On the Possible Conflict Between Economic Growth and Social Development*

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Abstract

The present contribution proposes a simple growth model with private and social capital accumulation. We investigate whether these two processes move together or not and show that both outcomes are possible, depending on the initial relative endowment of private and social capital, on the social technology and on the degree of individual impatience. Such dynamics affects and is affected by the choice of time allocation between labor and social participation and by the choice of consumption of both private and relational goods. Taking all these aspects into account allows us to study in an articulated way the interplay between the private and the social component of well-being.

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1 Introduction

Individual and aggregate well-being depend in the long run both on material growth and on social and cultural development. While this has perhaps always been true, for most of human history material growth has played no significant role: it has been most of the times absent, with some positive and negative exceptions [see e.g. Goodfriend and McDermott (1995)]. Since the Industrial Revolution, on the contrary, a significant fraction of the world has kept growing at a positive rate, accumulating physical capital, developing better and better technologies, and accumulating human capital. Indeed, these processes have captured the most part of economists' attention, whereas social and cultural dynamics have remained at the margin of economic analysis. In recent years, however, an increasing number of economists have begun to pay attention to the interplay between these two broad aspects.

Where material needs have been satisfied to a substantial degree, as it is the case in advanced economies, well-being depends to an increasing extent upon social factors, like social environment, individual relative position and social status, ability to construct and enjoy meaningful and satisfactory relations with other people, and so on: in one word, well-being also becomes a matter of building up a satisfactory individual and social identity[1]. Social status has already received a great deal of attention by economists. Here we rather focus on the social environment and on the enjoyment of social relations, building on the notions of 'social capital' and of 'relational goods'.

The present contribution proposes a simple model of growth with private and social capital accumulation. We investigate whether these two processes are positively correlated or not and show that both outcomes are possible, depending on the parameters of the economy. Taking into account the effects of such dynamics on consumption of both private and relational goods, we draw conclusions about well-being that apply to advanced economies. Section 2 clarifies the concepts and motivates our setup. Section 3 introduces the model. Section 4 concludes.

[1] An important case for the relevance of identity issues in economic analysis has recently been made by Akerlof and Kranton (2000). It is not our purpose here to specify the notion of identity in a deeper way.

2 Motivation

Social capital is the collection of those productive assets that are incorporated in the social structure of a group (rather than in physical goods and in single human beings, like physical and human capital) and that allow cooperation among its members to reach common goals. If we keep in mind that the group considered may be very small as well as include the whole society, this definition of social capital encompasses most of those to be found in the literature[2]. At one extreme, some scholars even define social capital as an individual asset, but we prefer to focus on groups[3]. Examples of social capital range from trust to effective civic norms and to the networks of voluntary associations typical of the civil society. A peculiar feature of social capital is that it is not accumulated through a standard mechanism of individual investment, since most of its benefits are not privately appropriable[4]. Rather, or at least to a much greater extent, it is accumulated through social participation to group activities. Such participation may only partially be regarded as an investment, since it is, perhaps mostly, an activity that entails the simultaneous production and consumption of a particular kind of goods, namely, relational goods.

Relational goods display the two peculiar features that they cannot be enjoyed alone, but exist only inasmuch as they are shared, and that very often their production and their consumption cannot be distinguished: relational goods are produced and consumed at the same time through the participation to some social activity with other people[5]. Examples range

[2]Seminal contributions are Coleman (1988 and 1990) and Putnam (1993). Since then, the literature on social capital has grown widely and we do not attempt to review it here.

[3]Glaeser, Laibson and Sacerdote (2000) call 'social capital' the 'social' component of human capital. Since we distinguish social capital from human capital, we do not follow their approach. DiPasquale and Glaeser (1999) define individual social capital as an individual's connections to others and argue that it matters much for private provision of local amenities and of local public goods. This is in line with our focus, although we emphasize more the role of aggregate social participation.

[4]Glaeser, Laibson and Sacerdote (2000) make the opposite point, namely that social capital accumulation responds to incentives to investment in exactly the same way as human capital. Indeed, this result is natural if one defines social capital as a component of human capital, but it does not hold anymore if one considers social capital as a group asset rather than as an individual asset.

[5]The notion of relational goods is due to Uhlaner (1989). Corneo and Jeanne (1999) refer to them as to socially provided private goods and study their interplay with social status and growth. Gui (2000) provides a number of interesting contributions on the

from going out with friends to participating in a choir, a football club, a voluntary organization, and so on.

We focus on two aspects of the relationship between relational goods and social capital. On one side, a higher social capital increases the returns to the time spent in social participation. For instance, it is easier and more rewarding to participate in an association in a social context characterized by a rich network of associative opportunities, as well as going out with friends in a context that offers many options for socially enjoyed leisure. In other words, social capital may be seen as an improvement in the technology of production of relational goods[6]. On the other side, a higher social participation brings about social capital accumulation as a byproduct. For instance, trust (or empathy) may be reinforced and generalized through social interactions (if individuals do not behave opportunistically). Likewise, a high social participation may lead to the formation of new associations, while still feeding the existing ones.

Social participation is an activity intrinsically characterized by external effects (generally speaking, there is no market where other people's participation may be bought and even less there is a market for social capital). If other people's participation is low, or if the level of social capital is low, the time spent in participating is unsatisfactorily productive and it becomes worthwhile to shift to private activities, that is, to activities that yield private goods. For instance, if my friends do not have time to go out together, or if they do, but the environment does not offer any interesting social opportunity, I may decide to spend my time watching television or reading a book: partly as the result of a substitution against suboptimal allocations of time (i.e. opting against well known, 'bad' social opportunities), partly as a form of defensive behavior against the risk of finding myself again in a boring and frustrating social situation (i.e. opting against unknown social opportunities with uncertain characteristics). Indeed, Corneo (2001) presents striking empirical evidence that the time devoted to watch television and to work are positively correlated across countries and explains this evidence through a model based on the substitution between privately enjoyed and socially en-

interpersonal dimension of economic interaction.

[6]Much of the literature on social capital also stresses its positive impact on the productivity of traditional private goods. We ignore this effect here, thus making our point sharper: if in our framework a problem of under-accumulation of social capital exists, such problem is going to be even worse if we also consider the effect of social capital on private production. We discuss this point in more detail in the concluding section.

joyed leisure (i.e., between some private goods and relational goods). While our work is quite close in the spirit to Corneo's paper, the main difference is that we analyze the dynamics of private and social capital accumulation, whereas he displays a simple static model with multiple equilibria.

The general point that relational goods and some private goods are substitutes need not be restricted to the examples of going out with friends and watching television, but may of course be applied to other examples of both private and relational goods as well.

In general, it appears natural to study a model with three goods: a private consumption good used to satisfy basic needs (say, food and clothes), a relational good (say, an evening out with friends) and a private consumption good that serves as a substitute of the relational good (say, collecting stamps). The key point is how individuals decide to allocate time between social participation, labor and private consumption, besides the allocation of the latter between the two private goods. Since our focus is on private and social capital accumulation, we disregard the precise allocation of time between the two forms of private consumption, simply assuming that both require income but not time.

Thus, a reduction in social participation implies at the same time an increase in labor supply and a substitution of private for relational goods. In general, this amounts to an individualization of the sources of well-being. On one side, such a shift stimulates the economy, since both production and consumption of private goods increase, and hence the GDP rises[7]; on the other side, it generates a negative externality on the productivity (in terms of relational goods) of social participation. Dynamically, this change has a negative effect on social capital accumulation, whereas the sign of the effect on private capital accumulation depends on whether production and consumption (of private goods) increase proportionally or disproportionately, i.e., on whether total savings increase together with consumption or decrease[8]. Theoretically, private and social capital may be both positively or negatively correlated. This is in line, for instance, with Putnam's (2000) empirical finding of a de-

[7]Observe that, while most private goods enter in the GDP, most relational goods do not.

