Competition and cooperation in natural resources exploitation: An evolutionary game approach

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Abstract. We propose a dynamic model to describe the commercial exploitation, by a population of strategically interacting agents, of a common property renewable resource. The population of players is assumed to be divided into two groups: defectors, that just maximize their own profit, and cooperators, that decide their harvesting policy by maximizing the overall profit of their group. An evolutionary mechanism, based on the replicator dynamics, is introduced to describe the time changes in the proportions of defectors and cooperators within the population. This leads us to the qualitative study of a two dimensional nonlinear dynamical system that describes the time evolution of the resource stock and the population share between cooperators and defectors. The long run evolution of this dynamical system is analyzed by analytical and numerical methods, and the role of some economic and ecologic parameters is investigated.

Keywords. Bioeconomic Models, Oligopoly Games, Resource Exploitation, Replicator Dynamics.

1   Introduction

Since the pioneering work of Gordon (1954), many bioeconomic models for the description of the commercial exploitation of common property renewable resources, such as fisheries, stressed the problem known as "the tragedy of the commons" (Hardin, 1968, see also Clark, 1990). This problem can be basically identified with a prisoner's dilemma (see e.g. Mesterton-Gibbons, 1993) because the presence of firms playing their dominant strategy which maximizes their own profit (disregarding competitors' profits) leads to severe depletion of the resource, and consequently to low profits for all. On the other hand if firms cooperate to maximize the total profit, then a sustainable exploitation is more likely to be obtained, which implies higher profits for all in the long run.

Dynamic models based on Cournot oligopoly games have been recently proposed by Szidarovszky and Okuguchi (1998, 2000) to describe commercial fishing, where the strategic interaction among players is related not only to the selling price, determined by the total harvesting quantity through a given demand function, but also to a cost externality, since resource stock reductions, as consequence of players' harvesting, lead to higher unitary fishing costs (see also Bischi and Kopel, 2002). In Szidarovszky and Okuguchi (1998) every player is assumed to decide his/her harvesting activity solving a profit maximization problem, without any concern with the implications of this activity on the

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depletion of the natural resource. Instead, in Szidarovszky and Okuguchi (2000) it is assumed that the fishermen form a grand-coalition (i.e. a cooperative venture) and each player determines his/her harvesting activity such that the *joint profit of all players* is maximized. In both cases, the solution of the optimization problem leads to harvesting functions that depend on fish stock, whose dynamics is governed by a biological growth function with an extra mortality term due to the harvesting activity.

A discrete time version of the oligopoly model proposed in Szidarovszky and Okuguchi (1998) is given in Bischi and Kopel (2002), where dynamics with perfect foresight (i.e. fishermen are assumed to know the fish stock at each time when they solve their maximization problems) are compared with dynamics with limited knowledge of the fish stock and its estimate is obtained by adaptive expectations. In Bischi et al. (2002), discrete time versions of both models (the one with non cooperative oligopoly competition of Szidarovszky and Okuguchi, 1998, and the one with total cooperation, where all the players form a unique cooperative venture, so that they behave like a sole owner, like in Szidarovszky and Okuguchi, 2000) are considered with adaptive expectations.

In the model proposed in this paper, the players (e.g. fishermen) have access to a common property resource (e.g. a sea where a given fish stock is available) and sell the harvested resource in the same market, but, differently from the above cited literature, both competitors and cooperators are present. In fact a fraction s of fishermen behaves as cooperators, and form a cooperative venture where each one decides the harvesting quota by maximizing the profit of the coalition, whereas the complementary fraction $(1 - s)$ of fishermen behaves as competitors (or "defectors" with respect to cooperative behavior) each of which decides harvesting quotas by maximizing his/her own profit (disregarding competitors' profits). Following the terminology typical of prisoner's dilemma (see also Sethi and Somanathan, 1996) we call the latter group defectors, because they deviate from the socially optimal attitude of cooperating, and consequently they produce a negative externality on the rest of the community, in terms of potential severe depletion of the common property resource.

Furthermore we introduce the possibility to impose sanctions to punish defectors. As in Szidarovszky and Okuguchi (1998, 2000), Bischi and Kopel (2002), the harvesting of each group depends on strategic interactions related not only to the influence of total supply on the market prices, but also to the dependence of harvesting cost on available fish stock, whose evolution is governed by biological laws as well as harvesting activities. Moreover, following Sethi and Somanathan (1996) we propose an evolutionary mechanism that describes how the population share is updated over time. This evolutionary mechanism, based on replicator dynamics, is based on the principle, typical of evolutionary games, that the fraction of agents playing the strategy that earns higher payoffs, increases in the next period.

However, differently from the model proposed in Sethi and Somanathan (1996), where the harvesting behavior of cooperators and defectors is assumed to be described by general functions that satisfy some formal assumptions, we explicitly deduce the harvesting decisions as the Nash equilibrium of the profit maximizing game that cooperators and defectors are playing, founded on explicitly defined demand and cost functions. This allows us to study, by theoretical and numerical methods, the effects of some economic parameters, such as the market price (i.e. the parameters that characterize the demand function) or the cost parameters (related to the technology adopted), on the long-run evolution of the resource stock and the population shares between the two groups. In particular, the proposed model lets detect what economic parameters may determine not only the extinction of the resource, but also the extinction of a given behavior.

The paper is organized as follows. In section 2 we propose a static game where a population of

profit maximizing agents decide the quantities to harvest on the basis of two different behaviors: a fraction of players form a coalition, each of them trying to maximize the overall profit, whereas the complementary fraction behave as "selfish" profit maximizers. The reaction functions are obtained and the unique Nash equilibrium of the game is computed. In section 3 a growth equation, governing the dynamics of the resource stock with harvesting, is introduced, the harvesting being decided by the two groups by choosing the Nash equilibrium quantities according to the game analyzed in section 2. In this section the population share between the two kinds of agents is assumed to be a parameter, so the existence and stability of the steady states are studied as well as how these are influenced by population share. In section 4 we introduce an evolutionary mechanism that, at each time period, describes how the population share is updated, based on replicator dynamics, and we study the problems of existence and stability of the equilibria. In particular a distinction is introduced between boundary equilibria, where all the players behave as cooperators or as defectors, and inner equilibria, where cooperators and defectors coexist in the long run. A short discussion of the results, a description of open problems and possible further developments are given in section 5.

2 The static game

Let us assume that a population of n agents harvest from a common property renewable resource stock, and sell the harvested resource at a price p determined by the total harvested quantity according to a given demand function. For example, we may imagine that the agents are fishermen that harvest fish from a sea where a given fish stock X is present, however similar considerations may be applied to the harvesting of different renewable resources, such as forests or others. The agents decide the quantities to harvest on the basis of profit maximization problems. However, we assume that a fraction s of them, denoted as "cooperators" form a coalition (a cooperative venture) and consequently each of them tries to maximize the overall profit of the coalition, whereas the remaining fraction $(1 - s)$ behave as "selfish" profit maximizers, and are denoted as "defectors" (with respect to the socially more desirable cooperative behavior).

In this section we consider X and s as fixed parameters[1], with $X > 0$ and $0 \leq s \leq 1$. Let x_c^i be the quantity harvested by the cooperator i, $i = 1, ..., ns$, and let x_d^i be the quantity harvested by defector i, $i = 1, ..., n(1 - s)$. Then the total fish which is supplied and sold in the market is

$$H = \sum_{i=1}^{ns} x_c^i + \sum_{i=1}^{n(1-s)} x_d^i$$

We assume that the selling price p is determined by the linear demand function

$$p = a - bH \tag{1}$$

where a and b are positive constants, and the cost function of player i for harvesting a quantity x when a fish stock X is present is given by

$$C(x, X) = \gamma \frac{x^2}{X}. \tag{2}$$

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[1]Since n is finite, admissible values of s should be discrete, i.e. $s = k/n$ with $k = 0, 1, ..., n$. However, as usual in population dynamics and evolutionary games modelling, we abstract from this and we allow s to be a real number in the interval $[0, 1]$, even if we shall consider ns and $n(1 - s)$ integers that sum to n, by assuming some approximation of ns to the nearest integer.

