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R&D Cooperation with Network Externalities in Real Option Analysis

Giovanni Villani

Department of Economics, Mathematics and Statistics
University of Foggia

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The Basic Model

- Two firms (A and B) that have the option to realize their R&D investment at initial time t_0 or to postpone their decision at time t_1.
- We state as Leader the pionner firm (A or B) that invests in R&D at time t_0 earlier than other one, namely the Follower that postpones the R&D investment decision at time t_1.
- The R&D projects are characterized by “Information Revelation” (see Dias 2004) and “Positive Network Externalities ” (see Huisman and Kort).

Information Revelation (Dias 2004)

The R&D success or failure of one firm generates an information revelation that influences the investment decision of the other firm.
Assuming by q and p the R&D success probability of firms A and B respectively, we have that:

- If firm A's R&D is successful, the firm B's probability p changes in positive information revelation p^+;
- In case of firm A's failure, p changes in negative information revelation p^-.
- The firm A's R&D success changes in q^+ or in q^- in case of success or failure of firm B at time t_0.

Following Dias model's about the information revelation process, it results that:

$$p^{+} = p + \sqrt{\frac{1-q}{q}} \cdot \sqrt{p(1-p)} \cdot \rho(B, A) \tag{1a}$$

$$p^{-} = p - \sqrt{\frac{q}{1-q}} \cdot \sqrt{p(1-p)} \cdot \rho(B, A) \tag{1b}$$

$$q^{+} = q + \sqrt{\frac{1-p}{p}} \cdot \sqrt{q(1-q)} \cdot \rho(A, B) \tag{1c}$$

$$q^{-} = q - \sqrt{\frac{p}{1-p}} \cdot \sqrt{q(1-q)} \cdot \rho(A, B) \tag{1d}$$

where the correlations $\rho(B, A)$ and $\rho(A, B)$ are the measures of information revelation from A to B and from B to A, respectively.

Growth Market Coefficients

We denote by:

- $$K_{0_S 0_S}, \quad K_{0_S 1_S}, \quad K_{1_S 0_S}, \quad K_{1_S 1_S}$$
  the growth market coefficients in case of A and B success.
- $$K_{0_S 0_F} = K_{0_S 1_F} \equiv K_{0_S}; \quad K_{1_S 0_F} = K_{1_S 1_F} \equiv K_{1_S}$$
  the market coefficients for the winning firm assuming the failure by the other player.
- $$K_{0_F 0_S} = 0, \quad K_{0_F 1_S} = 0, \quad K_{1_F 0_S} = 0, \quad K_{1_F 1_S} = 0$$
  $$K_{0_F 0_F} = 0, \quad K_{0_F 1_F} = 0, \quad K_{1_F 0_F} = 0, \quad K_{1_F 1_F} = 0$$
  the market coefficients for the losing firm.

- Positive Network Externality: the growth market coefficient in case of both R&D success will be bigger than the situation in which only one firm invests successfully:

$$K_{SS} > K_S \tag{2}$$

- R&D Success Time: the market coefficient increases if the reciprocal R&D success is realized at time t_0 rather than t_1, because there is more time to benefit both network externalities and R&D innovations. In the situation in which only one firm invests successfully, the market coefficient enlarges if the success is realized at time t_0 rather than t_1:

$$K_{0_S 0_S} > K_{1_S 1_S}; \quad K_{0_S} > K_{1_S} \tag{3}$$

- First Mover's Advantage: the firm that realizes with success the R&D investment at time t_0 will receive an higher market coefficient than other player that postpones successfully the project at time t_1:

$$K_{0_S 1_S} > K_{1_S 0_S}; \tag{4}$$

We denote by:

- R is the R&D investment for the development of a new product. This investment can be realized at initial time $t_0 = 0$ or at time t_1.
- D is the investment cost to realize the new product:

$$\frac{dD}{D} = (\mu_d - \delta_d)dt + \sigma_d dZ_d;$$

- V is the overall market value deriving by R&D innovation;

$$\frac{dV}{V} = (\mu_v - \delta_v)dt + \sigma_v dZ_v$$

- T is the expiration time to realize the investment D and so to obtain the market value V.
- The production investment is proportional to market share using the Growth Market Coefficient.

