Behavioral Portfolio Choice and Disappointment Aversion

An Analytical Solution with "Small" Risks

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Aims and scope

1. EUT predicts a **large** equity position for most households.
2. Anomalies: empirical evidence show small percentage of risky assets in financial portfolio

3. **Puzzling aspect:** *Excess return* on equities has been positive and even large over the last century.
4. The puzzle is the following: given that equities yield such a high risk premium, why do households buy so few stocks?

Aims and scope

5. Obviously, the evolution of the excess return is also characterized by its **volatility**; and the volatility of the excess return has been high.

6. Thus, there is evidence to suggest that the undersized proportion of equities in the household's portfolio depends on how a risk-averse agent **perceives** the *trade-off* between expected returns and riskiness

Aims and scope

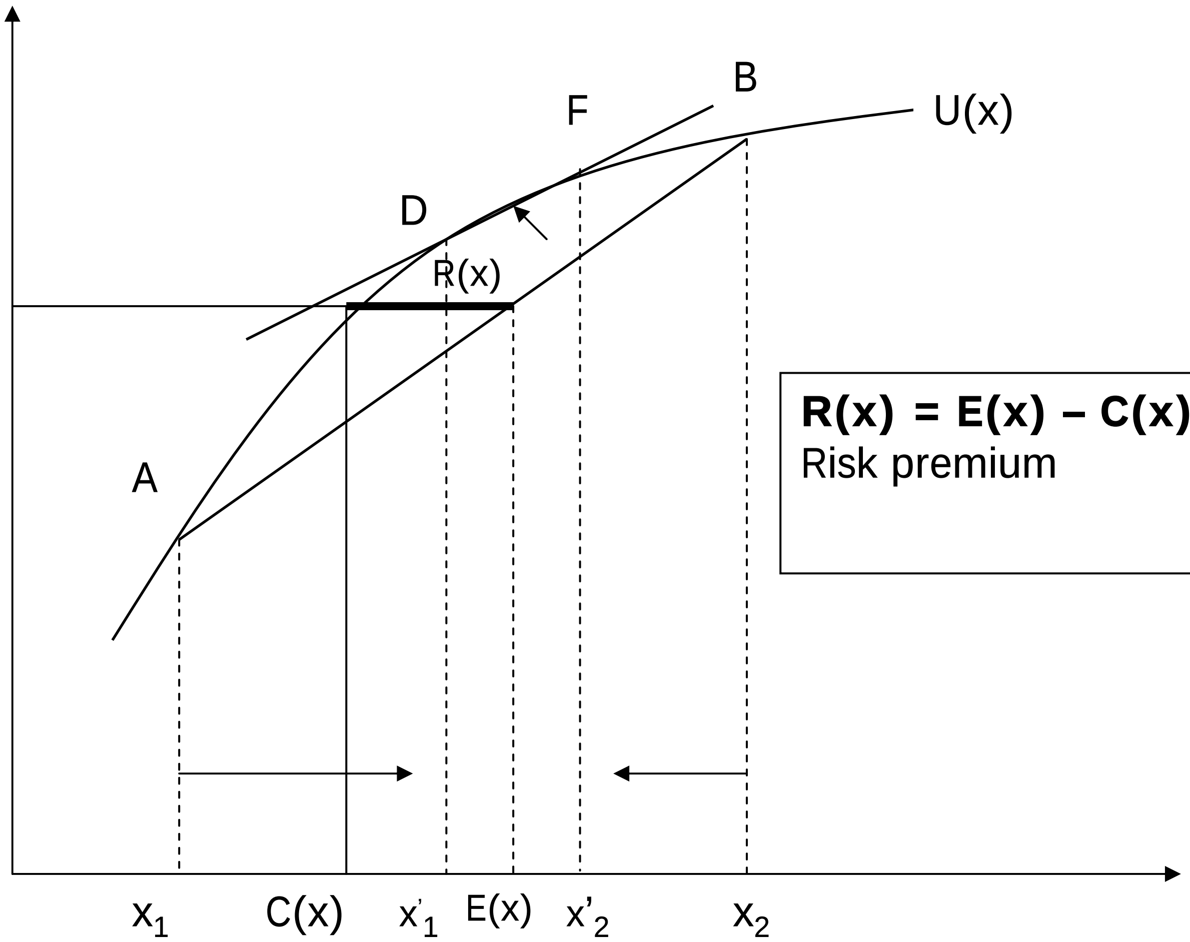
- We focus on behavioral finance

- Our aim is to provide an **analytical** solution to the portfolio choice with **Disappointment Aversion (Gul 1991)** and "small" risks.