This cost function can be derived from a Cobb-Douglas-type "production function" with fishing effort (labor) and fish biomass (capital) as production inputs (see Clark 1990, Szidarovszky and Okuguchi, 1998). It captures the fact that it is easier and less expensive to catch fish, if the fish population is large.

Following Sethi and Somanathan (1996) we shall also consider an extra-cost due to the presence of social norms that are intended to punish fishermen that behave as defectors, i.e. self-interested profit maximizers without any care of social optimum. Like in Sethi and Somanathan (1996) we assume that cooperators are entrusted to punish defectors by applying sanctions. This may be done directly by exerting a direct punishment, such as social disapproval damage or physical damage or destruction of equipment, as observed in less developed societies, or by alerting authorities so that they can impose sanctions according to the laws in force. Such punishment is costly for the defectors, the cost being $ns\xi$, where ξ is the amount of the sanction and ns represents the probability that a defector is notified by a cooperator. However, in general, this kind of punishment is also costly for the cooperators, the cost being proportional to the number of defectors. We shall represent by $n(1-s)\psi$ this extra cost for cooperators (of course $\xi > \psi$, and we shall often consider $\psi = 0$ in the following). All in all, the profit of i-th cooperator is

$$\pi_c^i = x_c^i (a - bH) - \gamma_c \frac{\left(x_c^i\right)^2}{X} - n(1-s)\psi \tag{3}$$

where γ_c is the fishing technology coefficient of cooperators and $n(1-s)\psi$ represents the cost that cooperators have to face in order to punish defectors, and the profit of i-th defector is

$$\pi_d^i = x_d^i (a - bH) - \gamma_d \frac{\left(x_d^i\right)^2}{X} - ns\xi \tag{4}$$

where γ_d is the fishing technology coefficient of defectors and $ns\xi$ represents the punishment that defectors have to bear for causing the negative externality in the community.

Each cooperator determines x_c^i by solving the optimization problem

$$\max_{x_c^i} \pi^V = \max_{x_c^i} \sum_{i=1}^{ns} \pi_c^i \tag{5}$$

where π^V, which is a concave function in the variables x_c^i, denotes the total profit of the cooperative venture. Assuming interior optimum, the first order conditions give a system of linear equations in the unknowns x_c^i

$$\frac{\partial \pi^V}{\partial x_c^i} = a - 2b \sum_{k=1}^{ns} x_c^k - b \sum_{k=1}^{n(1-s)} x_d^k - \frac{2\gamma_c}{X} x_c^i = 0 \tag{6}$$

Each defector determines x_d^i by solving the optimization problem

$$\max_{x_d^i} \pi_d^i \tag{7}$$

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Assuming, again, interior optimum, the first order conditions give a system of linear equations in the unknowns x_d^i

$$\frac{\partial \pi_d^i}{\partial x_d^i} = a - b \sum_{k=1}^{sn} x_c^k - b \sum_{k=1}^{(1-s)n} x_d^k - bx_d^i - \frac{2\gamma_d}{X} x_d^i = 0 \tag{8}$$

The equations (6) and (8) give a linear system of n equations with n unknowns. However, it is straightforward to see that any cooperator faces the same optimization problem, and analogously for the defectors. In fact, if we denote by $x_c^{TOT} = \sum_{k=1}^{sn} x_c^k$ the total harvest of the cooperators and by $x_d^{TOT} = \sum_{k=1}^{(1-s)n} x_d^k$ the total harvest of the defectors, from (6) we get

$$x_c^i = \frac{X}{2\gamma_c} \left(a - 2bx_c^{TOT} - bx_d^{TOT} \right) \quad \forall i = 1, ..., sn$$

and from (8) we get

$$x_d^i = \frac{X}{bX + 2\gamma_d} \left(a - b(x_c^{TOT} + bx_d^{TOT}) \right) \quad \forall i = 1, ..., (1-s)n$$

So, denoting the optimal harvesting decision of each cooperator by x_c, and the optimal harvesting decision of each defector by x_d, these quantities are obtained by solving the two linear equations

$$a - 2 \left(bsn + \frac{\gamma_c}{X} \right) x_c - b (1-s) n x_d = 0$$
$$a - bsnx_c - \left[b (1 + (1-s)n) + \frac{2\gamma_d}{X} \right] x_d = 0$$

from which the two linear *reaction functions* are obtained

$$x_c = h_c (x_d) = \frac{aX}{2 (bsnX + \gamma_c)} - \frac{b (1-s) nX}{2 (bsnX + \gamma_c)} x_d$$

$$x_d = h_d (x_c) = \frac{aX}{b (1 + (1-s)n) X + 2\gamma_d} - \frac{bsnX}{b (1 + (1-s)n) X + 2\gamma_d} x_c$$

These reaction functions allow one to compute, respectively, the optimal harvesting decision of a "representative cooperator", given the harvesting decision of a representative defector, and the optimal harvesting decision of a "representative defector", given the harvesting decision of a representative cooperator.

As $h_d (0) = \frac{aX}{b(1+(1-s)X+2\gamma_d)} < h_c^{-1}(0) = \frac{a}{b(1-s)n}$, being equivalent to $bX + 2\gamma_d > 0$, and $h_c (0) = \frac{aX}{2(bsnX+\gamma_c)} < h_d^{-1}(0) = \frac{a}{bsn}$, being equivalent to $bsnX + 2\gamma_c > 0$, the two reaction curves always intersect in a unique point (x_d^*, x_c^*) with positive coordinates, which represents the unique Nash equilibrium of the game. Of course, the harvesting strategies of cooperators and defectors at the Nash equilibrium are functions of fish stock X and the fraction of cooperators s, as well as of the cost parameters γ_c, γ_d and the market price parameters a and b, given by

$$x_d^* (X, s) = aX \frac{bsnX + 2\gamma_c}{b^2 sn (n (1-s) + 2) X^2 + 2b (2ns\gamma_d + \gamma_c + \gamma_c n(1-s)) X + 4\gamma_c\gamma_d} \tag{9}$$

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and

$$x_c^*(X, s) = \frac{X}{2} \frac{a - b(1 - s) n x_d^*(X, s)}{bsnX + \gamma_c} \tag{10}$$

Given $0 \leq s \leq 1$, these per-capita optimal harvested quantities both vanish for $X = 0$, i.e.

$$x_d^*(0, s) = x_c^*(0, s) = 0$$

are both positive for $X > 0$, have slopes at $X = 0$ given by

$$\frac{\partial x_d^*(X, s)}{\partial X}\Big|_{X=0} = \frac{a}{2\gamma_d}; \qquad \frac{\partial x_c^*(X, s)}{\partial X}\Big|_{X=0} = \frac{a}{2\gamma_c}$$

and both tend to horizontal asymptotes as $X \to +\infty$, given by

$$x_d^*(+\infty, s) = \frac{a}{b[(1 - s) n + 2]} \quad \text{and} \quad x_c^*(+\infty, s) = \frac{a}{bsn[(1 - s) n + 2]}$$

respectively. Moreover, x_d^* is an increasing and concave functions with respect to $X > 0$, whereas x_c^* is downward sloping for high values of X. Typical graphs of x_d^* and x_c^* are shown in fig. 1a. Notice that, for $sn \geq 2$ (i.e. if at least two cooperators exist) then $x_c^* < x_d^*$ for large values of fish stock, whereas for small values of X the comparison between x_d^* and x_c^* depends on the respective cost parameters. It is also worth to notice that both x_d^* and x_c^* increase if the selling price increases, i.e. a increases and/or b decreases in the demand function (1). Moreover, also the gap between x_d^* and x_c^*, for large values of the fish stock, increases with increasing prices and with the number ns of cooperators, being

$$x_d^*(+\infty, s) - x_c^*(+\infty, s) = \frac{a}{b[(1 - s) n + 2]}\left(1 - \frac{1}{sn}\right)$$

