IPRs, Knowledge Production, and Endogenous Growth

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MDEF2008
Fifth Workshop on Dynamic Models in Economics and Finance

Urbino (Italy), September 27, 2008

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- We build on a competitive endogenous growth model introduced by Tsur and Zemel (JEDC, 2007), in which (new) knowledge is produced according to Weitzman's (QJE, 1998) *recombinant expansion process*, and modify such framework by replacing the "competitive" (in a sense to be specified) R&D sector with a unique monopolist producing new knowledge, to whom IPRs are granted.
- Our goal is to compare the first-best solution of the competitive model to the solution of the monopolistic version, with special attention to conditions under which the economy sustains *endogenous growth*.

Our main results

1. If some conditions are satisfied, *in the long run* an economy *with monopolized R&D* sector grows along the same balanced growth path as in a *competitive economy*; both are characterized by the same *asymptotic turnpike*.
   Otherwise, the economy is driven toward *stagnation*.

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   Otherwise, the economy is driven toward *stagnation*.
2. For a given stock of knowledge, the *social unit cost of knowledge growth* for the economy with the *R&D monopolist is strictly larger than* the same social unit cost when the *R&D sector is competitive*.

   The last result affects *transitory dynamics* and *initial conditions*.

Recombinant knowledge
Seed ideas

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- Let $A(t)$ be the stock of knowledge at time t (the total number of ideas) and $C_m[A(t)] = A(t)!/\{m![A(t)-m]!\}$ be the number of different combinations of m elements of $A(t)$ [*e.g.*, if $m = 2$, $C_2(A) = A(A-1)/2$]; then the number of *new seed ideas* is

$$H(t) = C_m[A(t)] - C_m[A(t-1)].$$

Knowledge dynamics

- Let π be the *probability of obtaining a successful idea* from each matching. Then the *number of new successful ideas* at time t, $A(t)$, is a *second order process*:

  $$A(t+1) - A(t) = \pi H(t) = \pi \{C_m [A(t)] - C_m [A(t-1)]\}.$$

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  Thus, knowledge may *potentially* grow at *increasing rates*.
- Actually, *scarcity of resources* precludes explosive growth and leads to the following *production function for new knowledge*:

$$\Delta A = A(t+1) - A(t) = H\pi \left(\frac{J}{H}\right),$$

  H is the number of *seed ideas* and J is a measure of physical resources employed in matching ideas.

Main result

Assumption

Probability $\pi : \mathbb{R}_+ \rightarrow [0,1]$ *is independent of time;* $\pi' > 0$, $\pi'' < 0$, $\pi(0) = 0$, $\pi(\infty) \leq 1$.

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Theorem (Weitzman, 1998)

If J is a constant fraction of the total output y produced by the economy,

$$J = sy,$$

with s exogenously determined, then in the long run the asymptotic growth rate is a positive constant depending on the saving rate s.

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- *Output producing firms* operate in a competitive environment.
- The *regulator* levies sy as a *tax on the representative consumer*, through which *finances the R&D firms*.
- *R&D firms*, generate *new useful knowledge* according to $\Delta A = H\pi\left(sy/H\right)$, which is freely passed to the output producing sector.

Interpreting "competitive" R&D

Remark

Although R&D firms are rewarded by the regulator, both the competitive nature of the R&D market (zero profits) and the public good nature of knowledge are preserved ("competitive" R&D should be seen as an abstraction).

Continuous time setting

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- Thus:

$$H(t) = C_m' [A(t)] \dot{A}(t),$$

where $\dot{A}(t)$ is the time-derivative of the stock of knowledge at instant t, $A(t)$, and the new knowledge production function becomes:

$$\dot{A}(t) = H(t)\, \pi \left[\frac{J(t)}{H(t)} \right],$$

where probability π satisfies Weitzman's Assumption.

Unit cost of knowledge production

Letting $J(t) = s(t)\, y(t)$, the law of motion for $A(t)$ is:

$$\dot{A}(t) = \frac{J(t)}{\varphi[A(t)]} = \frac{s(t)\, y(t)}{\varphi[A(t)]},$$

where

$$\varphi(A) = C'_m(A)\, \pi^{-1}\left[\frac{1}{C'_m(A)}\right]$$

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Remark

$\varphi(\cdot)$ *is decreasing and* $\lim_{A\to\infty}\varphi(A) = \dfrac{1}{\pi'(0)}$.

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- Since a fraction $s(t)$ of $y(t)$ finances R&D:

$$\dot{k}(t) = [1 - s(t)]\, y(t) - c(t),$$

where $c(t)$ is instantaneous consumption.

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  where $c(t)$ is instantaneous consumption.
- For simplicity, *capital does not depreciate*.

