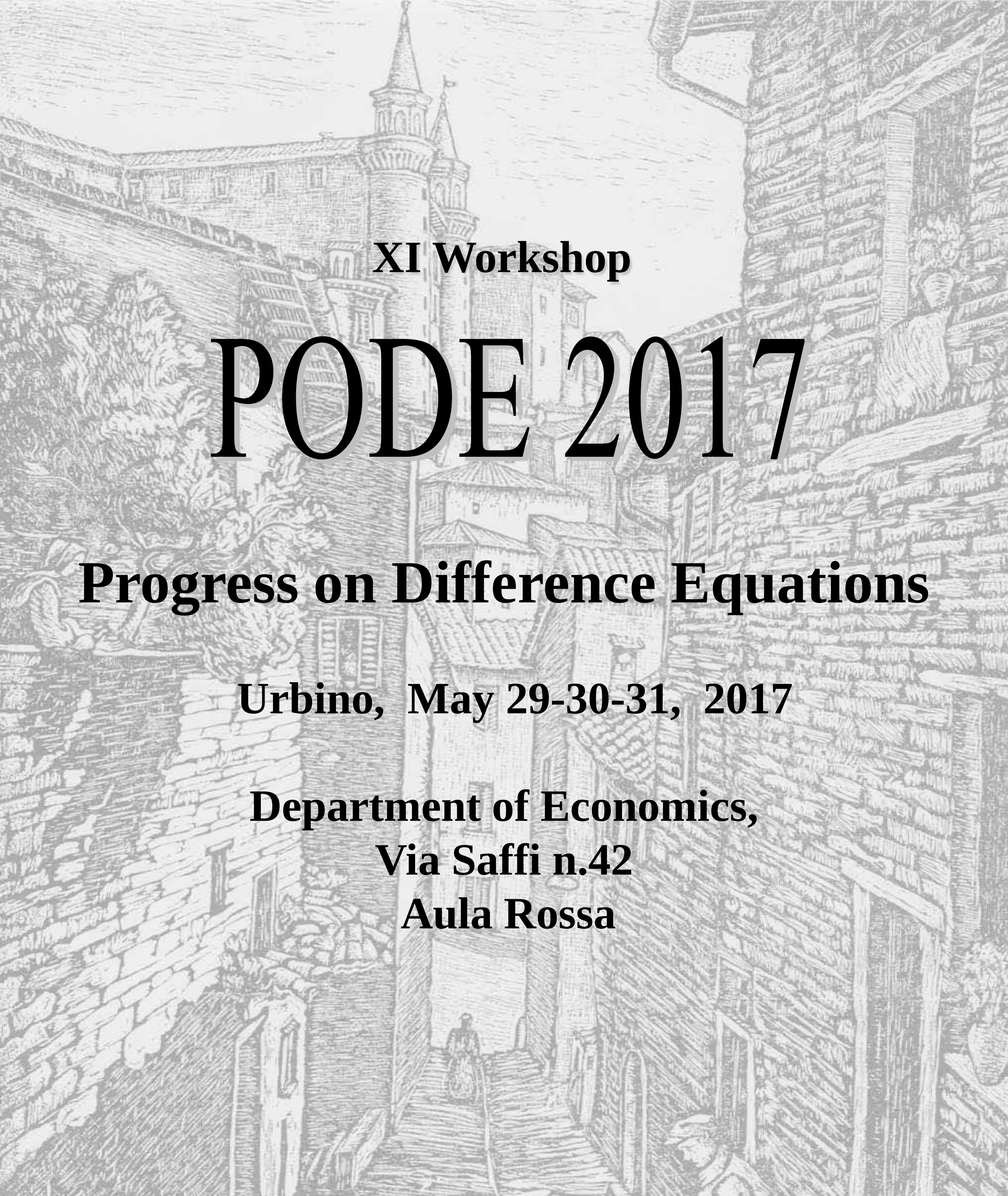

XI Workshop

PODE 2017

Progress on Difference Equations

Urbino, May 29-30-31, 2017

Department of Economics,
Via Saffi n.42
Aula Rossa

1506 UNIVERSITÀ DEGLI STUDI DI URBINO CARLO BO

DESP DIPARTIMENTO DI ECONOMIA, SOCIETÀ, POLITICA

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Caos e Complessità

International Society of Difference Equations

11th PODE International Conference.
Urbino (Italy) May 29-31, 2017

Conference web page:
http://www.mdef.it/meetings-workshops/pode-2017/

May 28, h. 19.30
Welcome Party
at Bar Centrale "Caffè Basili"
Piazza della Repubblica, Center of Urbino

Scientific Committee
Stephen Baigent (UK)
Saber Elaydi (USA)
Laura Gardini (Italy)
Malgosia Guzowska (Poland)
Stephen Siegmund (Germany)

Local Organizing Committee
Gian Italo Bischi (Università di Urbino)
Laura Gardini (Università di Urbino)
Fabio Tramontana (Università Cattolica di Milano)

May 29-31 Palazzo Battiferri, via Saffi n. 42, Aula Rossa.

Program

Monday, May 29

8:30-9:00 *Registration*

9:00-9.10 *Opening Section*

9:10-10:50 Chair: Saber Elaydi

- Andrejs Reinfelds: Dynamic equivalence of dynamic systems on time scales
- Agnese Šuste: On local Stability of Some Exponential-Type Difference Equations
- Inese Bula: Periodic and eventually periodic solutions of a single neuron model
- Saber Elaydi: A Dynamically consistent discretization method

10:50-11:20 *Coffee-Break*

11:20-13:00 Chair: Stephen Baigent

- Daniel Franco: Harvest timing effect on discrete population models
- Juan Segura Salinas: Population control through adaptive limiters
- Antonin Slavik: Discrete Bessel functions and partial difference equations
- Stephen Baigent: Global stability in competitive population models

13:00 -15:00 *Lunch at University Guest House*

15:10-16:50 Chair: Anna Agliari

- Nicolò Pecora: A monopoly model with memory: analysis of 1:4 resonance
- Fausto Cavalli (with A. Naimzada): Pattern formation and periodic attractors in a spatially extended consumer model
- Lorenzo Cerboni Baiardi: (with G.I. Bischi and D. Radi): Asymptotic dynamics of a 2D map modelling renewable resource exploitation
- Anna Agliari (with N. Pecora): Dynamics inside periodicity regions: co-dimension 2 and global bifurcations

16:50-17:20 *Coffee-Break*

17:20-18:35 Chair: Marek Zdun

- Hikmet Kemaloğlu: Some spectral results of Finite Difference Sturm-Liouville Problem
- Zbigniew Leśniak: On topological properties of Brouwer homeomorphisms
- Grzegorz Guzik: Asymptotic properties of cocycles contracting on fibers
- Marek Zdun: On singular iteration groups of monotonic functions

19:30 *Dinner at the University Guest House*

Tuesday, May 30

9:10-10:50 Chair: Robert Sacker

- Cristina Serpa (with J. Buescu): Systems of iterative functional equations: some constructive examples
- Jonas Volek: Landesman-Lazer conditions for difference equations with graph Laplacian
- Armengol Gasull: A dynamic type Parrondo paradox
- Robert Sacker (with G.R. Sell): Almost Periodicity, Ricker Map, Beverton-Holt Map and Others, a General Method

10:50-11:20 *Coffee-Break*

11:20-13:00 Chair: Fabio Lamantia

- Guzowska Malgorzata (with E. Michetti): Local and Global Dynamics in a Discrete Ramsey Model
- Roya Makrooni (with L. Gardini) Piecewise monotone maps chaotic in the whole interval
- Anastasiia Panchuk (with I. Sushko and F. Westerhoff): Bifurcation structures related to chaotic attractors in a 1D PWL map defined on three partitions
- Fabio Lamantia (with M. Pezzino): The effects of tax competition on compliance and migration: an evolutionary analysis through piecewise-smooth maps

13:00 -15:00 *Lunch at University Guest House*

15:10-16:50 Chair: Iryna Sushko

- Francesca Grassetti (with C. Mammana and E. Michetti): Qualitative dynamics of Solow-Swan growth model
- Andrea Caravaggio (with M. Sodini): Multiple attractors and dynamics in an OLG model with productive environment
- Davide Radi (with L. Gardini and P. Harting): An evolutionary generalization of the Shelling's neighborhood tipping model: Inequality and segregation
- Iryna Sushko (With L. Gardini): Smale Horseshoe in 2D Noninvertible Maps

16:50-17:20 *Coffee-Break*

17:20-19:00 Chair: Witold Jarczyk

- Petr Stehlík: Spectra of indefinite perturbations of discrete operators
- Dorota Głazowska and Justyna Jarczyk (with W. Jarczyk): Embeddability of pairs of weighted quasi-arithmetic means in a semiflow
- Mariusz Sudzik: On a problem of Derfel
- Witold Jarczyk: Iterated procedure of joining means

20:00 *Social Dinner at the University College "Il Colle"*

Wednesday, May 31

9:10-10:50 Chair: Francisco Balibrea

- Maria Teresa Silva (with L. Silva and S. Fernandes): Convergence time to equilibrium distributions of autonomous and periodic non autonomous graphs
- Luis Silva: Periodic attractors on a family of nonautonomous dynamical systems generated by stunted tent maps
- João Ferreira Alves: Spectral invariants of Markov Periodic Systems
- Francisco Balibrea: The logistic two delays difference equation, revisited

10:50-11:20 *Coffee-Break*

11:20-13:00 Chair: Gian Italo Bischi

- Mauro Sodini (with L. Gori): Price competition in a nonlinear differentiated duopoly with isoelastic demand
- Fabio Tramontana (with A. Naimzada and N. Pecora): A behavioral cobweb model
- Giovanni Campisi (with S. Brianzoni and A. Russo): Dynamical Analysis of a Financial Model in Discrete Time with Heterogeneous Agents
- Gian Italo Bischi: Some recent global results for recurrences arising from dynamic economic modelling

13:00 -15:00 *Lunch at University Guest House*

15:00-16:40 Chair: Henrique Oliveira

- Emma D'Aniello: Classes of functions and their invariant sets (attractors)
- Peter Raith: Mixing properties for monotonic mod one transformations with two pieces
- Laura Tedeschini Lalli (with C. Falcolini) "Dribbling Method" for continuation of bifurcation curves from conservative into dissipative systems
- Henrique Oliveira: Bifurcation equations for periodic orbits of implicit discrete dynamical systems

16:40-17:10 *Coffee-Break*

17:10-18:25 Chair: Laura Gardini

- Elias Camouzis (with H. Kollias and I. Leventidis): Construction of an infinite path towards perfect market
- Eros Pruscini (with G.I. Bischi and U. Merlone): Evolutionary dynamics in threshold binary games
- Laura Gardini (with Z. Du): Notes on the anharmonic routes in piecewise monotone maps

18:25-19:00 *Open Problems and Remarks*

19:00-19:10 *Closing section*

19:30 *Dinner at the University Guest House*

ABSTRACTS

In alphabetic order of the speaker

Dynamics inside periodicity regions: co-dimension 2 and global bifurcations

Anna Agliari*[1] and Nicolò Pecora†[1]

[1] *Dept. of Economics and Social Science, Catholic University, Piacenza, Italy*

Abstract

We study a simple model based on the cobweb demand-supply framework with costly innovators and free imitators. The evolutionary selection between technologies depends on a performance measure which is related to the degree of memory (see [1], [2], and [3]).

The resulting dynamics is described by a two-dimensional discrete map. The local stability analysis of the map shows that its unique fixed point may lose stability either via flip bifurcation or supercritical Neimark-Sacker bifurcation. This latter is the starting point of the present talk, in which we investigate some different situations associated with its occurrence.

In particular, through numerical simulations of the map we investigate the dynamics associated with different eventualities, such as the occurrence of global bifurcations and codimension-2 bifurcations (1:3 resonance). Furthermore, some periodicity regions are numerically investigated. In fact, a typical structure of the bifurcation diagram, in a two-dimensional parameter plane, is given by the so-called "Arnold's tongues" issuing from the Neimark-Sacker bifurcation curve. It is well known that the boundaries of a p/q tongue are saddle-node bifurcation curves of a cycle of period q, and inside the tongue (in a neighborhood of the Neimark-Sacker bifurcation curve) we generally have an attracting closed invariant set formed by a saddle-node connection, that is, the unstable set of the saddle q-cycle reaches the node (or focus) q-cycle, thus forming a closed attracting curve. Through the analysis of a particular periodicity region we shall show that as long as we move inside the tongue, there exist peculiar situations in which an attracting closed curve coexists with a periodic orbit. Such coexistence eventuality first appears and then disappears due to global bifurcations occurring within the periodicity region. We shall show that the bifurcation mechanism leading to invariant closed curves may be associated with a pair of cycles, a saddle cycle and an attracting one (node or focus), and the appearance/disappearance may be related to a saddle-connection, also called homoclinic connection ([4]).

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References

[1] Agliari, A., Naimzada, A., & Pecora, N. (2017). Bifurcation structures of a cobweb model with memory and competing technologies. Commun Nonlinear Sci Numer Simulat, Doi: 10.1016/j.cnsns.2017.04.008.

[2] Diks, C., Hommes, C., & Zeppini, P. (2013). More memory under evolutionary learning may lead to chaos. Physica A: Statistical Mechanics and its Applications, 392(4), 808-812.

[3] Hommes, C., & Zeppini, P. (2014). Innovate or Imitate? Behavioural technological change. Journal of Economic Dynamics and Control, 48, 308-324.

[4] Agliari, A., Bischi, G. I., Dieci, R., & Gardini, L. (2005). Global bifurcations of closed invariant curves in two-dimensional maps: a computer assisted study. International Journal of Bifurcation and Chaos, 15(04), 1285-1328.

Spectral Invariants of Markov Periodic Systems

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Center for Mathematical Analysis, Geometry and Dynamical Systems
Instituto Superior Técnico, Universidade de Lisboa, Portugal

Consider a nonautonomous dynamical system determined by a p-periodic sequence, $(f_i)_{i\in\mathbb{N}}$, of continuos self maps on $I=[a,b]$. Such a system is called Markov if there exists a finite partition of I,

$$I_1=[c_0=a,c_1]\,,I_2=[c_1,c_2]\,,...,I_k=[c_{k-1},c_k=b]\,,$$

satisfying the following:

1) f_i is monotone on I_j, for $i=0,...,p-1$; $j=1,...,k$.