- $s(V, D, T)$ is the value of a Simple European Exchange Option assuming that the initial time $t_0 = 0$:

$$s(V, D, T) = Ve^{-\delta_v T} N(d_1(P, T)) - De^{-\delta_d T} N(d_2(P, T))$$

- $c(s(V, D, T), \varphi D, t_1)$ is the value of a Compound European Exchange Option assuming that the initial time $t_0 = 0$:

$$\begin{aligned} c(s(V, D, T), \varphi D, t_1) &= Ve^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*}, t_1\right), d_1(P, T); \rho\right) \\ &\quad - De^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*}, t_1\right), d_2(P, T); \rho\right) \\ &\quad - \varphi De^{-\delta_d t_1} N_1\left(d_2\left(\frac{P}{P^*}, t_1\right)\right) \end{aligned}$$

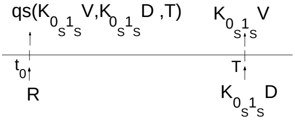
To determine the growth market coefficients K, we assume that they depend by a parameter k involving the R&D innovation and by length of R&D benefits. For the network externality, we take into account two times the one firm market coefficient.

$$K_{0_S} = kT \tag{5a}$$
$$K_{0_S 0_S} = 2kT \tag{5b}$$
$$K_{1_S} = k(T - t_1) \tag{5c}$$
$$K_{1_S 1_S} = 2k(T - t_1) \tag{5d}$$
$$K_{0_S 1_S} = 2k(T - t_1) + kt_1 \tag{5e}$$
$$K_{1_S 0_S} = 2k(T - t_1) - kt_1 \tag{5f}$$

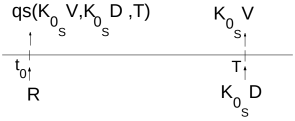
To ensure that the Positive Network Externaility holds, we need to impose that $t_1 < \frac{T}{3}$. This condition is reasonable with the consideration that the information revelation disappears in time and so, if one firm invests at time t_0, the other firm decision will be made within $t_1 < \frac{T}{3}$ to allow the realization of development phase in $T - t_1$.

Leader's Payoff

The firm A (Leader) invests in R&D at time t_0 while the firm B (Follower) decides to wait to invest.



(a) Follower's success situation



(b) Follower's failure situation

So the Leader's payoff in case of Follower's success is:

$$L_A^S(V,D) = -R + q \cdot s\left(K_{0_S 1_S} V, K_{0_S 1_S} D, T\right) \tag{6}$$

while the Leader's payoff in case of Follower's unsuccess is:

$$L_A^F(V,D) = -R + q \cdot s\left(K_{0_S} V, K_{0_S} D, T\right) \tag{7}$$

The probability to receive $L_A^S(V,D)$ is p^+ because the Follower receives the positive information revelation while the probability to obtain $L_A^F(V,D)$ is $1 - p^+$. Computing the expectation value, the Leader's payoff (firm A) is:

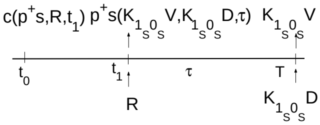
$$L_A(V,D) = p^+ \cdot L_A^S(V,D) + (1 - p^+) \cdot L_A^F(V,D) \tag{8}$$

Simmetrically, assuming that firm B (Leader) invests at time t_0 while firm A (Follower) decides to postpone its decision, the Leader's payoff became:

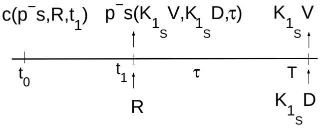
$$L_B(V,D) = q^+ \cdot L_B^S(V,D) + (1 - q^+) \cdot L_B^F(V,D) \tag{9}$$

Follower's Payoff

The firm B (Follower) decides to postpone its R&D investment decision at time t_1 and firm A invests at time t_0.



