value
beta	0.1
delta	0.8
af = ac	0.05
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
sigps	0.0
tau	0.0
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	1
swichf	0
swichm	0



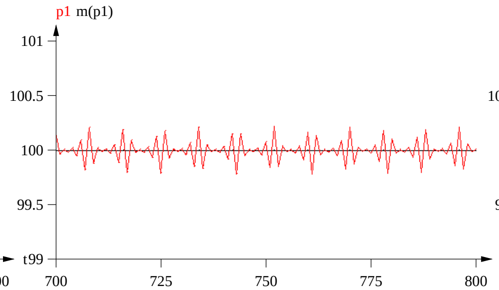
(a) volker4-1-p1-p2-beta-0.1-gf-0.47 (b) volker4-1-p1-p2-beta-0.1-gf-0.47337 (c) volker4-1-p1-p2-beta-0.1-gf-0.47369 (d) volker4-1-p1-p2-beta-0.1-gf-0.475

(e) volker4-1-p1-p2-beta-0.1-gf-0.48 (f) volker4-1-p1-p2-beta-0.1-gf-0.485 (g) volker4-1-p1-p2-beta-0.1-gf-0.49 (h) volker4-1-p1-p2-beta-0.1-gf-0.5

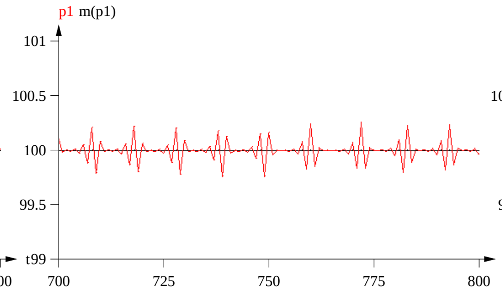
Figure 6.8: **switch - no noise with** $\beta = 0.1$ **:** Attractors



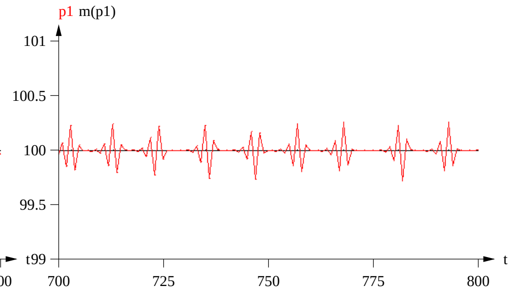
(a) volker16-1-p-beta-0.1-gf-0.47.xpm.eps



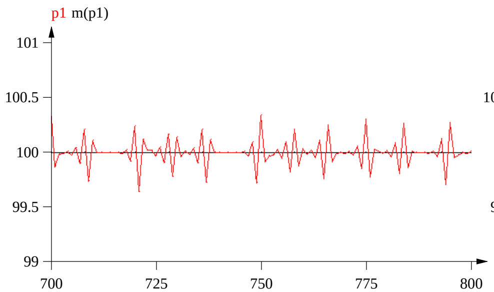
(b) volker16-1-p-beta-0.1-gf-0.47337.xpm.eps



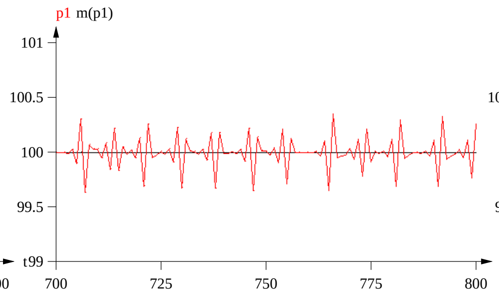
(c) volker16-1-p-beta-0.1-gf-0.47369.xpm.eps



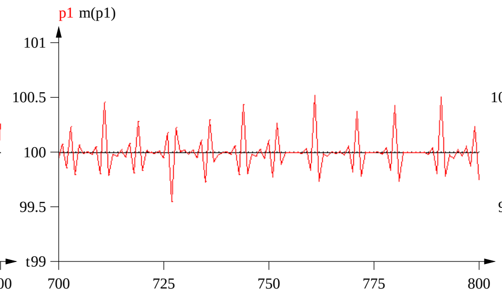
(d) volker16-1-p-beta-0.1-gf-0.475.xpm.eps



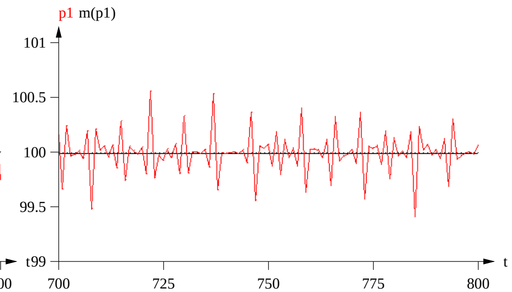
(e) volker16-1-p-beta-0.1-gf-0.48.xpm.eps



(f) volker16-1-p-beta-0.1-gf-0.485.xpm.eps

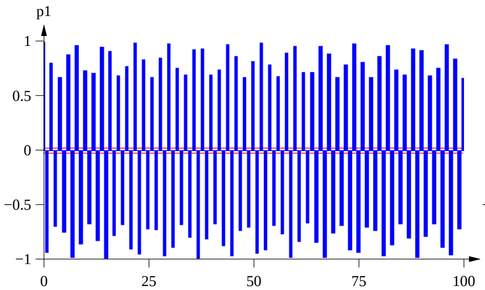


(g) volker16-1-p-beta-0.1-gf-0.49.xpm.eps

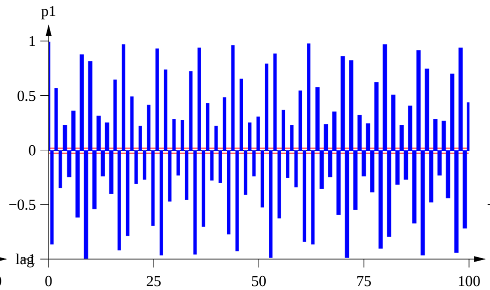


(h) volker16-1-p-beta-0.1-gf-0.5.xpm.eps

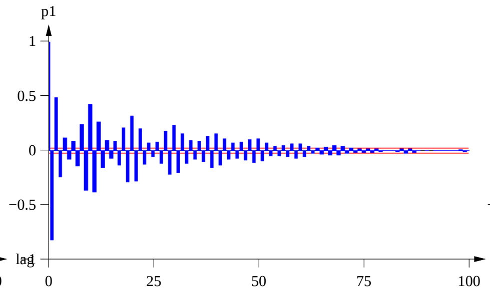
Figure 6.9: **switch - no noise with** $\beta = 0.1$ **:** time series



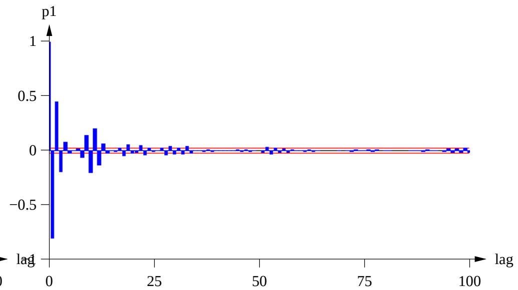
(a) volker87-1-p-beta-0.1-gf-0.47.xpm.eps



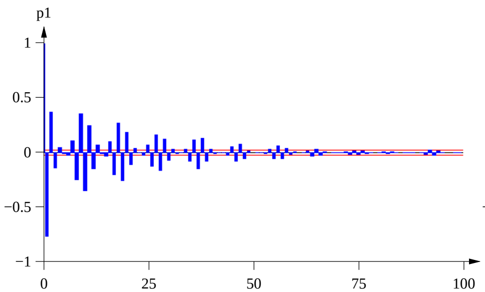
(b) volker87-1-p-beta-0.1-gf-0.47337.xpm.eps



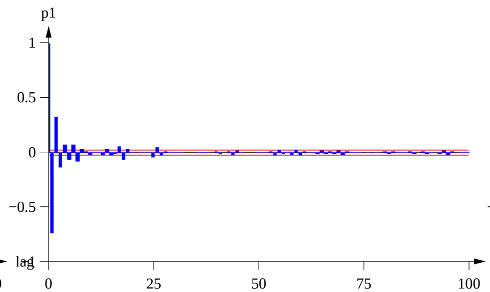
(c) volker87-1-p-beta-0.1-gf-0.47369.xpm.eps



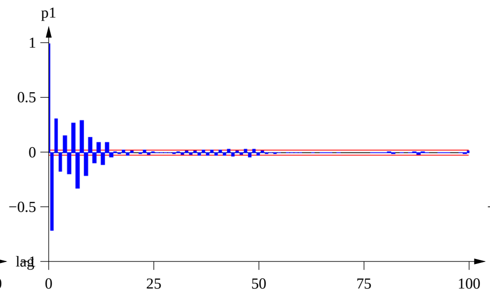
(d) volker87-1-p-beta-0.1-gf-0.475.xpm.eps



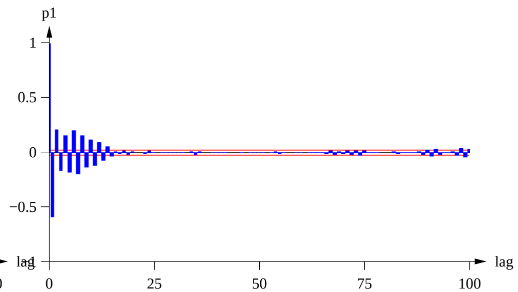
(e) volker87-1-p-beta-0.1-gf-0.48.xpm.eps



(f) volker87-1-p-beta-0.1-gf-0.485.xpm.eps

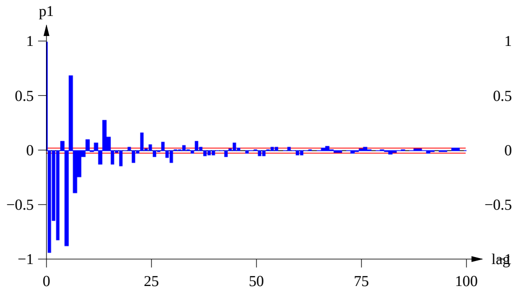


(g) volker87-1-p-beta-0.1-gf-0.49.xpm.eps

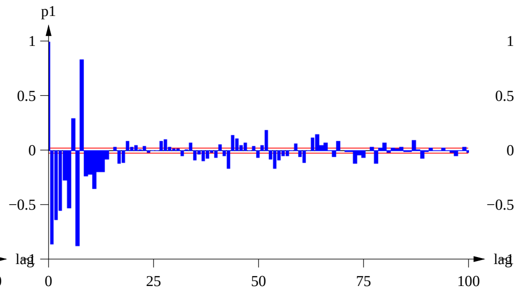


(h) volker87-1-p-beta-0.1-gf-0.5.xpm.eps

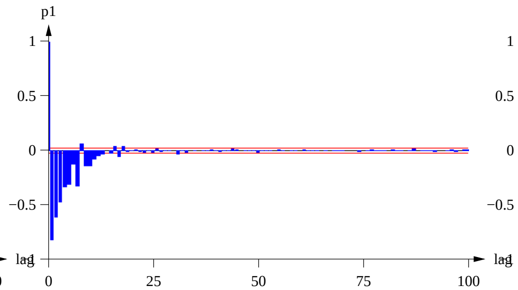
Figure 6.10: **switch - no noise with** $\beta = 0.1$ **:** ACF



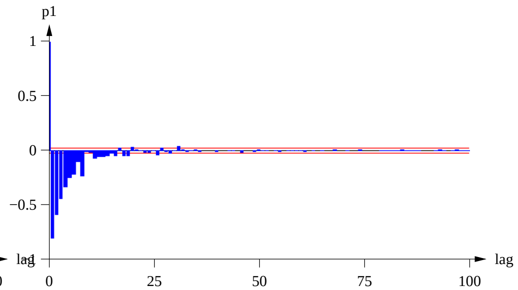
(a) volker88-1-p-beta-0.1-gf-0.47.xpm.eps



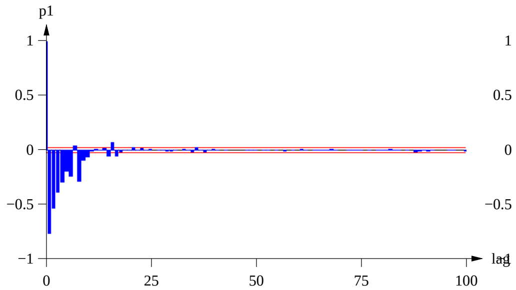
(b) volker88-1-p-beta-0.1-gf-0.47337.xpm.eps



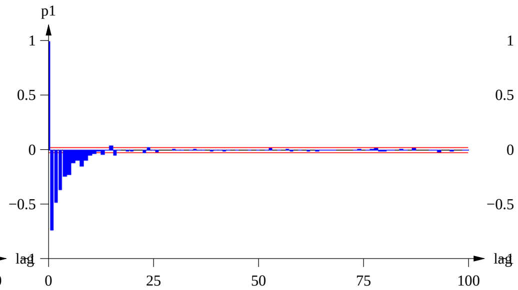
(c) volker88-1-p-beta-0.1-gf-0.47369.xpm.eps



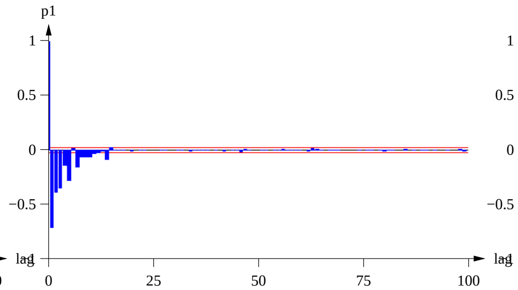
(d) volker88-1-p-beta-0.1-gf-0.475.xpm.eps



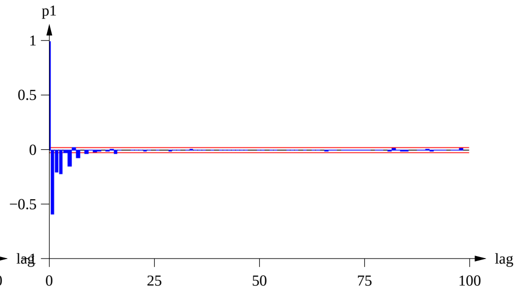
(e) volker88-1-p-beta-0.1-gf-0.48.xpm.eps