2) f_i is invariant in $C=\{c_0,...,c_k\}$, that is $f_i(C)\subset C$, for $i=0,...,p-1$.

The spectral invariants of a Markov periodic system $F=(f_i)_{i\in\mathbb{N}}$ are defined as the spectral invariants of a nonautonomous graph attached to F. Since these invariants characterize the periodic structure of the non-autonomous graph, it is natural to ask what role these invariants play in characterizing the periodic structure of F.

References

[1] Z. AlSharawi, J. Angelos, S. Elaydi, L. Rakesh, An extension of Sharkovsky's theorem to periodic difference equations, J. Math. Anal. Appl., 316 (2006), 128-141.

[2] Z. AlSharawi, J. Cánovas, A. Linero, Folding and unfolding in periodic difference equations, J. Math. Anal. Appl. 417 (2014) 643–659.

[3] J. F. Alves, L. Silva, Periodic paths on nonautonomous graphs, Lin. Algeb. Appl. 437 (2012) 1003–1015.

[4] J. F. Alves, M. Málek, L. Silva, Spectral invariants of periodic nonautonomous discrete dynamical systems, J. Math. Anal. Appl., 430 (2015), 85–97.

[5] F. Balibrea, P. Oprocha, Weak mixing and chaos in nonautonomous discrete systems, Appl. Math. Lett. 25(8) (2012) 1135–1141.

[6] S. Elaydi, R. Luís, H.M. Oliveira, Local bifurcation in one-dimensional nonautonomous periodic difference equations, Internat. J. Bifur. Chaos Appl. Sci. Engrg. 23(03) (2013).

[7] J. S. Cánovas, A. Linero, Periodic structure of alternating continuous interval maps, J. Difference Equ. Appl., 12 (2006), 847-858.

Global stability in competitive population models

Stephen Baigent

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Abstract

Local stability of fixed points of a map is usually simple to establish, but global stability is a much harder problem. Here we discuss a geometric method for proving global stability of many well-known population models. This method is then used to obtain the global stability of interior fixed points for some well-known competitive population models. Lastly we discuss how the method can be refined in competitive models if they are known to possess a carrying simplex, i.e. an invariant manifold of codimension-1 that attracts all points bar the origin.

The logistic two delays difference equation, revisited

Francisco Balibrea

Abstract. We analyze mainly the state of art of logistic second order difference equations with two delays. They model the evolution of populations with respect to seasons in time $n \in \mathbb{N}$. We will concentrate in cases when the population is composed of only one species. The appearance of two delays are usually related with the effect of food in the evolution of the population.

The equation to consider is

$$x_{n+1} = ax_n(1 - x_{n-1}) \quad a > 0$$

As an adequate tool to understand the different behaviours of solutions of the equation, we use an unfolding of it obtaining a discrete dynamical system of dimension two, defined in the unit square by the transformation

$$L_a(x, y) = (y, ay(1 - x))$$

where $(x, y) \in [0, 1]^2$

We review some dynamical properties already known like periodic solutions and local linear analysis around the fixed points of the unfolding. Besides we also introduce new results on the analysis of the behaviour of invariant curves. All analysis depend on the parameter a. Our study is mainly devoted to the range $a \in (0, 2]$, the setting where we will give some new results. Open problems remain for the range when $a \geq 2.27$ where 2.27 is a critical value. At approximately this value the transformation has infinite many periodic orbits

Reference:

F.Balibrea, *A logistic non-linear difference equation with two-delays*. To appear in a Special Issue devoted to Model in Biology in Springer

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Some recent global results for recurrences arising from dynamic economic modeling

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Abstract

In this lecture we give a summary of some nonlinear discrete-time dynamical systems recently studied in the framework of the research group on "Dynamic Models in Economics and Finance". Starting from these models, ranging from oligopoly games with boundedly rational agents to systems with expectatoins and learning with fading memory, we show how analytical, geometric and numerical methods have been combined to obtain a global qualitative analysis of some complex dynamic scenarios. Methods for the study of some nonlocal (or global) properties have been employed, including some global (or contact) bifurcations such as those based on the critical sets in the analysis of iterated noninvertible maps, and other ones have been created ad hoc, such as the study of focal points and prefocal sets, new kinds of singularities that characterize the global bifurcations of recurrences with a vanishing denominator. Chaos synchronization phenomena, with on-off intermittency, bubbling and riddled basins, observed in models with invariant submanifolds with lower dimension than the phase space, that naturally arise in symmetric models with identical agents and evolutionary games, are described in terms of Milnor attractors, natural transverse Lyapunov exponents and minimal absorbing areas.

Some references

Bischi G.I., Naimzada A. (1997). "Global analysis of a nonlinear model with learning". *Economic Notes,* **26** (3), 143-174.
Bischi G.I. and L. Gardini (1998). "Role of invariant and minimal absorbing areas in chaos synchronization". *Physical Review E*. **58** (5), 5710-5719.
Bischi G.I., L. Stefanini and L. Gardini (1998). "Synchronization, intermittency and critical curves in a duopoly game". *Mathematics and Computers in Simulation*. **44** 559-585
Bischi G.I., L. Gardini and C. Mira (1999). "Plane maps with denominator. Part 1: some generic properties". *International Journal of Bifurcation and Chaos*. **9** (1), 119-153.
Bischi G.I., C. Mira and L. Gardini (2000). "Unbounded sets of attraction". *International Journal of Bifurcation and Chaos*. **10** (6), 1437-1469.
Bischi G.I. and L. Gardini (2000). "Global Properties of Symmetric Competition Models with Riddling and Blowout Phenomena". *Discrete Dynamics in Nature and Society*. **5** 149-160.
Gardini L., Bischi G.I., Mira C. (2007). "Maps with Vanishing Denominators", *Scholarpedia http://www.scholarpedia.org/article/Maps_with_vanishing_denominators*
Bischi G.I., Cerboni Baiardi L. (2015). "Fallacies of composition in nonlinear marketing models". *Communications in Nonlinear Science and Numerical Simulation*. **20** 209-228.
Bischi G.I., Lamantia F., Radi D. (2015). "An evolutionary Cournot model with limited market knowledge". *Journal of Economic Behavior & Organization*. **116** 219-238
Bischi G.I., Cerboni Baiardi L., " Bubbling, Riddling, Blowout and Critical Curves", *Journal of Difference Equations and Applications,* 2017

Periodic and eventually periodic solutions of a single neuron model

INESE BULA

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The basic model of our investigation is a delayed differential equation $x'(t) = -g(x(t-\tau))$ that is used as a model for a single neuron with no internal decay ([3]), where $g : \mathbf{R} \to \mathbf{R}$ is either a sigmoid function or a piecewise linear signal function and $\tau \leq 0$ is a synaptic transmission delay. From this equation it is possible to obtain a discrete model in the form

$$x_{n+1} = \beta x_n - g(x_n), \quad n = 0, 1, 2, ...$$

In [4] a difference equation was analyzed as a single neuron model, where $\beta > 0$ is an internal decay rate and a signal function g is the following piecewise constant function with McCulloch-Pitts nonlinearity:

$$g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases} \tag{1}$$

Now we will study the following non-autonomous piecewise linear difference equation:

$$x_{n+1} = \beta_n x_n - g(x_n), \quad n = 0, 1, 2, ..., \tag{2}$$

where $(\beta_n)_{n=0}^{\infty}$ is a period two or three sequence

$$\beta_n = \begin{cases} \beta_0, & \text{if } n = 2k, \\ \beta_1, & \text{if } n = 2k+1, \end{cases} \quad \text{or} \quad \beta_n = \begin{cases} \beta_0, & \text{if } n = 3k, \\ \beta_1, & \text{if } n = 3k+1, \\ \beta_2, & \text{if } n = 3k+2, \end{cases} \quad k = 0, 1, 2, ...$$

and g is in the form (1).

In [1] and [2] we studied this model where $(\beta_n)_{n=0}^{\infty}$ is a period two and three sequences respectively. In our presentation we show that for model (2) there are many periodic and eventyally periodic solutions, but the order of period depend on period of coefficients β_n.

This work is collaboration with M.A. Radin, Rochester Institute of Technology, USA.

References

[1] I. Bula, M.A. Radin, *Periodic Orbits of a Neuron Model with Periodic Internal Decay Rate*, Applied Mathematics and Computation 266 (2015) 293–303.

[2] I. Bula, M.A. Radin, N. Wilkins, *Neuron model with a period three internal decay rate*, Electronic Journal of Qualitative Theory of Differential Equations (2017, to appear).

[3] J. Wu, Introduction to Neural Dynamics and Signal Transmission Delay. De Gruyter, Berlin (2001).

[4] Z. Zhou, *Periodic Orbits on Discrete Dynamical Systems*, Computers and Mathematics with Applications 45 (2003) 1155–1161.

Construction of an Infinite Path Towards Perfect Market

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May 7, 2017

Abstract

In this article we present a Cournot type market, with an infinite number of firms and variable marginal costs. At the equilibrium of this market the price of the homogeneous product is equal with the "average cost of the entire market",

$$\bar{c} = \lim_{n\to\infty} \frac{\sum_{j=1}^{n} c_j}{n},$$

where c_j is the cost of the firm j when firm j is active in the market. We present cases, for which the equilibrium market contains an infinite number of infinitesimal firms that coexist with a finite number of non-infinitesimal firms. In particular, when all marginal costs are identical, the equilibrium market is a perfectly competitive market with an infinite number of infinitesimal firms. We also establish that such markets might result as uniform limits of sequences of Cournot-type markets each with a finite number of firms.

Dynamical Analysis of a Financial Model in Discrete Time with Heterogeneous Agents

Giovanni Campisi,* Serena Brianzoni,† Alberto Russo ‡

Keywords
Piecewise Linear Maps, financial crises, heterogeneous agents, discrete dynamics.

In the present work we propose a financial model, following [9]. More precisely, we analyze a financial market populated by three types of agents – fundamentalists, chartists and imitators.The latter submit buying/selling orders according to different trading rules using a 2D Piecewise Linear Map (PWL).
Our aim is to extend analytically the model of [9] which provides mainly numerical results. In particular, we would like to better investigate the bifurcations shown by the model and the large variety of periodic cycles produced. Moreover we frame our work in the light of the dynamics of financial crises as described by [4] and [2]. There are only few papers analyzing different states of the market as proposed by previous authors, see [1], [2] and [6] for example. We think that the use of PWL maps can help to enlarge this strand of research. Finally, we provide numerical simulations in order to detect the different regimes of the market and to better understand the model.

References

[1] C. Chiarella, X.Z. He, W. Huang, H. Zheng, *Estimating behavioural heterogeneity under regime switching*, Journal of Economic Behavior Organization, Volume 83, Number 3, pages 446-460, Elsevier, 2012.

[2] W. Huang, H. Zheng, W.M. Chia, *Financial crises and interacting heterogeneous agents*, Journal of Economic Dynamics and Control, Volume 34, Number 6, pages 1105-1122, Elsevier, 2010.

[3] W. Huang, H. Zheng, *Financial crises and regime-dependent dynamics*, Journal of Economic Behavior & Organization, Volume 82, Number 2, pages 445-461, Elsevier, 2012.

[4] M. Gallegati, A. Palestrini, J.B. Rosser, *the period of financial distress in speculative markets: interacting heterogeneous agents and financial constraints*, Macroeconomic Dynamics, Volume 15,Number 1, pages 60-79, Cambridge Univ Press, 2011.

[5] L. Gardini, F. Tramontana, V. Avrutin, M. Schanz, *Border-collision bifurcations in 1D piecewise-linear maps and Leonov's approach*, International Journal of Bifurcation and Chaos, Volume 20,Number 10, pages 3085-3104, World Scientific, 2010.

[6] S. Manzan, F. Westerhoff, *Heterogeneous expectations, exchange rate dynamics and predictability*, Journal of Economic Behavior Organization, Volume 64, Number 1, pages 111-128, Elsevier, 2007.

[7] F. Tramontana, L. Gardini, F. Westerhoff, *On the complicated price dynamics of a simple one-dimensional discontinuous financial market model with heterogeneous interacting traders*, Journal of Economic Behavior & Organization, Volume 74,Number 3, pages 187-205, Elsevier, 2010.

[8] F. Tramontana, L. Gardini, V. Avrutin, M. Schanz, *Period adding in piecewise linear maps with two discontinuities*, International Journal of Bifurcation and Chaos, Volume 22,Number 3, World Scientific, 2012.

[9] F. Tramontana, *The role of cognitively biased imitators in a small scale agent based financial market*, DEM Working Papers Series, University of Pavia, Department of Economics and Management, 2013.

[10] F. Tramontana, F. Westerhoff, L. Gardini, *One-dimensional maps with two discontinuity points and three linear branches: mathematical lessons for understanding the dynamics of financial markets*, Decisions in Economics and Finance, Volume 37,Number 3, pages 27-51 Springer, 2014.

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Multiple attractors and dynamics in an OLG model with productive environment

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April, 2017

Abstract

This work analyses an overlapping generations model in which environment enters the production function as a productive input and environmental quality is assumed as a free public resource damaged by the economic activity. In addition, public expenditures for environmental maintenance, financed by a share of general labour income taxation are considered. By investigating some geometric properties of the map and performing numerical simulations, we investigate consequences of the interplay between environmental public expenditure and private sector, describing different scenarios and characterizing dynamical properties of the model. According to different parameters configurations, multiple equilibria as well as complex dynamics may appear.

References

[1] A. Antoci. Environmental degradation as engine of undesirable economic growth via self-protection consumption choices. Ecological Economics, 68(5):13851397, 2009.

[2] A. Antoci, M. Galeotti, and P. Russu. Poverty trap and global indeterminacy in a growth model with open-access natural resources. Journal of Economic Theory, 146(2):569591, 2011.

[3] A. Antoci, L. Gori, and M. Sodini. Nonlinear dynamics and global indeterminacy in an overlapping generations model with environmental resources. Communications in Nonlinear Science and Numerical Simulation, 38:5971, 2016.

[4] T. R. Barman and M. R. Gupta. Public expenditure, environment, and economic growth. Journal of Public Economic Theory, 12(6):11091134, 2010.

[5] R. H. Day. Irregular growth cycles. The American Economic Review, 72(3): 406414, 1982.

[6] P. A. Diamond. National debt in a neoclassical growth model. The American Economic Review, 55(5):11261150, 1965.