(c) Leader's success situation



(d) Leader's failure situation

The value of CEEO with positive information is:

$$
\begin{aligned}
c(p^{+}) &= p^{+}k(2T-3t_1)Ve^{-\delta_v T}N_2\left(d_1\left(\frac{P}{P^{*}_{upB}},t_1\right),d_1\left(P,T\right);\rho\right)\\
&\quad -p^{+}k(2T-3t_1)De^{-\delta_d T}N_2\left(d_2\left(\frac{P}{P^{*}_{upB}},t_1\right),d_2\left(P,T\right);\rho\right)\\
&\quad -\varphi De^{-\delta_d t_1}N_1\left(d_2\left(\frac{P}{P^{*}_{upB}},t_1\right)\right) \qquad (10)
\end{aligned}
$$

The value of CEEO with negative information is:

$$
\begin{aligned}
c(p^{-}) &= p^{-}k(T-t_1)Ve^{-\delta_v T}N_2\left(d_1\left(\frac{P}{P^{*}_{dwB}},t_1\right),d_1\left(P,T\right);\rho\right)\\
&\quad -p^{-}k(T-t_1)De^{-\delta_d T}N_2\left(d_2\left(\frac{P}{P^{*}_{dwB}},t_1\right),d_2\left(P,T\right);\rho\right)\\
&\quad -\varphi De^{-\delta_d t_1}N_1\left(d_2\left(\frac{P}{P^{*}_{dwB}},t_1\right)\right) \qquad (11)
\end{aligned}
$$

The Follower obtains the CEEO $c(p^+)$ in case of Leader's success with a probability q or the CEEO $c(p^-)$ in case of Leader's failure with a probability $1 - q$. Hence, the Follower's payoff at time t_0 is the expectation value:

$$F_B(V, D) = q\, c(p^+) + (1 - q)\, c(p^-) \tag{12}$$

Similary, if we consider that firm B (Leader) invests in R&D at time t_0 and firm A (Follower) decides to wait to invest we have that:

$$F_A(V, D) = p\, c(q^+) + (1 - p)\, c(q^-) \tag{13}$$

Simultaneous Investment Payoff

In this situation both firms invest in R&D at time t_0. We can assume that there is not information revelation and so $\rho(A, B) = \rho(B, A) = 0$. So, the firm's *A* payoff assuming the firm *B*'s R&D success will be:

$$S_A^S(V, D) = -R + q \cdot s\left(K_{0_S 0_S} V, K_{0_S 0_S} D, T\right) \tag{14}$$

otherwise, in case of firm *B* failure, the firm's A payoff will be:

$$S_A^F(V, D) = -R + q \cdot s\left(K_{0_S} V, K_{0_S} D, T\right) \tag{15}$$

So, recalling that firm B's probability success is equal to p, the firm's *A* payoff in case of simultaneous investment will be the expectation value:

$$S_A(V, D) = p \cdot S_A^S(V, D) + (1 - p) \cdot S_A^F(V, D) \tag{16}$$

Simmetrically, the firm's *B* payoff will be:

$$S_B(V, D) = q \cdot S_B^S(V, D) + (1 - q) \cdot S_B^F(V, D) \tag{17}$$

Waiting Payoff

Now we suppose that both firms decide to delay their R&D investment decision at time t_1 and we can setting that there is not information revelation. First of all, we analyse the situation of firm A. Assuming the R&D success of firm B, player A realizes the investment R at time t_1 and holds, with a probability q, the development option s with a market coefficient $K_{1_S 1_S}$. Then the firm's A payoff at time t_0 is a CEEO:

$$W_A^S(V, D) = c\left(q \cdot s(K_{1_S 1_S}V, K_{1_S 1_S}D, \tau), R, t_1\right) \tag{18}$$

and specifically, assuming that R is a fraction φ of asset D, we have:

$$\begin{aligned} W_A^S(V, D) \;=\; & q2k(T - t_1)Ve^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*_{wsA}}, t_1\right), d_1(P, T); \rho\right) \\ & - q2k(T - t_1)De^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*_{wsA}}, t_1\right), d_2(P, T); \rho\right) \\ & - \varphi De^{-\delta_d t_1} N_1\left(d_2\left(\frac{P}{P^*_{wsA}}, t_1\right)\right) \end{aligned} \tag{19}$$

But, in case of firm's B failure, the firm A growth market coefficient will be K_{1_S}. So, the firm' A payoff at time t_0 is the following CEEO:

$$W_A^F(V,D) = c\left(q\cdot s(K_{1_S}V, K_{1_S}D, \tau), R, t_1\right) \tag{20}$$

and specifically:

$$\begin{aligned} W_A^F(V,D) &= qk(T-t_1)Ve^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*_{wfA}}, t_1\right), d_1(P,T);\rho\right) \\ &\quad -qk(T-t_1)De^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*_{wfA}}, t_1\right), d_2(P,T);\rho\right) \\ &\quad -\varphi De^{-\delta_d t_1} N_1\left(d_2\left(\frac{P}{P^*_{wfA}}, t_1\right)\right) \end{aligned} \tag{21}$$