(f) volker88-1-p-beta-0.1-gf-0.485.xpm.eps

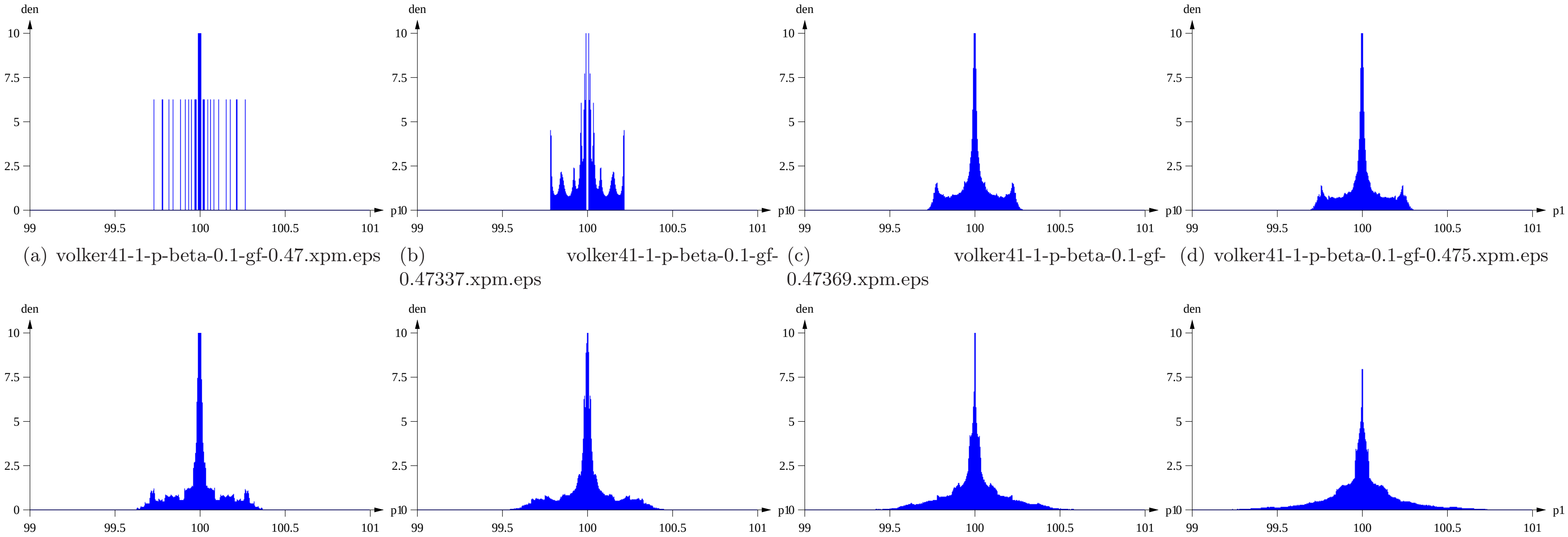


(g) volker88-1-p-beta-0.1-gf-0.49.xpm.eps



(h) volker88-1-p-beta-0.1-gf-0.5.xpm.eps

Figure 6.11: **switch - no noise with** $\beta = 0.1$ **:** PACF



(a) volker41-1-p-beta-0.1-gf-0.47.xpm.eps

(b) volker41-1-p-beta-0.1-gf-0.47337.xpm.eps

(c) volker41-1-p-beta-0.1-gf-0.47369.xpm.eps

(d) volker41-1-p-beta-0.1-gf-0.475.xpm.eps

(e) volker41-1-p-beta-0.1-gf-0.48.xpm.eps

(f) volker41-1-p-beta-0.1-gf-0.485.xpm.eps

(g) volker41-1-p-beta-0.1-gf-0.49.xpm.eps

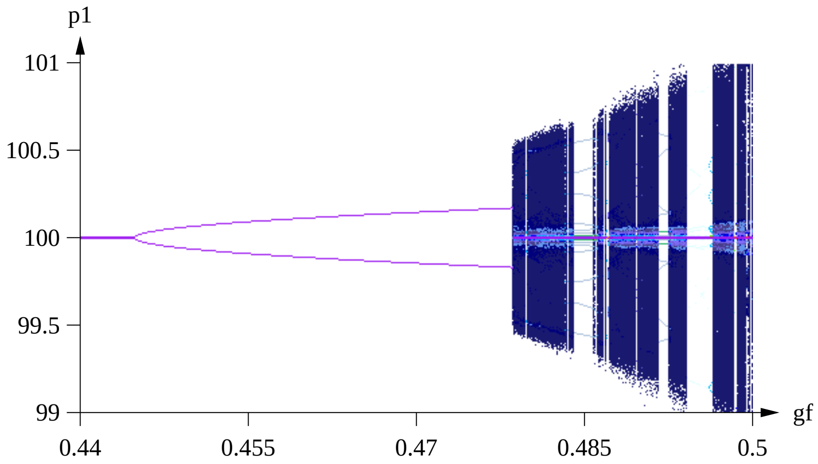
(h) volker41-1-p-beta-0.1-gf-0.5.xpm.eps

Figure 6.12: **switch - no noise with** $\beta = 0.1$ **:** Densities

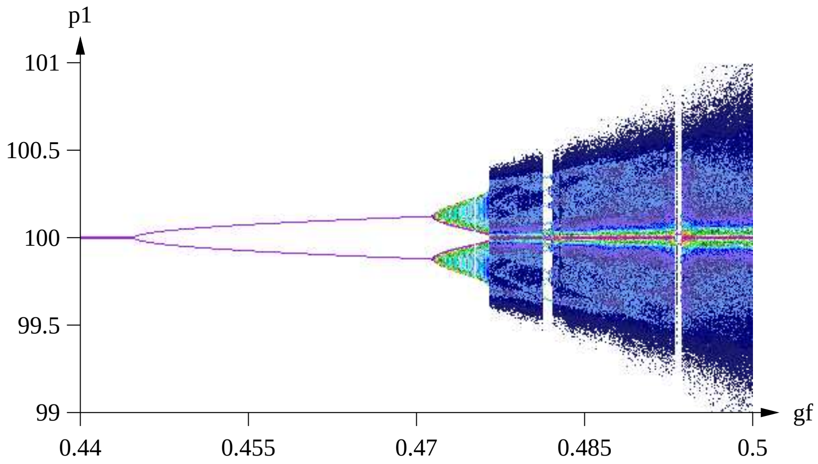
statistic	$g^f = 0.47$	$g^f = 0.47337$	$g^f = 0.47369$	$g^f = 0.475$
mean	100	100	100	100
variance	0.026446	0.027347	0.0283736	0.0287874
standard deviation	0.162622	0.165369	0.168445	0.169669
skewness	1.02088e-05	7.33859e-06	-0.000467131	-0.00760042
kurtosis	-1.98896	-2.00847	3.47773	3.3415
quantile (0.95)	100.273	100.278	100.242	100.247

statistic	$g^f = 0.48$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100	100	100	99.9999
variance	0.0302197	0.037511	0.0462322	0.0770309
standard deviation	0.173838	0.193678	0.215017	0.277544
skewness	0.00125277	9.56144e-06	0.000750769	-0.0232838
kurtosis	3.55228	3.1011	4.5675	7.90634
quantile (0.95)	100.247	100.29	100.256	100.211