- It is well known that in *EU theory*, the *Arrow-Pratt approximation* implies that risk yields a second-order effect on welfare.

→ if the risk is *small,* the major concern of the individual is the *expected value* of the lottery.

Second order risk aversion



R(x) = E(x) – C(x)
Risk premium

Aims and scope

➔ With a "small" risk the risk averse agent behaves as if he is a *risk neutral*!

- This implication of the EUT can explain the previous counterintuitive *predictions* about the:

1. large proportion of risky assets in financial portfolio
2. high partecipation rate of households
3. small effects of uncertainty on portfolio choice

EU model

- Consider the *standard portfolio problem*. Determine the composition of a portfolio containing a risk-free and a risky asset. (benchmark)

- W is wealth, r riskless interest rate, α is the **amount** of risky asset.
- $x = x_0 - r$ is the *excess return (equity premium)*
- The end period value of porfolio is

$$(W-\alpha)(1+r)+\alpha(1+\tilde{x}_0) = W(1+r)+\alpha(\tilde{x}_0 - r) = w_0 + \alpha\tilde{x}$$

EU model

- The aim of the risk averse agent is to choose α so as to maximize his expected utility $U(\alpha)$:

$$\max U(\alpha) = Eu(w_0 + \alpha x)$$

with $u'>0$ and $u''<0$.

- The FOC when $\alpha^* = 0$ is the optimal amount of the risky asset has the form:

EU model

$$U'(0) = u'(w_0)E(x)=0$$

Since $u'>0$, the condition is satisfied only when $E(x) \leq 0$.

Consequence: in the EU framework the risk averse agent will prefer the riskless asset ***if and only if*** the excess return is equal to zero.

- → $\alpha^* = 0$ if and only if $E(x) \leq 0$
- → the risky asset is always preferred $\alpha^* > 0$ if $E(x) > 0$

EU model: implications

1. In EU theory, the major concern of the decision maker will be the *expected value* of the excess return E(x) ………

2. ….. even when, ***with high uncertainty*** it would be better not to invest in the risky asset.

EU and small risks

- It is helpful to determine the solution to this problem when the *portfolio risk* is **"small".**

- The problem of this approach is that the size of the portfolio risk is *endogenous* in this problem because **α** * depends on the magnitude of the risk associated to the risky asset.

→ To escape this difficult define the **excess return** as:

$$x = k\mu + y$$

where *k,* **μ** >0, with mean *E(y)* =*0.* When *k*→ *0 then x* = *y* and since *y is a pure risk* ➔ *E(x)=E(y)=0*

EU and small risks

- Hence, the optimal investment in the risky asset is **$\alpha^*(k)$,** which is a function of k, with **$\alpha^*(0) = 0$**.

- When k is positive we obtain the solution of the optimal share solving the following FOC

$$E(k\mu + \tilde{y})u'(w_0 + \alpha^*(k)(k\mu + \tilde{y})) = 0$$

Small risks

→ Using the approximation of $\alpha^*(k)$ *around k=0* the optimal **amount** of risky assets is:

$$\alpha^*(k) = \frac{E(\tilde{x})}{var(\tilde{x})} \frac{1}{A(w_0)}$$

where $A(w_0)$ is the Arrow-Pratt coefficient of absolute risk aversion.

Optimal share

- The relative share is equal to:

$$\frac{\alpha^*(k)}{w_0} = \frac{E(\tilde{x})}{var(\tilde{x})} \frac{1}{R(w_0)}$$

- Example. Let us consider a logarithm investor (R=1). If E(X)=7% and std(x)=30% then:

$$\alpha^* = 0.77$$

- The optimal portolio contains around 7/9 in the risky asset
- this seems to be a **rather large proportion.**

Aims and scope

- How does this result change under **Disappointment Aversion preferences (DA)**?

- Basic properties:

  → it gives *more weight* to the *unfavorable* events and less weight to the favorable ones. Agent is less attracted by risky assets!

  → when the 'bad' outcome occurs, the agent is disappointed

  → his welfare is **reduced** by a term which depends on his degree of disappointment aversion.

Aims and scope

- Advantage of DA

→ It is an **axiomatic** and normative theory.

- Drawbacks of DA

→ DA **does not deliver closed form solution** to the optimal portfolio choice because of the endogenous reference point in the value function.

→ **Numerical solutions** are the standard tool for studying the properties and the implications of the DA preferences.

