International Environmental Agreement: a Dynamic Model of Emissions Reduction

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International Environmental Agreements - IEA

- Over the last two decades, the interest in international environmental problems such as climate change, ozone depletion, marine pollution has grown immensely and it has driven an increased sense of interdipendence between countries.
- Cooperation results in IEA such as:
  - Helsinky Protocol (1985);
  - Oslo Protocol (1994);
  - Montreal Protocol (1987);
  - Kyoto Protocol on the reduction of greenhouse gases causing global warming (1997).

In these IEAs, the number of signatories varies considerably.

- Why are IEAs ratified only by a fraction of the countries?
- Which strategies can increase the number of signatories?
- Will the agreement be stable?
- What does stable mean?

- Agreements must be self-enforcing (in the absence of any international authority, there must be incentives for countries to join and to remain in an agreement)
- Agreements must be profitable (there must be gains to all signatory countries)

IEA

Both Cooperative and Non-Cooperative game theory have been used to study coalition formation.

- In the Cooperative Game framework, using core concepts and implementing transfers to solve the heterogeneity of the countries, Chander and Tulkens (1995) reach the conclusion that the grand coalition is stable.
- In the Non-Cooperative Game framework, the concept of Internal and External stability has been applied to obtain the size of a coalition. The idea is to check for which size of a coalition an individual country is indifferent between remaining in the coalition or leaving it. For most models in literature the size of the stable coalition is very small.

These two approaches make opposite assumptions on what happens if a country leaves the coalition.

- In the Cooperative Game framework, it is assumed that the whole coalition breaks down.
- In the Non-Cooperative Game framework, it is assumed that the rest of the coalition remains intact.
- A bridge between these two extremes is given by the concept of farsightedness (Chwe, 1994).
  If a country leaves an agreement, it may trigger other countries to leave until a new stable situation is reached.
- The definition of farsightedness is a recursive one.

Following Rubio and Casino (2005) in a continuous time setting and, in a time-discrete setting de Zeeuw (2008), in this talk we study a two-stage game.

- In the first stage (the membership game) each country decides noncooperatively wether or not to join an IEA.
- In the second stage (the abatement differential game) signatories and nonsignatories determine their abatement levels in a dynamic continuous time setting.

- In the differential game proposed, Feedback Nash equilibria are calculated in order to determine the optimal paths of the abatement levels and of the stock pollutant.
- Rubio and Casino (2005) assumes a myopic behaviour of the countries and establish the result that only bilateral agreements are self-enforcing.
- We use farsighted stability concept showing as both large and small coalitions can be stable.

- Let us assume that n identical countries decide to abate emissions in order to reduce the environmental pollution.
- Initially the accumulated emissions are at a level s_0 and each country i chooses to abate the quantity of emissions $a_i(t)$ (for $i = 1, 2, ..., n$).

- The dynamic of accumulated emissions is given by the following differential equation

$$\dot{s}(t) = L - \sum_{i=1}^{n} a_i(t) - k\, s(t) \quad s(0) = s_0$$

$$0 \leq a_i(t) \leq \frac{L}{n}$$

- L represents a constant source of pollutant and k a positive rate of natural decay.
- The constraint tells us that a single country is allowed to abate only a fraction of the emissions produced by itself and it assures us that $s(t) \geq 0$.

- We assume that players minimize a cost function $c_i(a_i(t))$ which is the sum of two terms: the abatement costs and the costs due to the remaining pollution.
- $$c_i(a_i(t)) = \frac{1}{2}a_i(t)^2 + \frac{1}{2}p\,s(t)$$
- A major role is played by the parameter $p > 0$; it can be seen as a measure of the environmental awareness of the country; i.e. it denotes the relative weight attached to the damage costs as compared to the abatement costs.

Let assume that, as the outcome of the first stage game, there are m signatories ($i = 1, ..., m$) and $n - m$ non signatories ($j = m + 1, ..., n$).
So, we consider a simple structure in which there is only one coalition while the other countries play as individual outsiders.

The game is solved in a backward order.

- In the second stage non signatory countries choose their abatement levels acting noncooperatively and minimizing the discount present value of their costs taking as given the strategy of the other countries.
- Signatories choose their abatement levels acting noncooperatively against non signatories in order to minimize the discount present value of the aggregate costs of the m signatories. Signatories also take as given the strategies of non signatories.

- The optimal abatement levels and accumulated emission paths are given by the Nash equilibria of a differential game.
- From them it is possible to obtain the equilibrium discounted present value of the cost $C_i(m)$ of a signatory country and the cost $C_j(m)$ of a non signatory country.