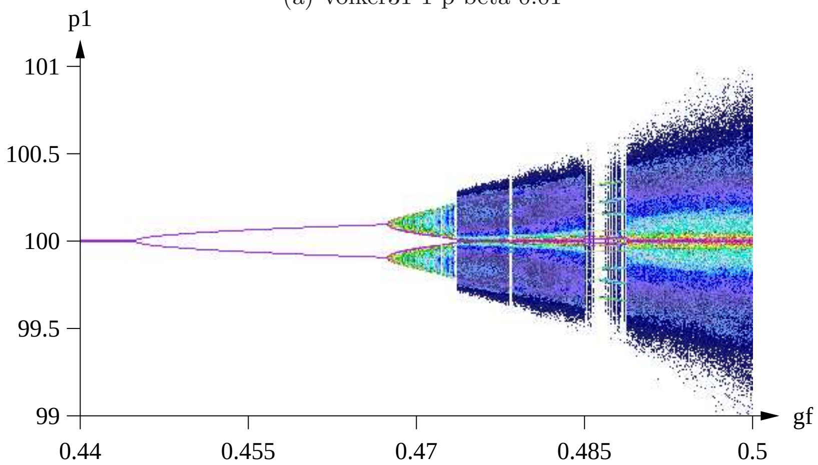
6.3 Role of the switching speed β



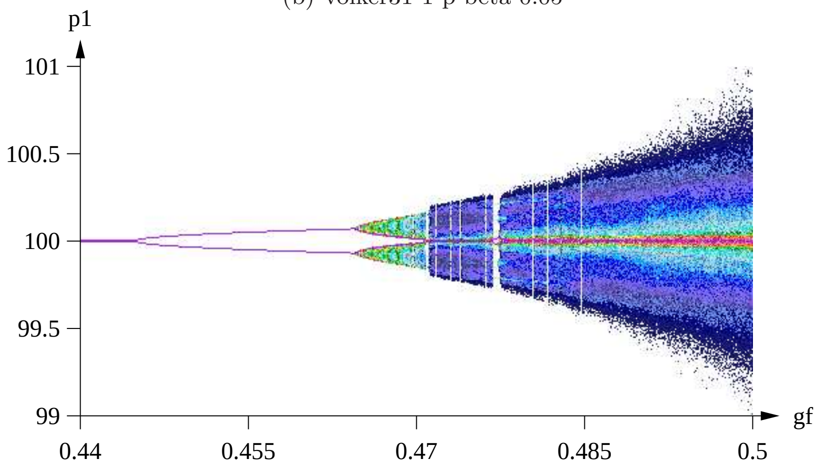
(a) volker31-1-p-beta-0.01



(b) volker31-1-p-beta-0.05

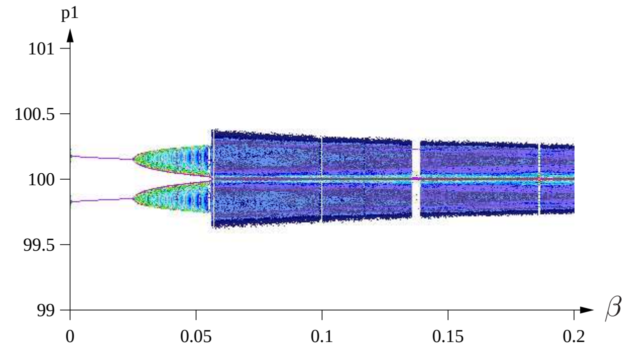


(c) volker31-1-p-beta-0.1

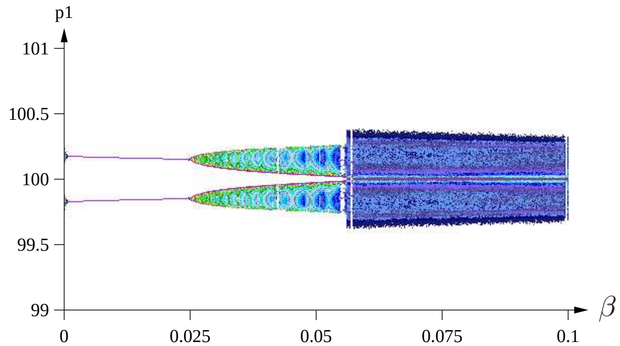


(d) volker31-1-p-beta-0.2

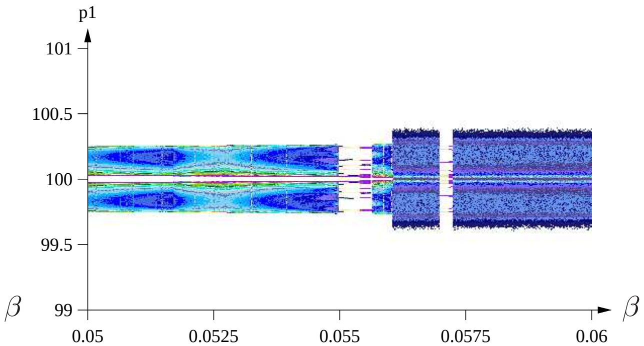
Figure 6.13: Bifurcation



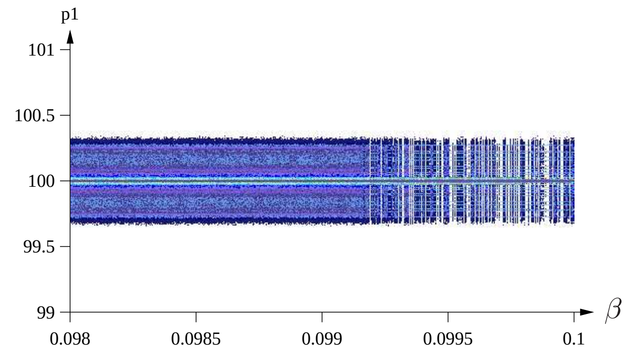
(a) volker31-1-beta-0-0.2-gf-0.476



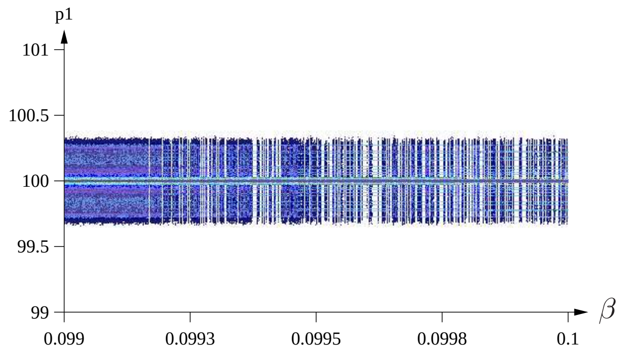
(b) volker31-1-beta-0-0.1-gf-0.476



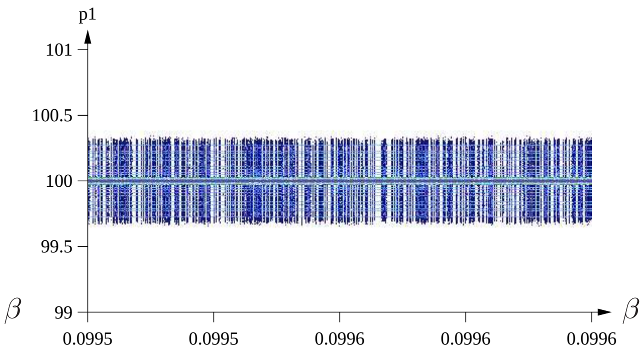
(c) volker31-1-beta-0.05-0.06-gf-0.476



(d) volker31-1-beta-0.098-0.1-gf-0.476

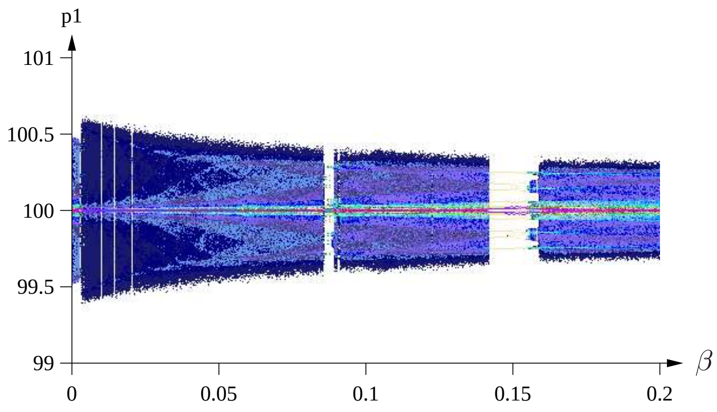


(e) volker31-1-beta-0.099-0.1-gf-0.476

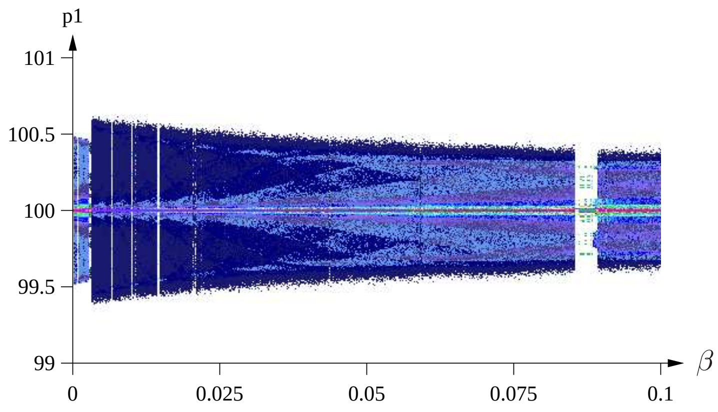


(f) volker31-1-beta-0.0995-0.0996-gf-0.476

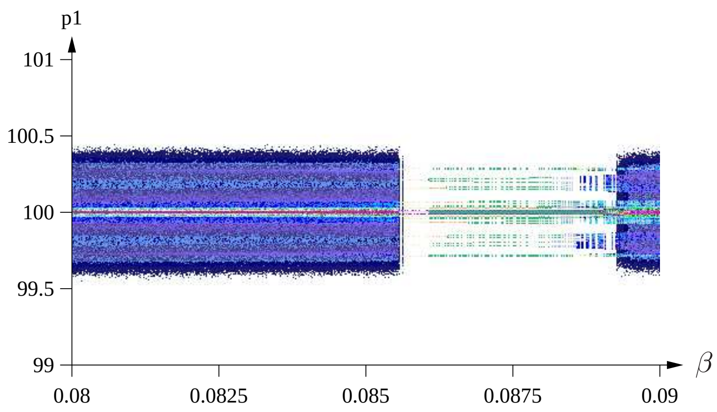
Figure 6.14: **switch - no noise with: Role of** β $g_f = 0.476$



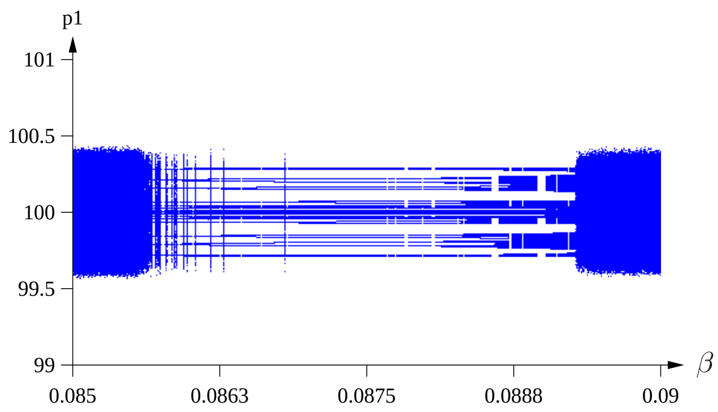
(a) volker31-1-beta-0.0-0.2-gf-0.4798



(b) volker31-1-beta-0.0-0.1-gf-0.4798

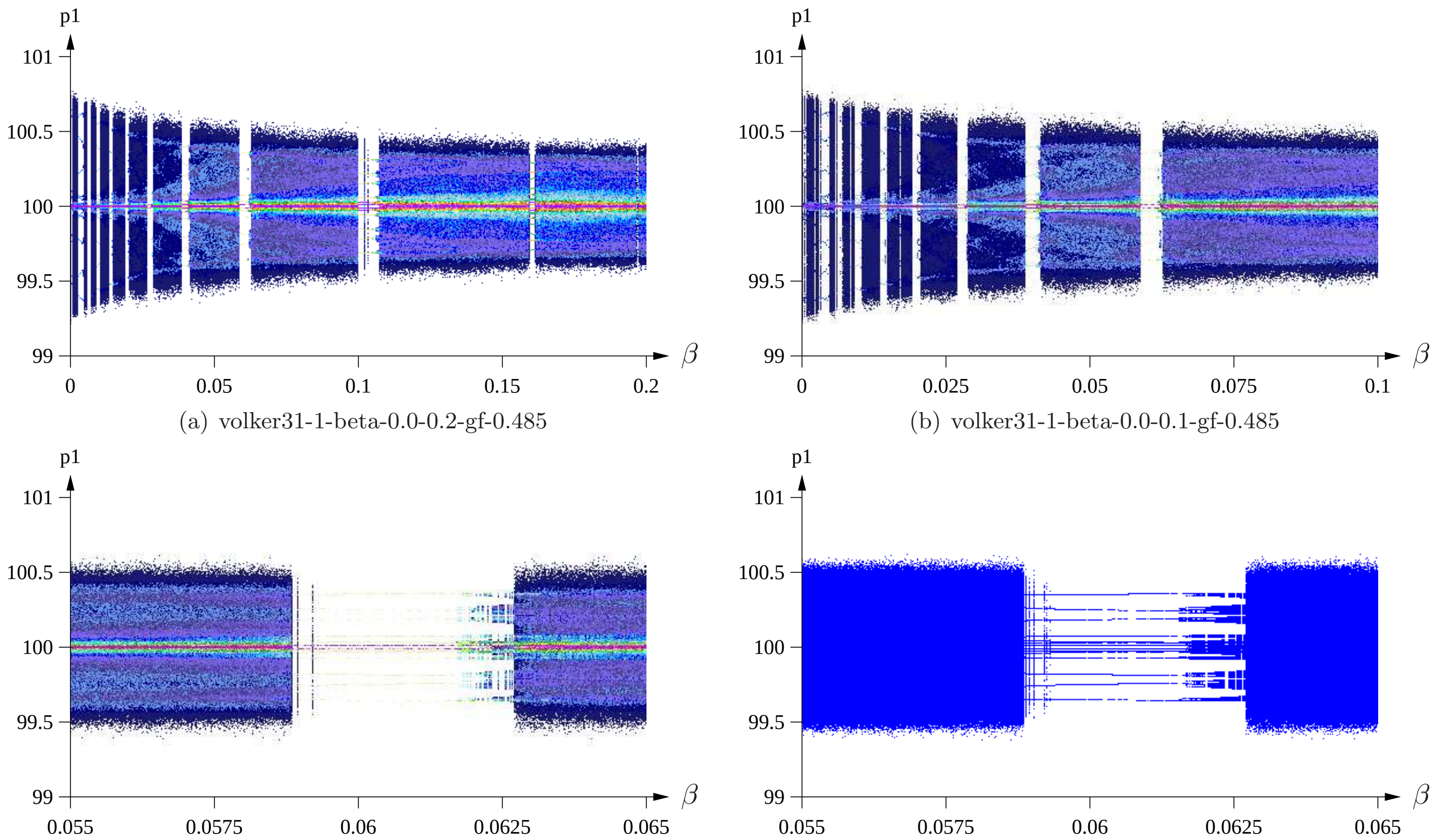


(c) volker31-1-beta-0.08-0.09-gf-0.4798



(d) volker1-1-beta-0.085-0.09-gf-0.4798

Figure 6.15: **switch - no noise with: Role of** β $g_f = 0.4798$



(a) volker31-1-beta-0.0-0.2-gf-0.485

(b) volker31-1-beta-0.0-0.1-gf-0.485

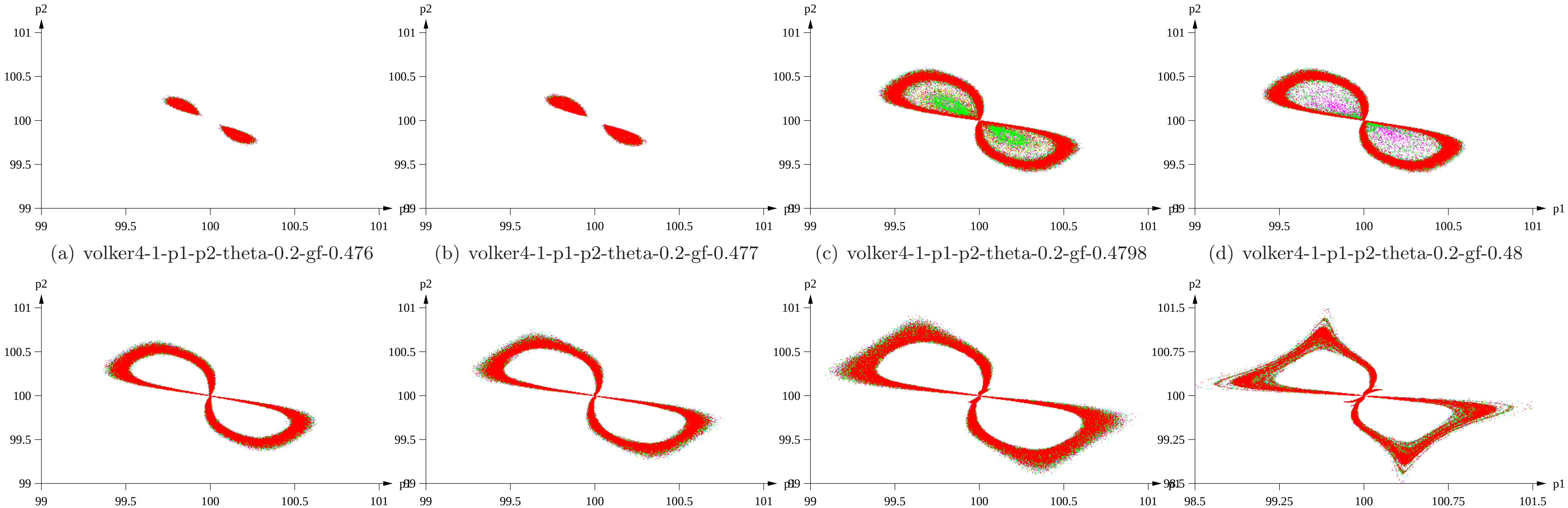
(c) volker31-1-beta-0.055-0.065-gf-0.485

(d) volker1-1-beta-0.05-0.065-gf-0.485

Figure 6.16: **switch - no noise with: Role of** β $g_f = 0.485$

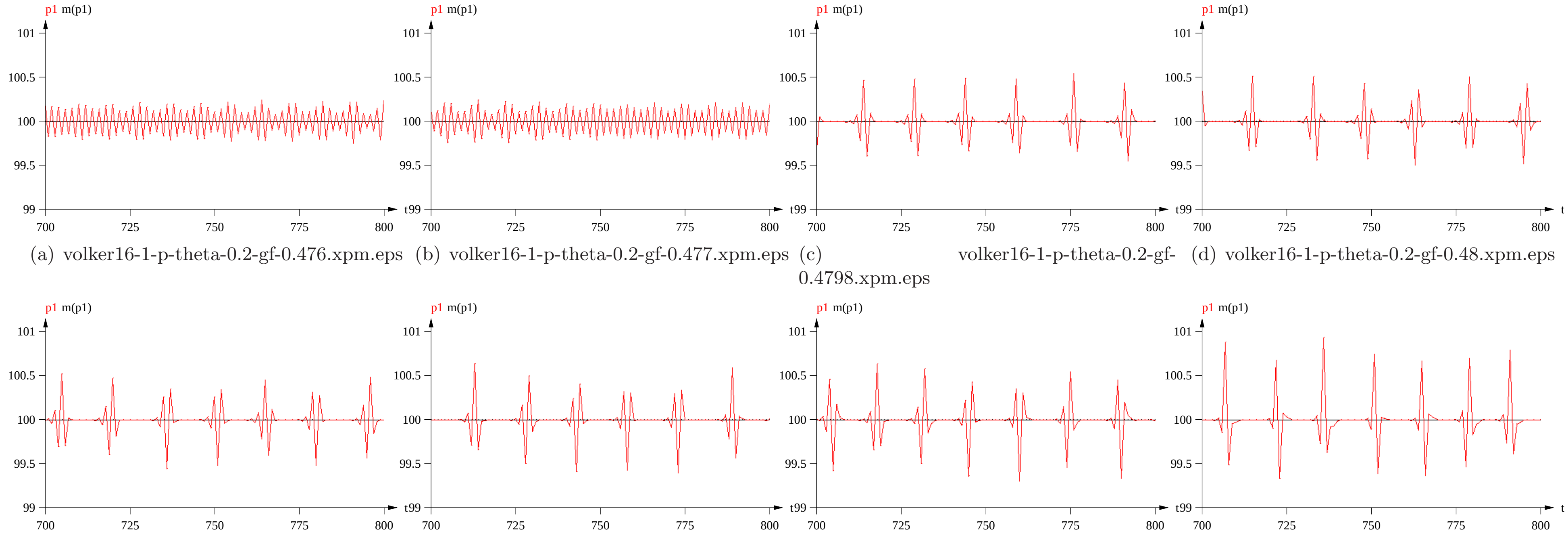
6.4 Variation of g^f - with switch and noisy dividends ($\theta = 0.2$)