[8]Notice that this is consistent with an interpretation of private capital in terms of physical capital. This interpretation will be held throughout our model, although we will keep speaking of private capital in general because we believe that a broad interpretation of private capital in terms both of physical and of human capital would not alter the picture significantly.

cline in US social capital followed by a revival, over a time horizon in which private growth has always been observed.

Both ideas, that private growth brings about social development, and that it generates social disruption, are supported by long-standing traditions of thought. We do not attempt to reconstruct this fascinating intellectual debate here and limit ourselves to refer to Hirsch (1976) as a representative of the view that private growth may entail negative social externalities. In particular, Hirsch argues that growth makes individual time constraints increasingly binding, thereby inducing a shift from time-intensive activities (among which there is indeed social participation) to time saving ones (among which there are many forms of private consumption – think e.g. of the fast-foods)[9]. We emphasize here that this kind of shift may even reinforce private growth.

The idea that negative externalities, either on the natural or on the social environment, might foster growth, in that they lead to an increase both in private (defensive) consumption and, at the same time, in labor supply and hence in production and savings, is studied within an evolutionary framework by Antoci (1996), Antoci and Bartolini (1999) and Antoci and Borghesi (2001). The same idea is further studied within a neoclassical framework by Antoci (1997a and 1997b), Bartolini and Bonatti (1997, 1998, 1999a and 1999b), Antoci, Borghesi and Galeotti (2002) and Antoci (2002). A common point is that negative externalities may be an engine of growth, but in this case growth results from a coordination failure and is not necessarily desirable; moreover, since impatience reduces private capital accumulation, it may increase steady state welfare. All of these contributions, although mentioning the possibility of a sociological interpretation, are indeed more focused on natural resources, which are typically subject to a spontaneous flow of renewal, but can be damaged by economic activities (think e.g. of the environmental effects of waste disposal and of pollution). In contrast, we focus here on social capital, whose accumulation dynamics is quite different and depends on individual choices of social participation.

In two companion papers [Antoci, Sacco and Vanin (2001 and 2002)] we explore a similar framework respectively with the tools of evolutionary game theory and of neoclassical economics. The broad result is that growing economies may fall into social poverty traps, defined as situations in which, although material wealth is high, social poverty drives down overall well-

[9]See Becker (1965) for a pioneering economic analysis of time allocation.

being. For the sake of simplicity, in those models we consider the dynamics of only one asset, namely, social capital. Here we extend such analysis to include the accumulation of private capital as well. One might expect that, once the latter is taken into account, possibly together with the positive externalities it brings about, material growth may be strong enough to more than compensate, from the point of view of well-being, its negative social externalities. Indeed, we show that this may but need not be the case and that whether it happens or not depends on the parameters of preferences and technology. Among other things, we also find that impatience may increase steady state well-being, since it reduces inefficient over-accumulation of private capital. This is true as long as positive externalities of private capital accumulation (of the kind studied in endogenous growth theory) are not too strong, i.e., as long as in equilibrium, including its external effects, private capital still has decreasing returns to scale, and as long as two further conditions are met: that social capital does not depreciate too fast and that the elasticity of relational goods to social capital is high enough.

3 Model

We present now a simple growth model with private and social capital accumulation. Since some of the basic insights may be appreciated even in a static framework, we first introduce a static version, in which private and social capital are considered as exogenously given in some strictly positive amount, and then introduce their dynamics (in continuous time).

3.1 Static specification

Preferences and technology

We model an economy populated by a continuum of identical, infinitely lived individuals, of size normalized to 1, whose utility depends on three goods: a private consumption good C used to satisfy basic needs, a relational good B and a private consumption good C_s that serves as a substitute of the relational good. Instantaneous preference are described by the utility function $u(C, B, C_s) = \ln C + a\ln(B + bC_s)$, where $a > 0$ is the elasticity of substitution between basic needs satisfied by C on one side and needs satisfied by either B or C_s on the other side, and $b > 0$ is the marginal rate of substitution between B and C_s[10].

We assume that private consumption (i.e., both C and C_s) does not require time. On the contrary, the relational good may only be enjoyed if an individual spends time in social participation. Individuals are endowed with a unit of time, which they allocate between social participation (fraction s) and labor (fraction $1-s$). A single individual considers average social participation $\bar{s} = \int_0^1 s(i)\mathrm{d}i$ in the economy as exogenously given.

Each individual produces private goods using labor and private capital K, according to the production function $Y = (1-s)^{\epsilon} K^{1-\epsilon} A$, where $\epsilon \in (0,1)$ is a parameter[11]. The term $A \equiv (1-\bar{s})^{\sigma} \bar{K}^{\vartheta}$ captures a positive externality in production. Average private capital $\bar{K} = \int_0^1 K(i)\mathrm{d}i$ is considered as exogenously given by each single individual and, consequently, the same is true for the whole term A (σ and ϑ are strictly positive parameters).

Besides private capital, our economy is characterized by the presence of social capital K_s. Social capital is not the private property of any individual, but is rather an endowment of the entire economy, that each single individual considers as exogenous.

The quantity of the relational good B enjoyed by the representative individual is a function of her own social participation, of average social participation and of social capital, all of which are necessary factors: $B = s^{\alpha} \bar{s}^{\beta} K_s^{\gamma}$, where $\alpha,\ \beta,\ \gamma > 0$.

Individual problem and symmetric Nash equilibria

The problem solved by the representative individual is

$$\max_{s,C,C_s} u(C, B, C_s) \quad \text{s.t.} \tag{1}$$

$$C + C_s = Y, \qquad C, C_s \geq 0, \quad s \in [0,1]. \tag{2}$$

A symmetric Nash equilibrium (SNE) is a triple (s^*, C^*, C_s^*) that solves problem (1), under constraints (2), given that every other individual in the economy chooses s^*, so that, in particular, $\bar{s} = s^*$.

[10]The assumption that B and C_s are perfect substitutes is made just for the sake of simplicity.

[11]We also assume that everybody has the same initial endowment of private capital.

Proposition 1 *Let* $\tilde{s} = 0$, $\tilde{C} = \frac{1}{1+a} K^{1+\vartheta-\epsilon}$, $\tilde{C}_s = \frac{a}{1+a} K^{1+\vartheta-\epsilon}$. *The triple* $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ *is always a SNE*[12].

In this equilibrium no time is devoted to social interaction, since each individual believes that every other one will spend her entire amount of time working, thus rendering social participation not worthwhile.

To be able to investigate analytically the existence of a SNE in which $s > 0$, we make the following simplifying assumption.

Assumption 1 $\alpha + \beta = \epsilon + \sigma = \varphi < 1$: *this means that, at a SNE, the elasticity of the relational good to social participation is the same as the elasticity of private production to labor; we call* φ *the common value and assume that it is smaller than one*[13].

In order to state the following proposition, let

$$\hat{s} = \frac{1}{1 + \left(\frac{b\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma} \right)^{\frac{1}{1-\varphi}}}, \tag{3}$$

$$\hat{C} = \frac{1}{b(1+a)} \hat{s}^\varphi K_s^\gamma + \frac{1}{(1+a)} (1-\hat{s})^\varphi K^{1+\vartheta-\epsilon}, \tag{4}$$

$$\hat{C}_s = \frac{a}{(1+a)} (1-\hat{s})^\varphi K^{1+\vartheta-\epsilon} - \frac{1}{b(1+a)} \hat{s}^\varphi K_s^\gamma. \tag{5}$$

Proposition 2 *Under Assumption 1, there exists a unique SNE with strictly positive social participation, namely, the triple* $(\hat{s}, \hat{C}, \hat{C}_s)$.