The regulator's problem

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- Then the regulator solves

$$\max_{\{c(t),s(t)\}} \int_0^\infty u\left[c\left(t\right)\right] e^{-\rho t} dt$$

$$\text{subject to} \begin{cases} \dot{A}\left(t\right) = s\left(t\right) F\left[k\left(t\right), A\left(t\right)\right] / \varphi\left[A\left(t\right)\right], \\ \dot{k}\left(t\right) = \left[1 - s\left(t\right)\right] F\left[k\left(t\right), A\left(t\right)\right] - c\left(t\right), \\ 0 \leq s\left(t\right) \leq 1, \\ k\left(t\right) \geq 0,\ A\left(t\right) \geq 0,\ c\left(t\right) \geq 0, \\ k\left(0\right) = k_0 > 0,\ A_0\left(0\right) = A_0 > 0 \end{cases}$$

Equilibria Characterization

- On the locus $F_k(k, A) - F_A(k, A)/\varphi(A) = 0$ on the state space (A, k) the marginal product of capital equals that of knowledge. It can be written as a function of A:

$$\tilde{k}(A) = f[\varphi(A)]A,$$

with $f(\cdot)$ increasing. We call $\tilde{k}(A)$ the *(transitory) turnpike*.

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- $\tilde{k}(A)$ becomes linear for larger A; we call the function

  $$\tilde{k}_\infty(A) = \tilde{\eta}A,$$

  where $\tilde{\eta} = f[1/\pi'(0)]$, the *asymptotic turnpike*. $\tilde{k}(A) > \tilde{k}_\infty(A)$ for all $A < \infty$, and approaches $\tilde{k}_\infty(A)$ as A increases.

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- The locus $F_k(k, A) = \rho$ defines the *stagnation line:*

$$\hat{k}(A) = \hat{\eta}A,$$

  where $\hat{\eta}$ is a constant.

Proposition (Tsur and Zemel, 2007)

1. *A necessary condition for long run growth is that the stagnation line lies above the asymptotic turnpike:* $\hat{\eta} > \tilde{\eta}$*.*
   If $\hat{\eta} \leq \tilde{\eta}$ *the economy eventually reaches a stagnation point on* $\hat{k}\,(A)$*, corresponding to zero growth.*

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2. *If $\hat{\eta} > \tilde{\eta}$, for any A_0 there is a threshold $k^{sk}\left(A_0\right)$ such that if $k_0 \geq k^{sk}\left(A_0\right)$ the economy in the long run grows along a balanced growth path with constant growth rate depending on the capital rental rate, on ρ and on the intertemporal elasticity of substitution of u.*
   The income shares devoted to investments in knowledge and capital are constant and can be explicitly calculated.
   If $k_0 < k^{sk}\left(A_0\right)$ the economy eventually stagnates.

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Weitzman's result is confirmed in a more general setting.

Introducing IPRs

- In the previous *regulated competitive economy* the regulator maximizes aggregate welfare by choosing the firms' rewards for R&D. The optimal saving rate $s(t)$ is thus a *first-best* solution corresponding to the minimum cost required for producing knowledge and R&D firms earn zero profit.

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- In our model the regulator still chooses the resources to be devoted to production of new knowledge and levies it as a tax on consumers. The *novelty* is that these resources are now used to *purchase new knowledge from a unique monopolistic firm* who represents the entire R&D sector.

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- *IPRs grant the R&D firm a monopoly power* over production of new knowledge. The regulator or anyone else have no access to such information without paying the monopoly price.

R&D monopolist profit

- At each instant t the *R&D monopolist profit* is

$$\psi \dot{A}(t) - J(t),$$

where ψ is the *price of knowledge production*, $\dot{A}(t)$ is the amount of new knowledge produced (and sold to the regulator) and $J(t)$ is the *actual cost* that the R&D monopolist bears in order to produce $\dot{A}(t)$.

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- ψ contains the monopolist's unit cost of knowledge production plus a mark-up, as required by IPRs incentive theory.
- ψ is the expected *social unit cost of knowledge production* with monopolized R&D, to be compared with the same unit cost φ in the model with competitive R&D.

Profit maximization

- Using the usual production function of knowledge, profit is

$$\psi H(t)\,\pi\left[\frac{J(t)}{H(t)}\right]-J(t)\,, \tag{1}$$

which is concave in J as π satisfies the usual Assumption.

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  which is concave in J as π satisfies the usual Assumption.
- (1) has a unique (interior) maximum if

$$\psi>\frac{1}{H\pi'\left(0\right)}. \tag{2}$$

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Remark

(2) becomes less restrictive for larger H, vanishing for $H \to \infty$.

Profit maximization

Using FOC on profit and the formula of $\dot{A}$, and letting

$$sy = \psi \dot{A},$$

yields the law of motion of A under monopolized production:

$$\dot{A} = \frac{s(t)\, y(t)}{\psi\, [A(t)]}$$

where

$$\psi(A) = \left(\pi^{-1}\right)' \left[\frac{1}{C'_m(A)}\right] \quad (3)$$

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Remark

The formula (3) for $\psi(A)$ is similar to that of $\varphi(A)$.