Standard Parameter Set	
Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigps	0.0
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.0
theta	0.2
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	1
swichf	0
swichm	0



(a) volker4-1-p1-p2-theta-0.2-gf-0.476
(b) volker4-1-p1-p2-theta-0.2-gf-0.477
(c) volker4-1-p1-p2-theta-0.2-gf-0.4798
(d) volker4-1-p1-p2-theta-0.2-gf-0.48
(e) volker4-1-p1-p2-theta-0.2-gf-0.481
(f) volker4-1-p1-p2-theta-0.2-gf-0.485
(g) volker4-1-p1-p2-theta-0.2-gf-0.49
(h) volker4-1-p1-p2-theta-0.2

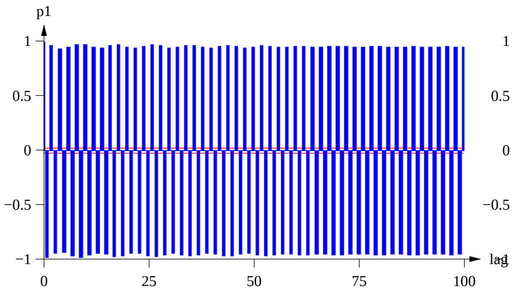
Figure 6.17: **switch - with noisy dividends**– $\theta = 0.2$ **:** Attractors



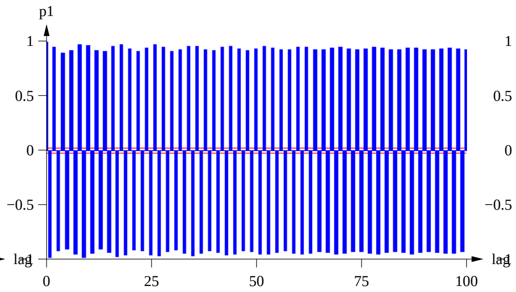
(a) volker16-1-p-theta-0.2-gf-0.476.xpm.eps (b) volker16-1-p-theta-0.2-gf-0.477.xpm.eps (c) volker16-1-p-theta-0.2-gf-0.4798.xpm.eps (d) volker16-1-p-theta-0.2-gf-0.48.xpm.eps

(e) volker16-1-p-theta-0.2-gf-0.481.xpm.eps (f) volker16-1-p-theta-0.2-gf-0.485.xpm.eps (g) volker16-1-p-theta-0.2-gf-0.49.xpm.eps (h) volker16-1-p-theta-0.2.xpm.eps

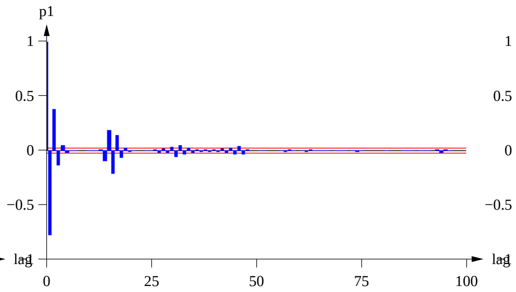
Figure 6.18: **switch - with noisy dividends**– $\theta = 0.2$ **:** time series



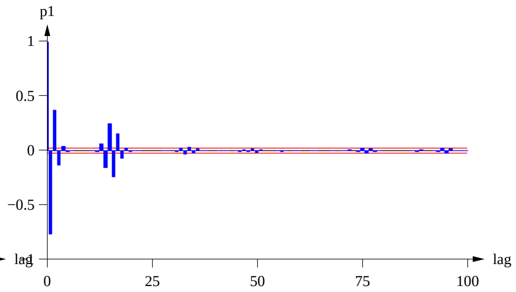
(a) volker87-1-p-theta-0.2-gf-0.476.xpm.eps



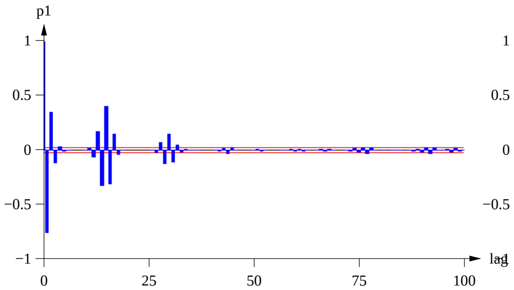
(b) volker87-1-p-theta-0.2-gf-0.477.xpm.eps



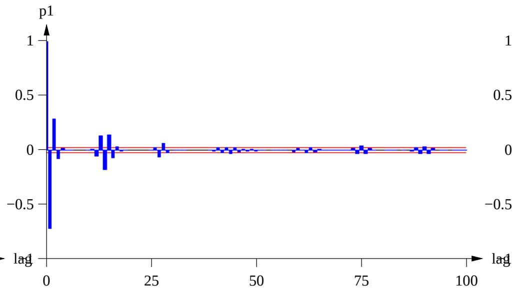
(c) volker87-1-p-theta-0.2-gf-0.4798.xpm.eps



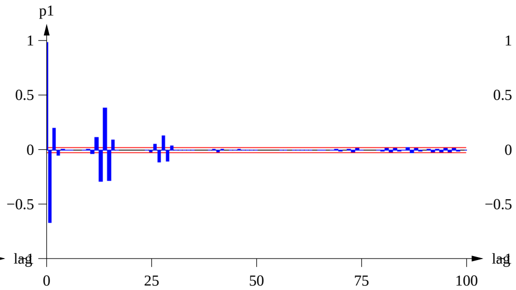
(d) volker87-1-p-theta-0.2-gf-0.48.xpm.eps



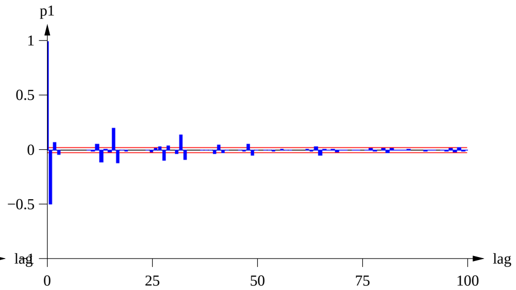
(e) volker87-1-p-theta-0.2-gf-0.481.xpm.eps



(f) volker87-1-p-theta-0.2-gf-0.485.xpm.eps

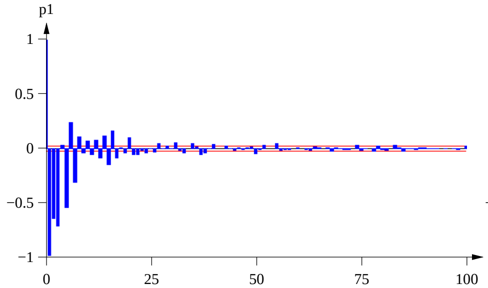


(g) volker87-1-p-theta-0.2-gf-0.49.xpm.eps

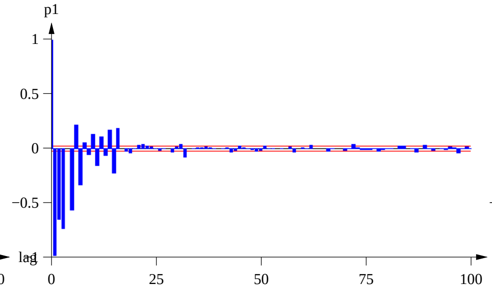


(h) volker87-1-p-theta-0.2.xpm.eps

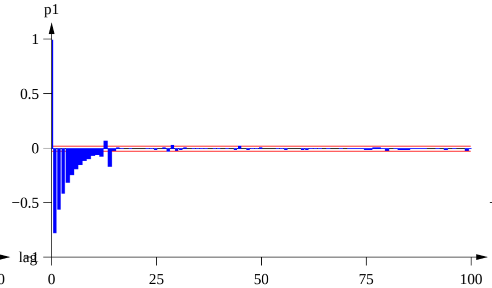
Figure 6.19: **switch - with noisy dividends**– $\theta = 0.2$ **:** ACF



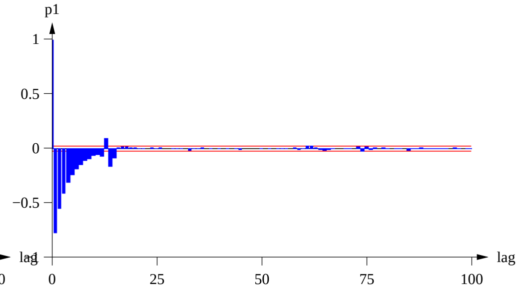
(a) volker88-1-p-theta-0.2-gf-0.476.xpm.eps



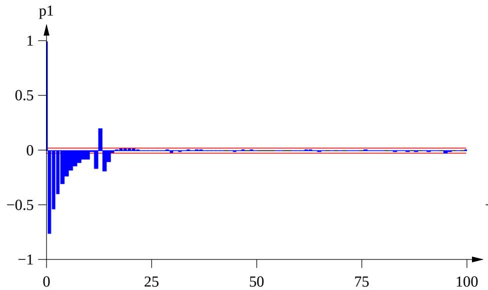
(b) volker88-1-p-theta-0.2-gf-0.477.xpm.eps



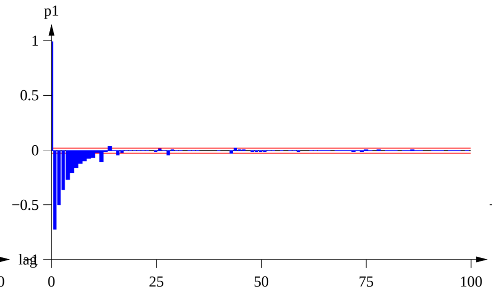
(c) volker88-1-p-theta-0.2-gf-0.4798.xpm.eps



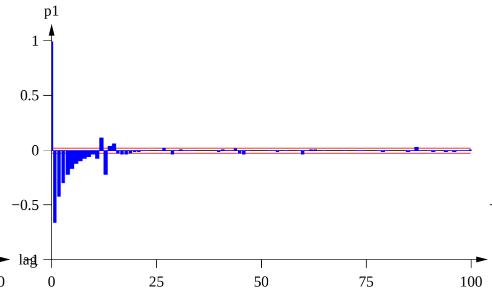
(d) volker88-1-p-theta-0.2-gf-0.48.xpm.eps



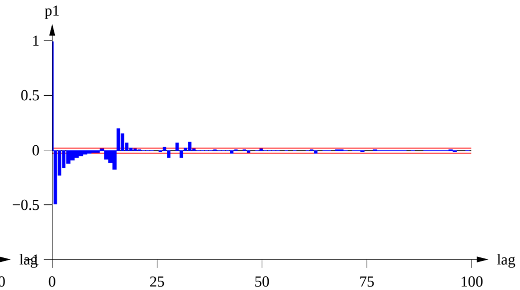
(e) volker88-1-p-theta-0.2-gf-0.481.xpm.eps



(f) volker88-1-p-theta-0.2-gf-0.485.xpm.eps

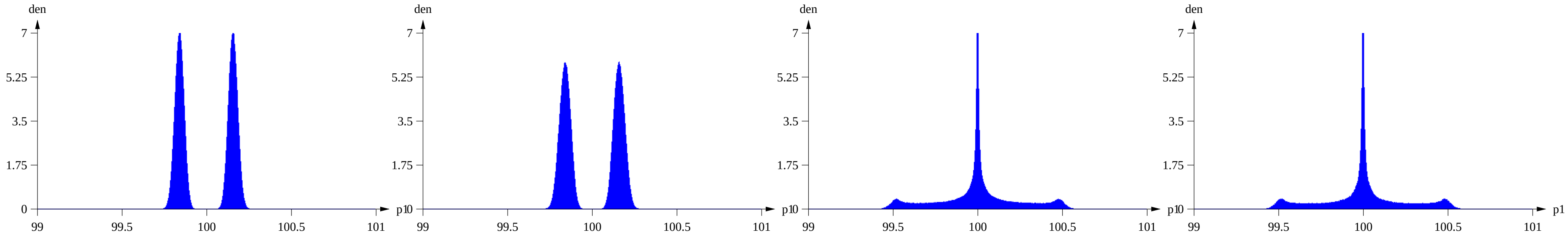


(g) volker88-1-p-theta-0.2-gf-0.49.xpm.eps

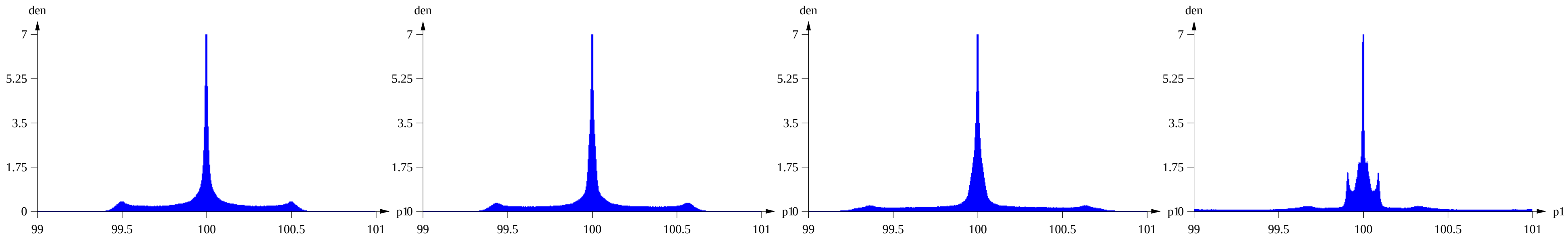


(h) volker88-1-p-theta-0.2.xpm.eps

Figure 6.20: **switch - with noisy dividends**– $\theta = 0.2$ **:** PACF



(a) volker41-1-p-theta-0.2-gf-0.476.xpm.eps (b) volker41-1-p-theta-0.2-gf-0.477.xpm.eps (c) volker41-1-p-theta-0.2-gf-0.4798.xpm.eps (d) volker41-1-p-theta-0.2-gf-0.48.xpm.eps