The total harvesting *at the Nash equilibrium* is given by

$$H^*(X, s) = n\left[sx_c^*(X, s) + (1 - s)x_d^*(X, s)\right]. \tag{11}$$

For each s, $H^*(X, s)$ is an increasing and concave function with respect to X, such that, $H^*(0, s) = 0$,

$$\frac{\partial H^*(X, s)}{\partial X}\Big|_{X=0} = \frac{na}{2}\left(\frac{s}{\gamma_c} + \frac{1 - s}{\gamma_d}\right)$$

and for $X \to \infty$ it approaches an horizontal asymptote.

$$H^*(+\infty, s) = na\left[\frac{1}{bn[(1 - s) n + 2]} + \frac{1 - s}{b[(1 - s) n + 2]}\right] =$$

$$= \frac{a(1 + n(1 - s))}{b[(1 - s) n + 2]} = \frac{a}{b}\left(1 - \frac{1}{2 + (1 - s) n}\right)$$

Notice that $H^* < a/b$ for each $X > 0$, so that prices are always positive. Moreover, if s is decreased, i.e. the number of defectors is increased, then $H^*(+\infty, s)$ increases, that is, as expected, in the presence of abundant resource the total harvest is greater if the number of defectors increases. In the limiting case $s = 0$ (all defectors) we have $H^*(+\infty, 0) = \frac{a}{b}\left(1 - \frac{1}{2+n}\right)$, and in the opposite limiting case $s = 1$

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(all cooperators, equivalent to the case of a sole owner) we get $H^*(+\infty, 1) = a/2b$, the monopolist optimum.

These results allow us to compute, at the Nash equilibrium, the profit of a representative defector and that of a representative cooperator, given by

$$\pi_d^* = x_d^* (a - bH^*) - \gamma_d \frac{(x_d^*)^2}{X} - ns\xi \quad \text{and} \quad \pi_c^* = x_c^* (a - bH^*) - \gamma_c \frac{(x_c^*)^2}{X} - n(1-s)\psi \quad (12)$$

respectively. Plugging the expressions of x_c^*, x_d^* and H^* into (12), π_d^* and π_c^* and can be rewritten as:

$$\pi_d^* = \frac{a^2 X (bnsX + 2\gamma_c)^2 (bX + \gamma_d)}{[bX (bn(-n(1-s)-2)) sX - 2(n(1-s)+1)\gamma_c - 4(bnsX + \gamma_c)\gamma_d]^2} - ns\xi \quad (13)$$

and

$$\pi_c^* = \frac{a^2 X (bnsX + \gamma_c)(bX + 2\gamma_d)^2}{[bX (bn(-n(1-s)-2)) sX - 2(n(1-s)+1)\gamma_c - 4(bnsX + \gamma_c)\gamma_d]^2} - (1-s)n\psi \quad (14)$$

respectively. These expressions show that, at the Nash equilibrium, the profits of defectors (cooperators) are positive provided that the applied sanctions (the costs to apply sanctions) are not too heavy. For example, under the assumption $\psi = 0$ and $\xi > 0$, that we shall often consider in the following, we have π_c^* always positive and π_d^* positive or negative according to the sanctions applied and to the number of cooperators.

FIG. 1 APPROXIMATELY HERE

3   The one-dimensional dynamics with fixed s

We now regard X as a dynamic variable and s as a parameter, that is, we consider the time evolution of the resource stock $X(t)$, that depends on its natural growth function and on the harvesting activity, and we assume a fixed share of fishermen population between cooperators and defectors.

Let $X(t)$ denote the fish stock at time period t. We consider the following discrete time equation to describe the time evolution of the fish stock

$$X(t+1) = F(X(t)) = X(t)(1 + \alpha - \beta X(t)) - H^*(X(t), s) \quad (15)$$

that is, we assume that in the absence of any harvesting the stock of the fish population in period t is determined by the discrete time logistic equation[2], with α and β *biological parameters* that characterize the fish population we are considering and the environment where it lives: α is the *intrinsic growth rate* and α/β the *carrying capacity*, i.e. the positive equilibrium value of the unharvested resource population, since for $H^* = 0$ and $X = \alpha/\beta$ (15) gives $X(t+1) = X(t)$. We also assume that the

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[2]A more general growth function $G(X) = XR(X)$, with $R(0) > 1$, $R(X) > 0$ for $0 < X < K$ and $R(X) < 0$ for $X > K$, $K > 0$ being the carrying capacity, may be considered. However, the choice $R(X) = 1 + \alpha - \beta X$, known as logistic growth, is one of the simplest and most commonly used for qualitative analysis purposes.

fish stock is harvested according to the Nash equilibrium of the static game described in the previous section.

The steady states of the model with harvesting are the non negative fixed points of the function F in (15), solutions of the equation

$$X\left(\alpha - \beta X\right) = H^*(X; s, \gamma_c, \gamma_d, a, b, n) \tag{16}$$

where the dependence on the parameters is emphasized. So, for any given value of s, the equilibria are located at the non negative intersections between a parabola and the concave function (11) (see fig.2).

FIG. 2 APPROXIMATELY HERE

The equilibrium $X_0 = 0$ (extinction of the resource) always exist. Furthermore, depending on the values of the parameters, two, one or no positive equilibria may exist (see fig.2). We now give some conditions for the existence of positive steady states. First of all, as the parabola, whose equation is given by the left hand side of (16), has slope α at $X = 0$, a necessary and sufficient condition for the existence of a unique positive steady state is (see curve a in fig. 2)

$$\frac{\partial H^*\left(X, s\right)}{\partial X}|_{X=0} = \frac{na}{2}\left(\frac{s}{\gamma_c} + \frac{1-s}{\gamma_d}\right) < \alpha. \tag{17}$$

In this case, a unique positive equilibrium exists, say X_1, with $0 < X_1 < \alpha/\beta$. If (17) holds, then $DF(0) > 1$ hence whenever a unique positive equilibrium exists, the fixed point $X_0 = 0$ is unstable (see fig. 3a). This means that if (17) is satisfied, i.e. the number of fishermen is not too high, or prices are not too high or cost parameters are not too small, then even if the resource stock is reduced at an arbitrary small positive value by some exogenous shock, the endogenous dynamics of the system is such that it spontaneously evolves to viable levels of the resource stock, close to X_1. The unique positive equilibrium X_1 may be stable, i.e. $|DF(X_1)| < 1$, or unstable, with $DF(X_1) < -1$. In the latter case, which occurs with high values of the parameter α, a more complex bounded attractor, which may be periodic or chaotic, may exist around X_1. The bounded attracting set is confined inside the trapping set $I = [c_1, c]$ where c is the maximum value of the function F and $c_1 = F(c)$. In any case, the basin of the bounded attractor is bounded by the unstable fixed point $X_0 = 0$ and its rank-1 preimage $X_{0_{-1}}$, i.e.

$$\mathcal{B} = \left(0, X_{0_{-1}}\right)$$

where $X_{0_{-1}}$ is the positive solution of the equation

$$1 + \alpha - \beta X = \frac{H^*(X)}{X} \tag{18}$$

It is straightforward to see that under the assumption (17) the equation (18) has a unique positive solution $X_{0_{-1}}$ such that $X_1 < X_{0_{-1}} < (1+\alpha)/\beta$. An initial condition with $X(0) > X_{0_{-1}}$ is mapped by the iterated function F to a negative value, so we consider as *unfeasible* such kind of trajectory[3].

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[3]This may be interpreted as a resource extinction due to overcrowding effects, a definitely unrealistic situation in the context we are considering.

It is worth to notice that the equilibrium value X_1 is influenced by the value of s. In fact, if s is increased, then the asymptote of H^* moves downwards, and this implies that X_1 increases with s, i.e. $X_1(s)$ is an increasing function. The intuition behind this is clear: more cooperators imply an higher resource stock at the long run equilibrium, due to a more conservative (or sustainable) resource exploitation.