[7] M. Fodha and T. Seegmuller. A note on environmental policy and public debt stabilization. Macroeconomic Dynamics, 16(03):477492, 2012.

[8] A. John and R. Pecchenino. An overlapping generations model of growth and the environment. The Economic Journal, pages 13931410, 1994.

[9] A. D. A. Le Kama. Sustainable growth, renewable resources and pollution. Journal of Economic Dynamics and Control, 25(12):19111918, 2001.

[10] A. Naimzada and M. Sodini. Multiple attractors and nonlinear dynamics in an overlapping generations model with environment. Discrete Dynamics in Nature and Society, 2010, 2010.

[11] J. B. Rosser. Complex ecologiceconomic dynamics and environmental policy. Ecological Economics, 37(1):2337, 2001.

[12] J. Zhang. Environmental sustainability, nonlinear dynamics and chaos. Eco- nomic Theory, 14(2):489500, 1999.

Pattern formation and periodic attractors in a spatially extended consumer model

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A. Naimzada
Dipartimento di Economia, Metodi Quantitativi e Strategie di impresa, Università di Milano-Bicocca.

In this talk, we present and study two families of high dimensional systems of difference equations for the modelling of an optimal choice consumer model with local interaction. In the first family of models, the equation governing the dynamics of each variable x_i is a convex combination of two non-decreasing one-dimensional maps h and f, respectively depending on x_i and on a reduced number of other variables x_j, where $j \in N(i)$ representing a neighborhood of i. The model describe a situation in which agents decide their optimal consumption level on the basis of their past consumption experience and on those of their neighboring agents. The parameter ν weighting the convex combination represents the degree of social interaction. We analytically characterize spatially homogeneous and heterogeneous (pattern formation) steady states and we prove, using Lyapunov stability, their local stability independently of ν. Moreover, despite h and f are both monotone, we prove that stable period-2 cycles, coexisting with stable steady states, can emerge depending on ν. We study and compare their basins of attraction and the probability to converge toward either a steady state or to a period-2 cycle.

In the second family, ν becomes a state variable and it is governed by an endogenous evolutionary mechanism. We characterize, both through analytical and numerical investigations, the possible steady states and we show the emergence of periodic attractors in which different periods coexist in different spatial regions.

F. Cavalli, A. Naimzada, M. Pireddu “Emergence of complex social behaviors from the canonical consumption model”, Mind & Society 15,71–81,2016

A. Bell, “Locally interdependent preferences in a general equilibrium environment” JEBO 47,309-333, 2002

On a discrete-time model with replicator dynamics in renewable resource exploitation

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Abstract

We consider a discrete time version of the model proposed by Lamantia and Radi (2015) to describe a fishery where a population regulated by a logistic growth function is exploited by a pool of agents that can choose, at each time period, between two different harvesting strategies according to a profit-driven evolutionary selection rule. The resulting discrete dynamical system, represented by a two-dimensional nonlinear map, is characterized by the presence of invariant lines on which the dynamics are governed by one-dimensional restrictions that represent pure, i.e. adopted by all players, strategies. However, interesting dynamics related to interior attractors, where players playing both strategies coexist, are evidenced by analytical as well as numerical methods that reveal local and global bifurcations.

Keywords: discrete-time population model, replicator dynamics, resource exploitation, attractors, bifurcations.

CLASSES OF FUNCTIONS AND THEIR INVARIANT SETS (ATTRACTORS)

EMMA D'ANIELLO

Abstract. Let X be a complete metric space with $\mathcal{S} = \{S_1, \dots, S_N\}$ a finite set of contraction maps from X to itself. We call a non-empty subset F of X *invariant*, or an *attractor*, for the iterated function system (IFS) [15] $\mathcal{S}$ if $F = \cup_{i=1}^{N} S_i(F) = \mathcal{S}(F)$. As an application of the Banach-Caccioppoli fixed point theorem, it turns out that, for a particular finite set of contraction maps $\mathcal{S}$, there exists a unique invariant compact set $F \subseteq X$. This and other results from [15] are summarized below.

Theorem 0.1. *Let (X, d) be a complete metric space with $\mathcal{S} = \{S_1, \dots, S_N\}$ a finite set of contraction maps from X to itself.*

a. *There exists a unique non-empty compact set $F \subseteq X$ such that $F = \mathcal{S}(F)$.*
b. *The set F is the closure of the set of fixed points $s_{i_1 \dots i_p}$ of finite compositions $S_{i_1} \circ \dots \circ S_{i_p}$ of members of $\mathcal{S}$.*
c. *If A is any compact set in X, then $\lim_{p \to \infty} \mathcal{S}^p(A) = F$ in the Hausdorff metric.*

When the contractions are similarities, so that $|S_i(x) - S_i(y)| = r_i|x - y|$ for all x, y in X, and $0 < r_i < 1$, then each S_i transforms subsets of X into geometrically similar sets, giving rise to invariant sets that are self-similar. When the images of the $S_i(F)$ do not overlap "too much" (that is, the *open set condition* is satisfied), then the self-similar set $F = \cup_{i=1}^{N} S_i(F)$ has Hausdorff dimension equal to the value of s satisfying $\sum_{i=1}^{N} {r_i}^s = 1$.
We investigate properties (the geometry, the Hausdorff dimension, etc...) of the attractors generated by various types of iterated function systems: those comprised of similarities, those which satisfy the open set condition, and those made up of weak contractions.
We furnish the class $\mathcal{K}$ of compact subsets of X with the Hausdorff metric $\mathcal{H}$; this space is complete, so that good use can be made of the Baire category theorem. We examine the collection of compact sets which are attractors and study how large (or small) it is. In particular, we focus our attention on the case when $X = [0, 1]^n$, $n \geq 1$.
Our study is motivated by researches dealing with the structure of attractive sets. Some of them are listed below.

References

[1] T. Banakh, M. Nowak, F. Strobin, *Detecting topological and Banach fractals among zero-dimensional spaces*, Topology Appl. 196 (2015), part A, 22-30.
[2] A.M. Bruckner, J. Smítal, *The structure of ω-limit sets for continuous maps of the interval*, Math. Bohem. 117 (1992), no. 1, 42-47.
[3] A.M. Bruckner, T.H. Steele, *The Lipschitz structure of continuous self-maps of generic compact sets*, J. Math. Anal. Appl. 188 (1994), no. 3, 798-808.
[4] E. D'Aniello, U.B. Darji, T.H. Steele, *Ubiquity of odometers in topological dynamical systems*, Topology Appl. 156 (2008), no. 2, 240-245.

[5] E. D'Aniello, P.D. Humke, T.H. Steele, *The space of adding machines generated by continuous self maps of manifolds*, Topology Appl. 157 (2010), no. 5, 954-960.
[6] E. D'Aniello, T.H. Steele, *Attractors for iterated function schemes on* $[0,1]^N$ *are exceptional*, J. Math. Anal. Appl. 424 (2015), no. 1, 537-541.
[7] E. D'Aniello, T.H. Steele, *Attractors for iterated function systems*, J. Fractal Geom. 3 (2016), no. 2, 95-117.
[8] E. D'Aniello, T.H. Steele, *Attractors for iterated function schemes on* $[0,1]^N$ *are exceptional*, J. Math. Anal. Appl. 424 (2015), 537-541.
[9] E. D'Aniello, T.H. Steele, *The persistence of* ω*-limit sets defined on compact spaces*, J. Math. Anal. Appl. 413 (2014), no. 2, 789-799.
[10] E. D'Aniello, T.H. Steele, *Stability in the family of* ω*-limit sets of alternating systems*, J. Math. Anal. Appl. 389 (2012), no. 2, 1191-1203.
[11] E. D'Aniello, T.H. Steele, *The* ω*-limit sets of alternating systems*, J. Difference Equ. Appl. 17 (2011), no. 12, 1793-1799.
[12] E. D'Aniello, T.H. Steele, *A* C^1 *function for which the* ω*-limit points are not contained in the closure of the periodic points*, Trans. Amer. Math. Soc. 355 (2003), no. 6, 2545-2556.
[13] Feng De-Jun, Wang Yang, *On the structures of generating iterated function systems of Cantor sets*, Adv. Math. 222 (2009), no. 6, 1964–1981.
[14] K. Falconer, *Fractal Geometry: Mathematical Foundations and Applications*, John Wiley and Sons, 1990.
[15] J. E. Hutchinson, *Fractals and self-similarity*, Indiana Univ. Math. J. 30 (1981), 713–747.
[16] M. Nowak, *Topological classification of scattered IFS-attractors*, Topology Appl. 160 (2013), no. 14, 1889–1901.
[17] A. Schief, *Separation properties for self-similar sets*, Proc. Amer. Math. Soc. 122 (1994), no. 1, 111-115.

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A Dynamically consistent discretization method

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Abstract

We present a nonstandard discretization method related to the methods of Mickens for converting single species population models from continuous time to discrete time.
The discretization method preserves the original dynamic properties of the continuous model, in the sense of equilibria and their stability and bifurcation characteristics, for a wider range of parameter values than do the common discretizations used in ecology. Furthermore, the discretization produces solution trajectories in remarkable agreement with those of the continuous model, so that the dynamics of the resulting difference equation is largely independent of the size of the step interval used. Examples of single and multi-species models with and without negative density dependence, with an Allee effect, and with an alternative positive stable equilibrium (predator pit) are studied.
We provide a comparative analysis of bifurcations of ODE and DE systems of some of these models. Partly out of historical tradition, ecological population models are frequently presented as ordinary differential equations (ODEs), but discretization of such models is generally necessary before such models can become useful scientific hypotheses to be tested with time series observations of population abundances. Results presented here will be important to future ecological studies that seek to evaluate the pervasiveness and strength of negative density dependence as well as Allee effects, along with the prospects of alternative stable states, in natural populations.

Harvest timing effect on discrete population models

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Abstract

We will present results obtained in [1, 2] on the impact of different harvest times on the stability of population dynamics using a discrete-time model proposed in [3]. Contrary to expectations, we will show that timing can be destabilizing in population models with overcompensation. Nevertheless, we will prove that high enough harvest intensities are always stabilizing and that timing can be stabilizing by itself.
We will consider the effect of timing on global stability as well. First, showing that strong enough removal efforts create a positive equilibrium that attracts all positive solutions under general conditions. Second, discussing how to obtain for the Ricker case a complementary global stability result independent of timing and valid for low-medium harvesting.

This talk is based on joint work with with H. Logemann (U. Bath, UK), J. Perán (UNED, Spain) and J. Segura (UNED and U. Pompeu-Fabra, Spain).

References

[1] D. Franco, H. Logemann, J. Perán, and J. Segura. Dynamics of the discrete Seno population model: Combined effects of harvest timing and intensity on population stability. *Applied Mathematical Modelling*, doi:10.1016/j.apm.2017.02.048, 2017.

[2] D. Franco, J. Perán, and J. Segura. Global stability for a discrete-time population model with harvesting (preprint).

[3] H. Seno. A paradox in discrete single species population dynamics with harvesting/thinning. *Mathematical Biosciences*, 214(1):63–69, 2008.

A dynamic type Parrondo paradox

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The study of periodic discrete dynamical systems is a classical topic that has attracted the researcher's interest in the last years, among other reasons, because they are good models for describing the dynamics of biological systems under periodic fluctuations whether due to external disturbances or effects of seasonality, see [3, 6] and the references therein.

These k-periodic systems can be written as

$$x_{n+1} = f_{n+1}(x_n), \tag{1}$$

with initial condition x_0, and a set of maps $\{f_m\}_{m\in\mathbb{N}}$ such that $f_m = f_\ell$ if $m \equiv \ell \,(\mathrm{mod}\ k)$. It is well-known that system (1) can be studied via the *composition map* $f_{k,k-1,\ldots,1} = f_k \circ f_{k-1} \circ \cdots \circ f_1$.

It can be seen that the fixed point of any composition map $f_{k,\ldots,1}$ in $\mathbb{R}^n$ resulting of the composition of k maps f_j with a common hyperbolic fixed point, which is repeller for all them, must be generically either repeller or a saddle, but it can never be a *local asymptotically stable* (LAS) fixed point. See for instance the examples of [1] or [5, p. 8]. We will show that this third possibility may happen dealing with non-hyperbolic fixed points. Our main result is:

Theorem. *The following statements hold:*

(a) For all $n \geq 1$ there exist $k \geq 3$ polynomial maps $f_i : \mathcal{U} \subseteq \mathbb{R}^n \to \mathbb{R}^n$, for $i \in \{1, \ldots, k\}$, sharing a common fixed point p which is repeller (resp. LAS) for each map, and such that p is LAS (resp. repeller) for the composition map $f_{k,k-1,\ldots,1}$. Furthermore, for one-dimensional maps ($n = 1$), this result is optimal on k, that is, it

is not possible to find only two of such maps such that the corresponding composition map $f_{2,1}$ satisfies the given properties.

(b) For all $n = 2m \geq 2$ there exist 2 polynomial maps $f_1, f_2 : \mathcal{U} \subseteq \mathbb{R}^{2m} \to \mathbb{R}^{2m}$, sharing a common fixed point p which is repeller (resp. LAS) for both maps, and such that p is LAS (resp. repeller) for the composition map $f_{2,1}$.

The so called *Parrondo's paradox* is a paradox in game theory, that essentially says that *a combination of losing strategies becomes a winning strategy*, see [4]. Our result implies that in the non-hyperbolic case the periodicity can destroy the repeller character of the common fixed points, giving rise to attracting points for the complete non-autonomous system, showing, in consequence, the existence of a kind of Parrondo's dynamic type paradox for periodic discrete dynamical systems.

This talk is based on the paper [2].

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References

[1] V.D. Blondel, J. Theys, J.N. Tsitsiklis. *When is a pair of matrices stable?*. In: V.D. Blondel, A. Megretski (eds.). Unsolved problems in Mathematical Systems and Control Theory. Princeton Univ. Press, NJ 2004.

[2] A. Cima, A. Gasull, V. Mañosa. *Parrondo's dynamic paradox for the stability of non-hyperbolic fixed points.* arXiv:1701.05816 [math.DS]

[3] S. Elaydi, R.J. Sacker. *Periodic difference equations, population biology and the Cushing-Henson conjectures.* Math. Biosci. 201 (2006), 195–207.