(e) volker41-1-p-theta-0.2-gf-0.481.xpm.eps (f) volker41-1-p-theta-0.2-gf-0.485.xpm.eps (g) volker41-1-p-theta-0.2-gf-0.49.xpm.eps (h) volker41-1-p-theta-0.2.xpm.eps

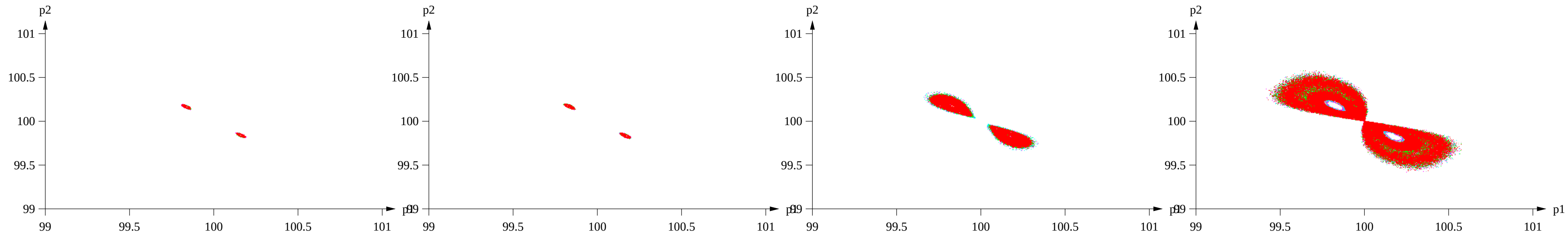
Figure 6.21: **switch - with noisy dividends**– $\theta = 0.2$ **:** Densities

statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	100	100	100	100
variance	0.0264769	0.0273911	0.028594	0.0288171
standard deviation	0.162717	0.165503	0.169098	0.169756
skewness	5.33309e-06	4.68452e-05	-0.000101558	-0.000589314
kurtosis	-1.88569	-1.85651	3.10875	3.29109
quantile (0.95)	100.273	100.278	100.253	100.249

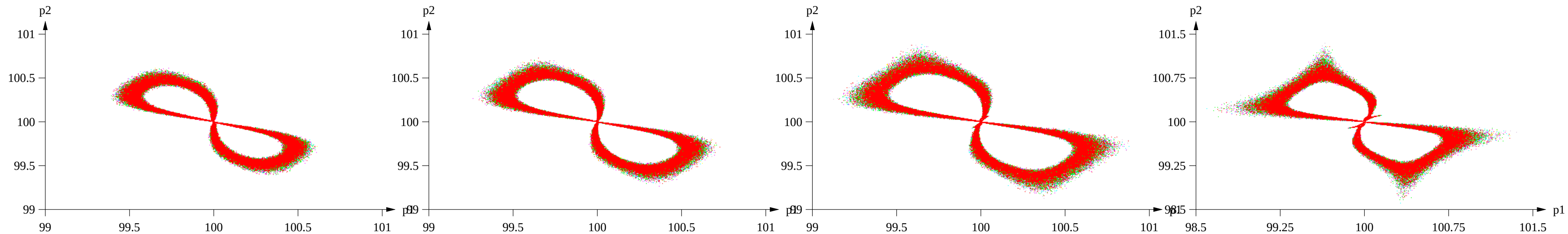
statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100	100	100	100
variance	0.0302867	0.0369311	0.0461944	0.0745183
standard deviation	0.174031	0.192175	0.214929	0.27298
skewness	0.00218371	0.00215292	0.00353685	-0.00226191
kurtosis	3.46947	3.69288	4.35971	7.44997
quantile (0.95)	100.25	100.267	100.266	100.218

6.5 Variation of g^f - with switch and noisy supply ($\tau = 10^-3$)

Standard Parameter Set	
Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigps	0.0
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.001
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	1
swichf	0
swichm	0

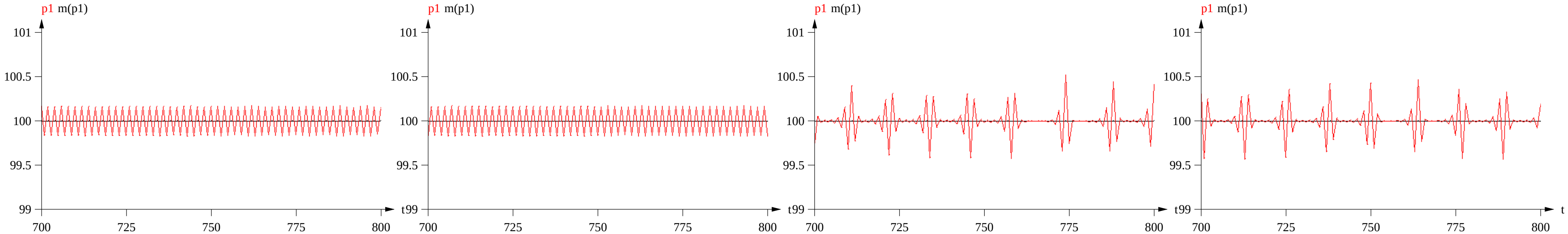


(a) volker4-1-p1-p2-tau-10h(-3)-gf-0.476 (b) volker4-1-p1-p2-tau-10h(-3)-gf-0.477 (c) volker4-1-p1-p2-tau-10h(-3)-gf-0.4798 (d) volker4-1-p1-p2-tau-10h(-3)-gf-0.48

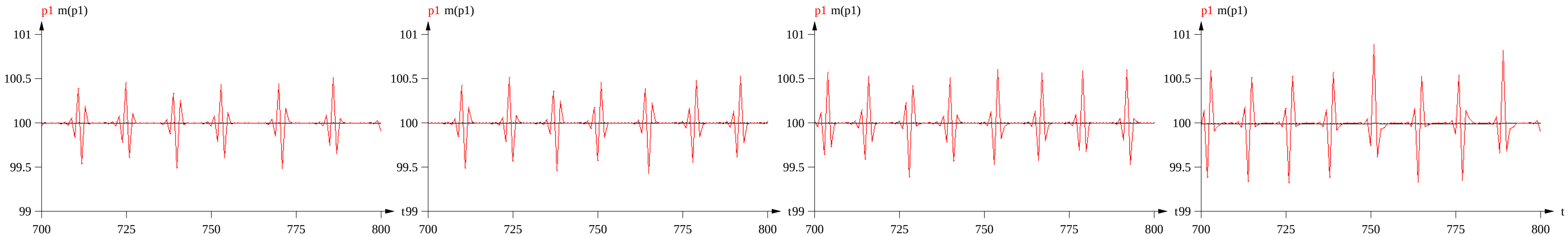


(e) volker4-1-p1-p2-tau-10h(-3)-gf-0.481 (f) volker4-1-p1-p2-tau-10h(-3)-gf-0.485 (g) volker4-1-p1-p2-tau-10h(-3)-gf-0.49 (h) volker4-1-p1-p2-tau-10h(-3)

Figure 6.22: **switch - with noisy supply** $- \tau = 10^-3$ **:** Attractors

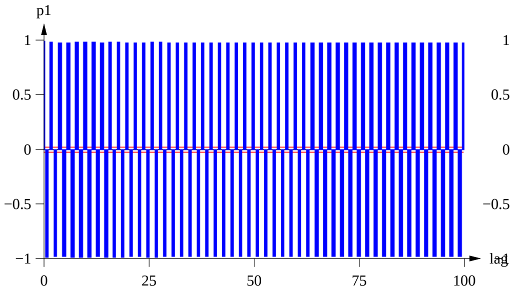


(a) volker16-1-p-tau-10h(-3)-gf-0.476.xpm.eps (b) volker16-1-p-tau-10h(-3)-gf-0.477.xpm.eps (c) volker16-1-p-tau-10h(-3)-gf-0.4798.xpm.eps (d) volker16-1-p-tau-10h(-3)-gf-0.48.xpm.eps

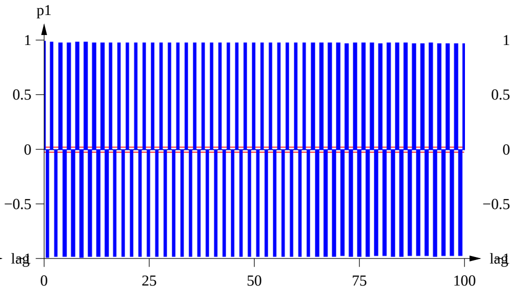


(e) volker16-1-p-tau-10h(-3)-gf-0.481.xpm.eps (f) volker16-1-p-tau-10h(-3)-gf-0.485.xpm.eps (g) volker16-1-p-tau-10h(-3)-gf-0.49.xpm.eps (h) volker16-1-p-tau-10h(-3).xpm.eps

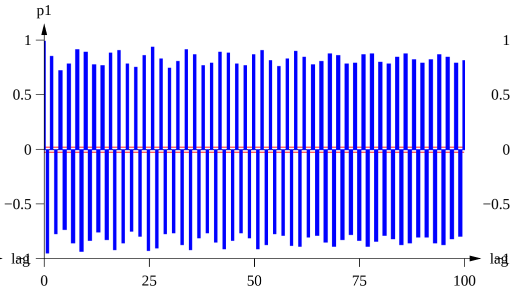
Figure 6.23: **switch - with noisy supply** $- \tau = 10^-3$ **:** time series



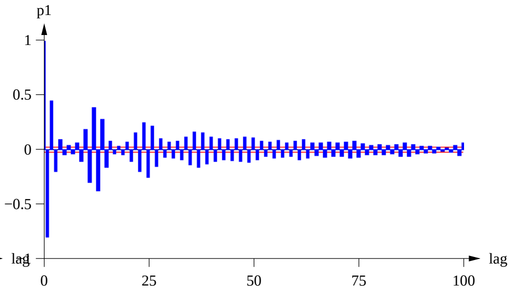
(a) volker87-1-p-tau-10h(-3)-gf-0.476.xpm.eps



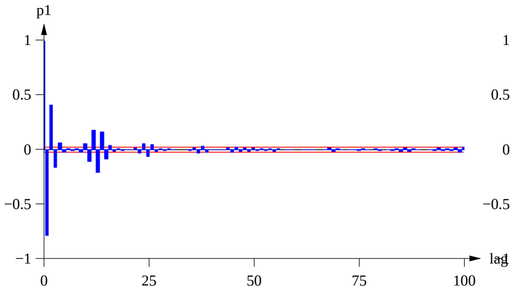
(b) volker87-1-p-tau-10h(-3)-gf-0.477.xpm.eps



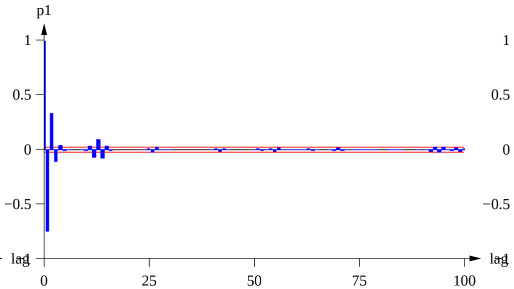
(c) volker87-1-p-tau-10h(-3)-gf-0.4798.xpm.eps



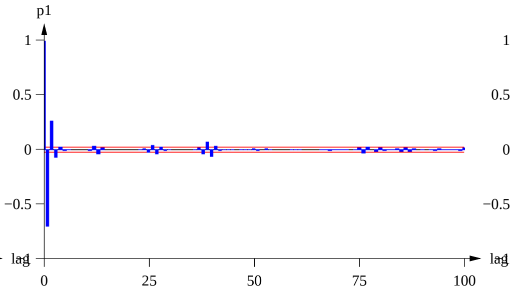
(d) volker87-1-p-tau-10h(-3)-gf-0.48.xpm.eps



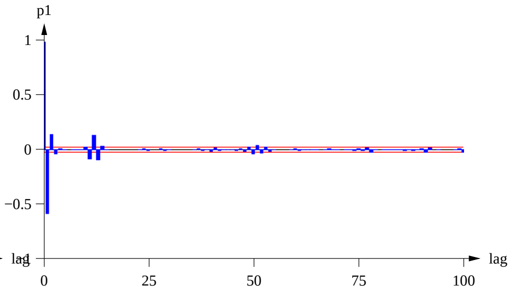
(e) volker87-1-p-tau-10h(-3)-gf-0.481.xpm.eps



(f) volker87-1-p-tau-10h(-3)-gf-0.485.xpm.eps

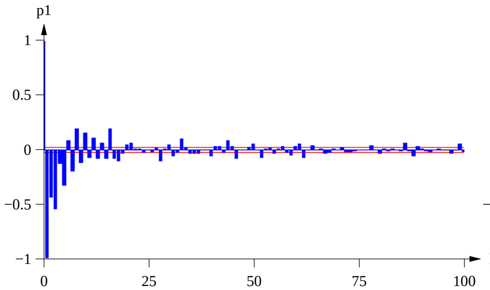


(g) volker87-1-p-tau-10h(-3)-gf-0.49.xpm.eps

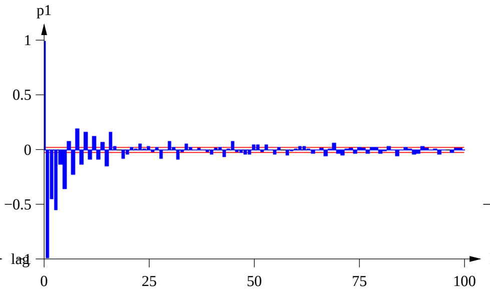


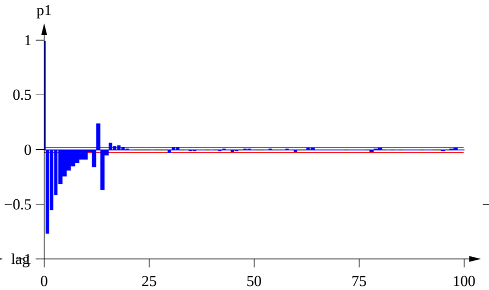
(h) volker87-1-p-tau-10h(-3).xpm.eps

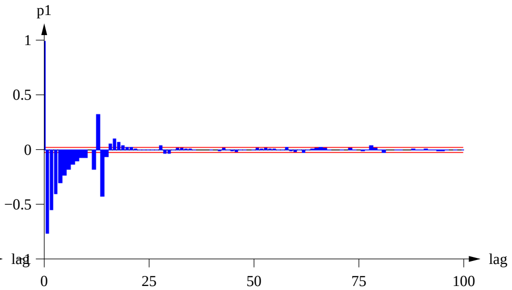
Figure 6.24: **switch - with noisy supply** $- \tau = 10^-3$ **:** ACF