If the aggregate parameter at the left hand side of (17) is increased, i.e. the number of fishermen and/or prices become higher compared with intrinsic growth of the fish species in the environment considered, and/or the fishing costs are lowered by using more sophisticated technologies, for

$$na \left(\frac{s}{\gamma_c} + \frac{1-s}{\gamma_d} \right) = 2\alpha \tag{19}$$

we have $DF(0) = 1$ and if $na \left(s/\gamma_c + (1-s)/\gamma_d \right)$ is further increased (or α decreased) then a transcritical (or stability exchange) bifurcation occurs after which the equilibrium $X_0 = 0$ becomes stable, i.e. $-1 < DF_n(0) < 1$, and a second fixed point X_2 enters the positive orthant, i.e. $0 < X_2 < X_1 < \alpha/\beta$. The new positive equilibrium X_2 is unstable, being $DF(X_2) > 1$, and belongs to the boundary which separates the basin of the stable equilibrium $X_0 = 0$ from the basin of the positive attractor. So, in this situation X_2 constitutes a threshold population level such that if the current population $X(t)$ falls below X_2 then the resource stock will spontaneously evolve towards extinction. It is worth to notice that if s is increased, so that the asymptote of H^* moves downwards, then threshold value X_2 moves to the left, i.e. it is a decreasing function of s. This means that more cooperators imply not only a higher resource stock at the long run equilibrium X_1, but also an enlargement of its basin of attraction.

As usual with noninvertible maps, all the rank-1 preimages of X_2 belong to basin boundaries, so the basin of the positive attractor is now given by

$$\mathcal{B} = \left(X_2, X_{2_{-1}} \right), \tag{20}$$

$X_{2_{-1}}$ being the rightmost solution of the equation $F(X) = X_2$. The set of positive initial conditions which asymptotically converge to $X_0 = 0$, and give rise to extinction in the long-run, is formed by the union of two disjoint intervals

$$\mathcal{BP}(0) = (0, X_2) \cup \left(X_{2_{-1}}, X_{0_{-1}} \right) \tag{21}$$

whereas the initial condition $X(0) > X_{0_{-1}}$ generate unfeasible trajectories.

In general, a sufficient condition for two equilibria is

$$\frac{na}{2} \left(\frac{s}{\gamma_c} + \frac{1-s}{\gamma_d} \right) > \alpha \text{ and } \frac{a}{b} \left(1 - \frac{1}{2 + (1-s)\,n} \right) < \frac{\alpha^2}{4\beta} \tag{22}$$

where the first inequality states that at $X = 0$ the curve $H^*(X)$ has slope greater than that of the parabola, and the second inequality states that the upper bound of H^* is below the vertex of the parabola (see the curve (b) of fig. 2).

When two positive equilibria exist, the dynamic scenario is the one described above, and represented in fig. 3b. With given values of the biological parameters α and β, so that the parabola in fig. 2 is fixed, if the other parameters are varied with the consequence that the asymptote of H^* moves upwards, the two positive equilibria become closer and closer, so that the basin of X_0 enlarges

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and, therefore, the basin of the viable equilibrium X_1 shrinks. This can be obtained, for example, by increasing prices (i.e. increasing a/b) or with decreasing values of s, i.e. by increasing the number of defectors. We are particularly interested in the latter effect: decreasing values of s imply less robustness of the viable equilibrium with respect to exogenous shocks. Of course, also a study of the effects of the parameter β may be interesting, as an higher value of β may be interpreted as the effect of a damaged environment, due to pollution or other factors.

If we reverse the second inequality in (22), i.e.

$$\frac{na}{2}\left(\frac{s}{\gamma_c}+\frac{1-s}{\gamma_d}\right) > \alpha \text{ and } \frac{a}{b}\left(1 - \frac{1}{2+(1-s)\,n}\right) > \frac{\alpha^2}{4\beta} \tag{23}$$

we obtain a necessary condition for the uniqueness of the extinction equilibrium $X_0 = 0$, i.e. no positive steady states. Such a situation may be obtained as the final effect of increasing $H^*(+\infty, s)$: The transition from a dynamic scenario characterized by two positive steady states to one with no positive steady states occurs via a fold (or tangent) bifurcation, due to a progressive decrease of X_1, increase of X_2 (so that the basin $\mathcal{B}$ becomes smaller and smaller) until they merge with $DF(X_1) = DF(X_2) = 1$, and then they disappear. It is trivial to prove that when X_0 is the unique equilibrium, then for every initial condition the system evolves towards extinction (see fig. 3c). Once more, we remark that such a sequence of bifurcations described above may occur for increasing prices, decreasing costs or decreasing values of s, i.e. by increasing the number of defectors.

FIG. 3 APPROXIMATELY HERE

4   The two-dimensional model with evolutionary dynamics of s

We now relax the assumption of a fixed population share between cooperators and defectors, and we introduce an evolutionary mechanism that, at each time period, describes how the population share is updated. In the spirit of evolutionary games, we assume that the fraction of agents playing a strategy that, with respect to the other strategies, earns higher payoffs, increases in the next period. In our case, the payoffs associated with the two available strategies of cooperation and defection are the profits $\pi_c^*(t)$ and $\pi_d^*(t)$ respectively, that, according to (14) and (13), depend on the current population share $s(t)$ as well as on the current resource stock $X(t)$. Since, as argued in the previous section, the dynamics of $X(t)$ are influenced by $s(t)$, this will give rise to a two-dimensional nonlinear dynamical system with dynamic variables $X(t)$ and $s(t)$, the study of which may give us information on the long-run evolution of the system. For example, we may ask not only if the resource stock will survive or become extinct in the long run, but we may also ask if some behavior (cooperation or defection) will survive or become extinct as the system evolves. Since, under the reasonable assumption $\gamma_c = \gamma_d$ (that means that the two groups adopt the same technology) the strategy chosen by defectors is dominant if no sanctions are applied, the assumption $\xi > \psi \geq 0$ will be crucial in order to obtain stable equilibria with a nonvanishing fraction s of cooperators.

4.1   Replicator dynamics

The simplest (and more frequently used) model proposed in the literature which gives an evolutionary pressure in favor of groups obtaining the highest payoffs is that of *replicator dynamics* (Taylor and

Jonker, 1978, see also Vega-Redondo, 1996, ch.3, Hofbauer and Sigmund, 1998, ch.7, Weibull, 1995, ch.3). The discrete time replicator dynamics for the fraction of cooperators can be written as

$$s\left(t+1\right) = s\left(t\right) \frac{\pi_c^*(t)}{\overline{\pi}} \tag{24}$$

where

$$\overline{\pi}(t) = s\pi_c^*(t) + (1-s)\pi_d^*(t) \tag{25}$$

represents the average profit observed at time t. So, (24) states that $s(t+1)$ will be greater than $s(t)$ if $\pi_c^*(t) > \overline{\pi}(t)$ whereas $s(t)$ will decrease if $\pi_c^*(t) < \overline{\pi}(t)$. As $\pi_c^*(t) > \overline{\pi}(t)$ if and only if $\pi_c^*(t) > \pi_d^*(t)$, it follows that the population share related to the better performing strategy at time period t increases in the next period.

Both $\pi_c^*(t)$ and $\overline{\pi}(t)$ depend on $s(t)$, as well as $X(t)$, so the difference equations (15) and (24) define a two-dimensional discrete dynamical system. Starting from a given initial biomass $X(0)$ and a given initial population share $s(0)$, the recurrences (15) and (24) allow one to obtain $X(t)$ and $s(t)$ for each $t \geq 0$, from which the corresponding values of $x_c^*(t)$, $x_d^*(t)$, $\pi_c^*(t)$, $\pi_d^*(t)$, $H^*(t)$ can be obtained.

It is interesting to study if the dynamic variables $X(t)$ and $s(t)$ converge to a given steady state in the long run, i.e. as $t \to \infty$, or if they exhibit some more complex time pattern.