Notice that, among other things, $\hat{s}$ is an increasing function of K_s and α and a decreasing function of K. We will come back to the interpretation of these findings in the context of the dynamic specification of the model.

Proposition 3 *For any parameter constellation there is an increasing function* g *such that the SNE* $(\hat{s}, \hat{C}, \hat{C}_s)$ *Pareto-dominates the SNE* $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ *if and only if* $K_s > g(K)$, *the reverse being true when* $K_s < g(K)$.

[12] All the proofs are in the Appendix.

[13] The equality plays no other role than enabling us to derive an analytic solution, whereas the assumption that $\varphi < 1$ rules out a possible indeterminacy of s at a SNE.

Proposition 3 is rather intuitive, since it tells us that it is comparatively efficient to specialize in private production for those economies that, having a relatively high stock of private capital, have a comparative advantage to do so, whereas it becomes more efficient to devote a certain fraction of time to social participation in those economies where the social environment is relatively rich. Since, though, both equilibria are present, it is possible that, due to coordination failure, an economy gets stuck in the Pareto-inferior equilibrium. The limit of Proposition 3 is that it does not tell us anything about the sources of the relative abundance of private versus social capital. To investigate this aspect, we have to turn to the dynamic specification of our model.

Before doing this, though, a further comment may be done about the externalities that drive the story of this static model. Since both average social participation and average labor time, which are here the complement to 1 of one another, are supposed to exert positive external effects (respectively, on the production of the relational good and of the private goods), it is not *a priori* clear whether, overall, social participation displays positive or negative spillovers[14]. In general, in this game there tend to be positive spillovers from social participation when social capital is high relative to private capital, whereas such spillovers are overall negative when the reverse is true[15].

Remark 1 *Under Assumption 1, since, generically, in the SNE* $(\hat{s}, \hat{C}, \hat{C}_s)$ *splillovers are present, such equilibrium is inefficient even when it Pareto-dominates the SNE* $(\tilde{s}, \tilde{C}, \tilde{C}_s)$[16].

Indeed, Remark 1 tells us that the common result that, in presence of non-internalized externalities, even the best SNE is generally inefficient, applies also to our case.

[14]According to Cooper and John's (1988) terminology, social participation has positive (negative) spillovers if an increase in average social participation raises (decreases) individual utility, i.e., if $\frac{\partial u(C,B,Y-C)}{\partial \bar{s}}$ is positive (negative).

[15]Formally, under the reasonable assumption that $\beta, \sigma < 1$, which is even weaker than Assumption 1, $\frac{\partial u(C,B,Y-C)}{\partial \bar{s}} > 0 \Leftrightarrow \beta s^{\alpha} \bar{s}^{\beta-1} K_s^{\gamma} > b\sigma(1-s)^{\epsilon}(1-\bar{s})^{\sigma-1} K^{1+\vartheta-\epsilon}$, i.e., when K_s is high relative to K, s is high and $\bar{s}$ is low.

[16]Precisely, in the SNE $(\hat{s}, \hat{C}, \hat{C}_s)$ there are positive spillovers when $\alpha < \frac{\beta\epsilon}{\sigma}$ and negative ones when the reverse is true. There are no spillovers only in the non-generic case in which $\alpha = \frac{\beta\epsilon}{\sigma}$. Remark 1 then follows from Proposition 2 of Cooper and John (1988).

3.2 Dynamic specification

In the dynamic specification of the model preferences and technology are the same as above, with the only difference that now private and social capital are endogenously determined. The dynamics of the representative individual's private capital is given by $\dot{K} = Y - C - C_s - \eta K$, where $\eta \geq 0$.

Social capital (which is still considered as exogenous by the representative individual at any given point in time) is not accumulated through a process of investment; rather, its stock increases when a high average social participation brings about a high average enjoyment of the relational good (denoted $\bar{B} = \int_0^1 B(i)\mathrm{d}i$). Since relations deteriorate over time if individuals do not actively take care of them, we also assume that K_s depreciates at a rate $\delta > 0$. We can thus summarize the dynamics of social capital as $\dot{K}_s = f(\bar{B}) - \delta K_s$, where f is a strictly increasing function. The idea that non-material forms of capital may be accumulated though a 'consumption' activity rather than through investment, although unconventional in economics, is neither new (it goes back to Aristotle's analysis of ethical virtues, whose influence is to be found in Nussbaum's (1986) discussion of relational goods) nor surprising (think, e.g., of knowledge, which is accumulated though the use of knowledge). Indeed, the engine of social capital accumulation is average social participation $\bar{s}$, but we specify 'gross investment' in social capital in terms of $\bar{B}$ in consideration of the fact that a given level of $\bar{s}$ is more effective at increasing K_s in an environment in which it also generates a greater amount of the relational good. This idea is particularly compelling if we think of trust and social norms as forms of social capital, which are evidently accumulated in accordance with the perceived results of social participation and not just as a consequence of social participation *per se*, but the same idea may also be extended to other forms of social capital, like association networks, whose ability to prosper and expand may be seen as a function of the amount of relational goods they are able to provide to the people involved.

For the sake of simplicity, we make the following assumptions.

Assumption 2 $\eta = 0$*: we ignore private capital depreciation.*

Assumption 3 $f(x) \equiv x$*: this means that* $\dot{K}_s = \bar{B} - \delta K_s$.

Assumption 4 $\epsilon > \vartheta$ *and* $\gamma < 1$*: this means that we do not allow either* K *or* K_s *to grow steadily at a strictly positive rate.*

Assumption 2 is an innocent one. Assumption 3 is made just for the sake of analytical simplicity[17]. Assumption 4 means that in our model there is no engine for endogenous growth.

Individual problem

Letting $r > 0$ be the intertemporal discount rate, the representative individual solves the following problem:

$$\max_{s,C,C_s} \int_0^\infty u(C, B, C_s)\mathrm{e}^{-rt}\mathrm{d}t = \int_0^\infty [\ln C + a\ln(s^\alpha \bar{s}^\beta K_s^\gamma + bC_s)]\mathrm{e}^{-rt}\mathrm{d}t \quad \text{s.t.} \tag{6}$$

$$\begin{aligned} \dot{K}_s &= \bar{s}^{\alpha+\beta}K_s^\gamma - \delta K_s, & (7)\\ \dot{K} &= (1-s)^\epsilon K^{1-\epsilon}A - C - C_s, \qquad A \equiv (1-\bar{s})^\sigma \bar{K}^\vartheta. & (8) \end{aligned}$$

The current value Hamiltonian function for this problem is

$$\begin{aligned} H = {} & \ln C + a\ln(s^\alpha \bar{s}^\beta K_s^\gamma + bC_s) + \lambda[(1-s)^\epsilon K^{1-\epsilon}A - C - C_s] + \\ & + \mu[\bar{s}^{\alpha+\beta}K_s^\gamma - \delta K_s]. \end{aligned} \tag{9}$$

For the maximum principle we have

$$\begin{aligned} \dot{K} &= \frac{\partial H}{\partial \lambda} = (1-s)^\epsilon K^{1-\epsilon}A - C - C_s, & (10)\\ \dot{\lambda} &= r\lambda - \frac{\partial H}{\partial K} = \lambda[r - (1-\epsilon)(1-s)^\epsilon K^{-\epsilon}A], & (11)\\ \dot{K}_s &= \frac{\partial H}{\partial \mu} = \bar{s}^{\alpha+\beta}K_s^\gamma - \delta K_s. & (12) \end{aligned}$$