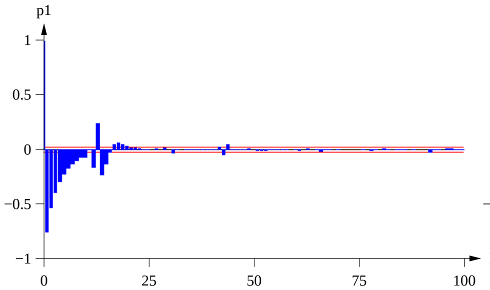


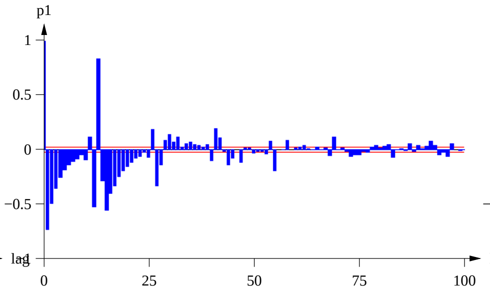
(a) volker88-1-p-tau-10h(-3)-gf-0.476.xpm.eps

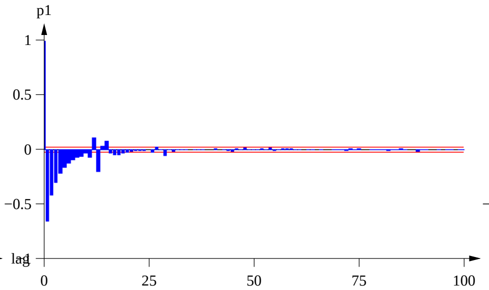

(b) volker88-1-p-tau-10h(-3)-gf-0.477.xpm.eps


(c) volker88-1-p-tau-10h(-3)-gf-0.4798.xpm.eps


(d) volker88-1-p-tau-10h(-3)-gf-0.48.xpm.eps


(e) volker88-1-p-tau-10h(-3)-gf-0.481.xpm.eps


(f) volker88-1-p-tau-10h(-3)-gf-0.485.xpm.eps


(g) volker88-1-p-tau-10h(-3)-gf-0.49.xpm.eps

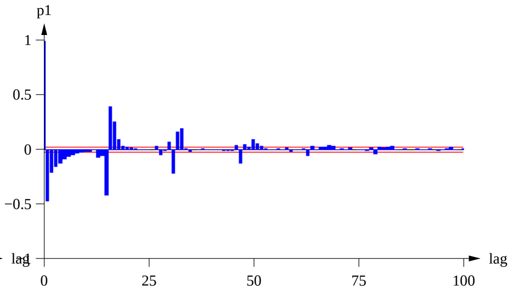
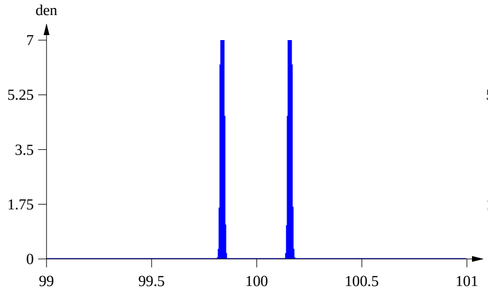
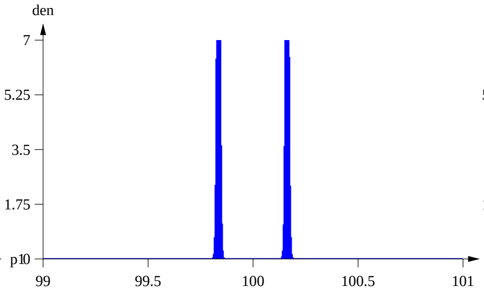

(h) volker88-1-p-tau-10h(-3).xpm.eps

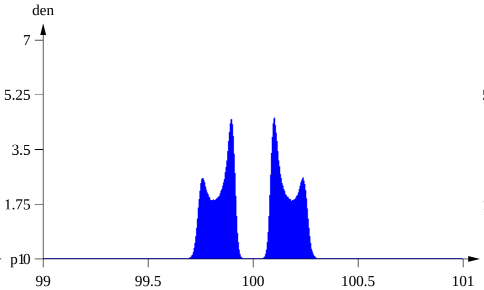
Figure 6.25: **switch - with noisy supply** $- \tau = 10^-3$ **:** PACF



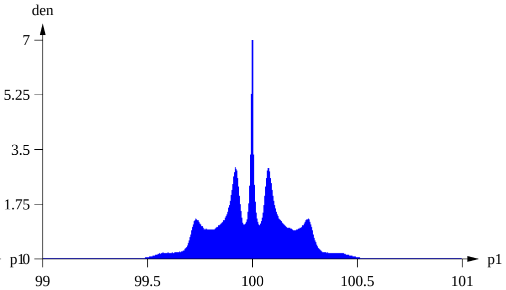
(a) volker41-1-p-tau-10h(-3)-gf-0.476.xpm.eps



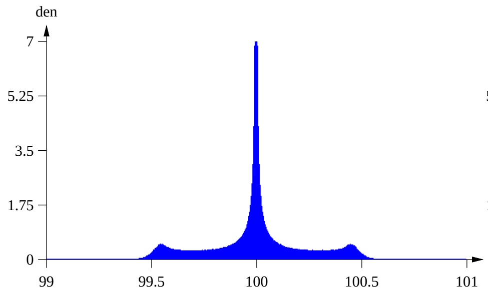
(b) volker41-1-p-tau-10h(-3)-gf-0.477.xpm.eps



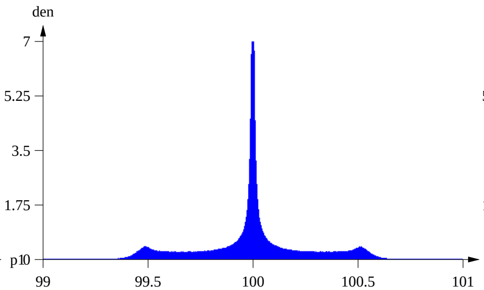
(c) volker41-1-p-tau-10h(-3)-gf-0.4798.xpm.eps



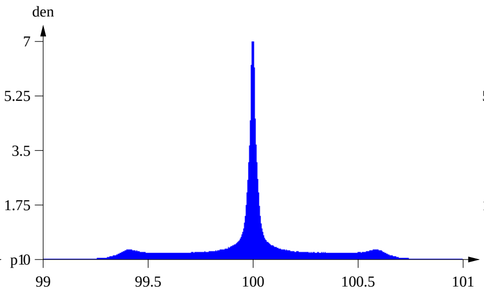
(d) volker41-1-p-tau-10h(-3)-gf-0.48.xpm.eps



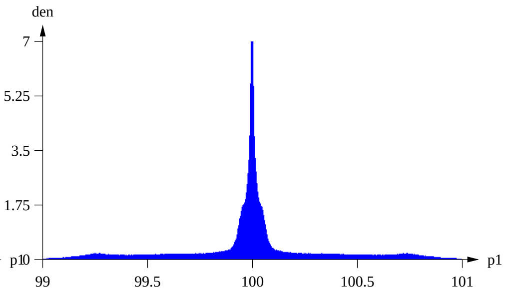
(e) volker41-1-p-tau-10h(-3)-gf-0.481.xpm.eps



(f) volker41-1-p-tau-10h(-3)-gf-0.485.xpm.eps



(g) volker41-1-p-tau-10h(-3)-gf-0.49.xpm.eps



(h) volker41-1-p-tau-10h(-3).xpm.eps

Figure 6.26: **switch - with noisy supply** $- \tau = 10^-3$ **:** Densities

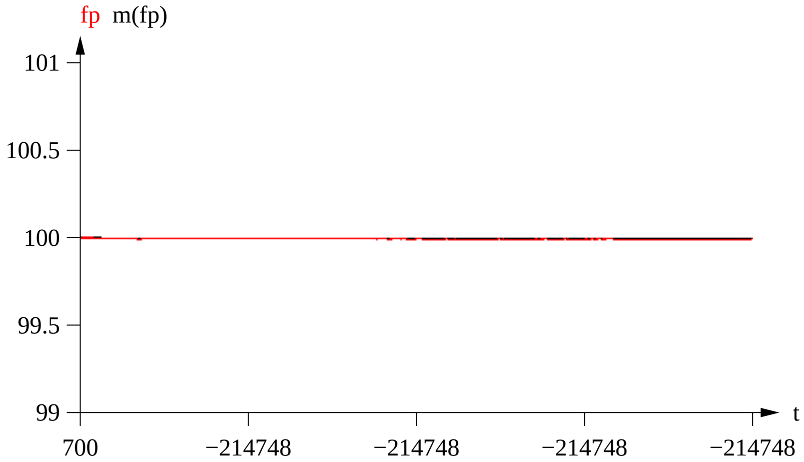
statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	100	100	100	100
variance	0.0264462	0.0273484	0.0300173	0.0301263
standard deviation	0.162623	0.165374	0.173255	0.173569
skewness	-6.30969e-05	6.09364e-06	-3.88673e-05	-0.000486063
kurtosis	-1.90883	-2.00224	-1.49735	-0.149682
quantile (0.95)	100.273	100.278	100.291	100.288

statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100	100	100	100
variance	0.0310712	0.0371514	0.0456911	0.0664172
standard deviation	0.17627	0.192747	0.213755	0.257715
skewness	-0.00302991	0.000552278	0.00111943	-0.00605965
kurtosis	2.2928	2.73394	3.1294	4.25531
quantile (0.95)	100.278	100.297	100.319	100.325

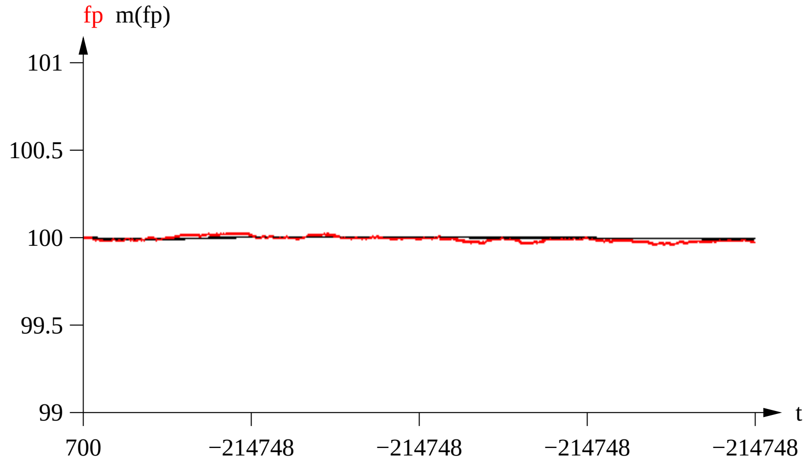
6.6 Variation of g^f - with switch and noisy fundamental ($\sigma_p = 10^-5$)

Parameter	Value
beta	0.01
delta	0.8
af = ac	0.05
sigps	0.00001
sigysquared	0.2
gc	0.5
b	2
C	0.0
ybar	0.02
zs	0.0
tau	0.0
theta	0.0
r	0.05
K	250
p0 = ps0	100
n0	0.0
swich1	1
swichf	0
swichm	0

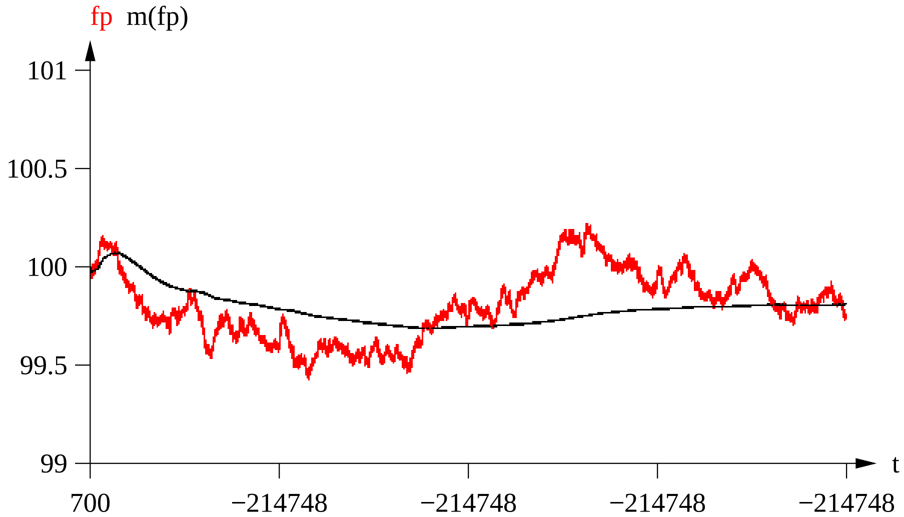
Role of noise on the fundamental price



(a) $\sigma_p = 10^{-6}$;volker16-1-fp-sigps-10h(-6)-gf-0.476

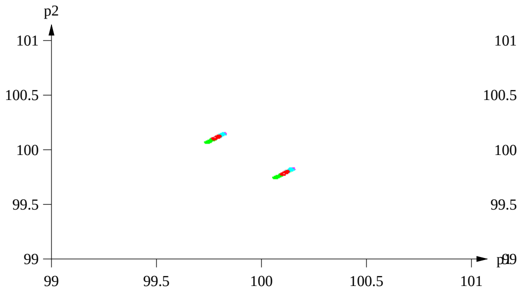


(b) $\sigma_p = 10^{-5}$; volker16-1-fp-sigps-10h(-5)-gf-0.476

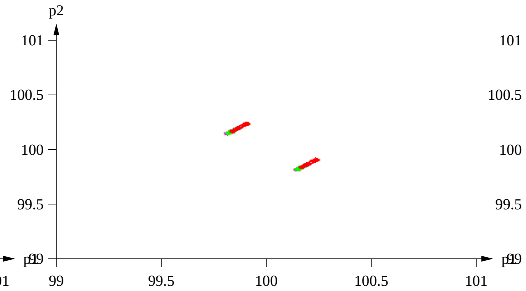


(c) $\sigma_p = 10^{-4}$; volker16-1-fp-sigps-10h(-4)-gf-0.476

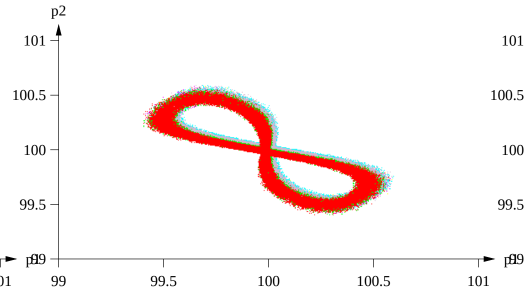
Figure 6.27: **Role of noise on the fundamental price:** time series of the fundamental price