4.2 Qualitative study of the two-dimensional dynamical system

The time evolution of the discrete time two-dimensional dynamical system in the dynamic variables X and s is obtained by the iteration of a map of the plane $T : (X(t), s(t)) \to (X(t+1), s(t+1))$ defined by

$$X(t+1) = X(t)\left(1 + \alpha - \beta X(t)\right) - H^*\left(X(t), s(t)\right)$$

$$\tag{26}$$

$$s\left(t+1\right) = s\left(t\right) \frac{\pi_c^*(X(t), s(t))}{s\pi_c^*(X(t), s(t)) + (1-s)\pi_d^*(X(t), s(t))}$$

where $H^*(X, s)$ is given by (11), with x_c^* and x_d^* defined in (10) and (9) respectively, and π_c^*, π_d^* are given in (14) and (13) respectively.

It is straightforward to see that if $\pi_c^*(t) > 0$ and $\pi_d^*(t) > 0$ then $s(t) \in (0,1)$ implies $s(t+1) \in (0,1)$ as well. However, negative profits may arise if the parameters ξ and/or ψ are positive. As the influence of these parameters is always related to the difference $\xi - \psi$, without loss of generality we shall assume $\psi = 0$, and whenever $\pi_d^*(t) < 0$ we shall assume $s(t+1) = 1$ (instead of the meaningless $s(t+1) > 1$, as obtained by simply applying (26)).

4.3 Two benchmark cases

It is important to notice that if $s(t) = 0$ then $s(t+1) = 0$ for each $t \geq 0$, and if $s(t) = 1$ then $s(t+1) = 1$ for each $t \geq 0$, i.e. the two boundary lines $s = 0$ and $s = 1$ are trapping lines, on which the dynamics are governed by one-dimensional unimodal maps, given by the restrictions of the two-dimensional map (26) to them. These two cases correspond to particular benchmark cases, where we have all cooperators and all defectors respectively, i.e. the cases considered in Szidarovszky and Okuguchi

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(1998) and Szidarovszky and Okuguchi (2000) respectively, see also Bischi, Kopel and Szidarovszky (2002). The properties of these one-dimensional dynamical systems can be easily obtained on the basis of the results of section 3. For example, the dynamics along the invariant edge $s = 0$, where all players are defectors, are governed by the one-dimensional map (15) with

$$H^*(X, 0) = nx_d^*(X, 0) = \frac{naX}{b(n+1)X + 2\gamma_d}$$

So the steady states are $X = 0$ and the solutions (if any) of the equation

$$b\beta(n+1)X^2 + (2\beta\gamma_d - \alpha b(n+1))X + na - 2\alpha\gamma_d = 0 \tag{27}$$

The analysis is the same as in Szidarovszky and Okuguchi (1998) or in Bischi et al. (2002).

On the other invariant edge $s = 1$, where all players are cooperators, the dynamics are governed by (15) with

$$H^*(X, 1) = nx_c^*(X, 1) = \frac{naX}{2(bnX + \gamma_c)}$$

So the fixed points are $X = 0$ and the solutions (if any) of the equation

$$2b\beta nX^2 + 2(\beta\gamma_c - \alpha bn)X + na - 2\alpha\gamma_c = 0 \tag{28}$$

The analysis is the same as in Szidarovszky and Okuguchi (2000) or in Bischi et al. (2002).

4.4 Steady states

As usual, the starting point for the qualitative analysis of a nonlinear dynamical system is the localization of the steady states and the study of their local stability. The steady states of the two-dimensional dynamical system (26) are the fixed points of the map T, solutions of the system $T(X, s) = (X, s)$. It is straightforward to see that two *corner equilibria* always exist, given by $E_0 = (0, 0)$ and $E_1 = (0, 1)$, characterized by extinction of the resource. Other boundary equilibria may exist along the invariant lines $s = 0$ and $s = 1$, given by the solutions, if any, of (27) and (28) respectively. If two equilibria with positive fish stock exist both on the invariant edge $s = 0$ and on invariant edge $s = 1$, say $X_2(0)$, $X_1(0)$ and $X_2(1)$, $X_1(1)$ respectively, then, on the basis on the arguments of section 3, the following relation must hold: $X_2(1) < X_2(0) < X_1(0) < X_1(1)$. A necessary condition for the existence of two positive equilibria along $s = 0$ is that two positive equilibria exist along $s = 1$. However, it may happen that two positive equilibria exist along $s = 1$ and no positive equilibria exist along $s = 0$. Of course, sufficient conditions for the existence of two positive boundary equilibria along the invariant edges are obtained from (22) with $s = 0$ and $s = 1$ respectively.

Instead, if (17) is satisfied for $s = 1$, i.e. $na < 2\alpha\gamma_c$, then a unique equilibrium exists along the edge with only cooperators, and the same holds on the other invariant edge if $na < 2\alpha\gamma_d$. These two conditions are equivalent under the reasonable assumption $\gamma_c = \gamma_d$. However, even when a unique equilibrium exists, the inequality $X_1(0) < X_1(1)$ holds, i.e. the long-run equilibrium under sustainable fishing is characterized by an higher values of resource stock in the limiting case of all cooperators than in the opposite limiting case of all defectors.

The stability of these equilibria with respect to the one-dimensional dynamics trapped inside the invariant edges can be easily deduced from the discussion on the one-dimensional dynamics given in

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section 3 applied to the particular benchmark cases $s = 0$ and $s = 1$. However, we are now mainly interested in the stability with respect to perturbations transverse to the invariant edges, i.e. what happens if a few defectors appear starting from a situation with all cooperators, or what happens if a few cooperators appear starting from a situation with all defectors: Are such small mutations eliminated by the evolutionary dynamics, so that the original benchmark case is restored (case of transverse stability) or do they grow up thus causing an irreversible departure from the original benchmark case?

An answer to these questions requires the study of the local stability of the boundary equilibria, i.e. the localization, in the complex plane, of the eigenvalues of the Jacobian matrix of (26) computed at the boundary steady states. This is not difficult in principle, as eigenvalues are always real because the Jacobian matrix of (26) is a triangular matrix along the invariant edges. This implies that we can only have nodes or saddle equilibria on the boundaries. However, the expressions of the eigenvalues are quite involved, and the stability conditions obtained are not easy to be interpreted.

So, in the following we prefer to follow a numerical and graphical method in order to obtain a global view of the dynamic properties of the dynamical system $(26)^4$.

First of all, we consider the question of the existence of inner equilibria, i.e. steady states characterized by coexistence of cooperators and defectors. These are obtained solving the nonlinear system

$$
\begin{aligned}
X(\alpha - \beta X) &= H^*(X, s) \\
\pi_c^*(X, s) &= \pi_d^*(X, s)
\end{aligned}
\tag{29}
$$

with $0 < s < 1$. The set of points of the plane (X, s) that satisfy the first equation represent locus of points that give one-period stationary resource stock, i.e. $X(t+1) = X(t)^5$. For each s in the range $[0, 1]$ the X coordinates of these points can be computed by solving the equation (16), already analyzed in section 3. So, this set of points may be formed by two branches, say $X_1(s)$ and $X_2(s)$, with $X_2(s) < X_1(s)$ for each s. Moreover, from the results of section 3, $X_2(s)$ is a decreasing function and $X_1(s)$ is increasing, so the branch $X_1(s)$ has positive slope and the branch $X_2(s)$ has negative slope in the plane (X, s) (see figures 4a and 5a). The intersection of the branch $X_1(s)$ $(X_2(s))$ with the invariant edge $s = 1$ gives the boundary equilibrium of X coordinate $X_1(1)$ $(X_2(1))$, and the same holds for the intersections (if any) with the other invariant edge $s = 0$. However, it may happen that the two branches intersect $s = 1$ but have no intersections with $s = 0$, because they may merge for $s > 0$ (see fig. 6a). If the condition (17) is satisfied for all $s \in [0, 1]$, then the locus of points such that $X(t+1) = X(t)$ is only formed by the branch $X_1(s)$. The knowledge of these curves give us the following information: Starting from a given point (X, s), a one-step iteration of (26) generates a new point $(X', s') = T$, rank-1 image of (X, s) by T, with $X' > X$ if (X, s) is in the strip between the curves $X_2(s)$ and $X_1(s)$ (or between the axis $X = 0$ and the curve $X_1(s)$ if only the branch $X_1(s)$ exists) and with $X' < X$ if (X, s) is on the left of the curves $X_2(s)$ (provided it exists) or on the right of the curve $X_1(s)$.

