Political Cycles: A Stochastic Control Approach

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MDEF2008, Urbino, September 2008

Motivations, aims and literature

- Elections proximity may affect politician's behaviour, thus generating a *political cycle*
  - Rogoff (1990), Ashworth (2005)
- We study an agency model of electoral competition when politician's competence is unobserved (*filtering theory*)
  - Karatzas and Zhao (2001), Rieder and Bäuerle (2005), Pham (2008), Björk, Davis and Landén (2008)

The model

Outline

- Time is continuous and divided into two periods of constant lenght $T > 0$
- Elections are held at the end of the first period
- An incumbent politician chooses
  - the level of public intervention in the economy
  - a rent seeking behavior

  and runs for reelection against an opponent randomly chosen among the population
- Voters are risk neutral w.r.t. the electoral choice

The model

Economy sectors dynamics

- $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$ filtered probability space, with $\mathbb{F} := (\mathcal{F}_t)_{0 \leq t \leq 2T}$
- $\mathbb{W} := (W_t)_{0 \leq t \leq 2T}$ an $\mathbb{F}$-adapted Brownian motion
- Public sector value process G satisfies

$$dG_t = G_t \left[\varepsilon dt + \sigma dW_t \right],$$

  - ε is a r.v. with values in $\{\varepsilon_L, \varepsilon_H\}$, $0 \leq \varepsilon_L < \varepsilon_H$, independent of $\mathbb{W}$
- $\mathbb{P}(\{\varepsilon = \varepsilon_H\}) = p$ common prior probability
- Private sector value process B satisfies

$$dB_t = rB_t dt,$$

$0 \leq r \leq \varepsilon_L < \varepsilon_H$

The model

Economy value and preferences

- Economy wealth:

$$dX_t = X_t \left[r dt + u_t (\varepsilon - r) dt + u_t \sigma dW_t \right] - k_t dt$$

  - k: rent flow
  - u: public sector size

- Citizens' expected utility:

$$\mathbb{E}\left[\frac{(X_T)^\alpha}{\alpha}\right], \quad 0 < \alpha < 1$$

- Politician's expected utility:

$$\mathbb{E}\left[\int_0^T \frac{k_s^\alpha}{\alpha} ds + \frac{(X_T)^\alpha}{\alpha}\right]$$

The model

Information and admissible controls

- Neither the Brownian motion $\mathbb{W}$ nor the random variable ε is observed, but the process G is
- $\mathbb{F}^G$: $\mathbb{P}$-augmented filtration generated by G
- $\mathcal{F}_t^G \subset \mathcal{F}_t$ (incomplete information)
- (k, u) is admissible if it is $\mathbb{F}^G$-progressively measurable

The analysis

We proceed backwards

Second period

The incumbent maximizes

$$J(t, x; k, u) := \mathbb{E}\left[\int_t^{2T} \frac{k_s^\alpha}{\alpha} ds + \frac{(X_{2T})^\alpha}{\alpha}\right]$$

over all admissible (k, u), where

$$dX_s = X_s[rds + u_s(\varepsilon - r)ds + u_s \sigma dW_s] - k_s ds, \quad X_t = x$$

The value function is

$$v(t, x) := \sup_{u,k} J(t, x; k, u)$$

This problem is not recursive as $\mathbb{W}$ is not a Brownian motion w.r.t. $\mathbb{F}^G$

The complete information problem

We can transform the problem into a two-dimensional Markov problem by introducing

- a new probability measure $\tilde{\mathbb{P}} \sim \mathbb{P}$
- a process $Y_t = \frac{\varepsilon}{\sigma} t + W_t$, which is a BM under $\tilde{\mathbb{P}}$ w.r.t. $\mathbb{F}^G$
- a new state variable Q (unnormalized conditional probability that $\varepsilon = \varepsilon_H$)