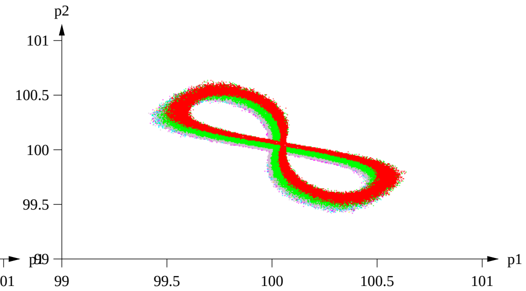
(a) volker4-1-p1-p2-sigps-10h(-5)-gf-0.476



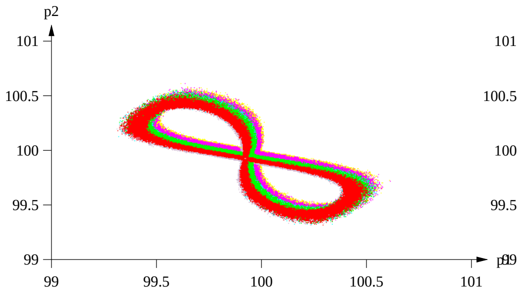
(b) volker4-1-p1-p2-sigps-10h(-5)-gf-0.477



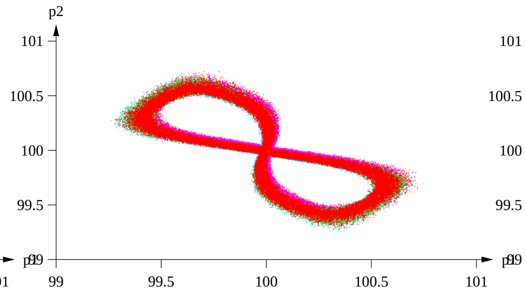
(c) volker4-1-p1-p2-sigps-10h(-5)-gf-0.4798



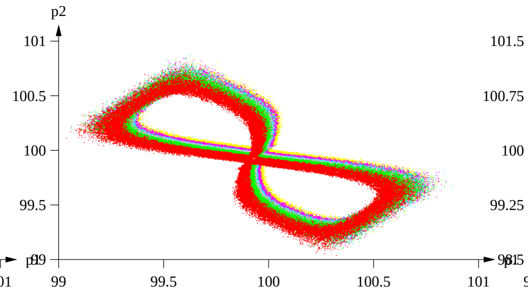
(d) volker4-1-p1-p2-sigps-10h(-5)-gf-0.48



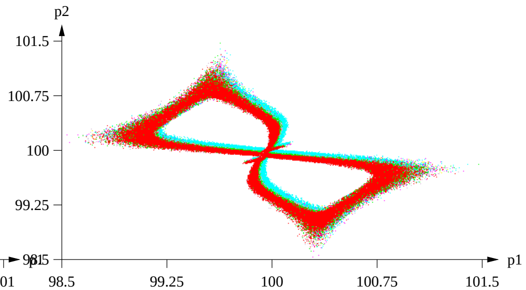
(e) volker4-1-p1-p2-sigps-10h(-5)-gf-0.481



(f) volker4-1-p1-p2-sigps-10h(-5)-gf-0.485

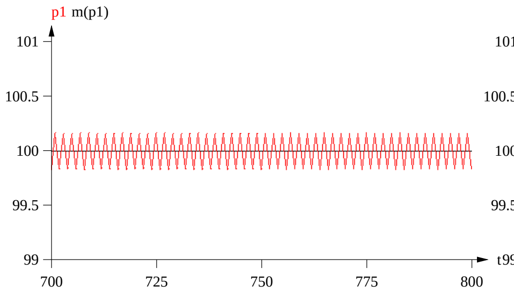


(g) volker4-1-p1-p2-sigps-10h(-5)-gf-0.49

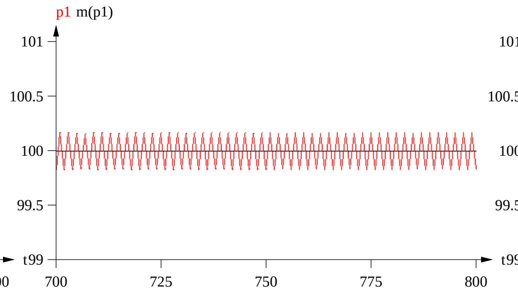


(h) volker4-1-p1-p2-sigps-10h(-5)

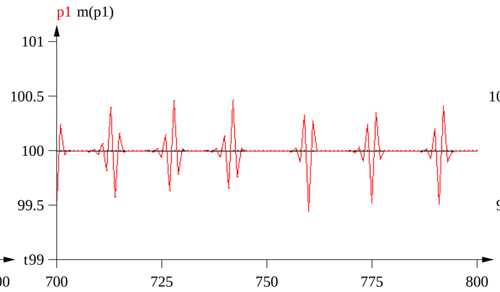
Figure 6.28: **switch - with noisy fundamental** – $\sigma_p = 10^-5$ **:** Attractors



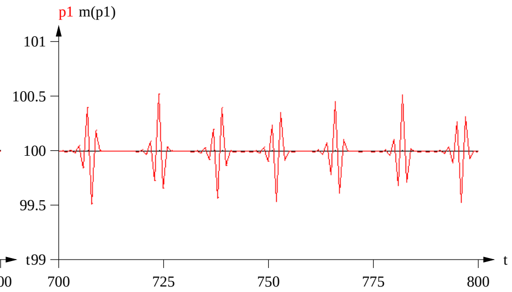
(a) volker16-1-p-sigps-10h(-5)-gf-0.476.xpm.eps



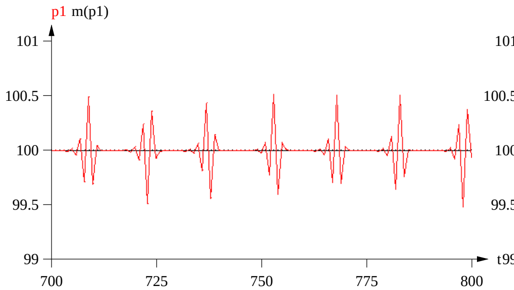
(b) volker16-1-p-sigps-10h(-5)-gf-0.477.xpm.eps



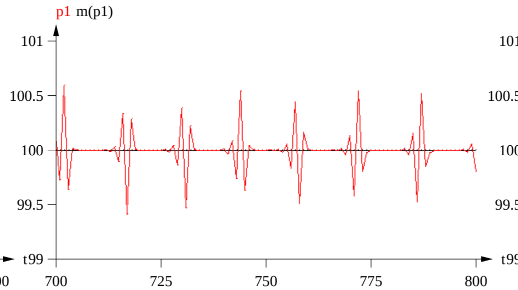
(c) volker16-1-p-sigps-10h(-5)-gf-0.4798.xpm.eps



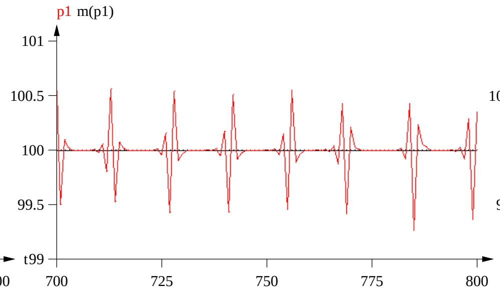
(d) volker16-1-p-sigps-10h(-5)-gf-0.48.xpm.eps



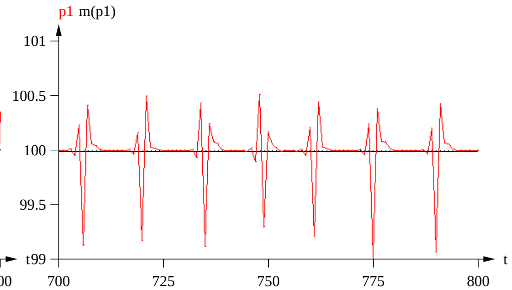
(e) volker16-1-p-sigps-10h(-5)-gf-0.481.xpm.eps



(f) volker16-1-p-sigps-10h(-5)-gf-0.485.xpm.eps

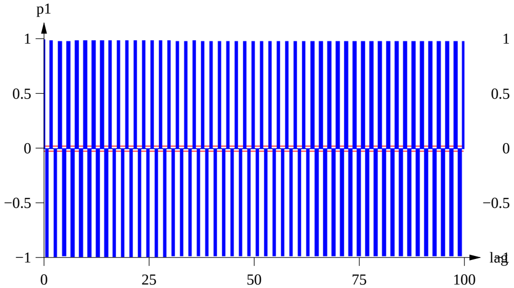


(g) volker16-1-p-sigps-10h(-5)-gf-0.49.xpm.eps

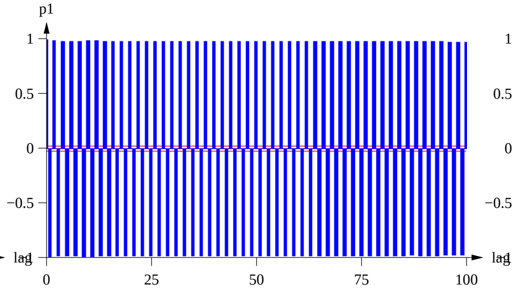


(h) volker16-1-p-sigps-10h(-5).xpm.eps

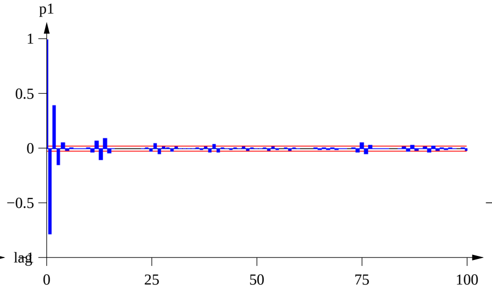
Figure 6.29: **switch - with noisy fundamental** $- \sigma_p = 10^-5$ **:** time series



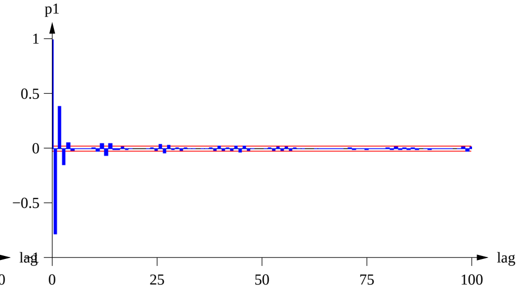
(a) volker87-1-p-sigps-10h(-5)-gf-0.476.xpm.eps



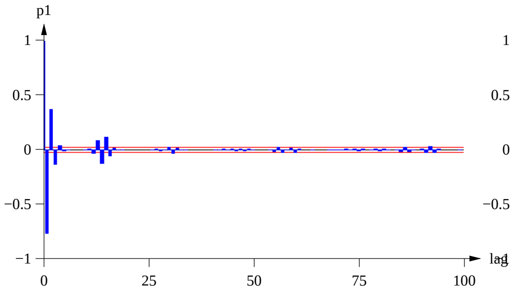
(b) volker87-1-p-sigps-10h(-5)-gf-0.477.xpm.eps



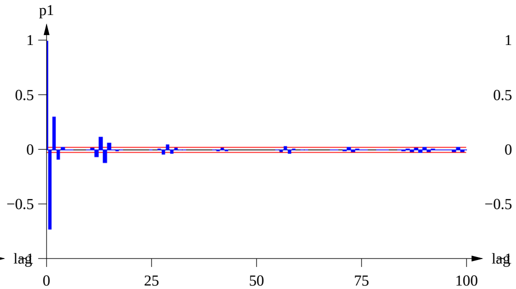
(c) volker87-1-p-sigps-10h(-5)-gf-0.4798.xpm.eps



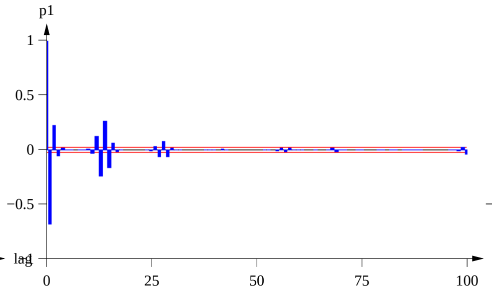
(d) volker87-1-p-sigps-10h(-5)-gf-0.48.xpm.eps



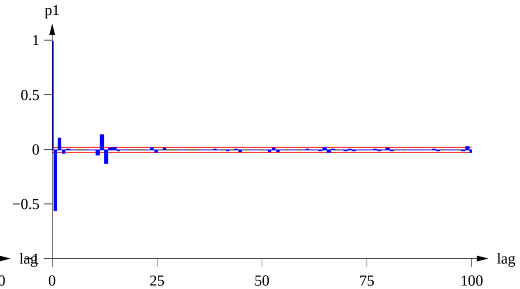
(e) volker87-1-p-sigps-10h(-5)-gf-0.481.xpm.eps