<div align="center">
FIG. 4 APPROXIMATELY HERE
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[4] A more standard analytical study of the stability of the boundary equilibria, obtained with the help of software packages for symbolic algebraic manipulation, is in progress.

[5] Of course, this does not mean that the dynamic system is in equilibrium, unless also $s(t)$ is stationary, i.e. the second equation (29) is satisfied as well.

A similar reasoning can be applied to the set of points that satisfy the second equation (29), that represent the locus of points that give one-period stationary population share, i.e. $s(t+1) = s(t)$. A qualitative study of this curve is more difficult, due to the complicated expressions of $\pi_c^*(X,s)$ and $\pi_d^*(X,s)$. However, the numerical solution of the equation $\pi_c^*(X,s) = \pi_d^*(X,s)$ for different sets of parameters gives rise to decreasing curves in the plane (X,s), as shown in the figures (4a, 5a, 6a). The points above the curve $\pi_c^*(X,s) = \pi_d^*(X,s)$ are points where $\pi_c^*(X,s) < \pi_d^*(X,s)$, hence the rank-1 images $(X',s') = T(X,s)$ of points (X,s) above the curve have $s' < s$. Of course, the points below the curve are characterized by $\pi_c^*(X,s) > \pi_d^*(X,s)$, hence $(X',s') = T(X,s)$ are such that $s' > s$. These arguments allow us to obtain a global qualitative picture of the dynamic behavior of the dynamical system (26), as shown by the arrows represented in fig.4a.

Of course, any intersection between the curve $\pi_c^*(X,s) = \pi_d^*(X,s)$ and one of the curves $X_i(s)$, $i = 1, 2$, represents a steady state of the dynamical system. Of course, all the steady states along the branch $X_2(s)$ cannot be stable, because all the points of that branch behave as repelling points along the X direction. Instead, steady states located along the branch $X_1(s)$ are candidates to be stable equilibria. For example, in fig. 4a, obtained with the set of parameters $n = 10$, $\alpha = 3$, $\beta = 1$, $a = 1.05$, $b = 0.3$, $\gamma_c = \gamma_d = 1$, $\psi = 0$, $\xi = 0.01$, the two equilibria on $X_2(s)$ are unstable, whereas the equilibrium point $E = (1.48, 0.15)$, located on $X_1(s)$ is stable. In fig. 4b, the numerically computed basin of attraction of the stable equilibrium E is represented by the white region, whereas the grey region represent the initial conditions leading to extinction of the resource. Two typical trajectories are also represented as sequences of black dots. From this picture the role of the curve $X_2(s)$ is quite evident: like in the one-dimensional model studied in section 3, the locus points $X_2(s)$ constitutes the boundary that separates the basin of attraction of the stable positive equilibrium from the basin of the trajectories leading to the extinction of the resource.

In the situation shown in fig. 4, the long run evolution of the system leads to an equilibrium situation characterized by a small fraction of cooperators (only 15% of the fishermen population). The amount of cooperators at the stable equilibrium can be increased by varying some parameters, for example by increasing ξ, i.e. more heavy sanctions, or by decreasing a/b, i.e. lower prices. Both these variations cause an upward movement of the curve $\pi_c^*(X,s) = \pi_d^*(X,s)$. For example, fig. 5 has been obtained by increasing the parameter ξ from 0.01 to 0.05. The consequence is that the new stable equilibrium is $E = (1.91, 0.62)$ i.e. 62% of fishermen are cooperators and, consequently, an higher resource stock is present at the equilibrium. If, starting from this situation, the parameter ξ is further increased, or a/b is decreased, the equilibrium E moves upwards along the curve $X_1(s)$ until it merges with the boundary equilibrium $X_1(1)$ through a transcritical (or stability exchange) bifurcation. After this, the boundary equilibrium $X_1(1)$ becomes the unique stable steady state, where only cooperators exist. This means that prices are so low that it is not convenient to be defectors, even if very few defectors are present, i.e. even if a defector appear (a mutation in the population composition) it is eliminated by the evolutionary mechanism.

FIG. 5 APPROXIMATELY HERE

It is also interesting to see what happens when prices are very high, like in the situation shown in fig. 6a, where the same parameters as in fig. 4 have been used, except the parameter $a = 1.2$. In this case no positive boundary equilibria exist along the invariant boundary $s = 0$ with all defectors,

and two positive equilibria, $X_2(1) < X_1(1)$ exist along the edge $s = 1$. In this situation all the initial conditions generate trajectories that approach the boundary $s = 0$ and then converge to the unique stable equilibrium $E_0 = (0, 0)$. This means that the prices are so high that defectors prevail, but their harvesting is so high that the resource is exhausted. However, if we constrain the system to start with $s = 1$ (all cooperators) and sufficient initial resource stock, i.e. $X(0) > X_2(1)$, then the system evolves towards the equilibrium $X_1(1)$, according to the one dimensional dynamics shown in fig. 6b. However, the equilibrium is not transversely stable. This implies that if just one defector appears, the endogenous evolutive dynamics will create more and more defectors, and the system will irreversibly evolve towards a situation with all defectors and extinction of the resource.

FIG. 6 APPROXIMATELY HERE

5   Conclusions

In this paper we have presented a model that constitutes an attempt to put together two opposite approaches to commercial fishing: cooperation (i.e. harvesting decisions obtained through the maximization of the overall profit, that lead to a more conservative resource harvesting) and the non cooperative, or defective, attitude (i.e. harvesting decisions obtained through the maximization of the personal profit without any concern with social welfare). Oligopoly models based on one of these opposite attitudes, that is all agents behaving as cooperators or all agents behaving as defectors, have been recently proposed by Szidarovszky and Okuguchi, 2000 and 1998 respectively (see also Bischi et al., 2002, for a comparison between the two cases). The model we propose describes a common property resource exploitation with a population of agents switching between a non-cooperative and a cooperative behavior regulated by an evolutive mechanism, known as replicator dynamics, based on the evolutionary idea that the fraction of agents playing the strategy that earns higher payoffs will increases in the next period (as proposed in Sethi and Somanathan, 1996).

Our work is mainly methodological, as its primary goal is the setup of the dynamic model: starting from a static game where the harvested quantities are decided by the two groups by choosing the Nash equilibrium, which depends on the available resource stock and the population share, we then introduce the dynamic equations that govern the growth of the renewable resource and then we introduce the replicator dynamics that governs the time evolution of the population share between cooperators and defectors. This leads us to the study of a nonlinear two-dimensional discrete time dynamical system. The results given in this paper constitute only a first step towards a full understanding of the dynamic behaviors of the model, as we only gave a numerical and graphical characterization of the equilibria and their stability.

We can summarize our results as follows. First, the presence of many cooperators always leads to a relatively high level of the resource stock, hence wealth in the long run, whereas many defectors can cause a severe depletion of the resource, influencing also the increment of the basin of attraction of steady states with extinction of the resource. Second, from the point of view of a regulator, not only the cooperative behavior can be supported increasing the level of sanctions, or lowering the prices, but also it is possible to reach steady states characterized by the presence of only cooperators which are transversely stable, i.e. even if a defector enters the market this behavior does not spread over the population. Otherwise, low sanctions and/or high prices could lead to steady states with only

defectors, and the potential depletion (even extinction) of the resource. In this case, even if the system starts from an initial condition with only cooperators, the non-cooperative behavior will prevail if only one agent decides to defect. Even if the qualitative analysis of the model can be carried out with the methods discussed in this paper, a more complete analysis of the effects of the parameters on the kinds of long-run evolution, as well as on the transverse stability of corner equilibria, is under development. Further enhancements of this model could also relax the assumption of perfect foresight about the next period fish stock replacing it by a weaker (and more realistic) assumption on expectations formations (e.g. assuming that agents have adaptive expectations on the available resource stock).