Hence, recalling that the firm B success is equal to p , we can compute the firm A payoff as the expectation value:

$$W_A(V,D) = p\,W_A^S(V,D) + (1-p)\,W_A^F(V,D) \tag{22}$$

Similary, the firm B payoff is:

$$W_B(V,D) = q\,W_B^S(V,D) + (1-q)\,W_B^F(V,D) \tag{23}$$

We can observe that:

1. $L_i(0) = -R; \quad W_i(0) = 0;$
2. $\dfrac{\partial L_i}{\partial V} > \dfrac{\partial W_i}{\partial V} > 0;$

for $i = A, B$. Then, the following proposition holds:

Proposition (1)

There exists, for each firm $i = A, B$, a unique critical market value V_i^W that makes $L_i(V_i^W) = W_i(V_i^W)$. Denoting by $V_W^* = \min(V_A^W, V_B^W)$ and $V_Q^* = \max(V_A^W, V_B^W)$, it results that:

$$\begin{array}{lll} L_i(V) < W_i(V) & \textit{for} & V < V_W^* \\ L_i(V) > W_i(V) & \textit{for} & V > V_Q^* \end{array}$$

If A's success probability q is higher than B, for $V \in]V_W^*, V_Q^*[$ it results:

$$L_A(V) > W_A(V); \quad L_B(V) < W_B(V)$$

otherwise $\qquad L_A(V) < W_A(V); \quad L_B(V) > W_B(V)$

We can observe that:

1. $F_i(0) = 0; \quad S_i(0) = -R;$
2. $\frac{\partial F_i}{\partial V} > 0; \quad \frac{\partial S_i}{\partial V} > 0$

for $i = A, B$. So the following proposition holds:

Proposition (2)

If $\frac{\partial S_A}{\partial V} > \frac{\partial F_A}{\partial V}$ *then there exists a unique critical market value* V_P^*

$$\begin{array}{lll} S_A(V) < F_A(V) & \text{for} & V < V_P^* \\ S_A(V) > F_A(V) & \text{for} & V > V_P^* \end{array}$$

otherwise, if $\frac{\partial S_A}{\partial V} \leq \frac{\partial F_A}{\partial V}$ *then* $S_A(V) < F_A(V)$ *for every value of* V.

If $\frac{\partial S_B}{\partial V} > \frac{\partial F_B}{\partial V}$ *then there exists a unique critical market value* V_S^*

$$\begin{array}{lll} S_B(V) < F_B(V) & \text{for} & V < V_S^* \\ S_B(V) > F_B(V) & \text{for} & V > V_S^* \end{array}$$

otherwise, if $\frac{\partial S_B}{\partial V} \leq \frac{\partial F_B}{\partial V}$ *then* $S_B(V) < F_B(V)$ *for every value of* V.

Cooperation value $C(A \cup B)$

The condition to respect to have $0 \leq p^+ \leq 1$ and $0 \leq p^- \leq 1$ is that:

$$0 \leq \rho(A, B) \leq \min\left\{\sqrt{\frac{p(1-q)}{q(1-p)}}, \sqrt{\frac{q(1-p)}{p(1-q)}}\right\} \tag{24}$$

With the alliance between A and B, we can assume that information is wholly revealed and we can setting that the cooperative information $\rho_{\max}$ is equal to:

$$\rho_{\max} = \min\left\{\sqrt{\frac{p(1-q)}{q(1-p)}}, \sqrt{\frac{q(1-p)}{p(1-q)}}\right\} \tag{25}$$

We denote by $C(A \cup B)$ the feasible set for the coalition: it is the sum of two firm's payoffs using the whole information revelation $\rho_{\max}$.