(f) volker87-1-p-sigps-10h(-5)-gf-0.485.xpm.eps

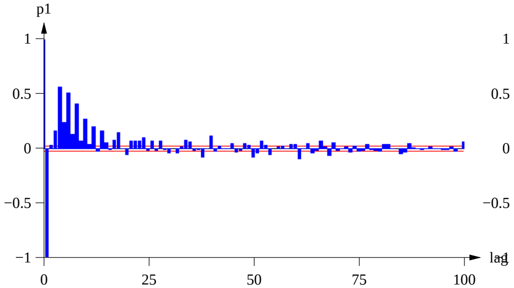


(g) volker87-1-p-sigps-10h(-5)-gf-0.49.xpm.eps

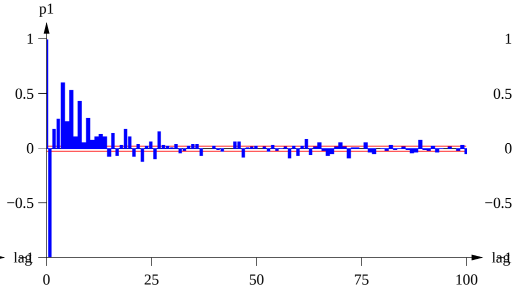


(h) volker87-1-p-sigps-10h(-5).xpm.eps

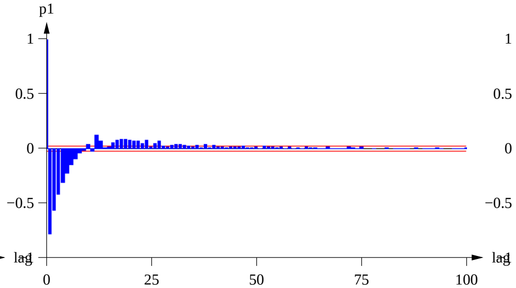
Figure 6.30: **switch - with noisy fundamental** – $\sigma_p = 10^-5$ **:** ACF



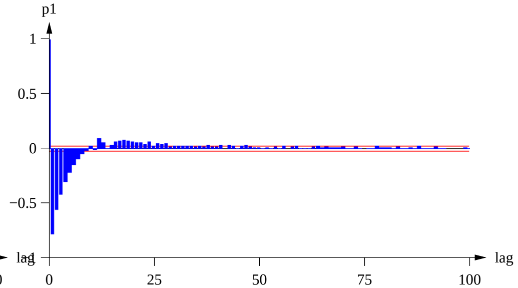
(a) volker88-1-p-sigps-10h(-5)-gf-0.476.xpm.eps



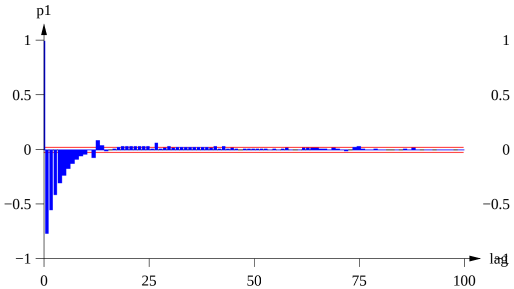
(b) volker88-1-p-sigps-10h(-5)-gf-0.477.xpm.eps



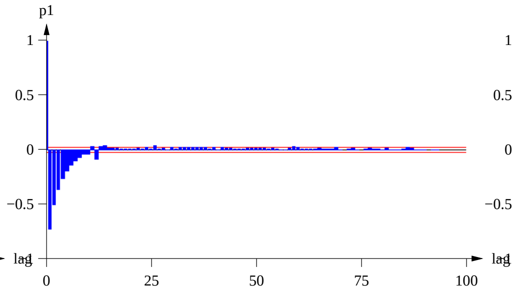
(c) volker88-1-p-sigps-10h(-5)-gf-0.4798.xpm.eps



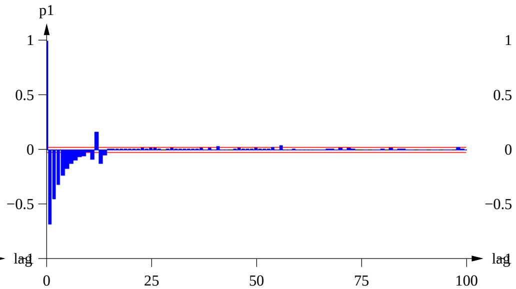
(d) volker88-1-p-sigps-10h(-5)-gf-0.48.xpm.eps



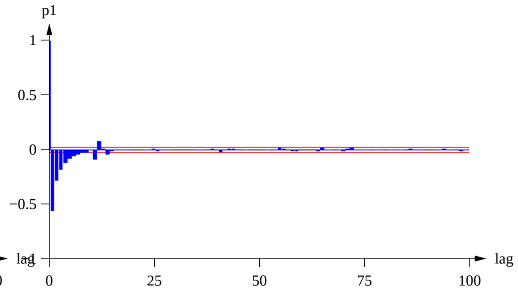
(e) volker88-1-p-sigps-10h(-5)-gf-0.481.xpm.eps



(f) volker88-1-p-sigps-10h(-5)-gf-0.485.xpm.eps

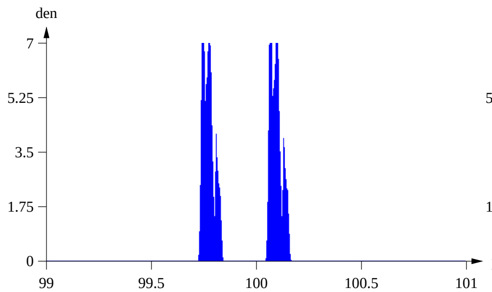


(g) volker88-1-p-sigps-10h(-5)-gf-0.49.xpm.eps

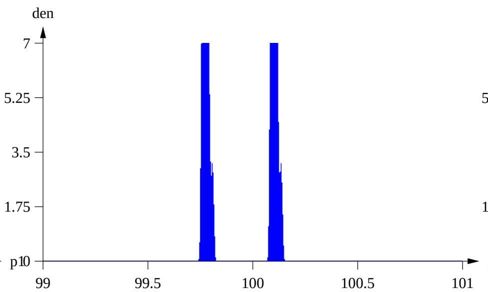


(h) volker88-1-p-sigps-10h(-5).xpm.eps

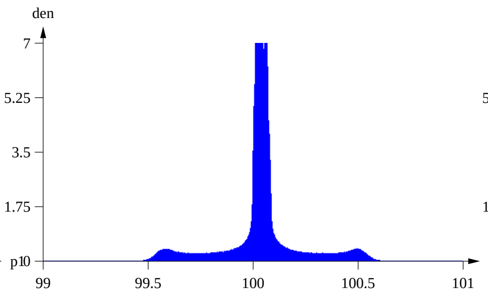
Figure 6.31: **switch - with noisy fundamental** $- \sigma_p = 10^-5$ **:** PACF



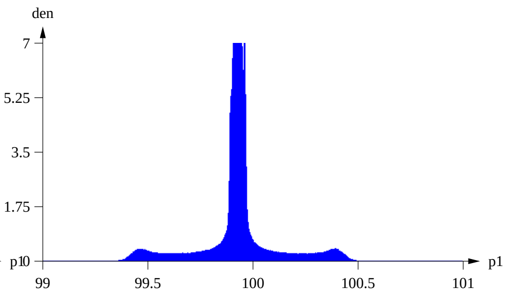
(a) volker41-1-p-sigps-10h(-5)-gf-0.476.xpm.eps



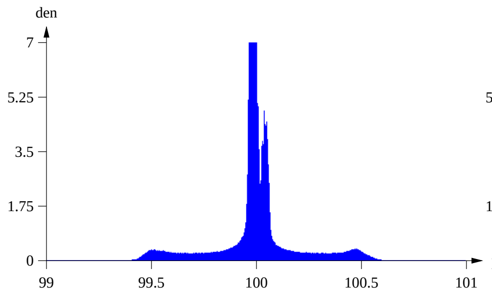
(b) volker41-1-p-sigps-10h(-5)-gf-0.477.xpm.eps



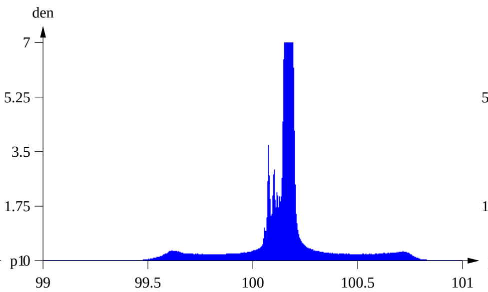
(c) volker41-1-p-sigps-10h(-5)-gf-0.4798.xpm.eps



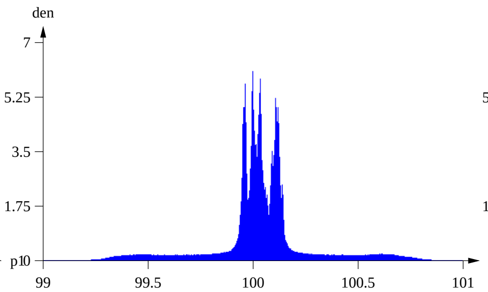
(d) volker41-1-p-sigps-10h(-5)-gf-0.48.xpm.eps



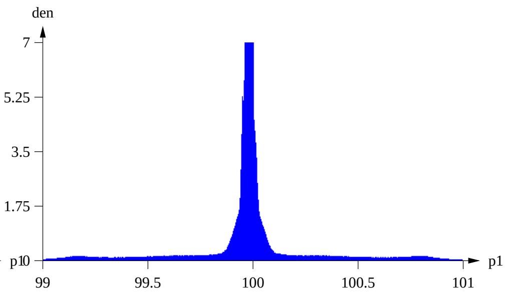
(e) volker41-1-p-sigps-10h(-5)-gf-0.481.xpm.eps



(f) volker41-1-p-sigps-10h(-5)-gf-0.485.xpm.eps



(g) volker41-1-p-sigps-10h(-5)-gf-0.49.xpm.eps



(h) volker41-1-p-sigps-10h(-5).xpm.eps

Figure 6.32: **switch - with noisy fundamental** – $\sigma_p = 10^{-5}$ **:** Densities

statistic	$g^f = 0.476$	$g^f = 0.477$	$g^f = 0.4798$	$g^f = 0.48$
mean	99.9466	99.948	100.032	99.938
variance	0.0274151	0.027706	0.0296933	0.0299625
standard deviation	0.165575	0.166451	0.172317	0.173097
skewness	0.00123602	0.000531702	0.00106596	0.00408892
kurtosis	-1.81466	-1.9584	2.62852	2.64323
quantile (0.95)	100.225	100.228	100.3	100.207

statistic	$g^f = 0.481$	$g^f = 0.485$	$g^f = 0.49$	$g^f = 0.5$
mean	100.003	100.131	100.025	99.9801
variance	0.0313598	0.0412066	0.0498726	0.0700015
standard deviation	0.177087	0.202994	0.223322	0.264578
skewness	0.00288821	-0.033321	0.00770037	0.0117907
kurtosis	2.89428	2.70688	3.19875	4.93859
quantile (0.95)	100.273	100.441	100.357	100.275

7 Summary – Conclusions – Further Observations

ROLE OF NON LINEARITIES

1. Subjective speed of reversion induces
   - bifurcations – Neimark Sacker – periodicities under all scenarios
   - long autocorrelation patterns disappear with higher periodicities
   - drastic changes of KURTOSIS for all sets of parameters
   - no evidence found for skewness under all scenarios
2. switching induces additional mean reversion and variance reversion
3. switching may cause non stationary orbits/non ergodicity
   (for example Böhm & Wenzelburger (2005) or Horst & Wenzelburger (2006))

ROLE OF NOISE

1. Additive i. i. d. random supply has no decisive influence
2. i. i. d. dividends have little decisive influence under switching
3. noise of fundamental causes non stationarity/non ergodicity ?
4. more complex noise requires more investigations

NEED FOR MORE BIFURCATION THEORY

A Univariate random variables: Definitions and computation of moments

– Let X denote a random variable in $\mathbb{R}$ and $\{x_t\}_{t=1}^T$ denote T realizations of the associated dynamic (stochastic) process.

– The so called raw moments of order r, $r = 1, 2, 3, \ldots$ of X are defined as[6]

$$\mu_r := \mathbb{E}[X^r],$$

while the r^{th} *central moment* of X about $\mathbb{E}(X) = \mu_1$ is denoted

$$\mu_{(r)} := \mathbb{E}[(X - \mu_1)^r].$$

Statistic	Definition	Estimator
Mean	$\mathbb{E}[X] = \mu_1 \equiv \mu$	$\hat{\mu} = \frac{1}{T}\sum_1^T x_t$
Variance	$\mathbb{V}[X] = \mu_{(2)} = \mathbb{E}[(X - \mu_1)^2]$	$\hat{\sigma}^2 = \frac{1}{T}\sum_1^T (x_t - \hat{\mu})^2$
Skewness	$\frac{\mu_{(3)}}{(\mu_{(2)})^{3/2}}$	$\hat{\gamma}_1 = \frac{\sqrt{T}\sum(x_t-\hat{\mu})^3}{\left[\sum_1^T (x_t-\hat{\mu})\right]^{\frac{3}{2}}}$
Kurtosis	$\frac{\mu_{(4)}}{\mu_{(2)}^2} - 3$	$\hat{\gamma}_2 = \frac{1}{T}\frac{\sum_1^T (x_t-\hat{\mu})^4}{\left[\sum_1^T (x_t-\hat{\mu})^2\right]^2} - 3$

Table 1: Descriptive Statistics and their estimators

[6] (see Jungeilges 2003)

where

$$\mu_{(2)} = \mathbb{E}[X^2] - \mu^2 \tag{A.1}$$

$$\mu_{(3)} = \mathbb{E}[X^3] - 3\mu\mathbb{E}[X^2] + 2\mu^3 \tag{A.2}$$

$$\mu_{(4)} = \mathbb{E}[X^4] - 4\mu\mathbb{E}[X^3] + 6\mu^2\mathbb{E}[X^2] - 3\mu^4 \tag{A.3}$$

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