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Figure captions

Fig. 1. (a) Typical graphs of harvesting quantities of defectors, x_d^*, and cooperators, x_c^*, at the Nash equilibrium, plotted as functions of the available resource stock X. (b) The corresponding total harvesting $H^*(X, s)$.

Fig. 2. Qualitative graphical representation of the equation (16) with fixed values of the parameters a and β and three different functions H^*. The curves denoted by a, b, c can be obtained, respectively, by decreasing values of s or increasing values of a or decreasing values of b.

Fig. 3. Qualitative representation of the function F, that governs the one-dimensional dynamics of the resource stock X according to (15), in the three different cases represented in fig.2: (a) One positive equilibrium (b) Two positive equilibria (c) No positive equilibria.

Fig. 4. (a) Numerical graphical representation of the equation (29) obtained with the set of parameters $n = 10$, $\alpha = 3$, $\beta = 1$, $a = 1.05$, $b = 0.3$, $\gamma_c = \gamma_d = 1$, $\psi = 0$, $\xi = 0.01$ The arrows give a qualitative indication of the directions of one-step advancement of the discrete dynamical system (26) (b) For the same parameters of as those used in (a) the basins of attraction are represented: the white region represents the basin of the inner equilibrium E, the grey region is the set of points that generate trajectories leading to extinction (i.e. $X \leq 0$). Two trajectories, starting from initial conditions $(2, 0.9)$ and $(2, 0.05)$, are also represented by sequences of black dots.

Fig. 5. The same as fig. 4, the only difference being the value of the parameter $\xi = 0.05$.