We omit the dynamics of μ, the 'shadow price' of social capital, since equations (10) to (12) are independent of it, due to the fact that K_s is entirely treated as an externality. The first order conditions are

[17]In principle, there is no reason for the 'gross investment' in social capital to be exactly equal to the average benefit from social participation, even if it is an increasing function of the latter; though, the identical specification is by far the easiest one and has the advantage of a straightforward interpretation: it lets us think of social capital just in terms of accumulated relational goods.

$$\frac{\partial H}{\partial C} = \frac{1}{C} - \lambda = 0, \qquad C > 0, \tag{13}$$

$$\frac{\partial H}{\partial C_s} = \frac{ab}{s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + bC_s} - \lambda \leq 0, \qquad C_s \frac{\partial H}{\partial C_s} = 0, \qquad C_s \geq 0, \tag{14}$$

$$\frac{\partial H}{\partial s} = \frac{a\alpha s^{\alpha-1}\bar{s}^{\beta}K_s^{\gamma}}{s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + bC_s} - \epsilon\lambda(1-s)^{\epsilon-1}K^{1-\epsilon}A \leq 0, \tag{15}$$

$$s\frac{\partial H}{\partial s} = 0, \qquad s \in [0,1].$$

Notice that s and C_s cannot be chosen both equal to zero. Thus, either condition (14) or condition (15) must hold with equality.

Symmetric Nash equilibrium

A SNE of this economy is now a triple (s^*, C^*, C_s^*) that solves problem (6), under constraints (7)-(8), given that every other individual in the economy chooses (s^*, C^*, C_s^*), so that, even if for the representative individual *ex ante* $\bar{s}$ and $\bar{K}$ are considered as exogenous, *ex post* they turn out to be equal, respectively, to s^* and to K (the representative individual's own capital stock).

In order to maintain in the dynamic version of the model the analytical tractability of the static version, we modify Assumption 1 into the following one.

Assumption 5 $\alpha + \beta = \epsilon + \sigma = \varphi = 1$*: this means that in equilibrium* B *is a linear function of* s *and* Y *is a linear function of* $1-s$*.*

Proposition 4 *At a SNE, the curve*

$$K_s = \left(\frac{\epsilon b}{\alpha}K^{1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}, \tag{16}$$

separates in the (K, K_s) *plane the region in which* $s > 0$ *and* $C_s = 0$ *from the one in which* $s = 0$ *and* $C_s > 0$ *(see figure 1).*

Precisely, in the two regions s *and* C_s *are chosen as follows:*

$$\text{Case (a)}: K_s < \left(\frac{\epsilon b}{\alpha}K^{1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}: \qquad \begin{cases} s = 0 \\ C_s = \frac{a}{\lambda} \end{cases}, \tag{17}$$

$$\text{Case (b)} : K_s > \left(\frac{\epsilon b}{\alpha} K^{1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}} : \qquad \begin{cases} s = \min\left\{1, \frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}}\right\} \\ C_s = 0 \end{cases} . \quad (18)$$

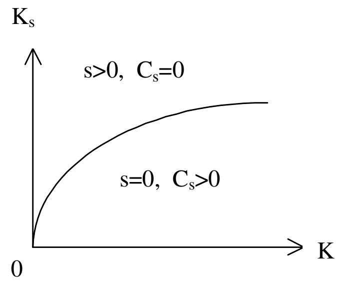

Figure 1

Case (a) identifies a situation in which social capital is scarce relative to private capital, so that, rather than spending time in social participation, whose returns are low, in equilibrium it is better to choose a high labor supply, which has a high return, and to substitute a high consumption of private goods for the relational good.

On the contrary, case (b) captures a situation of relative scarcity of private capital, as compared to social capital. In equilibrium social interaction (besides basic, subsistence consumption) is the basic source of individual well-being. On one side labor productivity is too low to make it worthwhile to work more in order to substitute some private consumption for the relational good; on the other side, the social environment is rich of opportunities and makes returns to social participation high. The difference between case (a) and (b) may help to understand why we observe big differences in the patterns of time allocation among different activities across countries with comparable size and private capital stock: indeed, such difference may be due to the presence of different relative stocks of private and social capital.

Fixed points

Exploiting Proposition 4, we are now able to characterize the dynamic properties of our economy. In particular, we focus attention on the fixed points by stating the next proposition[18], where we let

[18]For expositional purposes we do not mention here the steady state values of λ, that are in any case uniquely determined.

$$
\begin{aligned}
K^* &= \left(\frac{1-\epsilon}{r}\right)^{\frac{1}{\epsilon-\vartheta}}, && (19)\\
K_s^* &= 0, && (20)\\
K^{**} &= \left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{1}{\epsilon-\vartheta}}, && (21)\\
K_s^{**} &= \left[\frac{a\alpha}{\delta(\epsilon+a\alpha)}\right]^{\frac{1}{1-\gamma}}. && (22)
\end{aligned}
$$

Proposition 5 *In the plane (K, K_s) the point (K^*, K_s^*) is always a fixed point of the economy. Such point is locally saddle-path stable.*

There exists at most one more fixed point, namely (K^{**}, K_s^{**}). The latter is a fixed point if and only if

$$
\frac{a\alpha}{\delta(\epsilon+a\alpha)} > \left(\frac{\epsilon b}{\alpha}\right)^{\frac{1-\gamma}{\gamma}} \left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{(1-\gamma)(1+\vartheta-\epsilon)}{\gamma(\epsilon-\vartheta)}}. \qquad (23)
$$

If this condition is met, (K^{**}, K_s^{**}) is locally saddle-path stable.

Remark 2 *It is immediate to check that $K^{**} < K^*$.*

Remark 2 emphasizes the fact that, when both fixed points are present, private capital is lower in the fixed point in which social capital is higher.

Remark 3 *Ceteris paribus, condition (23) holds if δ and b are low enough and r, α and a are high enough.*

Remark 3 tells us that the fixed point in which social capital is higher (indeed, positive at all) exists when

δ **is low:** social capital does not depreciate too fast (an intuitive condition);

r **is high:** individuals are not too patient, i.e., they prefer to enjoy the relational good got through social participation today rather than to work and save more in the prospect of a higher future private consumption;

α **is high:** they have indeed an incentive to spend time in social participation, i.e., the amount of relational good they enjoy is sensitive enough to their own time spent in social participation (in other words, the relational good is enough a private good and not too much a public good);

a **is high:** they attribute enough weight to the needs satisfied by either the relational good or its private substitute (again an intuitive condition);

b **is low:** the balance between the relational good and its private substitute as a means of satisfying individual preferences is not too much in favor of the latter[19].

It is interesting to speculate on the meaning of such parameters in terms of real world examples. One might argue, for instance, that a high degree of individual mobility gives rise to many 'weak' ties[20]. If we think of such ties in terms of social capital, then individual mobility will be positively correlated with δ, the social capital depreciation rate, for the simple reason that weak ties tend to go lost more quickly in absence of a positive effort to keep them alive[21]. From this point view, we might speculate that a steady state with high social capital is more likely to exist in Europe than in the US, exactly because individual mobility is lower in the former than in the latter country.