Assuming $\varepsilon_H = 1$ and $\varepsilon_L = 0$, the problem becomes:

$$\sup_{k,u} \tilde{\mathbb{E}} \left[\int_t^{2T} \frac{k_s^\alpha}{\alpha} (Q_s + 1 - p)\, ds + \frac{X_{2T}^\alpha}{\alpha} (Q_{2T} + 1 - p) \right] =: \tilde{v}(t, x, q),$$

where

$$dX_s = X_s \left[(1 - u_s)\, r dt + u_s \sigma dY_s \right] - k_s ds, \quad X_t = x > 0$$

$$dQ_s = \frac{1}{\sigma} Q_s dY_s, \quad Q_t = q > 0$$

The *Dynamic Programming Principle* holds for this problem and yields the following HJB equation for $\tilde{v}$:

$$0 = h_t + \sup_{u,k} \left\{ \mathbb{A}^{u,k} [h] (x, q) + \frac{k^\alpha}{\alpha} (q + 1 - p) \right\},$$

with boundary condition

$$h (2T, x, q) = \frac{x^\alpha}{\alpha} (q + 1 - p)$$

where

$$\mathbb{A}^{u,k} [h] (x, q) := \frac{1}{2}\sigma^2 u^2 x^2 h_{xx} + xuqh_{xq} + \frac{1}{2}\frac{q^2}{\sigma^2} h_{qq} + x (1 - u) rh_x - kh_x$$

Standard homogeneity arguments and a power transformation enable us to characterize the solutions of the HJB equation as follows:

$$h(t, x, q) = \frac{x^{\alpha}}{\alpha} w(t, q)^{1-\alpha},$$

where $w(t, q)$ solves a *non-homogeneous linear parabolic equation* that can be represented in stochastic form by means of the Feynman-Kac formula

In particular, if $\alpha = 1/2$ and $\delta = 0$, w takes the form

$$\hat{w}(t,q) = a(t)\, q^2 + b(t)\, q + c(t)$$

where $a(t)$, $b(t)$ and $c(t)$ are solutions of a system of ODEs

Verification arguments prove that $\dfrac{x^\alpha}{\alpha}\hat{w}(t,q)^{1-\alpha}$ is the value function and

$$u^* = \frac{q\hat{w}_q}{\sigma^2 \hat{w}}, \qquad k_2^* = x\frac{(q+1-p)^2}{\hat{w}}$$

are the optimal (markov) controls

- The rent k is increasing in time (first source of cycles)
- The public sector proportion is independent of the economy size

Reelection rule

Voters' expected utility

$$\tilde{\mathbb{E}}\left[\frac{(X_{2T})^{\alpha}}{\alpha}(Q_{2T}+1-p)\right],$$

is increasing in Q, hence

- society reelects the incumbent politician $\Leftrightarrow Q_T \geq p$

First period

The politician maximizes

$$\tilde{\mathbb{E}}\left[\int_t^T \frac{k_s^\alpha}{\alpha}(Q_s+1-p)\,ds+\frac{X_T^\alpha}{\alpha}(Q_T+1-p)+\chi_{\{Q_T\geq p\}}\tilde{v}(T,X_T,Q_T)\right]$$

over (u,k), where χ_A is the characteristic function of $A\subset\Omega$

- The same HJB equation as period two but with the following boundary condition

$$w(T,x,q)=\begin{cases}\frac{x^\alpha}{\alpha}(q+1-p)+\tilde{v}(T,x,q)\,, & q\geq p\\ \frac{x^\alpha}{\alpha}(q+1-p)\,, & q<p\end{cases}$$

We reduce the HJB equation to a pair of *linear parabolic equations*

This reduction enable us to prove that

$$k_1^* < k_2^*$$

where k_i^* is the optimal (markov) rent extraction of period i

- opportunistic behavior (Rogoff 90, Ashworth 05)

Conclusions

- Without any electoral constraint, incumbent politician's rent seeking behavior gets worse over time (political cycles)
- With an electoral constraint we observe an opportunistic behavior so that the rent is lower (all other things the same)