We assume that the payoff obtained by cooperation can be transferred from one player to other. For istance firms A and B can share equitably the surplus of cooperation using the Shapley values:

$$Sh_A = v(A) + \frac{C(A \cup B) - (v(A) + v(B))}{2} \quad (26a)$$

$$Sh_B = v(B) + \frac{C(A \cup B) - (v(A) + v(B))}{2} \quad (26b)$$

where

- $C(A \cup B) - (v(A) + v(B))$ is the surplus of cooperation;
- $v(A)$ and $v(B)$ are the non cooperative Nash equilibriums.

We can assume also asymmetric shares to split the surplus of cooperation value. For istance:

$$P_A = v(A) + \frac{q}{p+q}(C(A \cup B) - (v(A) + v(B))) \quad (27a)$$

$$P_B = v(B) + \frac{p}{p+q}(C(A \cup B) - (v(A) + v(B))) \quad (27b)$$

The four possible cooperation strategies are:

1. Both players decide to wait to invest at time t_0:

$$C(A \cup B) = W_A(V) + W_B(V) \equiv W_C(V)$$

2. The firm A invests at time t_0 while the firm B delays its decision at time t_1. The firm B obtains the overall information revelation ρ_{max}:

$$C(A \cup B) = L_A^C(V) + F_B^C(V) \equiv LF_C(V)$$

3. Simmetrically, the firm B invests at time t_0 and the firm A delays its decision at time t_1:

$$C(A \cup B) = F_A^C(V) + L_B^C(V) \equiv FL_C(V)$$

4. Both players decide to invest at time t_0:

$$C(A \cup B) = S_A(V) + S_B(V) \equiv S_C(V)$$

Final payoffs at time t_0

FIRM A \ FIRM B	*Wait*	*Invest*
Wait	Non−Cooperation (W_A, W_B) Cooperation W_C	Non−Cooperation (F_A, L_B) Cooperation FL_C
Invest	Non−Cooperation (L_A, F_B) Cooperation LF_C	Non−Cooperation (S_A, S_B) Cooperation S_C

The aim of two firms acting together is to improve their position compared with no partnership. To realize this objective, we have to determine the maximum value among the four cooperation strategies according to several expected market values V at time t_0.

Relations among the cooperation strategic values

It results that:

1. $W_C(0) = 0; \qquad S_C(0) = -2R;$
2. $LF_C(0) = -R; \quad FL_C(0) = -R;$
3. $\dfrac{\partial S_C}{\partial V} > \dfrac{\partial W_C}{\partial V} > 0.$
4. If $q = p$ then $LF_C(V) = FL_C(V)$;
5. If $q < p$ then $LF_C(V) < FL_C(V)$;
6. If $q > p$ then $LF_C(V) > FL_C(V)$;
7. Assuming $q \geq p$ we have: $\dfrac{\partial LF_C}{\partial V} > \dfrac{\partial W_C}{\partial V} > 0.$

Proposition (3)

There exists a unique critical market value V_C^* such that:

$$\begin{array}{lll} LF_C(V) < W_C(V) & \text{for} & V < V_C^* \\ LF_C(V) > W_C(V) & \text{for} & V > V_C^* \end{array}$$

First case: $\frac{\partial LF_C}{\partial V} \geq \frac{\partial S_C}{\partial V}$

If $\frac{\partial LF_C}{\partial V} \geq \frac{\partial S_C}{\partial V}$ then there is not intersection between the functions LF_C and S_C. Moreover, the intersection LF_C and W_C occurs before than S_C and W_C. So, in this case, we have to consider only the critical market value V_C^* given by proposition 3 and we can state that:

- If $V < V_C^*$ the maximum payoff attainable cooperating is

$$C(A \cup B) = W_C(V)$$

- If $V > V_C^*$ the maximum payoff attainable cooperating is

$$C(A \cup B) = LF_C(V)$$

In this case, the best strategic cooperation is the waiting policy (W_C) until the expected market value V is below the critical value V_C^* and, if $V > V_C^*$, the optimal strategy is the Leader-Follower one (LF_C) in which the firm with higher success probability realizes the R&D investment at time t_0 and the other player postpones its decision at time t_1.