On the other side, one might argue that α is also positively related to individual mobility, so that, from this point of view, the previous conclusion would be reversed. The reason would be in this case that individual mobility renders social spheres more open and thus reduces the weight of the externality represented by average participation in determining an individual's relational good, making the latter more a private good, i.e., bringing it more

[19]To have a numerical feeling, let us parameterize the model in a simple way, so that $a = b = 1,\ \alpha = \epsilon = 0.5,\ \vartheta = 0.1,\ \gamma = 0.8$. In this case, if social capital depreciation rate δ is, say, 10%, then condition (23) is met even with a discount rate r of 1%. If we lower γ to 0.5, then, with the same $\delta = 10\%$, condition (23) fails to be met up to a discount rate r of 8%, whereas it is met for $r \geq 9\%$.

[20]Granovetter (1973) makes the point that weak ties may be economically very important, since they are often the vehicle of new information, not yet available to an individual or to her close social neighborhood.

[21]Schiff (1999 and 2002) analyzes the sharp difference between the two traditional forms of factor mobility, namely migration and trade, that become apparent once we consider their different impact on social capital.

fully under individual control[22]

Well-being analysis

Let us now consider, when both fixed points exist, i.e. under condition (23), which one is Pareto-superior. Let u^* and u^{**} be the representative individual's utility in the fixed points (K^*, K_s^*) and (K^{**}, K_s^{**}), respectively.

Proposition 6 *Suppose condition (23) is satisfied and $\delta < \frac{a\alpha}{\epsilon + a\alpha}$. Then the fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*), i.e., $u^{**} > u^*$, if, ceteris paribus, δ is low enough and r and γ are high enough. The reverse is true if δ is high enough and r and γ are low enough.*

Proposition 6 tells us that the same two forces, namely impatience and low social capital depreciation rate, that let (K^{**}, K_s^{**}) be a fixed point, also make it Pareto-superior. Moreover, as it is natural to expect, a high elasticity γ of the relational good to social capital contributes to the comparative efficiency of the fixed point with positive social capital[23].

When the fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*) and the economy gets stuck in the latter, this one may be described as a social poverty trap. The convergence to such a trap may have two basic causes. On one side, it may be due to the fact that the initial endowment of the two forms of capital is close to the inefficient fixed point. This is the case of advanced economies with very low social capital: for instance, one might think of Russia in the last decade[24]. On the other side, there is the general problem posed by externalities: individuals are not able to recognize that, if everybody were to participate more, everybody would also be better off in the long run. On

[22] In general, relational goods are an intermediate case between public and private goods. In our model, B is a pure public good if $\alpha = 0$, in which case any private incentive to social participation is absent. On the other side, B is a pure private good if $\beta = 0$, that is, under Assumption 5, if $\alpha = 1$.

[23] To get a feeling, consider again the simple parametrization $a = b = 1$, $\alpha = \epsilon = 0.5$, $\vartheta = 0.1$, $\gamma = 0.8$. In this case, $u^{**} - u^* = \frac{3}{2} \ln r - 4 \ln \delta - 4 \ln 2$, which, for instance, is positive for $\delta = 10\%$ and $r = 3\%$, as well as for any lower social capital depreciation rate and higher discount rate. If $\delta = 5\%$, then $u^{**} > u^*$ even with a discount rate of 1%. If we lower γ to 0.5, then $u^* > u^{**}$ for any reasonable value of δ and r.

[24] Rose (1998) considers in detail how the centralization of the Soviet Union may have eroded wide forms of social capital, inducing individuals to rely on a narrow circle of family ties, which represents at the same time a response to the state of affairs and a social trap, that inhibits the mechanism of social development.

the contrary, not taking into account the immediate and cumulative external effects of social participation, each individual reacts privately, trying to work and save more, in order to compensate for a poor social sphere through a higher future private consumption. Such private, defensive choice may thus lead to an inefficient overaccumulation of private capital, at the expenses of social capital and of individual and social well-being[25]. In this latter case, we may say that private growth and social development conflict with each other, and that it would be efficient to increase social participation and decrease labor supply, sacrificing some accumulation of private capital, but gaining in terms of an improved social environment. Of course, this remains true only if the fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*) and the economy gets stuck in the latter; since the former fixed point is as well locally stable, the economy will converge to it if its initial endowment of social capital is high enough[26]. If we assume that convergence to the fixed point (K^{**}, K_s^{**}) takes place from below along both dimensions, then, in this latter case, social development and economic growth move together[27].

On the other hand, we have seen that (K^*, K_s^*) may Pareto-dominate the fixed point (K^{**}, K_s^{**}) if the social technology is 'bad' and if individuals are very patient. Moreover, we have shown that under the same conditions, (K^{**}, K_s^{**}) may even fail to be a fixed point. In the first case, (K^{**}, K_s^{**}) should be regarded as a situation in which individuals devote too much time to socially enjoyed leisure, while working and saving too little to reach a more efficient steady state. In the second case, since there is no alternative, there is no comparative discussion.

4 Conclusion

The present contribution sheds light on the interplay between the private and the social component of well-being in a scenario in which both private and social capital are present, relational goods play a role and their substitutability with some private goods is taken into account.

[25]This enables, for instance, to make sense of the strange phenomenon, observed in many advanced societies, of brilliant professionals whose social life is quite poor and whose satisfaction, despite material wealth, remains low.

[26]Precisely, if the initial endowment (K^0, K_s^0) is close enough to (K^{**}, K_s^{**}). Notice that even this case, although more favorable, does not solve the problem of the externalities.

[27]Remember though that, because of Assumption 4, neither private growth nor social development may be endogenously sustained forever.

We first present a static model, which displays two equilibria: a private-oriented one, in which labor time and private production are high and relational goods are substituted for by private goods, and a social-oriented one, in which labor supply is low and social participation high, so that, besides private consumption, relational goods become a key determinant of well-being. Which of the two equilibria Pareto-dominates the other crucially depends on the relative endowment of social and private capital: if social capital is low relative to private capital, the private-oriented equilibrium is Pareto-superior; if the reverse is true, the social-oriented equilibrium is more efficient. Since equilibrium selection is a matter of coordination, it is possible for the economy to get stuck in the Pareto-inferior equilibrium.

The static model does not explain the determinants of the relative endowment of social and private capital. Therefore, we next introduce a dynamic version of the model, in which private capital is accumulated in a standard way through savings, and social participation, generating relational goods, is the engine of social capital accumulation. If social capital does not depreciate too fast, individuals are not too patient and relational goods are privately appropriable to some degree, the dynamics admits two fixed points: one in which there is only private capital and one in which both forms of capital are present (in which case private capital is lower than at the first equilibrium). The same factors that cause the latter point to be a steady state also make it Pareto-dominant against the former one. When this is the case, since both equilibria are saddle-path stable, it is possible that the economy converges to the Pareto-inferior state, where only private capital is observed. Along the convergence path, we may witness a conflict between economic growth and social development, since growth drives the economy to a social poverty trap. If, in turn, the economy converges to the Pareto-superior fixed point, we may have economic growth and social development moving in the same direction. The distinction between these two cases depends once again upon the initial relative endowment of private and social capital, but also upon the social technology and the degree of individual impatience.