[4] G.P. Harmer and D. Abbott. *Losing strategies can win by Parrondo's paradox.* Nature (London), Vol. 402, No. 6764 (1999) p. 864.

[5] R. Jungers. The Joint Spectral Radius. Spinger, Berlin 2009.

[6] J.F. Selgrade, J.H. Roberds. *On the structure of attractors for discrete, periodically forced systems with applications to population models.* Physica D 158 (2001), 69–82.

Notes on the anharmonic routes in piecewise monotone maps

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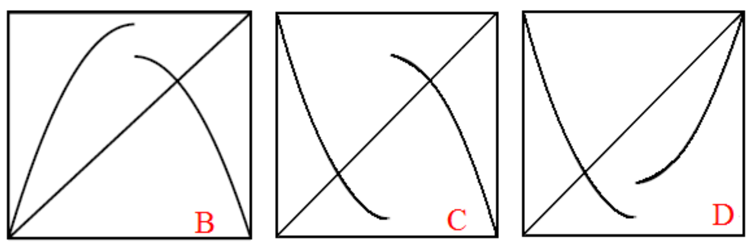
Abstract

In this paper we describe the mechanism of the anharmonic routes to chaos in one-dimensional piecewise monotonic maps with a single discontinuity. This kind of route is via a sequence of period doubling bifurcations and border collision bifurcations which generate periodic orbits of period p_n for $n \geq 1$, on the boundary of chaos, with

$$p_{n+1} = 2p_n + (-1)^n k \quad (1)$$

where k is a nonzero integer, which exists for classes of maps having the shapes as in the following figure:



We analyze the border collision bifurcations of the periodic orbits of the maps and explain the sequences leading to the appearances and disappearances of periodic orbits in terms of bifurcations. An important related question is if there exist maps having anharmonic cascade with $|k| > 1$. Glendinning has claimed that the answer to this question is positive by considering larger classes of maps. But to date, no concrete examples of such maps have been found. In this work we present a counterexample and motivate why such an anharmonic sequence can exist only with $|k| = 1$.

References

Glendinning, P. Robust new routes to chaos in differential equations. Phys. Lett. A 168, 40–46 (1992)

Glendinning, P. The anharmonic route to chaos: kneading theory. Nonlinearity 6, 349–367 (1993)

Glendinning, P. Renormalization for the boundary of chaos in piecewise monotonic maps with a single discontinuity. Nonlinearity 27, 143–162 (2014)

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Embeddability of pairs of weighted quasi-arithmetic means in a semiflow

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This is a joint work with Witold Jarczyk.

For any arbitrary set X we consider *semiflows* $\Phi\colon X\times(0,+\infty)\to X$, that is solutions of the celebrated *translation equation*

$$\Phi\left(\Phi(x,s),t\right)=\Phi(x,s+t).$$

Φ is also called an *iteration semigroup*. If, in addition, $\phi\colon X\to X$ and $\Phi(\cdot,1)=\phi$, then we say that Φ is an *iteration semigroup of the function* ϕ. In the case when X is a topological space and $\Phi(x,\cdot)$ is a continuous function for all $x\in X$, then the semiflow Φ is called *continuous*.

Given an interval I by $\mathcal{CM}(I)$ we denote the set of all continuous strictly monotonic functions defined on I. If $f\in\mathcal{CM}(I)$ and $p\in(0,1)$, then A_p^f denotes a mean on I given by the equality

$$A_p^f(x,y)=f^{-1}\left(pf(x)+(1-p)f(y)\right).$$

It is so-called *weighted quasi-arithmetic mean* generated by f with *the weight* p.

Given functions $f,g\in\mathcal{CM}(I)$ and numbers $p,q\in(0,1)$ the pair $\left(A_p^f,A_q^g\right)$ is said to be *embeddable* if there exists a continuous semigroup $\Phi\colon I^2\times(0,+\infty)\to I^2$ of the pair $\left(A_p^f,A_q^g\right)$ such that for every $t\in(0,+\infty)$ the function $\Phi(\cdot,t)$ is a pair of weighted quasi-arithmetic means.

We find all pairs of weighted quasi-arithmetic means on I, which are embeddable in a continuous semiflow of pair of such means. Moreover, we prove that such embedding is uniquely determined.

QUALITATIVE DYNAMICS OF SOLOW-SWAN GROWTH MODEL

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Abstract. The present work investigates the qualitative dynamics of a Solow-Swan growth model with differential saving as proposed by Böhm and Kaas [1] assuming the Shifted Cobb-Douglas production function as given by Capasso et al. [7]. The resulting model is a discontinuous map generating a poverty trap. We show that, as in Brianzoni et al. [2, 5, 3, 6, 4], fluctuation may arise. Moreover multistability and complex basins emerge.

References

[1] V. Böhm and L. Kaas. "Differential savings, factor shares, and en- dogenous growth cycles". In: *Journal of Economic Dynamics and Control* 24 (2000), pp. 965–980.

[2] S. Brianzoni, C. Mammana, and E. Michetti. "Complex Dynamics in the Neoclassical Growth Model with Differential Savings and Non-Constant Labor Force Growth". In: *Studies in Nonlinear Dynamics and Econometrics* 11.3 (2007).

[3] S. Brianzoni, C. Mammana, and E. Michetti. "Local and Global Dynamics in a Discrete Time Growth Model with Nonconcave Production Function". In: *Discrete Dynamics in Nature and Society* (2012).

[4] S. Brianzoni, C. Mammana, and E. Michetti. “Local and Global Dynamics in a Neoclassical Growth Model with Nonconcave Production Function and Nonconstant Population Growth Rate”. In: *SIAM Journal of Applied Mathematics* 75.1 (2015), pp. 61–74.

[5] S. Brianzoni, C. Mammana, and E. Michetti. “Nonlinear dynamics in a business-cycle model with logistic population growth”. In: *Chaos, Solitons and Fractals* 40 (2009), pp. 717–730.

[6] S. Brianzoni, C. Mammana, and E. Michetti. “Variable elasticity of substituition in a discrete time Solow–Swan growth model with differential saving”. In: *Chaos, Solitons and Fractals* 45 (2012), pp. 98–108.

[7] V. Capasso, R. Engbers, and D. La Torre. “On a spatial Solow model with technological diffusion and nonconcave production function”. In: *Nonlinear Anal.* 11 (2010), pp. 3858–3876.

Local and Global Dynamics in a Discrete Ramsey Model

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Abstract

The choice of time as a discrete or continuous variable may radically affect the stability of equilibrium in an endogenous growth model with durable consumption. In the continuous-time model the steady state is locally saddle-path stable with monotonic convergence. However, in the discrete-time model the steady state may be unstable or saddle-path stable with monotonic or oscillatory convergence.
In this paper, we study general polynomial discretization in backward and forward looking, and the preservation of stability properties. We apply these results to the Ramsey model. Finally, in this paper, we study the local and global dynamics of a new discrete Ramsey model

References:

1. Bosi S, Ragot L. Time representation in economics. Theor. Econ. Lett. 2012,2,10–15.
2. Hall RE. Intertemporal substitution in consumption. J. Polit. Econ. 1988,96,339–357.
3. Krivine H, Lesne A, Treiner J. Discrete-time and continuous-time modelling: some bridges and gaps. Math. Struct. Comp. Sci. 2007,17,261–276.
4. Lesne A. The discrete versus continuous controversy in physics. Math. Struct. Comp. Sci. 2007,17,185–223.
5. Licandro O, Puch LA. Is discrete time a good representation of continuous time? EUI Working Paper ECO 2006/28. European University Institute, San Domenico di Fiesole (FI), Italy, 2006.
6. Mitra T, Sorger G. On Ramsey's conjecture. J. Econ. Theory 2013,148,1953–1976.
7. Ramsey FP. A mathematical theory of saving. Econ. J. 1928,38,543–559.

Asymptotic properties of cocycles contracting on fibers

Grzegorz Guzik

We deal with cocycle mappings on general metric spaces which are uniformly contractive on fibers. Namely, let (X, ϱ) be a metric space (a phase space) and Ω be a non–void set (a parameter space), moreover let $\mathbb{T}$ be a set of all possible 'times' (some nontrivial subgroups of reals). Suppose that $\theta = \{\theta_t : \Omega \to \Omega : \; t \in \mathbb{T}\}$ is a group of bijective transformations (a base flow) and the mapping $\varphi : \mathbb{T}^+ \times \Omega \to X^X$ satisfies the following equation

$$\varphi(s+t, \omega) = \varphi(t, \theta_s\omega) \circ \varphi(s, \omega) \quad \text{for} \;\; s, t \in \mathbb{T}^+ \quad \text{and} \quad \omega \in \Omega.$$

We assume that every function $\varphi(t, \omega) : X \to X$ is continuous. A pair (θ, φ) is called a *cocycle* (over θ).

Given a cocycle (θ, φ) for $\omega \in \Omega$ and $D \subset X$ we define the limit set

$$\mathcal{L}(\omega, D) := \bigcap_{t \in \mathbb{T}^+} \mathrm{cl}\big(\bigcup_{s \geq t} \varphi(s, \theta_{-s}\omega)(D)\big).$$

Then define a family $\mathcal{A} = \{\mathcal{A}_\omega : \; \omega \in \Omega\}$ by

$$\mathcal{A}_\omega := \mathrm{cl} \bigcup_D \mathcal{L}(\omega, D) \quad \text{for} \;\; \omega \in \Omega,$$

where the sum on the right–hand side is taken over all bounded subsets D of X.

The cocycle (θ, φ) is said to be *pullback uniformly contractive on fibers* if for every $\omega \in \Omega$, every nonempty bounded subset D of X and every $\varepsilon > 0$ there is a $t_0 = t_0(\varepsilon, \omega, D) \in \mathbb{T}^+$ such that for every $t \geq t_0$ condition

$$diam(\varphi(t, \theta_{-t}\omega)(D)) < \varepsilon$$

is satisfied.

Our main result is as follows.

Theorem 0.1. *Let (X, ϱ) be a complete metric space and (θ, φ) be a cocycle pullback uniformly contractive on fibers. Suppose that there exists a nonempty bounded subset A of X such that*

$$\varphi(t, \omega)(A) \subset A \quad \textit{for}\, t \in \mathbb{T}^+ \; \textit{and}\, \omega \in \Omega.$$

Then for every $\omega \in \Omega$ there is a unique point $x_\omega \in X$ such that $\mathcal{A}_\omega = \{x_\omega\}$.

In fact, this says that under assumption of uniform contractivity on fibers one can obtain a (pullback) cocycle attractor which consists of singletons. Since the parameter space Ω is supposed to be a nonempty set only, both cases of random as well as nonautonomous dynamical systems are covered. Moreover, we can obtain a vast class of iterated function systems with so–called point–fibred attractors.

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Iterated procedure of joining means

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This is a joint work with Justyna Jarczyk.

Let I be an interval of reals. A function $M : I \times I \to I$, satisfying the condition

$$\min\{x, y\} \leq M(x, y) \leq \max\{x, y\}, \qquad x, y \in I,$$

is called a *mean on* I. If the above inequalities are sharp whenever $x, y \in I$, $x \neq y$, then the mean M is said to be *strict*.

Take any interior point ξ of I and define $I_\xi := \{x \in I\colon\ x \leq \xi\}$ and ${}_\xi I := \{x \in I:\ \xi \leq x\}$. Given means M and N on the intervals I_ξ and ${}_\xi I$, respectively, the problem is to find a mean $M \oplus N$ on I extending both M and N:

$$M \oplus N|_{I_\xi \times I_\xi} = M \quad \text{and} \quad M \oplus N|_{{}_\xi I \times {}_\xi I} = N.$$

We present one of possible approach to the problem, studying the limit behaviour of a simple dynamical system built by use of the *limit functions* $h_1, h_2\colon I \to I$ defined by

$$h_1(x) = \begin{cases} M(x, \xi), & \text{if } x \in I_\xi, \\ N(x, \xi), & \text{if } x \in {}_\xi I, \end{cases} \qquad h_2(y) = \begin{cases} M(\xi, y), & \text{if } y \in I_\xi, \\ N(\xi, y), & \text{if } y \in {}_\xi I. \end{cases}$$

The iterated procedure of joining M and N, presented in the talk, makes use of some properties of the iterates $(h_1 \times h_2)^n$, where $h_1 \times h_2\colon I \times I \to I \times I$ is given by

$$(h_1 \times h_2)(x, y) = (h_1(x), h_2(y)).$$

In particular, the following facts are crucial.

1. Assume that I is compact and the functions h_1, h_2 are continuous. Then
(i) $(h_1^n(I))_{n\in\mathbb{N}}$ and $(h_2^n(I))_{n\in\mathbb{N}}$ are decreasing (in the sense of inclusion) sequences of compact intervals containing ξ;
(ii) $J_1 := \bigcap_{n=1}^{\infty} h_1^n(I)$ and $J_2 := \bigcap_{n=1}^{\infty} h_2^n(I)$ are compact intervals containing ξ;
(iii) $J_1 \times J_2$ is an attractor of the dynamical system $((h_1 \times h_2)^n)_{n\in\mathbb{N}}$: for every neighbourhood U of $J_1 \times J_2$ there is an $n_0 \in \mathbb{N}$ such that

$$((h_1 \times h_2)^n)(I \times I) \subset U, \qquad n \geq n_0.$$

2. Assume that I is compact, the functions h_1, h_2 are continuous and the means M and N are strict. Then $J_1 \times J_2 = \{(\xi, \xi)\}$, that is

$$\lim_{n\to+\infty} (h_1 \times h_2)^n (x, y) = (\xi, \xi)$$

uniformly in $I \times I$.

Some Spectral Results on Finitie Difference Sturm-Liouville Problem

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Let N be positive integer and $h = \frac{\pi}{N}$. Also, $x_i = nh$, $q_n = q(nh)$ for $n = 0, 1, 2, ...N-1$. In this talking, we will give some spectral results as eigenvalues, eigenfunctions, norming constants and zeros of eigenfunctions. Also, by comparing continuous and discrete case for classical Ambarzumyan theorem [4], we will give similar and different points for difference Sturm-Liouville problem [1-3]

$$-\Delta^2 y(n) + q(n)y(n+1) = \lambda y(n+1), \tag{1}$$

$$y(0) = y(N) = 0 \tag{2}$$

where the sequence $q = [q(n)]$ is referred to as the potential. As usual, Δ is the forward difference operator defined by

$$\Delta y(n) = \frac{y(n+1) - y(n)}{h},\ \Delta^2 y(n) = \frac{y(n+2) - 2y(n+1) + y(n)}{h^2}.$$

It is well known that the problem (1),(2) has simple real eigenvalues with corresponding orthogonal eigenfunctions.