Second case: $\frac{\partial LF_C}{\partial V} < \frac{\partial S_C}{\partial V}$

If $\frac{\partial LF_C}{\partial V} < \frac{\partial S_C}{\partial V}$ then there is intersection between the functions LF_C and S_C. So the following proposition holds:

Proposition (4)

If $\frac{\partial LF_C}{\partial V} < \frac{\partial S_C}{\partial V}$ then there exists a unique critical market value V_G^*

$$\begin{array}{lll} S_C(V) < LF_C(V) & \textit{for} & V < V_G^* \\ S_C(V) > LF_C(V) & \textit{for} & V > V_G^* \end{array}$$

Moreover, it results $V_C^* < V_G^*$. So, using the Propositions 3 and 4 we observe that:

- If $V < V_C^*$ the maximum payoff is $C(A \cup B) = W_C(V)$
- If $V_C^* < V < V_G^*$ the maximum payoff is $C(A \cup B) = LF_C(V)$
- If $V > V_G^*$ the maximum payoff is $C(A \cup B) = S_C(V)$

Assumptions and Inputs

We develop two numerical examples for the cooperative R&D game between firms A and B with the following parameters:

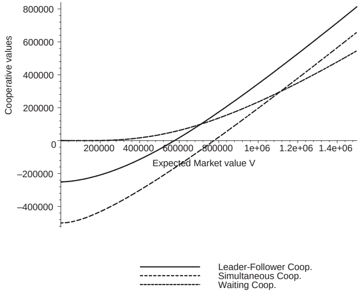
- R&D Investment: R= 250 000 $;
- Development Investment: D= 400 000 $;
- Market and Costs Volatility: $\sigma_v = 0.93$; $\sigma_d = 0.23$;
- Fraction of D required for R: $\varphi = \frac{R}{D} = 0.625$;
- Correlation between V and D: $\rho_{vd} = 0.15$;
- Dividend-Yelds of V and D: $\delta_v = 0.15$; $\delta_d = 0$;
- R&D innovation parameter $k = 0.30$;
- Expiration Time of Simple Option: $T = 3$ years;
- A and B success probability: $q = 0.60$; $p = 0.55$;
- Non Coop. Information Revelation: $\rho(A, B) = \rho(B, A) = 0.40$;
- Cooperative Information Revelation: $\rho_{max} = 0.9026$;

First Case: $\frac{\partial LF_C}{\partial V} \geq \frac{\partial S_C}{\partial V}$

Assuming that the R&D investment decision can be delay at time $t_1 = 0.5$ year, we obtain the following growth market coefficients:

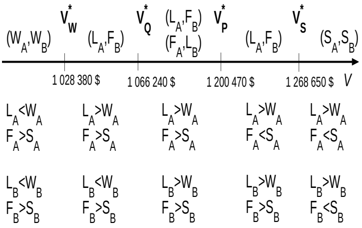
$$K_{0_S 0_S} = 1.8; K_{0_S 1_S} = 1.65; K_{1_S 1_S} = 1.50;$$
$$K_{1_S 0_S} = 1.35; \; K_{0_S} = 0.90; \; K_{1_S} = 0.75$$

Moreover, for our adapted number it results that $V_C^* = 700\,037$.



The non cooperative critical market values are:

$$V_W^* = 1\,028\,380;\ V_Q^* = 1\,066\,240;\ V_P^* = 1\,200\,470;\ V_S^* = 1\,268\,650;$$



If $V < 700\,037 \Rightarrow C(A \cup B) = W_C(V)$ and the alliance does not add value because the surplus of cooperation

$$W_C(V) - (W_A(V) + W_B(V)) = 0$$

If $V > 700\,037 \Rightarrow C(A \cup B) = LF_C(V)$ and the cooperation is favourable since the cooperation gain $C(A \cup B) - (v(A) + v(B))$ is positive.

For istance, when the expected market value $V = 1\,400\,000\,\$$ we have:

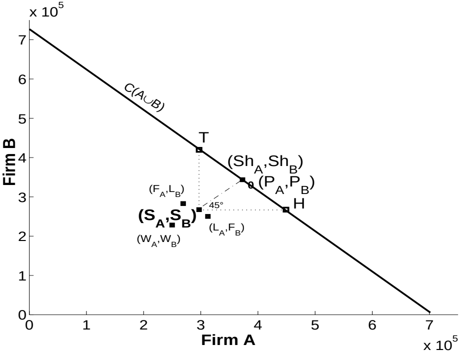
		FIRM B Wait	FIRM B Invest
FIRM A	*Wait*	Non−Cooperation (249.879, 228.291) Cooperation 478.170	Non−Cooperation (269.253, 282984) Cooperation 698.053
FIRM A	*Invest*	Non−Cooperation (312.391, 250.310) Cooperation 716.600	Non−Cooperation (296.958, 267.552) Cooperation 564.510

