Heterogeneous adaptive expectations and cobweb phenomena

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Questions

- How does the evolution in the number / types of agents affect the long run dynamics of a given economy?

- How does expectations' heterogeneity influence local stability?
- What can we expect when markets integrate?
- Can we make predictions on stability when only the probability distribution of types is known?
- Can we say anything about transitional dynamics / speed of convergence based on the "amount" of expectations' heterogeneity?

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Preview of results

- Two sources of (potential) instability are identified:
  - a structural source, linked to the market's fundamentals
  - a behavioural source, embedded in the average profile of expectations.
- We find a simple relation connecting these factors to stability/instability
- Can predict outcome of market integration, under (stronger than elsewhere in the paper) qualifications
- Study random selection of firms from a continuous distribution and document a form of polarisation of convergence probabilities when number of market's participants is increased
- Give conditions that ensure monotone and fastest convergence towards steady state

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- M. Nerlove - *QJE 1958*: introduced adaptive exp. into Cobweb model
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- ARED stream of literature - Brock-Hommes *ECONOMETRICA 1997*
- Lasselle et al. - *MACRO. DYN. 2005*
- T. Puu - *JEBO 2008*

In a nutshell

- A standard Cobweb model with n firms
- Firms supply a commodity with a one-period production lag
- Output decisions are based on expectations about future prices
- At each period, given aggregate supply, the price is determined by the demand

The model: details

- Supply and demand are monotonic
- The optimal supply is proportional to firm's size, $\psi_i > 0$ hence $S_i\left(p_i^e\right) = \psi_i s\left(p_i^e\right)$
- All form adaptive expectations, gain parameter is firm-specific

$$p^e_{t+1,i} = p^e_{t,i} + \alpha_i(p_t - p^e_{t,i}) \qquad i = 1, \ldots n$$

- Demand, $D\left(p\right)$ and aggregate supply are smooth and intersecting at a point p^*

Price equation

- Let $\Psi = \sum_i \psi_i$ the industry scale factor, $S(\cdot) = \Psi s(\cdot)$ and $\phi_i = \frac{\psi_i}{\Psi}$ the firm's relative weight
- Market clearing requires that $D(p_t) = \sum_{i=1}^n \phi_i S\left(p^e_{t,i}\right)$ hence

$$p_t = D^{-1}\left(\sum_{i=1}^n \phi_i S\left(p^e_{t,i}\right)\right) = F\left(p^e_{t,1}, \ldots, p^e_{t,n}\right)$$

with the property $p^* = F(p^*, \ldots, p^*)$

Expectations dynamics

- Plugging price equation into expectations gives the following system of n difference equations

$$\begin{aligned} p^e_{t+1,1} &= p^e_{t,1} + \alpha_1 \left(F\left(p^e_{t,1}, \dots, p^e_{t,n} \right) - p^e_{t,1} \right) \\ \cdots &= \cdots \\ p^e_{t+1,n} &= p^e_{t,n} + \alpha_n \left(F\left(p^e_{t,1}, \dots, p^e_{t,n} \right) - p^e_{t,n} \right) \end{aligned}$$

- Point p^* is unique steady state

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Special case: one representative firm

- With a single firm price equation reduces to

$$p_t = D^{-1}\left(\Psi s\left(p_t^e\right)\right) = D^{-1}\left(S\left(p_t^e\right)\right)$$

- Therefore the system evolves according to

$$p_{t+1}^e = p_t^e + \alpha\left(D^{-1}\left(S\left(p_t^e\right)\right) - p_t^e\right)$$

- and stability requires $-1 < 1 - \alpha + \alpha\frac{S'(p^*)}{D'(p^*)} < 1$
- Defining $\delta = -\frac{S'(p^*)}{D'(p^*)}$ and $\beta = \frac{\alpha}{2-\alpha}$, can write this as

$$\delta\beta < 1$$

- Label δ as *structural degree of instability* and β as *behavioural degree of instability*

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Conditions for stability - 1