- In the first stage countries play a simultaneous open membership game.
- The strategies for each countries are to sign or not an agreement and any player is free to join it.
- The choice between the two different kinds of behaviour are simultaneous and the agreement is formed by all players that have choosen to cooperate, the others are non signatories.

- The Hamilton-Jacobi-Bellman equations for signatories and for non signatories are, respectively,

$$\delta V_i = \max_{a_i} \left\{ -\sum_{h=1}^{m} \left(\frac{1}{2} a_h^2 + \frac{1}{2} p\, s \right) + V_i' \left(L - \sum_{h=1}^{m} a_h - \sum_{j=m+1}^{n} a_j - ks \right) \right\}$$

$$\delta V_j = \max_{a_j} \left\{ - \left(\frac{1}{2} a_j^2 + \frac{1}{2} p\, s \right) + V_j' \left(L - \sum_{i=1}^{m} a_i - \sum_{j=m+1}^{n} a_j - ks \right) \right\}$$

- $V_i(s)$ and $V_j(s)$ represent the optimal control value functions of the coalition and of a non signatory, V_i' and V_j' are the first derivative
- $\delta > 0$ is the discount rate.

We obtain the following set of necessary conditions for an interior feedback Nash equilibrium

$$-a_i - V_i' = 0 \qquad i = 1, ..., m$$

$$-a_j - V_j' = 0 \qquad j = m + 1, ..., n$$

These conditions define the optimal strategies for abatements as functions of accumulated emissions.

The constraints on the control variables lead to the following conditions on the abatement levels, respectively, for a signatory country and for a non signatory one

$$a_i = \begin{cases} 0 & \text{if} \quad -V_i' < 0 \\ -V_i' & \text{if} \quad 0 \le -V_i' \le \dfrac{L}{n} \\ \dfrac{L}{n} & \text{if} \quad -V_i' > \dfrac{L}{n} \end{cases} \qquad a_j = \begin{cases} 0 & \text{if} \quad -V_j' < 0 \\ -V_j' & \text{if} \quad 0 \le -V_j' \le \dfrac{L}{n} \\ \dfrac{L}{n} & \text{if} \quad -V_j' > \dfrac{L}{n} \end{cases}$$

where $i = 1, ..., m \;\; j = m+1, ..., n$

- We have analysed all possible combinations between interior and boundary a_i and a_j values.
- If $0 \leq -V_i' \leq \frac{L}{n}$ and $0 \leq -V_j' \leq \frac{L}{n}$, then $a_i = -V_i'$ and $a_j = -V_j'$.

  Substituting these abatement level expressions in HJB equations, we obtain the following nonlinear differential equations

$$\delta V_i = \frac{m}{2}(V_i')^2 + V_i'(L + (n-m)V_j' - ks) - \frac{1}{2}mps$$

$$\delta V_j = \left(\frac{2n-2m-1}{2}\right)(V_j')^2 + V_j'(L + mV_i' - ks) - \frac{1}{2}ps$$

- Given the linear quadratic structure of the game, we guess that the optimal value functions are quadratic.
- The equilibrium strategies are linear in respect to the state variable.
- We postulate quadratic value functions of this form

$$V_i = \frac{1}{2}\alpha_i s^2 + \beta_i s + \mu_i \quad V_j = \frac{1}{2}\alpha_j s^2 + \beta_j s + \mu_j$$

  where α, β, μ are constant parameters which are to be determined.
- Substituting V_i, V_j, V_i' and V_j' in the above differential equations, we obtain a system of algebraic Riccati equations for the coefficients of the value functions

$$\begin{cases} \frac{1}{2}\alpha_i\delta = \dfrac{m}{2}\alpha_i^2 + (n-m)\alpha_i\alpha_j - k\alpha_i \\ \beta_i\delta = m\alpha_i\beta_i + L\alpha_i + (n-m)\alpha_i\beta_j + (n-m)\beta_i\alpha_j - k\beta_i - \frac{1}{2}mp \\ \mu_i\delta = \beta_i\left[\dfrac{m}{2}\beta_i + L + (n-m)\beta_j\right] \\ \frac{1}{2}\alpha_j\delta = \left(\dfrac{2n-2m-1}{2}\right)\alpha_j^2 + m\alpha_i\alpha_j - k\alpha_j \\ \beta_j\delta = (2n-2m-1)\alpha_j\beta_j + L\alpha_j - k\beta_j + m\alpha_j\beta_i + m\alpha_i\beta_j - \frac{1}{2}p \\ \mu_j\delta = \beta_j\left[\left(\dfrac{2n-2m-1}{2}\right)\beta_j + L + m\beta_i\right] \end{cases}$$