Our analytical results are derived under some assumptions, that deserve some discussion here. First of all, we assume that the relational good has a perfect private substitute. Relaxing this hypothesis would not add much in terms of economic content of the model, but would complicate the mathematics. Second, we assume that neither social capital matters for the production of private goods, nor private capital for relational goods. Both these cross relations might indeed be somewhat relevant, but we believe that they are of secondary importance when compared to the causal links included in the model. Nevertheless, this might be a possible future extension. Third, while we consider positive learning-by-doing externalities in private production, we do not allow them to be so strong as to generate endogenous growth. This is another possible extension of the model. Fourth, we assume that private consumption does not require time, so that all leisure time is devoted to social participation. Although unrealistic, we make this modeling choice because, generally speaking, social participation is a more time-intensive activity than private consumption. Clearly, the consumption of some private substitutes of the relational good (think e.g. of watching television) is also time-intensive to a degree, so that an interesting extension would be to take this into account, along the lines set by Corneo (2001). Fifth, the assumption that the 'gross investment' in social capital is exactly equal to the average production of the relational good could easily be generalized (for instance by assuming that just a fraction of the relational good produced accumulates as social capital), without changing any of the results of the model. The assumption has simply been dictated by notational economizing. Finally, Assumption 1 and Assumption 5 are crucial to obtain simple analytical solutions. Relaxing the former to some extent would not alter the results of the static model, although it would preclude the possibility to express them in closed form[28]. As far as the latter is concerned, a comparison with Antoci, Sacco and Vanin (2002) lets us conjecture that its main effect is to rule out a repulsive fixed point that separates the two stable ones. Since our mathematical findings are supported by a clear economic intuition, we are rather confident of their general validity.

Appendix

Proof of Proposition 1

Using the production function and the budget constraint to substitute for C_s, and calling $v(s,C) = u(C, B, Y - C)$, we can re-write problem (1)-(2) as

$$\max_{s,C} \; v(s,C) = \tag{24}$$
$$= \ln C + a \ln\{s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + b[(1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon} - C]\} \quad \text{s.t.}$$

[28]Precisely, this would be the case if one just assumed $\alpha + \beta < 1$ and $\epsilon + \sigma < 1$ without requiring them to be equal.

$$C \geq 0, \qquad (1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon} - C \geq 0, \qquad s \in [0,1]. \tag{25}$$

The FOC's of this problem are

$$\begin{aligned} \frac{\partial v}{\partial C} &= 0, \qquad 0 \leq C \leq Y, & (26) \\ \frac{\partial v}{\partial s} &\leq 0, \qquad s\frac{\partial v}{\partial s} = 0, \qquad 0 \leq s \leq 1. & (27) \end{aligned}$$

Equation (26) yields immediately

$$C = \frac{1}{b(1+a)}[s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + b(1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon}], \tag{28}$$

which, plugged in inequality (27), yields, after rearranging,

$$\frac{(1-s)^{1-\epsilon}}{s^{1-\alpha}} \leq \frac{b\epsilon(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon}}{\alpha\bar{s}^{\beta}K_s^{\gamma}}, \text{ with equality if s} > 0, \quad 0 \leq s \leq 1. \tag{29}$$

When $\bar{s} = 0$, the relational good is zero whatever the individual choice of s. Hence, the optimal individual response to $\bar{s} = 0$ is to choose $s = 0$. The rest of the proposition follows from equation (28) and from the production function.

Proof of Proposition 2

The value of $\hat{s}$ follows from equation (29) after plugging the SNE condition $\bar{s} = s$ and Assumption 1. The values of $\hat{C}$ and of $\hat{C}_s$ then follow from equation (28) and from the budget constraint.

Proof of Proposition 3

Let $\tilde{u}$ and $\hat{u}$ be the representative individual's utility in the two SNE $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ and $(\hat{s}, \hat{C}, \hat{C}_s)$, respectively. Then,

$$\begin{aligned} \tilde{u} &= (1+\vartheta-\epsilon)\ln K - \ln(1+a) + a\ln ab - a\ln(1+a) + \\ &\quad + a(1+\vartheta-\epsilon)\ln K = \\ &= (1+a)(1+\vartheta-\epsilon)\ln K - (1+a)\ln(1+a) + a\ln a + a\ln b, \\ \hat{u} &= \ln[\hat{s}^{\varphi}K_s^{\gamma} + b(1-\hat{s})^{\varphi}K^{1+\vartheta-\epsilon}] - \ln b(1+a) + a\ln(\hat{s}^{\varphi}K_s^{\gamma} + b\hat{C}_s) = \end{aligned}$$

$$
\begin{aligned}
&= \ln[\hat{s}^{\varphi} K_s^{\gamma} + b(1-\hat{s})^{\varphi} K^{1+\vartheta-\epsilon}] - \ln b(1+a) + a \ln \frac{a}{1+a} + \\
&\quad + a \ln[\hat{s}^{\varphi} K_s^{\gamma} + b(1-\hat{s})^{\varphi} K^{1+\vartheta-\epsilon}] = \\
&= (1+a)\ln[\hat{s}^{\varphi} K_s^{\gamma} + b(1-\hat{s})^{\varphi} K^{1+\vartheta-\epsilon}] - (1+a)\ln(1+a) + a \ln \frac{a}{b}, \\
\hat{u} - \tilde{u} &= (1+a)\{\ln[\hat{s}^{\varphi} K_s^{\gamma} + b(1-\hat{s})^{\varphi} K^{1+\vartheta-\epsilon}] - (1+\vartheta-\epsilon)\ln K\} - 2a \ln b = \\
&= (1+a)\ln\left[\hat{s}^{\varphi} \frac{K_s^{\gamma}}{K^{1+\vartheta-\epsilon}} + (1-\hat{s})^{\varphi} b\right] - 2a \ln b.
\end{aligned}
$$

Remembering that $\hat{s}$ is increasing in K_s and decreasing in K, it is easy to see that the last expression becomes definitely positive as soon as K_s is large enough relative to K, and thus implicitly defines the increasing function g.

Proof of Proposition 4

Plugging Assumption 5 and the equilibrium conditions $\bar{s} = s$ and $\bar{K} = K$ into equations (13) to (15), we get

$$
\begin{aligned}
C &= \frac{1}{\lambda}, && (30) \\
\frac{\partial H}{\partial C_s} &= \frac{ab}{sK_s^{\gamma} + bC_s} - \lambda \leq 0, \qquad C_s \frac{\partial H}{\partial C_s} = 0, \qquad C_s \geq 0, && (31) \\
\frac{\partial H}{\partial s} &= \frac{a\alpha K_s^{\gamma}}{sK_s^{\gamma} + bC_s} - \epsilon \lambda K^{1+\vartheta-\epsilon} \leq 0, \qquad s \frac{\partial H}{\partial s} = 0, \qquad s \in [0,1]. && (32)
\end{aligned}
$$

The inequality $\frac{\partial H}{\partial C_s} \leq 0$ may be re-written in the form $\frac{a}{sK_s^{\gamma}+bC_s} - \frac{\lambda}{b} \leq 0$.

For $K_s > 0$ the inequality $\frac{\partial H}{\partial s} \leq 0$ may be re-written in the form $\frac{a}{sK_s^{\gamma}+bC_s} - \frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^{\gamma}} \lambda \leq 0$.

Hence, if $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^{\gamma}} > \frac{1}{b}$, it holds $\frac{\partial H}{\partial C_s} = 0$ and $\frac{\partial H}{\partial s} < 0$, so that the representative individual's equilibrium choice is such that $C_s > 0$ and $s = 0$. If, on the contrary, $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^{\gamma}} < \frac{1}{b}$, then we have $C_s = 0$ and $s > 0$. If, finally, $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^{\gamma}} = \frac{1}{b}$, we remain with one equation for two unknowns and the choice of C_s and s is indeterminate. The remainder of Proposition 4 follows from a straightforward substitution in equations (31) and (32).