- The Jacobian matrix of the system evaluated at steady state is

$$\begin{bmatrix} 1-\alpha_1\left(\phi_1\delta+1\right) & -\alpha_1\phi_1\delta & \cdots & -\alpha_1\phi_1\delta \\ -\alpha_2\phi_2\delta & 1-\alpha_2\left(\phi_2\delta+1\right) & \cdots & -\alpha_2\phi_2\delta \\ \cdots & \cdots & \cdots & \cdots \\ -\alpha_n\phi_n\delta & -\alpha_n\phi_n\delta & \cdots & 1-\alpha_n\left(\phi_n\delta+1\right) \end{bmatrix}$$

- Define $\bar{\beta}_n = \sum_{i=1}^{n} \phi_i\beta_i$ the *market degree of behavioural instability* for the n heterogeneous firms case
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Conditions for stability - 2

- How does this compare with the $n = 1$ case?
- **Proposition 2**: Consider an n-firms market with gains $\alpha_1, \ldots, \alpha_n$ and weights $\phi_1, \ldots, \phi_n$ and an average-single-firm market with gain $\alpha = \sum_{i=1}^{n} \phi_i \alpha_i$. Conditions for stability in the heterogeneous market are sufficient but not necessary for the average homogeneous market.
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Comparative statics on n

- What is the role of n ceteris paribus?
- **Proposition 3**: Consider economy A and economy B where B has some extra firms in the supply side, given the same industry scale factor. Economy B's extra firms have a weight $1-\rho$ and a given $\bar{\beta}_{extra}$. Then if economy A is stable so is economy B if $\delta\bar{\beta}_{extra} < 1$. If instead $\delta\bar{\beta}_{extra} > 1$ then economy B is stable if and only if $\rho > \frac{\delta\bar{\beta}_{extra}-1}{\delta\bar{\beta}_{extra}-\delta\bar{\beta}_A}$.
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- (In progress) Basically things are straightforward if steady state does not move
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Transitional dynamics

- How about the path of convergence to the steady state?
- **Propositions 4-5**: The system shows monotonic local convergence to the steady state if and only if $\sum_{i=1}^{n} \phi_i \frac{\alpha_i}{(1-\alpha_i)} < \frac{1}{\delta}$. If $\phi_1 = \cdots = \phi_n = 1/n$ then the maximum speed of convergence to the steady state is $\ln\left(\frac{\delta+2}{\delta}\right)$ and it is attained if and only if $\alpha_1 = \cdots = \alpha_n = \frac{2}{\delta+2}$.

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- Know very little about actual expectations
- Assume firms' behavioural parameter results from a random choice, given a distribution, e.g. uniform on unit interval
- Define a *stable sample* of behavioural parameters one for which the corresponding system has a locally stable steady state
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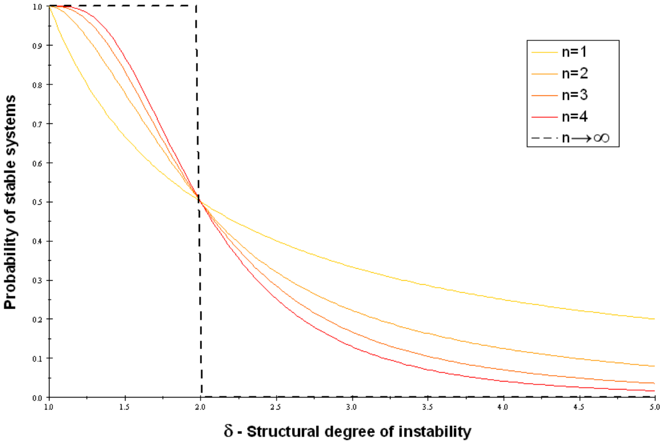
Probabilities as δ varies



Conclusions

- We study the effect of a varying the level of market's heterogeneity in a Cobweb model with adaptive expectations
- We fully characterize the local stability properties for the generic *n*-firms case
- We discuss the case of market integration giving conditions which grant stability in the resulting, integrated, market
- We study the possibility of making predictions about the properties of market dynamics when firm's types are unknown. We show that when types are uniformly distributed the probability of having a stable system polarizes towards 0 or 1 depending on the structural characteristics of the market