Results

1. We provide an **analytical** solution to the portfolio choice in presence of DA utility and "small" risks.

2. Under DA the optimal portfolio choice is proportional to the ratio between the ***adjusted*** mean and the variance of the excess return.

3. The original probabilities are *adjusted* by the degree of disappointment β. We call these new probabilities disappointing probabilities.

Results

→ DA has some helpful implications for asset pricing:

- When the risk is "small" the DA allows to compute a **share** of risky assets which is order of magnitude less than the corresponding share under the EU theory.

DA preferences

- **Basic model** with only *two states of nature* with outcomes $x_1 > 0 > x_2$.

- The **DA expected utility** $V(\alpha)$ can be written as

$$V(\alpha) = p_1 u(w_0 + \alpha x_1) + p_2 u(w_0 + \alpha x_2) - \beta p_2 [V(\alpha) - u(x_2)]$$

- The last term captures the effect of the disappointment.
- β is the unit value of disappointment.

DA

- Rewrite this equation as:

$$V(\alpha) = p_1 \frac{1}{1+p_2\beta} u(w_0 + \alpha x_1) + p_2 \frac{1+\beta}{1+p_2\beta} u(w_0 + \alpha x_2)$$

$$V(\alpha) = q_1 u(w_0 + \alpha x_1) + q_2 u(w_0 + \alpha x_2)$$

- where:

$$q_1 = p_1 \frac{1}{1+p_2\beta} \qquad \text{and} \qquad q_2 = 1 - q_1$$

- We shall call q_1 and q_2 **disappointing** probabilities

DA

- Now, the corresponding FOC when the optimal share is $\alpha_D{}^* = 0$ is given by the expression:

$$V'(0) = u'(w_0)E_D(x) = 0$$

where $E_D(x)$ is the expected value of x computed using the **disappointing probabilities**.

DA:implication

- As for EU theory the risk averse agent will prefer the riskless asset *if and only if* the excess return is equal to zero, $E_D(x) = 0$

- **But** now $E_D(x) = 0$ implies that:

$$E_D(x) \equiv \left(p_1 \frac{1}{1+p_2\beta}\right)x_1 + \left(p_2 \frac{1+\beta}{1+p_2\beta}\right)x_2 = 0$$

DA

- That is

$$E(x) = -p_2 x_2 \beta > 0$$

- Hence, $\alpha_D^* = 0$ if and only if ***E(x)*** is equal to the *expected disappointment* $-p_2 x_2 > 0$, times the degree of disappointment aversion β.
- So, under DA it might be better **not to invest** in the risky asset even when the expected return of the gamble is positive $E(x) > 0$.

DA

- **DA implications**.

1. In DA theory, for a risk averse agent the portfolio choice depends *not only* on the expected values of the risky asset, *but also* on the probability of the bad outcome and on the disappointment degree β.

2. So, ***with very high*** β it may be better *not to invest* in the risky asset even when the expected return is positive $E(x)>0$.

DA and *small* risks

- As before, let's define the **risky return** as

$$\boldsymbol{x = k\mu + y}$$

where $k, \mu > 0$.

*But **now for small risk,*** we mean that when k tends to zero, the *expected excess return* tends to the value $-p_2 y_2 \beta > 0$, which is the measure of the disappointment.

DA and *small* risks

- As before expanding the FOC around ***k*=0,** the optimal amount of risky asset is:

$$\alpha_D^*(k) = \frac{E_D(x)}{var_D(x)} \frac{1}{A(w_0)}$$

but now the mean and the variance depend on the **"new"** probability distribution q_1 and q_2 .

Optimal shares

- Example. Utility is *CRRA:*

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma} \text{ with } 0 < \gamma < \infty \qquad \text{we assume } \gamma = 2$$

- The **excess return** is generated by the following binomial process

$$y_1 = 0.39,\ y_2 = -0.25 \quad \text{with} \quad p_1 = p_2 = 0.5$$

- $\beta=0.56$, $\mu=0.05$, and $w_0=1$

Optimal shares

- When $k=0$, under EUT the optimal share is:

$$\frac{\alpha^*(k)}{w_0} = \frac{0.07}{0.1024}\frac{1}{2} = 0.34$$

- Under DA the optimal share is:

$$\frac{\alpha_D^*(k)}{w_0} = \frac{0}{0.1073} = 0$$

Optimal shares

- Assume now *k=0.1*. This small change affects the expected returns, but does not affect the variance.