References

[1] Ambarzumyan, V. A., Über eine frage der eigenwerttheorie, *Zeitschrift für Physik*, **53,** 690-695, (1929).

[2] O. H. Hald, *Discrete inverse Sturm-Liouville problems. I. Uniqueness for Symmetric potentials*, Numer. Math. 27(2) 249-256, (1976-77).

[3] M. Bohner and H. Koyunbakan, *Inverse Sturm-Liouville difference equations,* Filomat,30(5), 1297-1304, (2016)

[4] W. Kelley and A. Peterson, Difference Equations: An Introduction with Applications, 2nd ed., Harcourt/Academic Press, San Diego, 2001.

The effects of tax competition on compliance and migration: an evolutionary analysis through piecewise-smooth maps

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Abstract

We study the dynamic effects of fiscal reforms on migration and tax evasion in an international context with two asymmetric countries. Given an initial international distribution of honest and dishonest tax payers, the tax system (e.g. tax rates and degrees of progressivity) and the salary in each country, individuals decide where to reside and how much time to spend working. The model allows us to study in an evolutionary dynamic setting how the distribution of honest and dishonest earners is geographically affected by fiscal reforms (e.g. variations in tax rates) and auditing efforts (e.g. probability of auditing and fines) of different countries. Due to progressivity of tax systems, the evolutionary system is modelled through a piecewise-smooth map. We show that various dynamic long term scenarios can be generated. The particular convergence of the model depends crucially on the initial geographic distribution of dishonest agents. This implies that tax reforms that have been successful in reducing tax evasion in one country may produce very different results in others, if initial conditions are significantly different. Chaotic cyclical behavior may also arise if individual propensity to migrate is sufficiently high.

Keywords: Migration; Tax evasion; Progressive vs. flat tax; Evolutionary dynamics; Piecewise-smooth maps.

JEL codes: H26; H31; F22.

On topological properties of Brouwer homeomorphisms

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We describe properties of Brouwer homeomorphisms that are not necessarily embeddable in a flow. The presented results can be treated as counterparts of results concerning Brouwer flows proved in [1] and [2].

We show results which concern invariant lines that are closed sets. Such lines play a similar role as trajectories in the case where a Brouwer homeomorphism is embeddable in a flow. In particular, using such lines we can describe the structure of equivalence classes of the codivergency relation.

References

1. Z. Leśniak, *On an equivalence relation for free mappings embeddeable in a flow*, Internat. J. Bifur. Chaos Appl. Sci. Engrg. 13, No. 7 (2003), 1911–1915.
2. Z. Leśniak, *On parallelizability of flows of free mappings*, Aequationes Math. 71, No. 3 (2006), 280–287.

Piecewise Monotone Maps Chaotic in the Whole Interval

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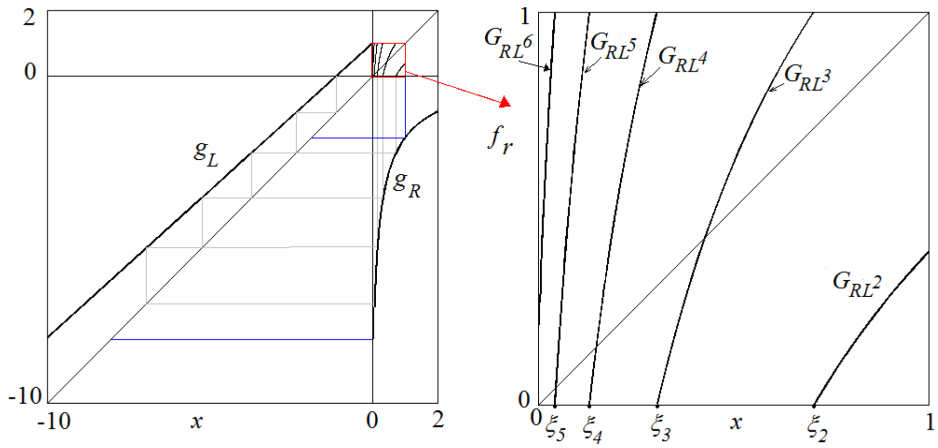
In this work we consider a discontinuous piecewise smooth *expanding* map f of an interval into itself, constituted by $N-$pieces with $N \geq 2$. It is expanding when $f'(x) > \lambda \geq 1$ in all the points of continuity. Our goal is to determine the necessary and sufficient conditions for the map to be chaotic in the sense of Devaney in the whole interval. This condition, which we call of full chaos, is very important in engineering applications, especially those related to grazing bifurcations, as well as in other applied fields. Without loss of generality we consider the unit interval, and a one-dimensional discontinuous piecewise smooth map $f : [0, 1] \longrightarrow [0, 1]$ defined as follows:

$$f(x) = \begin{cases} f_1(x) & \text{if } 0 \leq x < \xi_1 \\ f_2(x) & \text{if } \xi_1 \leq x < \xi_2 \\ \vdots & \vdots \\ f_N(x) & \text{if } \xi_{N-1} \leq x \leq 1 \end{cases} \tag{1}$$

where the strictly increasing functions $\{f_i\}_{i=1}^N$ defined in $I_i = [\xi_{i-1}, \xi_i]$ satisfy $f_i(\xi_i) = 1$ and $f_{i+1}(\xi_i) = 0$ for $1 < i < N-1$, while in the two extrema: $0 \leq f_1(0) < 1$, $0 < f_N(1) \leq 1$.

For $N = 2$ the considered map is the standard expanding *Lorenz* map, which has been studied by many authors. It is well known that for the expanding Lorenz map, $N = 2$, if the derivative satisfies $f'(x) > \sqrt{2}$ then the piecewise smooth map is chaotic in the whole interval. However, as it is known, this condition is not necessary, and the necessary and sufficient conditions are related to the existing basic cycles RL^n and R^nL, $n \geq 1$, which must be all homoclinic. New results are obtained considering the expanding map for $N \geq 3$, which we call Baker-like map, proving that for $N = 3$ the necessary and sufficient condition for chaos in $[0, 1]$ is that the internal fixed point is homoclinic, while for $N > 3$ it is always true, i.e. the map is always chaotic in the whole interval. Our main result is to show how this theorem can be used *to prove full chaos in non expanding Lorenz maps* (a discontinuous map with only two branches).

In fact, the obtained results are applicable to a non expanding Lorenz map of an interval onto itself, by use of a suitable first return map which is necessarily a Baker-like map. We show an example of a non expanding Lorenz map $g(x)$ from the engineering field, whose graph is shown in the following figure.



References

[1] P. Glendinning, C. Sparrow, Prime and renormalisable kneading invariants and the dynamics of expanding Lorenz maps, Physica D 62 (1993) 22-50.

[2] P. Glendinning, T. Hall, Zeros of the kneading invariant and topological entropy for Lorenz maps, Nonlinearity 9 (1996) 999-1014.

[3] F. Hofbauer, The maximal measure for linear mod one transformations. J. London Math. Soc. 23 (1981) 92-112. doi: 10.1112/jlms/s2-23.1.92

[4] A. J. Homburg, Global Aspects of Homoclinic Bifurcations of Vector Fields, Memoirs of the American Math. Soc., Vol. 121, 1996.

[5] J.H. Hubbard, C.T. Sparrow, The classification of topologically expanding Lorenz maps, Comm. Pure. Appl. Math. 43 (1990) 431-443.

Bifurcation equations for periodic orbits

of implicit discrete dynamical systems

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Abstract

Bifurcation equations, non-degeneracy and transversality conditions are obtained for the fold, transcritical, pitchfork and flip bifurcations for periodic points of one dimensional implicitly defined discrete dinamical systems.
The backward Euler method and the trapezoid method for numeric solutions of ordinary differential equations fall in the category of implicit dynamical systems. Examples of bifurcations are given for some implicit dynamical systems including bifurcations for the backward Euler method when the step size is changed.

Bifurcation structures related to chaotic attractors in a 1D PWL map defined on three partitions

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Let us consider a family of one-dimensional discontinuous piecewise linear maps defined as

$$f \,:\, x \to f(x) = \begin{cases} f_{\mathcal{L}}(x) = a_{\mathcal{L}}x + \mu_{\mathcal{L}} & \textit{if} \quad x \leq -z^-, \\ f_{\mathcal{M}}(x) = a_{\mathcal{M}}x & \textit{if} \quad -z^- < x < z^+, \\ f_{\mathcal{L}}(x) = a_{\mathcal{R}}x - \mu_{\mathcal{R}} & \textit{if} \quad x \geq z^+, \end{cases} \tag{1}$$

where $\mu_{\mathcal{L}}$, $\mu_{\mathcal{R}}$, z^- and z^+ are positive parameters, and $a_{\mathcal{M}} > 1$, $a_{\mathcal{L}}, a_{\mathcal{R}} \in \mathbb{R}$, $a_{\mathcal{L}} = a_{\mathcal{R}}$. The map of such a kind appears when modelling bull and bear asset market dynamics. For example, in [1] a symmetric case with $\mu_{\mathcal{L}} = \mu_{\mathcal{R}}$, $z^- = z^+$ was considered. And in [2] for a similar map bifurcation structures in the parameter space corresponding to attracting cycles were described.

The aim of the paper presented is studying bifurcation structures in the chaotic domain of the map (1) for the particular case with $a_{\mathcal{L}} = a_{\mathcal{M}} = a_{\mathcal{R}} =: a$. The map can have at most three fixed points: $x^*_{\mathcal{L}} = \mu_{\mathcal{L}}/(1-a)$, $x^*_{\mathcal{R}} = \mu_{\mathcal{R}}/(a-1)$, $x^*_{\mathcal{M}} = 0$. However, since all three slopes are greater than one the only possible asymptotic dynamics is a set of chaotic intervals or a single chaotic interval.

Since the function f has two discontinuity points there can be two coexisting disjoint absorbing intervals $J_{\mathcal{L}} := [c^-_{\mathcal{M}}, c^-_{\mathcal{L}}]$ and $J_{\mathcal{R}} := [c^+_{\mathcal{R}}, c^+_{\mathcal{M}}]$, or only one absorbing interval, namely, $J_{\mathcal{L}}$, $J_{\mathcal{R}}$, $J_{\mathcal{M}} := [c^-_{\mathcal{M}}, c^+_{\mathcal{M}}]$ or $J := [c^+_{\mathcal{R}}, c^-_{\mathcal{L}}]$. One can distinguish seven different cases:

1) $\{c^+_{\mathcal{M}} < x^*_{\mathcal{R}}, c^+_{\mathcal{R}} > x^*_{\mathcal{M}}, c^-_{\mathcal{M}} > x^*_{\mathcal{L}}, c^-_{\mathcal{L}} < x^*_{\mathcal{M}}\}$: the map f has two disjoint invariant absorbing intervals $J_{\mathcal{L}}$ and $J_{\mathcal{R}}$ each representing a chaotic attractor.

2) $\{c^+_{\mathcal{M}} < x^*_{\mathcal{R}}, c^+_{\mathcal{R}} > x^*_{\mathcal{M}}, c^-_{\mathcal{M}} < x^*_{\mathcal{L}}\}$ or $\{c^+_{\mathcal{M}} < x^*_{\mathcal{R}}, c^+_{\mathcal{R}} > x^*_{\mathcal{M}}, c^-_{\mathcal{L}} > x^*_{\mathcal{M}}\}$: the map f has an invariant absorbing interval $J_{\mathcal{R}}$.

3) $\{c^+_{\mathcal{M}} > x^*_{\mathcal{R}}, c^-_{\mathcal{M}} > x^*_{\mathcal{L}}, c^-_{\mathcal{L}} < x^*_{\mathcal{M}}\}$ or $\{c^+_{\mathcal{R}} < x^*_{\mathcal{M}}, c^-_{\mathcal{M}} > x^*_{\mathcal{L}}, c^-_{\mathcal{L}} < x^*_{\mathcal{M}}\}$: the map f has an invariant absorbing interval $J_{\mathcal{L}}$.

4) $\{c^-_{\mathcal{M}} < c^+_{\mathcal{R}} < x^*_{\mathcal{M}}, c^+_{\mathcal{M}} < x^*_{\mathcal{R}}, x^*_{\mathcal{M}} < c^-_{\mathcal{L}} < c^+_{\mathcal{M}}, c^-_{\mathcal{M}} > x^*_{\mathcal{L}}\}$: f has an invariant absorbing interval $J_{\mathcal{M}}$.

5) $\{c^-_{\mathcal{M}} < c^+_{\mathcal{R}} < x^*_{\mathcal{M}}, c^+_{\mathcal{M}} < c^-_{\mathcal{L}} < x^*_{\mathcal{R}}, c^-_{\mathcal{M}} > x^*_{\mathcal{L}}\}$: f has an invariant absorbing interval $J_{\mathcal{L}}$.

6) $\{x^*_{\mathcal{L}} < c^+_{\mathcal{R}} < c^-_{\mathcal{M}}, c^+_{\mathcal{M}} < x^*_{\mathcal{R}}, x^*_{\mathcal{M}} < c^-_{\mathcal{L}} < c^+_{\mathcal{M}}\}$: f has an invariant absorbing interval $J_{\mathcal{R}}$.

7) $\{x^*_{\mathcal{L}} < c^+_{\mathcal{R}} < c^-_{\mathcal{M}}, c^+_{\mathcal{M}} < c^-_{\mathcal{L}} < x^*_{\mathcal{R}}\}$: f has an invariant absorbing interval J.

For the cases 1)–3) the restriction of f on every its absorbing interval is represented by a discontinuous (piecewise increasing) map defined on two partitions. In the parameter space the associated chaoticity regions are organised in *bandcount adding bifurcation structure* (see, *e.g.*, [3]).

References

[1] Fabio Tramontana, Frank Westerhoff, and Laura Gardini. The bull and bear market model of Huang and Day: Some extensions and new results. *J. Econ. Dyn. Control*, 37:2351–2370, 2013.

[2] Iryna Sushko, Fabio Tramontana, Frank Westerhoff, and Viktor Avrutin. Symmetry breaking in a bull and bear financial market model. *Chaos Solitons Fract.*, 79:57–72, 2015.