- It has 4 solutions
- Only one produces value functions satisfying the stability condition $\frac{d\dot{s}}{ds} < 0$
- To obtain this condition we substitute, in the dynamical constraint of accumulated emissions, the linear strategies

$$a_i = -\alpha_i s - \beta_i \ , \ \ a_j = -\alpha_j s - \beta_j$$

- We obtain the following differential equation

$$\dot{s} = [m\alpha_i + (n-m)\alpha_j - k]s + L + m\beta_i + (n-m)\beta_j$$

- The stability condition is

$$\frac{d\dot{s}}{ds} = m\alpha_i + (n-m)\alpha_j - k < 0$$

- It is satisfied only by the following solution of the system

$$\alpha_i = \alpha_j = 0 \ , \ \beta_i = -\frac{mp}{2(k+\delta)} \ , \ \beta_j = -\frac{p}{2(k+\delta)}$$

$$\mu_i = -\frac{mp(4kL + 4L\delta - p(m^2 - 2m + 2n))}{8\delta(k+\delta)^2}$$

$$\mu_j = -\frac{p(4kL + 4L\delta - p(2m^2 - 2m + 2n - 1))}{8\delta(k+\delta)^2}$$

- This solution, combined with the constraints $0 \leq -V_i' \leq \dfrac{L}{n}$ and $0 \leq -V_j' \leq \dfrac{L}{n}$, gives us the optimal abatement levels:

$$a_i = \frac{mp}{2(\delta + k)} \quad \text{and} \quad a_j = \frac{p}{2(\delta + k)}$$

when the following condition on p is satisfied

$$p \leq \frac{2L(\delta + k)}{mn}$$

- If we suppose that $-V_i' > \frac{L}{n}$ and $0 \le -V_j' \le \frac{L}{n}$, then $a_i = \frac{L}{n}$ and $a_j = -V_j'$.

  Reasoning as above we obtain

$$a_i = \frac{L}{n} \quad \text{and} \quad a_j = \frac{p}{2(\delta + k)}$$

  when the following condition on p is satisfied

$$\frac{2L(\delta + k)}{mn} < p \le \frac{2L(\delta + k)}{n}$$

- If we suppose that $-V_i' > \frac{L}{n}$ and $-V_j' > \frac{L}{n}$, then $a_i = a_j = \frac{L}{n}$.

  Reasoning as above we obtain

$$a_i = a_j = \frac{L}{n}$$

  when the following condition on p is satisfied

$$p > \frac{2L(\delta + k)}{n}$$

- If we consider the remaining combinations between a_i and a_j values, solutions of the Riccati system don't satisfy the constraints.
- We have to distinguish three different cases which are characterized by different values of p.
  Let
  $$r = \frac{2L(\delta + k)}{np}$$

CASE I

- If $r \geq m$ then

$$a_i = \frac{mp}{2(\delta + k)} \quad \text{and} \quad a_j = \frac{p}{2(\delta + k)}$$

  and so every signatory country abates m times more than the non signatory country.
- The optimal path for the state variable s is

$$s(t) = s_0 e^{-kt} + \frac{1}{k}\left[L - \frac{m^2 p}{2(\delta + k)} - \frac{(n-m)p}{2(\delta + k)}\right](1 - e^{-kt})$$

CASE II

- If $1 \leq r < m$ then

$$a_i = \frac{L}{n} \quad \text{and} \quad a_j = \frac{p}{2(\delta + k)}$$

and so every cooperator reduces to zero its emission.

- The optimal path for the state variable s is

$$s(t) = s_0 e^{-kt} + \frac{(n-m)}{k}\left[\frac{L}{n} - \frac{p}{2(\delta + k)}\right](1 - e^{-kt})$$

CASE III

- If $r < 1$ then
$$a_i = a_j = \frac{L}{n}$$
and so cooperators and outsiders reduce to zero their emissions. In this case the role of a signatory and of a non signatory is the same.
- The optimal path for the state variable s is
$$s(t) = s_0 e^{-kt}$$

- The concept of *farsightedness* has been introduced in literature by Chwe (1994).
- A country belonging to a coalition of size m decides to abandon the coalition if its current cost $C_i(m)$ is higher than the cost he should pay going outside the coalition.
- Nevertheless, by the farsighted approach, he will take into account the possibility that if he leaves the coalition then other coalition members may find convenient to abandon the coalition, too.