Proof of Proposition 5

For case (a), i.e. under condition (17), the equilibrium dynamics of our economy is described by

$$\dot{K} = K^{1+\vartheta-\epsilon} - \frac{1+a}{\lambda}, \tag{33}$$
$$\dot{\lambda} = \lambda[r - (1-\epsilon)K^{\vartheta-\epsilon}], \tag{34}$$
$$\dot{K}_s = -\delta K_s. \tag{35}$$

For case (b), i.e. under condition (18), if $\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} \leq 1$,[29] the equilibrium dynamics is

$$\dot{K} = K^{1+\vartheta-\epsilon} - \left(1 + \frac{a\alpha}{\epsilon}\right)\frac{1}{\lambda}, \tag{36}$$
$$\dot{\lambda} = \lambda\left[r - (1-\epsilon)\left(K^{\vartheta-\epsilon} - \frac{a\alpha}{\epsilon\lambda K}\right)\right], \tag{37}$$
$$\dot{K}_s = K_s^{\gamma}\left(\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} - \delta K_s^{1-\gamma}\right). \tag{38}$$

The analytical determination of (K^*, K_s^*) and (K^{**}, K_s^{**}) follows from a straightforward substitution in the systems (33) to (35) and (36) to (38), setting the LHS of each equation equal to zero. (K^*, K_s^*) satisfies the condition of case (a): $K_s^* < \left(\frac{\epsilon b}{\alpha}K^{*1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}$ and is thus indeed a fixed point. (K^{**}, K_s^{**}) is a fixed point if and only if it satisfies the condition of case (b): $K_s^{**} > \left(\frac{\epsilon b}{\alpha}K^{**1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}$. Equation (23) is just a re-writing of this condition.

The stability properties are determined as follows. The Jacobian matrix of the system (33) to (35), evaluated at (K^*, K_s^*), is

$$A = \begin{bmatrix} (1+\vartheta-\epsilon)K^{\vartheta-\epsilon} & \frac{1+a}{\lambda^2} & 0 \\ (1-\epsilon)(\epsilon-\vartheta)\lambda K^{\vartheta-\epsilon-1} & 0 & 0 \\ 0 & 0 & -\delta \end{bmatrix}.$$

One eigenvalue is therefore $-\delta < 0$ and the other two have opposite sign, since negative is the determinant of the sub-matrix obtained from A by deleting the third row and the third column. Therefore, if (K, K_s) is initially

[29]Since we are interested in the fixed points of this dynamics, we do not consider, under case (b), the possibility that $\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} > 1$, since in this case $\dot{K} = -\frac{1}{\lambda}$ and there is no fixed point. Notice, moreover, that this possibility is not a relevant one, since it means that individuals do not work at all and derive their private consumption only from 'eating' their existing stock of private capital.

close enough to (K^*, K_s^*), there exists a unique initial value of λ that puts the representative agent on the stable arm (which, in turn, has dimension 2).

Observe now that the Jacobian matrix of the system (36) to (38), evaluated at (K^{**}, K_s^{**}), is such that $\frac{\partial \dot{K}}{\partial K_s} = \frac{\partial \dot{\lambda}}{\partial K_s} = 0$ and $\frac{\partial \dot{K}_s}{\partial K_s} = -\delta(1-\gamma) < 0$. Therefore, this latter value is one of the eigenvalues of the Jacobian and the other two ones have opposite sign, since negative is the determinant of the sub-matrix

$$B = \begin{bmatrix} \frac{\partial \dot{K}}{\partial K} & \frac{\partial \dot{K}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial K} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{bmatrix}.$$

To see this, one has to go through the following passages.

$$\begin{aligned}
\frac{\partial \dot{K}}{\partial K} &= (1+\vartheta-\epsilon)K^{\vartheta-\epsilon} > 0, \\
\frac{\partial \dot{K}}{\partial \lambda} &= \left(1+\frac{a\alpha}{\epsilon}\right)\frac{1}{\lambda^2} > 0, \\
\frac{\partial \dot{\lambda}}{\partial K} &= -(1-\epsilon)\lambda\left[-(\epsilon-\vartheta)K^{\vartheta-\epsilon-1} + \frac{a\alpha}{\epsilon\lambda K^2}\right], \\
\frac{\partial \dot{\lambda}}{\partial \lambda} &= -\frac{a\alpha(1-\epsilon)}{\epsilon K \lambda} < 0.
\end{aligned}$$

Remembering that in the fixed point $\lambda K^{**1+\vartheta-\epsilon} = 1 + \frac{a\alpha}{\epsilon}$, one gets

$$\begin{aligned}
\operatorname{Det} B &= -(1+\vartheta-\epsilon)K^{\vartheta-\epsilon}\frac{a\alpha(1-\epsilon)}{\epsilon K \lambda} + \\
&\quad + \left(1+\frac{a\alpha}{\epsilon}\right)(1-\epsilon)\frac{1}{\lambda}\left[-(\epsilon-\vartheta)K^{\vartheta-\epsilon-1} + \frac{a\alpha}{\epsilon\lambda K^2}\right] = \\
&= -(1+\vartheta-\epsilon)K^{\vartheta-\epsilon}\frac{a\alpha(1-\epsilon)}{\epsilon K \lambda} + \\
&\quad + \left(1+\frac{a\alpha}{\epsilon}\right)(1-\epsilon)\frac{1}{\lambda}\frac{a\alpha}{\epsilon\lambda K^2} + \\
&\quad - \left(1+\frac{a\alpha}{\epsilon}\right)(1-\epsilon)\frac{1}{\lambda}(\epsilon-\vartheta)K^{\vartheta-\epsilon-1} = \\
&= \frac{a\alpha(1-\epsilon)}{\epsilon\lambda^2 K^2}\left[1+\frac{a\alpha}{\epsilon} - (1+\vartheta-\epsilon)\lambda K^{1+\vartheta-\epsilon}\right] + \\
&\quad - \left(1+\frac{a\alpha}{\epsilon}\right)(1-\epsilon)\frac{1}{\lambda}(\epsilon-\vartheta)K^{\vartheta-\epsilon-1} =
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a\alpha(1-\epsilon)}{\epsilon\lambda^2 K^2}\left[1+\frac{a\alpha}{\epsilon}-(1+\vartheta-\epsilon)\left(1+\frac{a\alpha}{\epsilon}\right)\right]+\\
&\quad -\left(1+\frac{a\alpha}{\epsilon}\right)(1-\epsilon)\frac{1}{\lambda}(\epsilon-\vartheta)K^{\vartheta-\epsilon-1}=\\
&= \frac{a\alpha(1-\epsilon)}{\epsilon\lambda^2 K^2}\left(1+\frac{a\alpha}{\epsilon}\right)(\epsilon-\vartheta)+\\
&\quad -\left(1+\frac{a\alpha}{\epsilon}\right)(1-\epsilon)\frac{1}{\lambda}(\epsilon-\vartheta)K^{\vartheta-\epsilon-1}=\\
&= \frac{(1-\epsilon)(\epsilon-\vartheta)}{\lambda^2 K^2}\left(1+\frac{a\alpha}{\epsilon}\right)\left[\frac{a\alpha}{\epsilon}-\lambda K^{1+\vartheta-\epsilon}\right]=\\
&= -\frac{(1-\epsilon)(\epsilon-\vartheta)}{\lambda^2 K^2}\left(1+\frac{a\alpha}{\epsilon}\right)<0.
\end{aligned}
$$

Proof of Proposition 6

In order to calculate u^*, observe first that, since we are in case (a), $s=0$ and $u^*=\ln C+a\ln bC_s$. From equations (30) and (31), it follows immediately that $C=\frac{1}{\lambda}$ and $C_s=\frac{a}{\lambda}$, so that $C_s=aC$. Equations (33) and (19) then imply $C=\frac{1}{1+a}K^{*1+\vartheta-\epsilon}=\frac{1}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}$ and $C_s=\frac{a}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}$. Therefore,

$$
\begin{aligned}
u^* &= \ln C+a\ln bC_s=\ln\frac{1}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}+a\ln\frac{ab}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}=\\
&= \ln\frac{1}{1+a}+\ln\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}+a\ln\frac{ab}{1+a}+a\ln\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}=\\
&= \ln\frac{1}{1+a}+a\ln\frac{ab}{1+a}+(1+a)\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\ln\frac{1-\epsilon}{r} \qquad (39)
\end{aligned}
$$