[3] Viktor Avrutin and Michael Schanz. On the fully developed bandcount adding scenario. *Nonlinearity*, 21(5):1077–1103, 2008.

A monopoly model with memory: analysis of 1:4 resonance

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Abstract

This paper considers a monopoly model with gradient adjustment mechanism and log-concave demand function. The monopolist is assumed to be boundedly rational and memory is introduced to obtain information on output. Locally, the unique equilibrium may be destabilized through a period doubling or a supercritical Neimark-Sacker bifurcation.

The analysis is then carried on from a global perspective. We first study the delimitation of the set of feasible trajectories and we identify phenomena of multistability. The phenomenon of attractors coexistence is relevant for the structure of the basins of attraction, and the dependence of the basin boundaries to the parameters of the model plays a fundamental role in the long-run behavior. Then the two-parameter bifurcations of the model is discussed. It is shown that the system undergoes a 1:4 resonance by using a continuation procedure. The numerical simulations, including phase portraits, illustrate the theoretical features associated with the resonance and display interesting and complex dynamical behaviors, like the emergence of *square* and *clover* orbits.

References

[1] Elsadany, A. A., & Awad, A. M. (2016). Dynamical analysis of a delayed monopoly game with a log-concave demand function. Operations Research Letters, 44(1), 33-38.

[2] Kuznetsov, Y. A. (2013). Elements of applied bifurcation theory (Vol. 112). Springer Science & Business Media.

[3] Li, B., & He, Z. (2015). 1: 2 and 1: 4 resonances in a two-dimensional discrete Hindmarsh-Rose model. Nonlinear Dynamics, 79(1), 705-720.

[4] Matsumoto, A., & Szidarovszky, F. (2012). Nonlinear delay monopoly with bounded rationality. Chaos, Solitons & Fractals, 45(4), 507-519.

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Evolutionary dynamics in club goods binary games

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Many situations, in social sciences and economics in particular, can be modeled as binary choices with externalities, that is, as games that describe collective behavior when agents have two alternatives and the received payoff depends on how many other agents choose which alternative.
In this paper a dynamic adjustment mechanism, based on replicator dynamics in discrete time, is used to study the time evolution of a population of players facing a binary choice game with social influence characterized by payoff curves that intersect at two interior points, also denoted as thresholds, so that besides the boundary equilibria (in pure strategies) where all players make the same choice, there are two further equilibria where agents playing different strategies coexist and get identical payoffs. For such binary games both interior points have an economic interpretations: the first one being related to a cooperative sharing cost and the second one (typical of minority games) to congestion. These games, also denoted as club goods games, can be used (and indeed have been used in the literature) to represent several social and economic systems. Existence and stability of equilibrium points are studied, as well as the creation of more complex attractors (periodic or chaotic) related with overshooting effects. The study of some local and global dynamic properties of the evolutionary model proposed reveals that the presence of two thresholds causes the creation of complex topological structures of the basins of coexisting attracting sets, so that a strong path dependence is observed. The dynamic effects of memory, both in the form of convex combination of a finite number of previous observation (moving average) and in the form of memory with increasing length and exponentially fading weights are investigated as well.

Keywords: Binary games, Social externalities, Club goods, Discrete Dynamical Systems, Replicator Dynamics, Global bifurcations

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An evolutionary generalization of the Shelling's neighborhood tipping model: Inequality and segregation

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Abstract

The major urban centers exhibit extreme racial separation both in Europe and in the United States. The extreme levels of segregation have a negative impact on the economic system and represent a serious threat to the social stability. The problem has attracted scholars' attention and, already several decades ago, Shelling pointed out that the driving forces behind persistent residential segregation are many and include both ethnic reasons and economic aspects, see Schelling (1969) and Schelling (1971). Since then, a burgeoning area of research focused on studying the effect of ethnic factors, such as a limited level of tolerance towards other ethnic groups, to explain the phenomenon of residential segregation, see, e.g. Zhang (2004), Pancs and Vriend (2007), G. Fagiolo and Vriend (2007), Zhang (2011) and Bischi and Merlone (2011). Instead, rather neglected are the economic drivers of residential segregation, see, e.g., Sethi and Somanathan (2004). In this paper, we try to bridge the gap by developing a model that includes both economic and ethnic drivers. In particular, economic segregation, caused by income inequality, and ethnic segregation, caused by limited levels of tolerance, are combined together in a dynamic evolutionary game. The modeling framework consists of two populations that differ for ethnicity and income. The first population is the more wealthy while the second one is the more tolerant and all households have a strong preference for integrated over segregated neighborhoods as revealed by empirical observations, see, e.g., Clark. (1991) and H. Schuman and Krysan (1997). In defining the residential location, an household sorts between a totally segregated neighborhood, without any possibility of integration, and an integrated neighborhood in which the ethnic composition evolves over time according to a replicator dynamics. The price of the houses in a neighborhood depends on residents' income. In particular the housing market is cheaper when there are more residents of the second population. Thus, the members of the first population may want to choose an integrated neighborhood because attracted by a cheap residential housing market. On the another hand, the members of the second population want to live in a mixed ethnic neighborhood because of the high preference for integration, but the racial income disparity that affect the housing price may prevent this choice.

The analysis shows that ethnic factors and economic factors combined together may facilitate integration and narrowing racial income disparities results in *increasing* residential segregation. Thus, a right mix of racial income disparities and racial preference for integration disparities facilitate integration, although the risk of segregation cannot be eliminate. In fact, a stable equilibrium of integration coexists with a stable equilibria of segregation. To foster integration a policy measure is introduce to subsidize house purchase in integrated neighborhood. The dynamics of the model reveals that this policy increases the number of households of the first population in integrated neighborhood, favoring evenly integrated neighborhoods, but the risk of segregation increases.

The analysis is further extended considering, within an agent-based modeling framework, heterogeneous incomes within the same ethnic group. The results of the stylized model are confirmed.

Keywords: Residential segregation; Housing market; Racial income disparities; Evolutionary games.

References

G. I. Bischi and U. Merlone. *Nonlinear economic dynamics*, chapter An Adaptive dynamic model of segregation, pages 191–205. Nova Science Publisher, 2011.

W. A. V. Clark. Residential preferences and neighborhood racial segregation: A test of the Schelling segregation model. *Demography*, 28(1):1–19, 1991.

M. Valente G. Fagiolo and N. J. Vriend. Segregation in networks. *Journal of Economic Behavior & Organization*, 64 (3–4):316–336, 2007.

L. Bobo H. Schuman, C. Steeh and M. Krysan. *Racial attitudes in America: Trends and Interpretations.* Cambridge, MA: Harvard University Press, 1997.

R. Pancs and J. N. Vriend. Schelling's spatial proximity model of segregation revisited. *Journal of Public Economics*, 91(1):1–24, 2007.

T. C. Schelling. Models of segregation. *The American Economic Review*, 59(2):488–493, 1969.

T. C. Schelling. Dynamic models of segregation. *Journal of Mathematical Sociology*, 1(2):143–186, 1971.

R. Sethi and R. Somanathan. Inequality and segregation. *Journal of Political Economy*, 112(6):1296–1321, 2004.

J. Zhang. Residential segregation in an all-integrationist world. *Journal of Economic Behavior and Organization*, 54 (4):533–550, 2004.

J. Zhang. Tipping and residential segregation: a unified schelling model. *Journal of Regional Science*, 51(1):167–193, 2011.

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Mixing properties for monotonic mod one transformations with two pieces

Peter Raith

Abstract. Let $f : [0,1] \to [0,2]$ be a continuous strictly increasing function, and suppose that f is differentiable on $(0,1) \setminus F$ where F is a finite set. Furthermore assume that $\inf f' > 1$, where $\inf f' := \inf_{x \in (0,1) \setminus F} f'(x)$. Because of these assumptions there exists a unique $c \in (0,1)$ with $f(c) = 1$. Define $T_f x := f(x) - \lfloor f(x) \rfloor$, where $\lfloor y \rfloor$ is the largest smaller or equal to y. Then T_f is called a monotonic mod one transformation with two monotonic pieces. Such maps are also called Lorenz maps. Observe that these maps are piecewise monotonic maps but have a discontinuity at c. Moreover, $\lim_{x \to c^-} T_f x = 1$ and $\lim_{x \to c^+} T_f x = 0$. Using a standard doubling points construction one can apply all usual definitions of dynamical systems also to monotonic mod one transformations.
Important notions in chaotic dynamical systems are topological transitivity and different mixing properties. Assume that β is a real number satisfying $\inf f' > \beta$. If $\beta \geq \sqrt{2}$ or if $\beta \geq \sqrt[3]{2}$ and $f(0) \geq \frac{1}{\beta+1}$ or $f(1) \leq 2 - \frac{1}{\beta+1}$ then T_f is topologically transitive. Moreover, it is also topologically mixing except in the cases $f(x) = \sqrt{2}x + 1 - \frac{1}{\sqrt{2}}$, $f(x) = \sqrt[3]{2}x + \frac{2+\sqrt[3]{4}-2\sqrt[3]{2}}{2}$ and $f(x) = \sqrt[3]{2}x + \frac{2-\sqrt[3]{4}}{2}$.
Other notions often used in this context are locally eventually onto and renormalizability. Here according to classical definition T_f is called locally eventually onto if for any nonempty open set U there exist open subintervals U_1, U_2 and natural numbers n_1, n_2 such that ${T_f}^{n_1}$ maps U_1 homeomorphically to $(0,c)$ and ${T_f}^{n_2}$ maps U_1 homeomorphically to $(c,1)$. Every locally eventually onto map is topologically mixing but the converse need not be true. It was believed that T_f is locally eventually onto if and only if T_f is not renormalizable. Unfortunately this is not true. One has the improve the definition above to obtain this equivalence.

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Dynamic equivalence of dynamic systems on time scales

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In 1988 (Ph.D. thesis), Stefan Hilger introduced the calculus of time scale in order to unify continuous and discrete analysis (see [1]) Many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different in nature from their continuous counterparts. The study of dynamic equations on time scales reveals such discrepancies, and helps avoid proving results twice, once for differential equations and once for difference equations.

We consider the dynamic system in a Banach space on unbounded above and below time scale:

$$\begin{cases} x^{\Delta} &= A(t)x + f(t,x,y), \\ y^{\Delta} &= B(t)y + g(t,x,y). \end{cases} \tag{1}$$

This system satisfies the conditions of integral separation with the separation constant ν, the integral contraction with the integral contraction constant μ, nonlinear terms are ε-Lipshitz, and the system has a trivial solution. We find sufficient conditions under which the system (1) is locally dynamic equivalent

$$\begin{cases} x^{\Delta} &= A(t)x + f(t,x,u(t,x)), \\ y^{\Delta} &= B(t)y. \end{cases} \tag{2}$$

References

[1] M. Bohner and A. Peterson, Dynamic Equations on Time Scales. An Introduction with Applications, Birkhäuser, Boston, Basel, Berlin, 2001.

[2] S. Hilger. *Generalized theorem of Hartman-Grobman on measure chains*, J. Austral. Math. Soc. Ser A, 60, (1996), no. 2, 157-191.

[3] A. Reinfelds and Dz. Steinberga, *Dynamical equivalence of quasilinear dynamic equations on time scales*, Journal of Mathematical Analysis, 7, (2016), no. 1, 115-120.

ALMOST PERIODICITY, RICKER MAP, BEVERTON-HOLT MAP AND OTHERS, A GENERAL METHOD

ROBERT J. SACKER AND GEORGE R. SELL

It is known that under the certain conditions on the coefficient the Ricker difference equation (or map) has a fixed point that is globally asymptotically stable with respect to the positive reals. We show here that under the same conditions, the Ricker equation with almost periodic coefficient has a globally asymptotically stable almost periodic solution with the same frequency module as the coefficient. This is accomplished by showing that the omega limit set Ω of an asymptotically stable solution is a covering space of the omega limit set of the coefficients and the flow on Ω is uniquely reversible. We provide a unified framework that allows us to conclude that any system of maps in finite dimensions that has an orbit that is bounded and whose omega limit set is asymptotically stable, also has the property that certain attributes of the coefficients (periodicity, almost periodicity) can be carried over, or lifted to the solution. In particular if the successive compositions are bounded and have Fréchet derivatives with spectrum inside the unit circle in the complex plane then the above conclusions apply.

Population control through adaptive limiters

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Abstract

Due to their recurring and sometimes unpredictable ups and downs, fluctuations in population size pose several challenges for biological conservation and the management of wildlife and exploited populations. Control strategies are aimed to reduce the outbreak frequency and extinction probability, stabilize the fluctuations or maximize the yield of harvested populations. Limiter control methods have the advantage that no detailed information of the system is required, which is why they are easy and fast to implement.

The purpose of this talk is to present a new limiter strategy called adaptive threshold harvesting (ATH). This control method is the harvesting version of adaptive limiter control (ALC), and takes effect only if the population size has grown by at least a certain factor in comparison to the previous census. It differs from textbook strategies like constant-effort or constant-yield harvesting, as it responds only to population increases sufficiently large.

Results relative to the stabilizing properties of ATH and possible applications of both ATH and ALC to the control of realistic biological populations will be exposed.

This talk is based on a joint work with Daniel Franco (UNED, Spain) and Frank Hilker (U. Osnabrück, Germany).

References

[1] Juan Segura, Frank M Hilker, and Daniel Franco. Adaptive threshold harvesting and the suppression of transients. *Journal of theoretical biology*, 395:103–114, 2016.

[2] Juan Segura, Frank M Hilker, and Daniel Franco. Population control methods in stochastic extinction and outbreak scenarios. *PLoS One*, 12(2):e0170837, 2017.

Systems of iterative functional equations: some constructive examples

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Abstract

We consider a general system of iterative functional equations between general spaces X and Y. The most common and representative functions which are solutions of these systems are the so called fractal interpolation functions. A general theory may be developed for affine systems, which are the most frequently found in the literature. We present general formulas for solutions of this type of systems which covers non-affine cases and provide a list of examples, with graphic illustrations.

References

[1] Barnsley, M.: *Fractal functions and interpolation*, Constr. Approx. 2 (1986), 303-329.

[2] Girgensohn, R.: *Functional equations and nowhere differentiable functions*, Aequationes Math. 46 (1993), 243-256.