- A disgregation process of the coalition can arise and then a country which decides to abandon a coalition of size m must compare its cost as a member of the coalition with the cost it should pay as an outsider of the remaining coalition at the end of this disgregation process.
- If no country has an incentive to leave a coalition of size m, behaving in a farsighted way, then the coalition is said to be farsighted stable.

- In order to apply the stability conditions proposed in the above section, we need to calculate $C_i(m)$ and $C_j(m)$.
- Case I $r \geq m$

$$C_i(m) = \frac{m^2p^2}{8\delta(\delta+k)^2} + \frac{p}{2\delta(\delta+k)}\left[L - \frac{m^2p}{2(\delta+k)} - \frac{(n-m)p}{2(\delta+k)}\right] + \frac{ps_0}{2(\delta+k)}$$

$$C_j(m) = \frac{p^2}{8\delta(\delta+k)^2} + \frac{p}{2\delta(\delta+k)}\left[L - \frac{m^2p}{2(\delta+k)} - \frac{(n-m)p}{2(\delta+k)}\right] + \frac{ps_0}{2(\delta+k)}$$

Case II $1 \leq r < m$

- $$C_i(m) = \frac{L^2}{2\delta n^2} + \frac{p(n-m)}{2\delta(\delta+k)}\left[\frac{L}{n} - \frac{p}{2(\delta+k)}\right] + \frac{ps_0}{2(\delta+k)}$$

$$C_j(m) = \frac{p^2}{8\delta(\delta+k)^2} + \frac{p(n-m)}{2\delta(\delta+k)}\left[\frac{L}{n} - \frac{p}{2(\delta+k)}\right] + \frac{ps_0}{2(\delta+k)}$$

Case III $r < 1$

- $$C_i(m) = C_j(m) = \frac{L^2}{2\delta n^2} + \frac{ps_0}{2(\delta+k)}$$

- Let us assume that $r \geq 1$.
- Let us assume that a FSC of size $m \geq 1$ exists.

In order to have the smallest farsighted stable coalition larger than the coalition of size m, we need to find the smallest integer h ($1 \leq h \leq n - m$) such that

$$C_i(m + h) \leq C_j(m)$$

Before studying the conditions for which this inequality is satisfied, we have to characterize the costs, which depend on the relative positions of $m + h$ and r.

- Let $g(m)$ defined as the smallest integer greater than or equal to $\sqrt{2m(m-1)}+1$.
- Let $w(m)$ defined as the smallest integer greater than or equal to $\dfrac{2m^2-2m-1+r^2}{2(r-1)}$.
- Let $z(m)$ defined as the smallest integer greater than or equal to $m+\frac{1}{2}(1+r)$.

- If $g(m) \leq \min([r], n)$ then $g(m)$ is the size of the smallest FSC larger than the coalition of size m.
- If $w(m) \geq [r] + 1$ and $[r] + 1 \leq n$ then the size of the smallest FSC larger than the coalition of size m is $[r] + 1$.
- If $z(m) \leq n$ then $z(m)$ is the size of the smallest FSC larger than the coalition of size m.

Let

$n = 100$
$L = 100$
$k = 1$
$\delta = 1$
$s_0 = 0$

Let $p = 0.01$.
Then the following coalitions are FS.

$\boxed{m = 2}$
$\boxed{m = 3}$
$\boxed{m = 5}$
$\boxed{m = 8}$
$\boxed{m = 12}$
$\boxed{m = 18}$
$\boxed{m = 26}$
$\boxed{m = 38}$
$\boxed{m = 55}$
$\boxed{m = 79}$

Let $p = 0.1$.
Then the following coalitions are FS.

$m = 2$
$m = 3$
$m = 5$
$m = 8$
$m = 12$
$m = 18$
$m = 26$
$m = 38$
$m = 41$
$m = 62$
$m = 83$

- Let $p = 1$.
  Then coalitions of size $m = 2$ and $m = 3$ are FS.
  Moreover any coalition of size
  $m = 3t - 1 \qquad t = 2, 3, ..., 33$ is FS.
  The largest FSC is $m = 98$.
- If $p \geq 4$, then any coalition is FS.

- Small coalitions are stable, as obtained using internal and external stability
- Farsighted stability assures us that also large coalitions can be stable
- The role of p is also very important
- If p increases the size of coalitions which are stable increases, too
- It is possible to find p values for which any coalition is stable and in particular, the grand coalition is stable