Let us now calculate u^{**} in an analogous way. Since we are in case (b), $C_s=0$ and $u^{**}=\ln C+a\ln sK_s^{\gamma}$. Remembering that in the fixed point $\lambda K^{1+\vartheta-\epsilon}=1+\frac{a\alpha}{\epsilon}$, equations (21) and (30) yield $C=\frac{1}{\lambda}=\frac{K^{**1+\vartheta-\epsilon}}{1+\frac{a\alpha}{\epsilon}}=\frac{\left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}}{1+\frac{a\alpha}{\epsilon}}$ and equation (32) yields $s=\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}}=\frac{a\alpha}{\epsilon+a\alpha}$. Since K_s is given by equation (22), we can calculate u^{**} as

$$
\begin{aligned}
u^{**} &= \ln C+a\ln sK_s^{\gamma}=\ln\frac{\epsilon}{\epsilon+a\alpha}+\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\ln\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}+a\ln\frac{a\alpha}{\epsilon+a\alpha}+\\
&\quad +a\frac{\gamma}{1-\gamma}\ln\frac{a\alpha}{\delta(\epsilon+a\alpha)}. \qquad (40)
\end{aligned}
$$

Proposition 6 follows from an analysis of the following expression[30]:

$$\begin{aligned} u^* - u^{**} &= \ln \frac{\epsilon + a\alpha}{\epsilon + a\epsilon} + \frac{1 + \vartheta - \epsilon}{\epsilon - \vartheta} \left[a \ln(1-\epsilon) - a \ln r + \ln \frac{\epsilon + a\alpha}{\epsilon} \right] + \\ &\quad + a \ln \frac{\epsilon + a\alpha}{\alpha + a\alpha} + a \ln b + a \frac{\gamma}{1-\gamma} \left[\ln \delta + \ln \frac{\epsilon + a\alpha}{a\alpha} \right]. \end{aligned} \tag{41}$$

References

[1] Akerlof G., Kranton R. (2000), Economics and Identity, Quarterly Journal of Economics, Vol. CXV, N. 3, 715-753

[2] Antoci A. (1996), Negative Externalities and Growth of the Activity Level, Report n.9308, M.U.R.S.T. Research Project on Non-Linear Dynamics and Application to Economic and Social Sciences, University of Florence

[3] Antoci A. (1997a), Self-Protection Activities from Negative Externalities and Economic Growth, Discussion Paper n.18/97, Dimadefas, University of Florence

[4] Antoci A. (1997b), Negative Externalities, Consumption Patterns and Economic Growth, Report n.9349, M.U.R.S.T. Research Project on Non-Linear Dynamics and Application to Economic and Social Sciences, University of Florence

[5] Antoci A. (2002), Negative Externalities as an Equilibrium Selection Device in an Economic Development Model, Mimeo, University of Sassari

[6] Antoci A., Bartolini S. (1999), Negative Externalities as the Engine of Growth in an Evolutionary Context, Nota di lavoro 83/99, Fondazione Eni Enrico Mattei, Milan

[7] Antoci A., Borghesi S. (2001), Working Too Much in a Polluted World: A North-South Evolutionary Model, Department Economics, University of Siena, Quaderni di Dipartimento n.334

[30] Under the parametrization $a = b = 1$, $\alpha = \epsilon = 0.5$, $\vartheta = 0.1$, $\gamma = 0.8$, expression (41) reduces to $4\ln 2 + 4\ln\delta - \frac{3}{2}\ln r$. If $\gamma = 0.5$, then it reduces to $\ln 2 + \ln\delta - \frac{3}{2}\ln r$.

[8] Antoci A., Borghesi S., Galeotti M. (2002), Economic Growth in an Economy in Which There Are Private Substitutes for Environmental Goods, Mimeo, University of Florence

[9] Antoci A., Sacco P.L., Vanin P. (2002), Participation, Growth and Social Poverty: Social Capital in a Homogeneous Society, Mimeo, University of Bologna

[10] Antoci A., Sacco P.L., Vanin P. (2001), Economic Growth and Social Poverty: The Evolution of Social Participation, Bonn Graduate School of Economics Discussion Paper No. 13/2001

[11] Bartolini S., Bonatti L. (1997), Negative Externalities as the Cause of Growth in a Neoclassical Model, Department of Economics, University of Trento, Discussion Paper n.9

[12] Bartolini S., Bonatti L. (1998), Growth as a Coordination Failure, Department of Economics, University of Trento, Discussion Papers n.5

[13] Bartolini S., Bonatti L. (1999a), Endogenous Growth and Negative Externalities, Department of Economics, University of Siena, Quaderni di Dipartimento n.270

[14] Bartolini S., Bonatti L. (1999b), Growth and Accumulation as Coordination Failures, Department of Economics, University of Siena, Quaderni di Dipartimento n.271

[15] Becker G. (1965), A Theory of the Allocation of Time, Economic Journal, Vol.75, N.299 (September), 493-517

[16] DiPasquale D., Glaeser E.L. (1999), Incentives and Social Capital: Are Homeowners Better Citizens?, Journal of Urban Economics 45, 354-384

[17] Coleman J. (1990), Social Capital, in J. Coleman, Foundations of Social Theory, Cambridge, Mass., and London, England, The Belknap Press of Harvard University Press

[18] Coleman J. (1988), Social Capital in the Creation of Human Capital, American Journal of Sociology, 94S, S95-S120

[19] Corneo G. (2001), Work and Television, Institute for the Study of Labor (IZA), Bonn, Discussion Paper n.376

[20] Corneo G., Jeanne O. (1999), Social Organization in an Endogenous Growth Model, International Economic Review, Vol. 40, No. 3, 711-725

[21] Cooper R., John A. (1988), Coordinating Coordination Failures in Keynesian Models, The Quarterly Journal of Economics, Vol. CIII, N.3, 441-463

[22] Glaeser E.L., Laibson D., Sacerdote B. (2000), The Economic Approach to Social Capital, NBER (Cambridge MA) Working Paper 7728

[23] Goodfriend M., McDermott J. (1995), Early Development, American Economic Review, 85(1), 116-133

[24] Granovetter M. (1973), The Strength of Weak Ties, American Journal of Sociology, 78(6), 1360-1380

[25] Hirsch F. (1976), Social Limits to Growth, Cambridge, Mass. Harvard University Press

[26] Nussbaum M. (1986), The Fragility of Goodnes: Luck and Ethics in Greek Tragedy and Philosophy (chapter 12, The Vulnerability of the Good Life: Relational Goods), Cambridge (etc.), Cambridge University Press

[27] Putnam R. (1993a), Making Democracy Work: Civic Traditions in Modern Italy, Princeton, Princeton University Press

[28] Rose R. (1998) Getting Things Done in an Anti-Modern Society: Social Capital Networks in Russia, Social Capital Initiative Working Paper N.6, The World Bank

[29] Schiff M. (1999), Labor Market Integratin in the Presence of Social Capital, Development Research Group, World Bank

[30] Schiff M. (2002), Love Thy Neighbor: Trade, Migration and Social Capital, European Journal of Political Economy, 18(1), 87-107

[31] Uhlaner C.J. (1989), Relational Goods and Participation: Incorporating Sociability into a Theory of Rational Action, Public Choice n.62, 253-285