[3] de Rham, G.: *Sur quelques courbes définies par des équations fonctionnelles*, Rend. Sem. Math. Torino 16 (1956), 101-113.

[4] Serpa, C., Buescu, J., *Constructive solutions for systems of iterative functional equations*, Constr. Approx., (2017) 45:273–299.

[5] Serpa, C., Buescu, J.: *Explicitly defined fractal interpolation functions with variable parameters*, Chaos Solitons Fractals, 75 (2015), 76-83.

[6] Wang, H.-Y., Yu, J.-S.: *Fractal interpolation functions with variable parameters and their analytical properties*, J. Approx. Theory 175 (2013), 1-18.

[7] Zdun, M. C.: *On conjugacy of some systems of functions*, Aequationes Math. 61 (2001), 239-254.

Periodic attractors on a family of nonautonomous dynamical systems generated by stunted tent maps

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Abstract

Families of stunted sawtooth maps have been used as models to study related families of differentiable maps, since they are closely related with symbolic dynamics and are rich enough to encompass in a canonical way all possible kneading data and all possible itineraries, see [1] and [3].

In [4] we studied the bifurcation structure of a family of 2-periodic nonautonomous dynamical systems, generated by the alternate iteration of two stunted tent maps. However, when we get into the general nonautonomous context, usual notions from autonomous (and periodic nonautonomous) discrete dynamics, like fixed or periodic points, invariant sets, attractivity and repulsivity must be reinterpreted and reformulated. This is the core of nonautonomous bifurcation theory, that has been developed in recent years by various authors, see for example [2] and references therein. In this work we will consider a family of nonautonomous dynamical systems $x_{k+1} = f_k(x, \lambda)$, $\lambda \in [-1, 1]^{\mathbf{N}_0}$, generated by a one-parameter family of stunted tent maps $g_\alpha(x)$, i.e., $f_k(x, \lambda) = g_{\lambda_k}(x)$ for all $k \in \mathbf{N}_0$. We will reinterpret the concept of attractive periodic orbit in this context, through the existence of some periodically invariant attractive sets and establish sufficient conditions, based on symbolic dynamics, over the parameter sequences for the existence of such periodic attractors.

References

[1] Milnor, J. & Tresser, C., "On entropy and monotonicity for real cubic maps," *Comm. Math. Phys. 209* **1**, 2000, pp 123–178. With an appendix by Adrien Douady and Pierrette Sentenac.

[2] C. Pötzsche "Bifurcations in Nonautonomous Dynamical Systems: Results and tools in discrete time," Proceedings of the International Workshop Future Directions in Difference Equations. June 13-17, 2011, Vigo, Spain. pp 163–212.

[3] Rădulescu, A.] "The connected isentropes conjecture in a space of quartic polynomials," *Discrete Contin. Dyn. Syst.* **19** no. 1, 2007, pp 139–175.

[4] L. Silva, J. Leonel Rocha and M. T. Silva, "Bifurcations of 2-periodic nonautonomous stunted tent systems," to appear in International Journal of Bifurcation and Chaos.

CONVERGENCE TIME TO EQUILIBRIUM DISTRIBUTIONS OF AUTONOMOUS AND PERIODIC NON AUTONOMOUS GRAPHS

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Abstract

We present some estimates of the time of convergence to the equilibrium distribution in autonomous and periodic non autonomous graphs, with ergodic stochastic adjacency matrices, using the eigenvalues of these matrices. On this way we generalize previous results from several authors, that only considered reversible matrices.

It is known that a Markov chain represented by an ergodic stochastic matrix P converges to a stationary distribution, the equilibrium π, which means that powers P^k converge componentwise to a stochastic matrix W in which all rows are equal to π (see [Beh00]). We associate a matrix A, the adjacency matrix, to a weighted directed graph $\mathcal{G} = (V, E)$. Following the classical relation between Markov chains and graph theory described in [LM95], from A we define a stochastic matrix P. All eigenvalues of a stochastic matrix have modulus less than one, except the first one that has modulus one, and the time of convergence of P^k to W will depend on the modulus of the second eigenvalue, $\lambda^* = \max_j\{|\lambda_j| : \lambda_j \textit{ is an eigenvalue and } \lambda_j \neq 1\}$. Some bounds for the time of convergence are known (see [Beh00] and [LPW09] for example), nevertheless the bounds using eigenvalues are restricted to reversible matrices, i. e., such that P and the equilibrium distribution π are in *detailed balance*.

Periodic non autonomous graphs can be used to model periodic non autonomous discrete dynamical systems, see [AS15] and [SSF14]. On the other hand, discrete periodic non autonomous dynamical systems are being increasingly considered as good models in applications, namely in biology, for example in modeling periodically forced populations ([Hen00] and references therein). We extend the results given in [SSF14], where we introduced the notion of equilibrium distribution in periodic non autonomous graphs and gave an estimate for the convergence time to the equilibrium under certain conditions. Namely, given a p-periodic non autonomous graph $\mathcal{G} = (V, (E_i)_{i=0}^{\infty})$ with adjacency matrices $(P_i)_{i=0}^{\infty}$, consider the cyclic products $B_i = P_i \cdots P_{(i+p-1) \mod p}, i = 1, \ldots, p$. Requiring one of the matrices B_i to be reversible we provided an estimate for the maximal relative error using λ^*, the second eigenvalue in modulus associated to the matrices B_i.

However, in both autonomous and non autonomous case, that kind of restriction brings some issues: in applications we deal with adjacency matrices which are not necessarily reversible, for example, matrices obtained from restricting the Perron-Frobenius operator to a finite dimensional space, see [BG97]. On the other hand, in the non autonomous case, how to choose stochastic matrices $P_i, i = 0 \ldots p-1$, such that one of the cyclic products B_i, $i = 0 \ldots p-1$, is reversible?

The estimates presented concern both autonomous and non autonomous graphs, giving weaker conditions to have useful bounds for the time of convergence for the equilibrium distribution. Roughly, the bounds proposed depend on the modulus of the second eigenvalue and on the dimension of its Jordan blocks, generalizing to a wider class of matrices the results of [SSF14] and [Beh00].

References

[AS12] J. Alves and L. Silva. Periodic paths on nonautonomous graphs. *Linear Algebra and Its Applications*, 437(3):1003–1015, 2012.

[AS15] J. Alves and L. Silva. Nonautonomous graphs and topological entropy of nonautonomous lorenz systems. *International Journal of Bifurcation and Chaos*, 25(6), 2015.

[Beh00] E. Behrends. *Introduction to Markov chains with special emphasis on rapid mixing.* Vieweg, 2000.

[BG97] A. Boyarsky and P. Góra. *Laws of chaos.* Birkhauser, 1997.

[Hen00] S. M. Henson. Multiple attractors and resonance in periodically forced population models. *Phisica D*, 140:33–49, 2000.

[LM95] D. Lind and B. Marcus. *An Introduction to Symbolic Dynamics and Coding.* Cambridge University Press, 1995.

[LPW09] D. Levin, Y. Peres, and E. Wilmer. *Markov Chains and Mixing Times.* American Mathematical Society, 2009.

[Sil08] D. D. Siljak. Dynamic graphs. *Nonlinear Analysis: Hybrid Systems*, 2(2):544–567, 2008.

[SSF14] T. Silva, L. Silva, and S. Fernandes. Equilibrium distributions of discrete non-autonomous graphs. *Journal of Difference Equations And Applications*, 20(8):1190–1200, 2014.

[SSF16] T. Silva, L. Silva, and S. Fernandes. Convergence time to equilibrium distributions of autonomous and periodic non-autonomous graphs. *Linear Algebra and Its Applications*, (488):199–215, 2016.

[Str88] G. Strang. *Linear Algebra and its Applications.* Brooks/Cole, 1988.

[Wal82] Peter Walters. *Introduction to ergodic theory.* Springer-Verlag, 1982.

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Price competition in a nonlinear differentiated duopoly with isoelastic demand

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Abstract

This article represents the first attempt to characterise the dynamics of a nonlinear duopoly with price competition and horizontal product differentiation by accounting for non-negativity constraints (on profits and the market demand). The model is set up by following the tradition led by Bischi et al. (1998), according to which players have limited information. It shows several local and global phenomena of a two-dimensional discrete time system when the price demand elasticity varies. It also points out the differences from both a mathematical and economic point of views in the dynamics of the economy when the non-negativity constraints are not binding and when they are binding. This is done by combining mathematical techniques and simulative exercises.

Keywords Chaos; Local and global bifurcations; Price competition; Product differentiation

References

[1] Andaluz, J., Jarne, G., 2016. Stability of vertically differentiated Cournot and Bertrand-type models when firms are boundedly rational. Annals of Operations Research 238, 1–25.

[2] Bischi, G.I., Gallegati, M., Naimzada, A., 1999. Symmetry-breaking bifurcations and representative firm in dynamic duopoly games, Annals of Operations Research 89, 253–272.

[3] Bischi, G.I., Stefanini, L., Gardini, L., 1998. Synchronization, intermittency and critical curves in a duopoly game. Mathematics and Computers in Simulation 44, 559–585.

[4] Cánovas, J.S., Puu, T., Ruíz, M., 2008. The Cournot–Theocharis problem reconsidered. Chaos, Solitons & Fractals 37, 1025–1039.

[5] Fanti, L., Gori, L., Sodini, M., 2012. Nonlinear dynamics in a Cournot duopoly with relative profit delegation. Chaos, Solitons & Fractals 45, 1469–1478.

[6] Fanti, L., Gori, L., Sodini, M., 2015. Nonlinear dynamics in a Cournot duopoly with isoelastic demand. Mathematics and Computers in Simulation 108, 129–143.

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[7] Fanti, L., Gori, L., Mammana, C., Michetti, E., 2013a. The dynamics of a Bertrand duopoly with differentiated products: synchronization, intermittency and global dynamics. Chaos, Solitons & Fractals 52, 73–86.

[8] Sushko, I., Gardini, L., Matsuyama, K., 2014. Superstable credit cycles and U-sequence. Chaos, Solitons & Fractals 59, 13–27.

[9] Tramontana, F., Gardini, L., Puu, T., 2010. Global bifurcations in a piecewise-smooth Cournot duopoly game. Chaos, Solitons & Fractals 43, 15–24.

Discrete Bessel functions and partial difference equations

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In their paper [1], M. Bohner and T. Cuchta have proposed a new definition of the discrete Bessel function, which is different from discrete Bessel functions studied in earlier papers. Its advantage is that it shares many properties with the classical Bessel function, e.g., it satisfies the difference equation

$$t(t-1)\Delta^2 y(t-2) + t\Delta y(t-1) + t(t-1)y(t-2) - n^2 y(t) = 0,$$

which is a discrete analogue of the Bessel differential equation

$$t^2 y''(t) + ty'(t) + (t^2 - n^2)y(t) = 0.$$

Inspired by the paper [1], we have introduced a new class of discrete Bessel functions and discrete modified Bessel functions denoted by $\mathcal{J}_n^c$ and $\mathcal{I}_n^c$, respectively, where $n \in \mathbb{N}_0$ is the order and $c \in \mathbb{R}$ is a parameter [2]. These functions are the discrete analogues of the functions $t \mapsto J_n(ct)$ and $t \mapsto I_n(ct)$, where I_n and J_n are the classical Bessel function and modified Bessel function, respectively. If $c = 1$, then $\mathcal{J}_n^c$ reduces to the discrete Bessel function from [1].

Our motivation comes from the theory of lattice differential equations, i.e., equations with discrete space and continuous time. It is known that the fundamental solutions of the semidiscrete wave equation have the form $u_1(x,t) = J_{2x}(2ct)$ and $u_2(x,t) = \int_0^t J_{2x}(2cs)\,\mathrm{d}s$, where $x \in \mathbb{Z}$ and $t \geq 0$ (see [3]). The fundamental solution of the semidiscrete diffusion equation has the form $u(x,t) = e^{-2ct} I_x(2ct)$ (see [4]).

Using the new functions $\mathcal{J}_n^c$ and $\mathcal{I}_n^c$, we obtain similar formulas for the fundamental solutions of the purely discrete wave equation and diffusion equation. Formulas for fundamental solutions of these partial difference equations are already available in the existing literature (e.g., [3, 4]), but they have a different form. Expressing them in terms of the discrete Bessel functions can simplify the study of their properties, such as the oscillatory behavior.

References

[1] M. Bohner, T. Cuchta, *The Bessel difference equation*, Proc. Amer. Math. Soc. 145 (2017), 1567–1580.

[2] A. Slavík, *Discrete Bessel functions and partial difference equations*, submitted for publication.

[3] A. Slavík, *Discrete-space systems of partial dynamic equations and discrete-space wave equation*, Qual. Theory Dyn. Syst., in press, DOI: 10.1007/s12346-016-0193-0.

[4] A. Slavík, P. Stehlík, *Explicit solutions to dynamic diffusion-type equations and their time integrals*. Appl. Math. Comput. 234 (2014), 486–505.

Spectra of indefinite perturbations of discrete operators

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(joint work with Petr Vaněk)

In this talk we provide sufficient conditions for positive (semi)definiteness of sign-changing diagonal perturbations of positive semidefinite difference operators and their matrix representations, the graph Laplacian matrices. Our estimates arise from the discrete version of the Poincaré inequality and involve the algebraic connectivity, i.e., the second smallest eigenvalue of the graph Laplacian matrix. We illustrate our results by numerical experiments and discuss the optimality of our assumptions. We also discuss applicability to discrete boundary value problems, stability of stationary solutions of lattice and graph dynamical systems. Finally, we examine possible extensions to difference operators which cannot be represented by graph Laplacian matrices.

References

[1] S.-N. Chow, *Lattice dynamical systems*, Dynamical systems, 1–102, Lecture Notes in Math., 1822, Springer, Berlin, 2003.

[2] R. Merris, *Laplacian Matrices of Graphs: A Survey*, Linear Algebra Appl. 197-198(1994), 143–176.

[3] P. Stehlík, P. Vaněk, *Sign-changing diagonal perturbations of graph Laplacian matrices*. Submitted.

[4] P. Stehlík, On Variational Methods for Periodic Discrete Problems, J. Difference Eq. Appl. 3 (2008), 259–273.

[5] J. Volek, *Landesman-Lazer conditions for difference equations involving sublinear perturbations*, J. Difference Equ. Appl. 22 (2016), no. 11, 1698–1719.