- The optimal shares are respectively:

$$\frac{\alpha^*(k)}{w_0} = \frac{0.075}{0.1024}\frac{1}{2} = 0.36, \quad \frac{\alpha_D^*(k)}{w_0} = \frac{0.005}{0.1073}\frac{1}{2} = 0.023$$

→ Under UT Optimal portfolio contains around 4/10 in the risky asset!

Extentions: Continuos random variables

- The disappointment appears when the realization of the random variable x is **below** the certainty equivalent x_c

- The utility function under DA is:

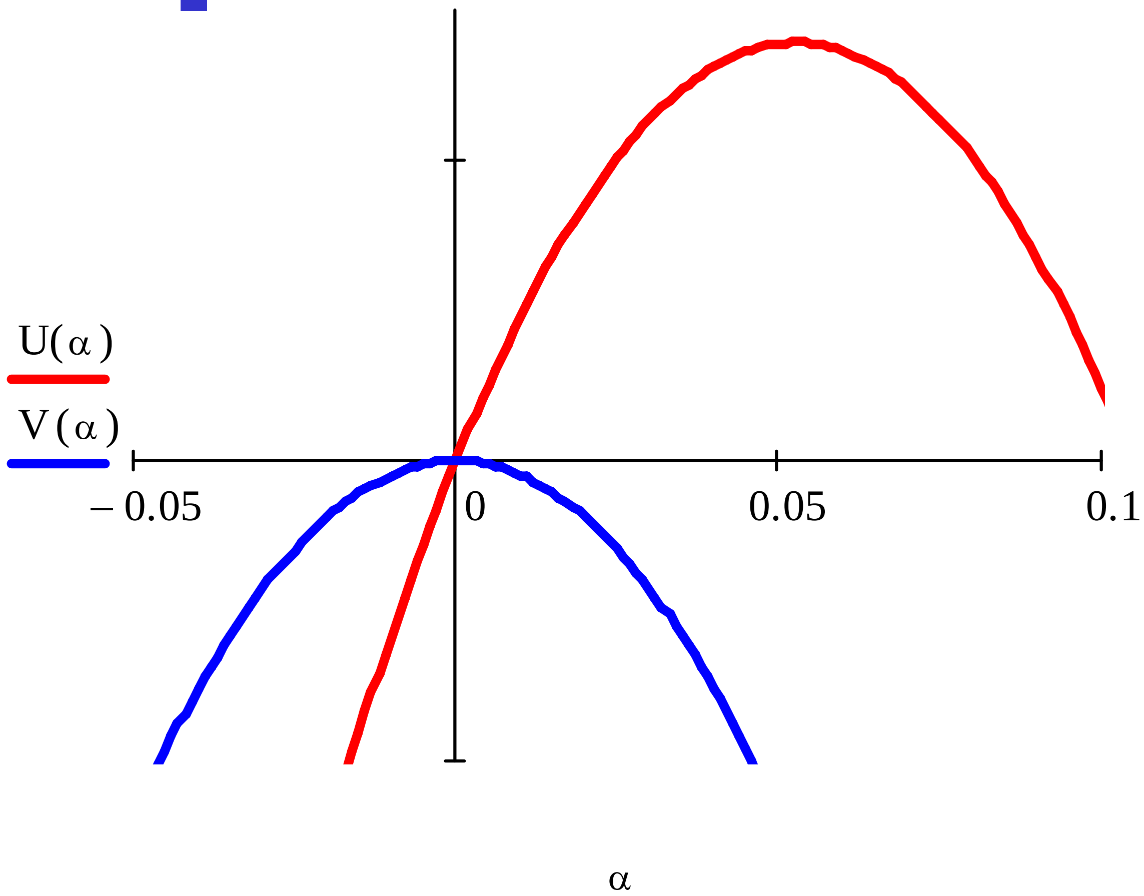
$$V(w) = E[u(w)] - \beta \int_{-\infty}^{x_c} [u(x_c) - u(w)] f(x) dx$$