Fig. 6. (a) The same as fig. 4a, the only difference being the value of the parameter $a = 1.2$. (b) For the same parameters of as those used in (a) the function $F(X)$ that governs the one dimensional dynamics along the invariant edge $s = 1$ is represented, with a trajectory starting from $X = 1$ and converging to the equilibrium $X_1(1)$.
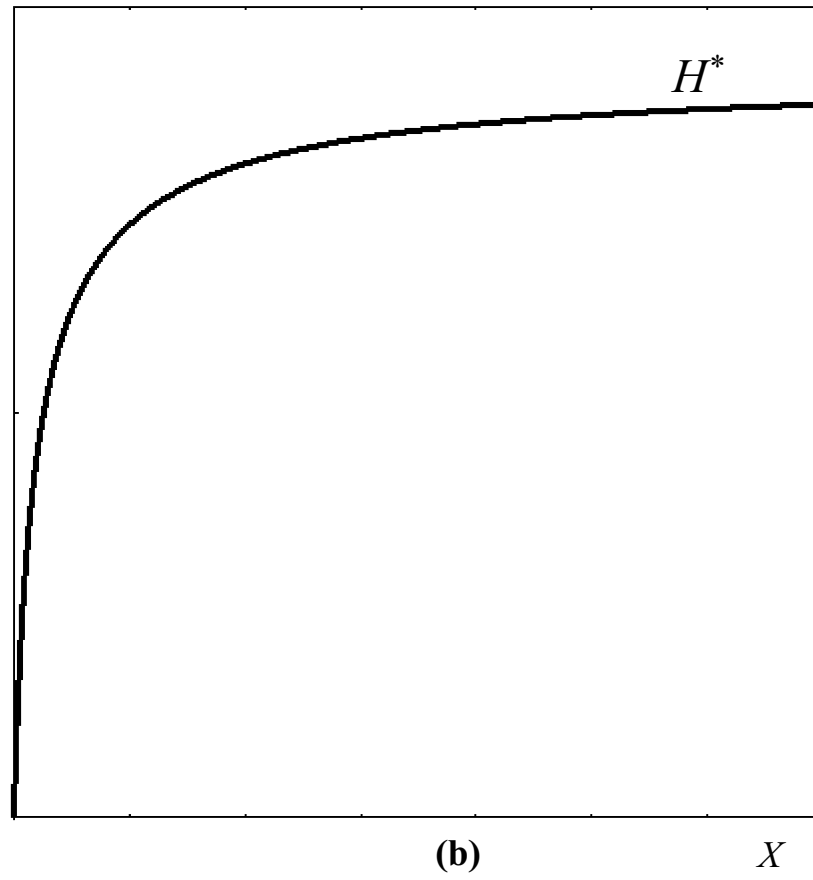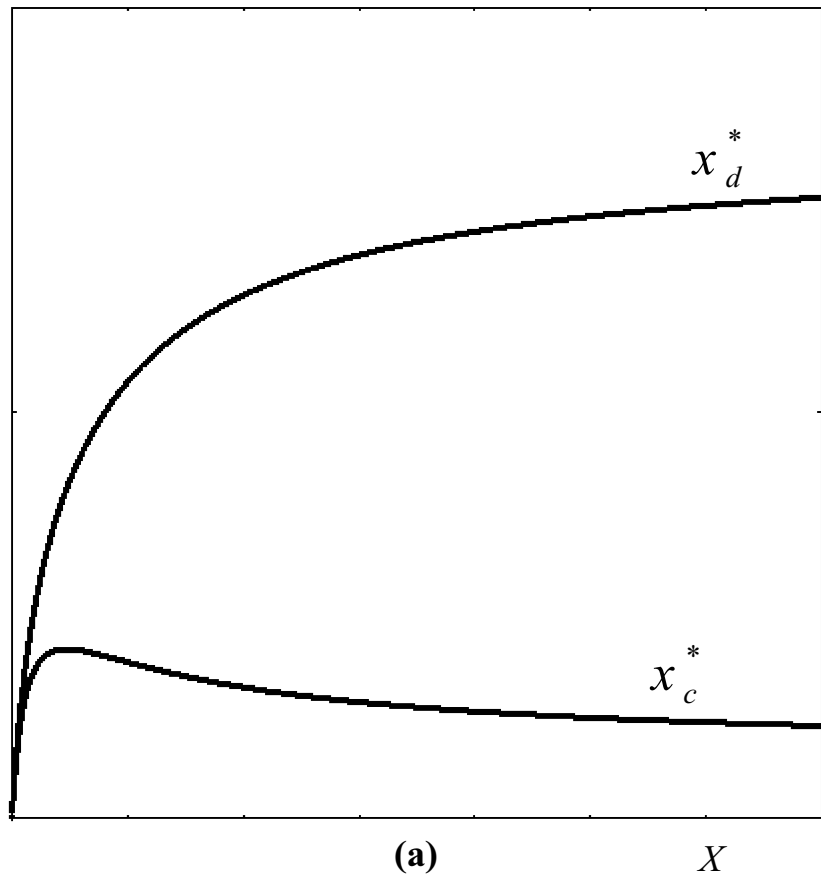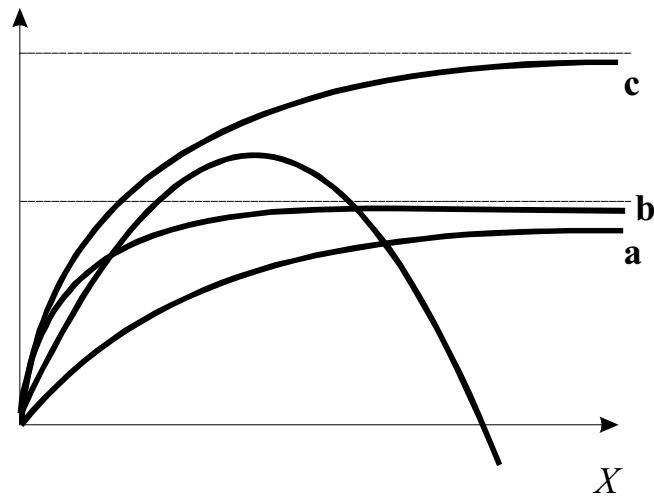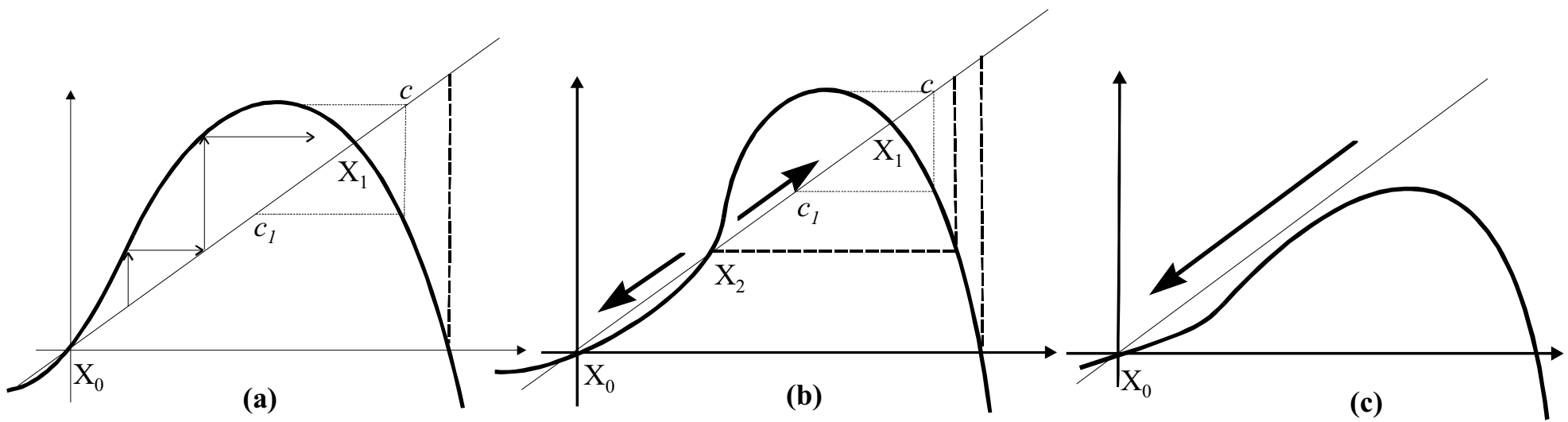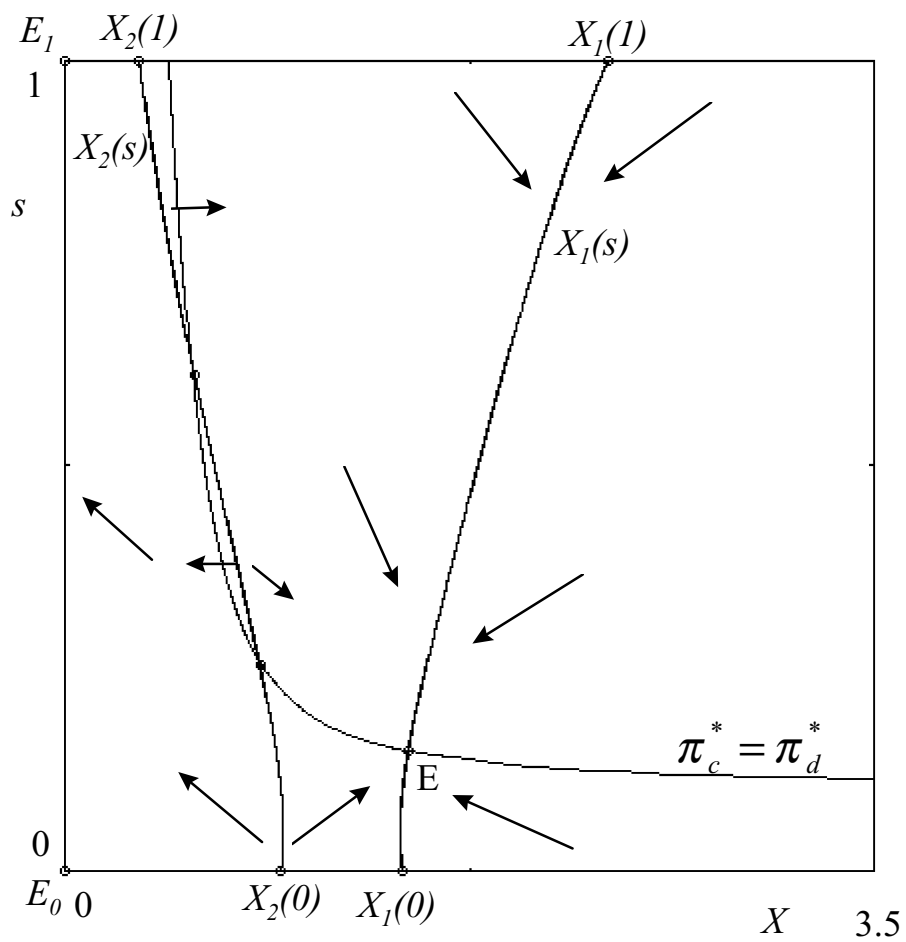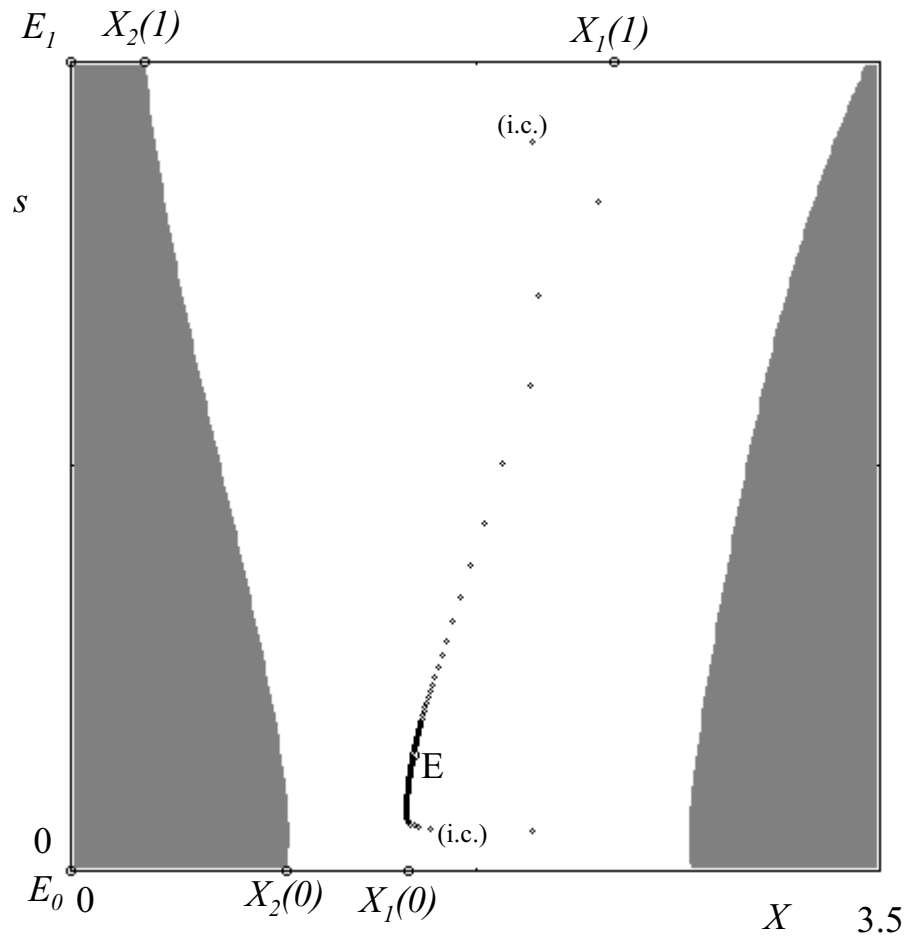
Fig. 1

Fig. 2

Fig. 3

(a)

(b)

Fig. 4

Fig. 5

Fig. 6