Spectra of indefinite perturbations of discrete operators

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In this talk we provide sufficient conditions for positive (semi)definiteness of sign-changing diagonal perturbations of positive semidefinite difference operators and their matrix representations, the graph Laplacian matrices. Our estimates arise from the discrete version of the Poincaré inequality and involve the algebraic connectivity, i.e., the second smallest eigenvalue of the graph Laplacian matrix. We illustrate our results by numerical experiments and discuss the optimality of our assumptions. We also discuss applicability to discrete boundary value problems, stability of stationary solutions of lattice and graph dynamical systems. Finally, we examine possible extensions to difference operators which cannot be represented by graph Laplacian matrices.

References

[1] S.-N. Chow, *Lattice dynamical systems*, Dynamical systems, 1–102, Lecture Notes in Math., 1822, Springer, Berlin, 2003.

[2] R. Merris, *Laplacian Matrices of Graphs: A Survey*, Linear Algebra Appl. 197-198(1994), 143–176.

[3] P. Stehlík, P. Vaněk, *Sign-changing diagonal perturbations of graph Laplacian matrices*. Submitted.

[4] P. Stehlík, On Variational Methods for Periodic Discrete Problems, J. Difference Eq. Appl. 3 (2008), 259–273.

[5] J. Volek, *Landesman-Lazer conditions for difference equations involving sublinear perturbations*, J. Difference Equ. Appl. 22 (2016), no. 11, 1698–1719.

On a problem of Derfel

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28 April 2017

The talk concerns the functional equation

$$f(x) = \frac{1}{2} f(x-1) + \frac{1}{2} f(-2x). \tag{1}$$

Last year, during *21st European Conference on Iteartion Theory* held in Innsbruck, Professor Gregory Derfel posed the following:

Problem *Is there a non-constant bounded continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfying equation* (1)?

I will show a partial solution to the Problem. The talk is divided into three parts. In the first part I will prove that every bounded continuous solution of equation (1) which attains its extreme value is constant. It is the main result. Its proof is quite elementary.

In the second part of my presentation I will discuss the properties of solutions of the equation (1) which do not attaining their extreme values. In this case the Problem is still open. I will show, for example, the following fact

Let $a, b \in \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be a bounded continuous solution of equation (1)*. If*

$$\inf\{f(t) : t \in \mathbb{R}\} < f(x) < \sup\{f(t) : t \in \mathbb{R}\} \quad \textit{for every } x \in \mathbb{R},$$

then

(*i*) $\inf\{f(x) : x \in \mathbb{R}\} = \inf\{f(x) : x < a\} = \inf\{f(x) : x > b\}$,
(*ii*) $\sup\{f(x) : x \in \mathbb{R}\} = \sup\{f(x) : x < a\} = \sup\{f(x) : x > b\}$.

The last part of my talk is connected with solutions of equation (1) in a smaller class of functions. For example, we can see that there are no non-trivial, bounded, continuous, with bounded variation solutions of equation (1).

Smale Horseshoe in 2D Noninvertible Maps

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Abstract

The original Smale horseshoe [5] has been constructed for *smooth diffeomorphisms.* Our aim is to discuss similar construction for *endomorphisms.* We first recall the characteristic features of noninvertible maps, associated with the structure of stable and unstable invariant sets of a saddle [7], [2], [1], [6], [4]. Then, as an example of the Smale horseshoe in a nonsmooth noninvertible map, we construct it for a 2D *piecewise linear* map $F : \mathbb{R}^2 \to \mathbb{R}^2$ which is known as *Border Collision Normal Form*, given by two linear maps F_L and F_R which are defined in two half planes denoted L and R:

$$F : (x, y) \mapsto \begin{cases} F_L(x, y) \text{ if } (x, y) \in L, \\ F_R(x, y) \text{ if } (x, y) \in R, \end{cases}$$

where

$$F_L \quad : \quad \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \tau_L x + y + \mu \\ -\delta_L x \end{pmatrix}, \; L = \{(x, y) : x \leq 0\},$$

$$F_R \quad : \quad \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \tau_R x + y + \mu \\ -\delta_R x \end{pmatrix}, \; R = \{(x, y) : x > 0\}.$$

Here τ_L, τ_R are the traces and δ_L, δ_R are the determinants of the Jacobian matrix of map F in the left and right halfplanes, i.e., in L and R, respectively. The dynamics of map F is nowadays quite intensively studied by many researchers not only due to its appearance in several applications, but also in order to classify border collision bifurcations in generic 2D piecewise smooth maps [3].

References

[1] Y. Maistrenko, I. Sushko, L. Gardini (1998) About two mechanisms of reunion of chaotic attractors, Chaos, Solitons & Fractals, 9(8), 1373-1390.

[2] C. Mira, L. Gardini, A. Barugola, J.C. Cathala (1996) Chaotic Dynamics in Two-Dimensional NonivertibIe Maps, World-Scientific Publ. Co., Nonlinear Sciences, Series A, Singapore.

[3] H. E. Nusse, J. A. Yorke (1992) Border-collision bifurcations including period two to period three for piecewise smooth systems, Physica D, 57, 39-57.

[4] E. Sander (2000) Homoclinic tangles for noninvertible maps, Nonlinear Analysis 41, 259-276.

[5] S. Smale (1963) Diffeomorphisms with many periodic points, Differential and Combinatorial Topology, ed. S. S. Cairns, Princeton, NJ: Princeton University Press, 63-80.

[6] L. P. Shilnikov, A. L. Shilnikov, D. V. Turaev L. O. Chua (1998, 2001), Methods of Qualitative Theory in Nonlinear Dynamics, Part I, II, World Scientific, Singapore.

[7] M. Shub (1969) Endomorphisms of compact differentiable manifolds, Amer. J. Math. 91, 175-199.

ON LOCAL STABILITY FOR SOME EXPONENTIAL-TYPE DIFFERENCE EQUATIONS

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In [1] was considered a first order difference equation

$$x_{n+1} = (1 - x_n)(1 - e^{-Ax_n}), \quad n = 0, 1, ... \tag{1}$$

that is a variation of the classical Reed-Frost continous-time model. In this model parameter A can be interpreted as infectivity of the disease. Authors of [1] also are interested in model

$$x_{n+1} = \left(1 - \sum_{j=0}^{k-1} x_{n-j}\right)(1 - e^{-Ax_n}), \quad n = 0, 1, ... \tag{2}$$

Authors of [3] proposed Research Project 6.7.1. and Research Project 6.7.2. about the difference equation (2).

In [4] was proposed Open Problem 6.10.14 about the difference equation

$$x_{n+1} = (1 - x_n - x_{n-1})(1 - e^{-Ax_n}), \quad n = 0, 1, ... \tag{3}$$

Authors of [9] study the oscillation, global asymptotic stability, and other properties of positive solutions of the difference equation (2). In [7] was investigated the global stability of the negative solutions of (2). In [6] was considered the fuzzy difference equation (2) where A and the initial values are in a class of fuzzy numbers. System of difference equations related to the model (2) are studied in [5] and [8]. Model (2) is extended in [2].

Accordingly to Open Problem 6.10.14 ([4]), we study the existence of positive solutions, the existence of a unique nonnegative equilibrium and the local asymptotic stability of the difference equation

$$x_{n+1} = (1 - x_n - x_{n-1})(1 - e^{-A(x_n + x_{n-1})}), \quad n = 0, 1, ..., \tag{4}$$

where $A \in (0, \infty)$ and the initial values x_{-1}, x_0 are arbitrary real positive numbers such that $x_{-1} + x_0 < 1$.

We also consider the difference equation

$$x_{n+1} = \left(1 - \sum_{j=0}^{k-1} x_{n-j}\right)\left(1 - e^{-A\sum_{j=0}^{k-1} x_{n-j}}\right) \tag{5}$$

where $A \in (0, \infty)$ and the initial values $x_{-k+1}, x_{-k+2}, ..., x_0$ are arbitrary real positive numbers such that $x_{-k+1} + x_{-k+2} + ... + x_0 < 1$.

This work is collaboration with I. Bula, University of Latvia.

REFERENCES

[1] K.L. Cooke, D.F. Calef, E.V. Level. Stability or chaos in discrete epidemic models. In: *Nonlinear Systems and Applications: An International Conference*, 73-93, 1977.

[2] E.-G. Gu. The nonlinear analysis on a discrete host-parasitoid model with pesticidal interference. In: *Commun Nonlinear Sci Numer Simulat*, 14, 27202727, 2009.

[3] V.L. Kocic, G. Ladas. *Global Behavior of Nonlinear Difference Equations of Higher Order with Applications*. Springer Netherlands, 1993.

[4] M.R.S. Kulenovic, G. Ladas. *Dynamics of Second Order Rational Difference Equations: With Open Problems and Conjectures*. Chapman and Hall/CRC, 2001.

[5] G. Papaschinopoulos, G. Stefanidou, K.B. Papadopoulos. On a modification of a discrete epidemic model. *Computers and Mathematics with Applications*, **59** 3559-3569, 2010.

[6] G. Stefanidou, G. Papaschinopoulos, C. J. Schinas. On an Exponential-Type Fuzzy Difference Equation. *Advances in Difference Equations*, **2010** 19 p., 2010.

[7] S. Stević. On a Discrete Epidemic Model. *Discrete Dynamics in Nature and Society*, **2007** 10 p., 2007.

[8] Z. Wei, M. Le. Existence and Convergence of the Positive Solutions of a Discrete Epidemic Model. *Discrete Dynamics in Nature and Society*, **2015** 10 p., 2015.

[9] D.C. Zhang, B. Shi. Oscillation and global asymptotic stability in a discrete epidemic model. *Journal of Mathematical Analysis and Applications*, **278** 194-202, 2003.

“Dribbling Method" for continuation of bifurcation curves from conservative into dissipative systems

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Abstract

In area-preserving maps of the plane, modelling conservative systems, it is customary to see a high coexistence of stable periodic solutions. In area-contracting maps, modelling dissipative systems, coexistence of stable (and attracting) periodic solutions is much more elusive. Homoclinic bifurcations result in infinite coexistence, albeit very difficult to detect. We study quasi-conservative maps, i.e. maps for which the area-contraction factor is almost 1. Using the area-contraction factor as parameter, we devised a numerical method to continue periodic solutions and their bifurcation values from the conservative case into the dissipative. Such bifurcation values are in general highly degenerate and allow no obvious continuation method. We devised one, based on topological considerations, and dribbling around the degeneracies.

This method allowed following entire families of coexisting elliptic periodic solutions, into families of coexisting sinks in the dissipative Henon map. In turn, some of the families coexist. All families arise via a rotatory homoclinic tangency.

A behavioral cobweb model

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Abstract

A cobweb model is extended in order to consider the notion of transaction utility (Thaler, R. 1985. Mental accounting and consumer choice. Marketing Science 4, 199-214). As a consequence, a psychological reference price enters directly into the demand function. Differently from the traditional cobweb model, the economy is described by a discontinuous map. Such a feature, deriving from the behavioral assumptions, directly influences the equilibrium price, its stability properties and the emerging dynamics. Keeping the essential underlying mechanics of the model intact, this behavioral feature covers the phenomenon of loss aversion manifested in agents' behavior and changes the model's dynamics considerably, permitting us to mimic some stylized facts concerning the dynamic evolution of prices and their volatility.

JEL codes: D03, E32

Keywords: Cobweb model; Reference price; Transaction utility; Behavioral economics; Discontinuous maps; Complex dynamics.

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Landesman-Lazer conditions for difference equations with graph Laplacian

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We study the existence and uniqueness for difference equations on graphs. This general setting involves, e.g., Neumann and periodic problems for both ordinary and partial difference equations. We consider semilinear problems with sublinear growth of the nonlinearity. All the proofs are based on reformulating these discrete problems as a general singular algebraic system. Firstly, we use variational techniques (specifically, the Saddle Point Theorem) and prove the existence result based on a type of Landesman-Lazer condition. Then we show that for a certain class of bounded nonlinearities this condition is even necessary and therefore, we specify also the cases in which there does not exist any solution. Finally, we study the uniqueness.

References

[1] J. Volek, *Landesman-Lazer conditions for difference equations involving sublinear perturbations*, J. Difference Eq. Appl. 22 (2016), 1698–1719.

On singular iteration groups of monotonic functions

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Let $I = (a, b)$ and $\varphi : I \to \mathbb{R}$ be a non-increasing continuous surjection. Denote by $\{I_\alpha, \alpha \in A\}$ the family of the intervals of constancy of φ. Put $L := I \setminus \bigcup_{\alpha \in A} \mathrm{Int}\, \mathrm{I}_\alpha$. Define the set-valued functions $F^t(x) = \varphi^{-1}[\{\varphi(x) + t\}],\ t \in \mathbb{R},\ x \in I$. The family $\{F^t : I \to 2^I\}$ forms a set-valued iteration group, i.e. $F^t \circ F^s = F^{t+s},\ \ t, s \in \mathbb{R}$, where $F^t \circ F^s(x) = \bigcup_{y \in F^s(x)} F^t(y)\ \ x \in I$. Define $f^t_-(x) := \inf\ F^t(x)$, $f^t_+(x) := \sup\ F^t(x)$ for $t \in R,\ x \in I$. The families $\{f^t_-, t \in R\}$ and $\{f^t_+, t \in R\}$ are iteration groups such that f^t_- and f^t_+ for $t \in R$ are non-decreasing discontinuous functions constant on the intervals I_α, moreover $f^t_-[I] \subset L$, $f^t_+[I] \subset L$, $t \in R$. Define $T := \{t \in \mathbb{R} : \varphi[I \setminus L^*] + t = \varphi[I \setminus L^*]\}$. If $T \neq \{0\}$, then T is a countable Abelian group and the set of intervals I_α is countable. If T is acyclic group then the set L is a Cantor set and φ is a singular Lebesgue function such that $\varphi[L] = \mathbb{R}$. If $T \neq \{0\}$ then there exists an iteration group $\{f^t, t \in T\}$ of homeomorphisms restricted to group T such that $f^t_- \leq f^t \leq f^t_+$ for $t \in T$ and T is a maximal group with this property.

References

1. Zdun M. C. *On singular interval-valued iteration groups*, Frontiers in Appl. Math. Stat. Sept. 2016, Vol. 3 Article 13.

2. Zdun M. C. *On set-valued iteration groups generated by commuting functions* J. Math. Annal. Appl. 398 (2013), 638-648.

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