Both players can split the surplus of cooperation

$$716\,600 - (296\,958 + 267\,552) = 152\,490$$

using the Shapley values: $(Sh_A, Sh_B) \Rightarrow (373\,003, 343\,597)$
or the Asimmetric values: $(P_A, P_B) \Rightarrow (376\,309, 340\,291)$

The black line denotes the the feasible set $C(A \cup B)$ of partnership. But only the combinations on the segment T-H are interesting otherwise firms have the incentive to deviate from cooperation. We can notice that the segment joins the couples (S_A, S_B) and (Sh_A, Sh_B) has a 45° slope since, by the Shapley value, A and B share equitably the surplus of cooperation.

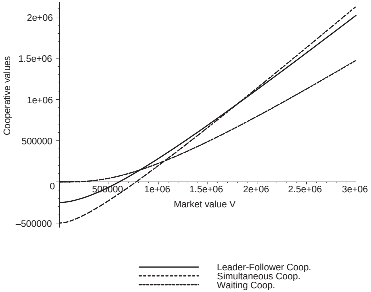


Second Case: $\frac{\partial LF_C}{\partial V} < \frac{\partial S_C}{\partial V}$

If we assume now that $t_1 = 0.8$ year, we have that the growth market coefficients are:

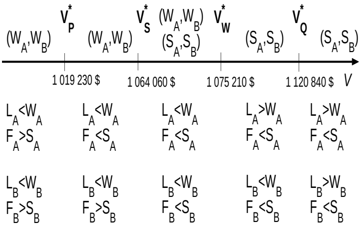
$$K_{0_S 0_S} = 1.8;\ K_{0_S 1_S} = 1.56;\ K_{1_S 1_S} = 1.32;$$
$$K_{1_S 0_S} = 1.08;\ K_{0_S} = 0.90;\ K_{1_S} = 0.66;$$

Numerically we that $V_C^* = 815\,710$ and $V_G^* = 1\,796\,130$.



The non cooperative critical market values are:

$V_P^* = 1\,019\,230$; $V_S^* = 1\,064\,060$; $V_W^* = 1\,075\,210$; $V_Q^* = 1\,120\,840$;



If $V < 815\,710 \Rightarrow C(A \cup B) = W_C(V)$ and the alliance does not add value because the surplus $W_C(V) - (W_A(V) + W_B(V)) = 0$;
If $815\,710 < V < 1\,796\,130 \Rightarrow C(A \cup B) = LF_C(V)$ and the cooperation is favourable since the gain $C(A \cup B) - (v(A) + v(B))$ is positive;
If $V > 1\,796\,130 \Rightarrow C(A \cup B) = S_C(V)$ and the alliance does not add value since the surplus of cooperation $S_C(V) - (S_A(V) + S_B(V)) = 0$.

For istance, when the expected market value $V = 1\,200\,000\,\$$ we have:

		FIRM B: *Wait*	FIRM B: *Invest*
FIRM A	*Wait*	Non−Cooperation (168.420, 153.905) Cooperation 387.553	Non−Cooperation (160.209, 169.129) Cooperation 423.726
FIRM A	*Invest*	Non−Cooperation (193.398, 147.452) Cooperation 440.190	Non−Cooperation (201.411, 177.142) Cooperation 378.553

Both players can split the surplus of cooperation

$$440\,190 - (201\,411 + 177\,142) = 61\,637$$

using the Shapley values: $(Sh_A, Sh_B) \Rightarrow (232\,229, 207\,960)$
or the Asimmetric values: $(P_A, P_B) \Rightarrow (233\,569, 206\,621)$

Also in this case we can observe that Shapley (Sh_A, Sh_B) and Asimmetric (P_A, P_B) values belong to the segment T-H. We can notice that the segment joins the couples (S_A, S_B) and (Sh_A, Sh_B) has a 45° slope since, by the Shapley value, A and B share equitably the surplus of cooperation.