Extentions

- The *new* probability distribution is given by

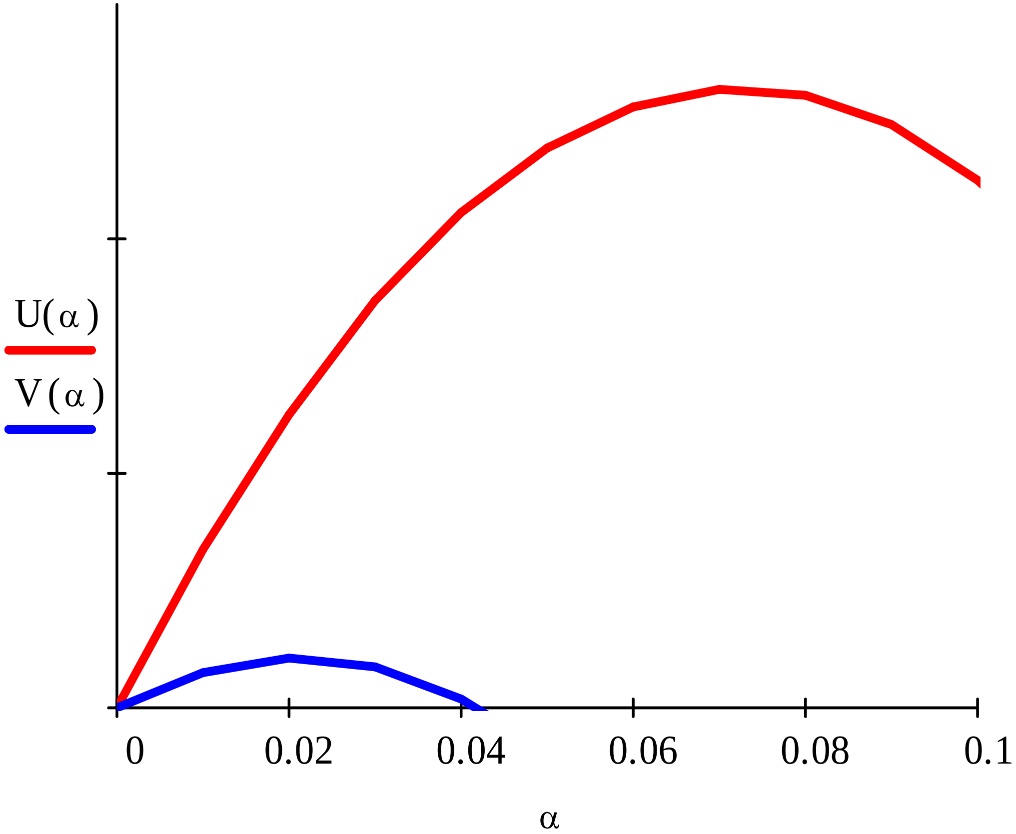
$$f_D(x) = \begin{cases} \dfrac{f(x)}{1+\beta\int_{-\infty}^{x_c} f(x)\,dx} & \text{if } x \geq x_c \\[2ex] \dfrac{(1+\beta)f(x)}{1+\beta\int_{-\infty}^{x_c} f(x)\,dx} & \text{if } x < x_c \end{cases}$$

Continuos random variables

- 

- The optimal share with *u(x) = ln(x)*, and when k = 0.

Continuos random variables



- The optimal share with *u(x) = ln(x)*, and *k* = *0.1*

Conclusions

1. We **provide an *analytical* solution with DA** when risk is small.

2. Under DA the optimal percentage of wealth invested in the risky asset has a **plausible size.**

3. It is proportional to the ratio of expectation and variance of the excess return, appropriately modified by **the degree of risk aversion**.

Conclusions

4. Under DA **the amount of risky asset** in the portfolio is *less* than the amount predicted by the EU.
5. Our future aim is to **extend** this basic model to the dynamic context**.**

Related literature

- There are innumerable papers dealing with the previous puzzling stylized facts.
- We divide these contributions in **three** main groups.

Related literature

1. Most papers emphasize the need for a purely new **descriptive** theory of decisions under uncertainty. *Psychological Models:* Kahneman Tversky (1979) prospect theory; Loomes and Sugden (1962) regret theory; Benartzi and Thaler (1995) myopic loss aversion.

→ Common element of these contributions is the emphasis on descriptive aspects and skepticism on normative theory

Related literature

2. A **second** group of paper tries to emend the standard portofolio choice. *Generalized Utility Function:* Epstein and Zin (1989), Weil (1989) recursive utility function; Costandinides (1990) *Habit formation*.

→ A more flexible version of the standard power utility

Related literature

3. A **third** strand of research focuses on the independence axiom of EU theory and on its violation (Allais paradox). *A new axiomatic Theory of behavioral finance*: Chew MacCrimmon (1979), Dekel (1986), Fishburn (1993), Yaari (1987), Gul (1991), Ang el Al. (2005).

→ Previous puzzles can be accomodated in a new framework where the *independence* axiom does not work.