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Corollary

If both economies are able to grow, in the long run they converge to the same asymptotic turnpike, $\tilde{k}_\infty(A)$. *Moreover:*
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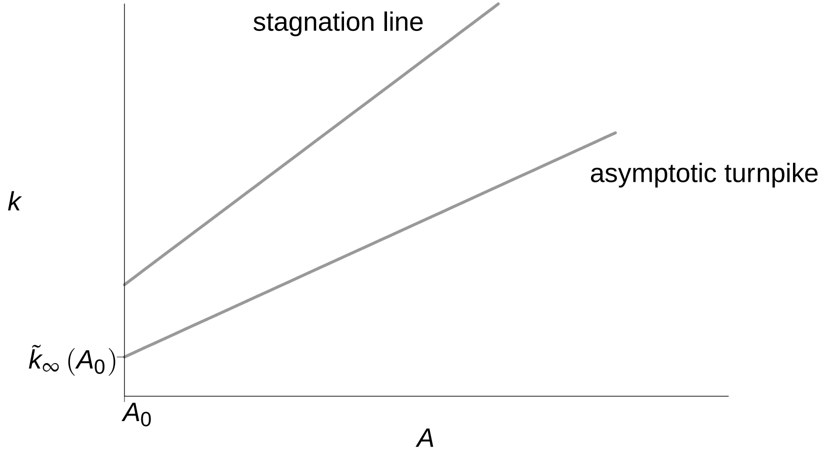
Given the same A, monopolized R&D requires larger capital and output to sustain growth, and the interest rate is smaller.

An example



$m = 2$, $\pi(x) = 1 - e^{-x}$, $F(k, A) = 0.00513k^{0.5}A^{0.5}$, $\rho = 0.002$. The growth condition, $\hat{\eta} > \tilde{\eta}$, holds as $\hat{\eta} = 1.65$ and $\tilde{\eta} = 1$.

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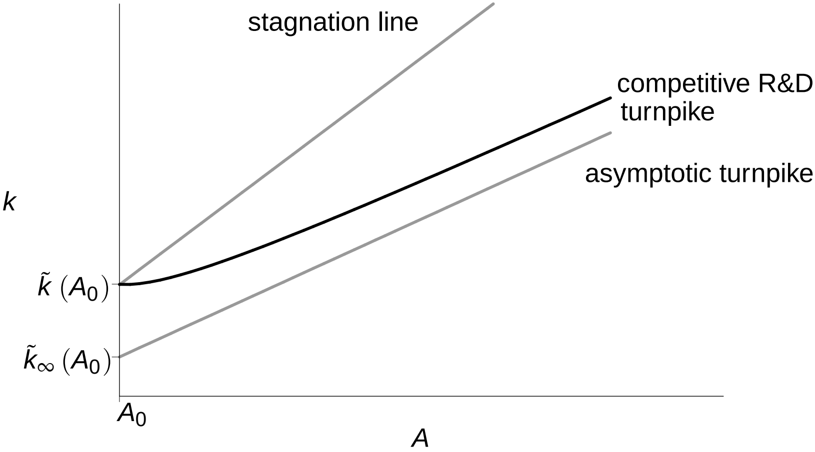

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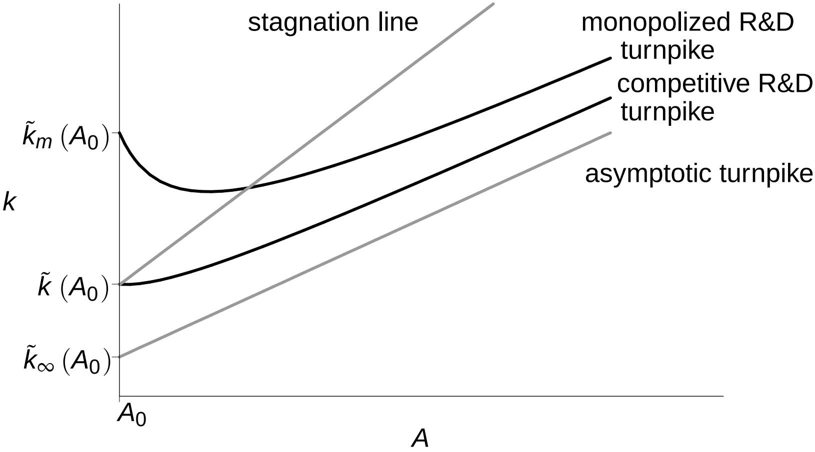

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Conclusions and plan for the future

- Our results are expressed in terms of *stocks* of A, k, y and r. To properly compare the behavior of the two economies we need to introduce *time*; that is, we must characterize the transitory dynamics, both toward the asymptotic turnpike $\tilde{k}_{\infty}(A)$ or toward a stagnation point on the line $\hat{k}(A)$.

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  2. whether the Skiba point $k^{sk}(A_0)$ increases by switching from competitive R&D to monopolized R&D.

Conjectures

1. *Conditions* for endogenous economic *growth* are *more restrictive* in the monopolized R&D economy than in the competitive economy. As a consequence, a (positive measure) set of economies with the opportunity to grow with competitive R&D may fail to grow under a IPRs regime.

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2. Provided that the economy grows along the *transitory turnpike*, its *transitory dynamics* are characterized by *larger stock values* and *smaller growth rates* for all variables at each instant in the monopolistic R&D version than in the competitive R&D model.