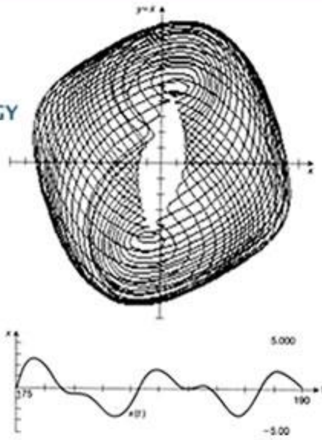
NED 2013

Dedicated to the memory of Richard Goodwin in the centennial of his birth

UNIVERSITÀ DI SIENA 1240

COST EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY



University of Siena, Siena, Italy, 4-6 July 2013

DIPARTIMENTO DI ECONOMIA POLITICA E STATISTICA

DEPARTMENT OF ECONOMICS & STATISTICS

ChiantiBanca

BCC

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A. Vercelli
F. Wagener

Local Organiser

S. Sordi

Keynote Speakers

R. Abraham
I. Kubin
K. Matsuyama

SOME USEFUL ADDRESSES AND MAPS

CONFERENCE LOCATION:

Department of Economics and Statistics
“Plesso di San Francesco” (Ex-Faculty of Economics “Richard M. Goodwin”)
Piazza San Francesco 7

- All plenary sessions take place in “Aula 9” (Ground floor)
- Parallel sessions take place in “Aula 9” and in “Aula Goodwin 2” (I floor)
- All meetings of COST Action IS1104 take place in “Aula Goodwin” (II floor)

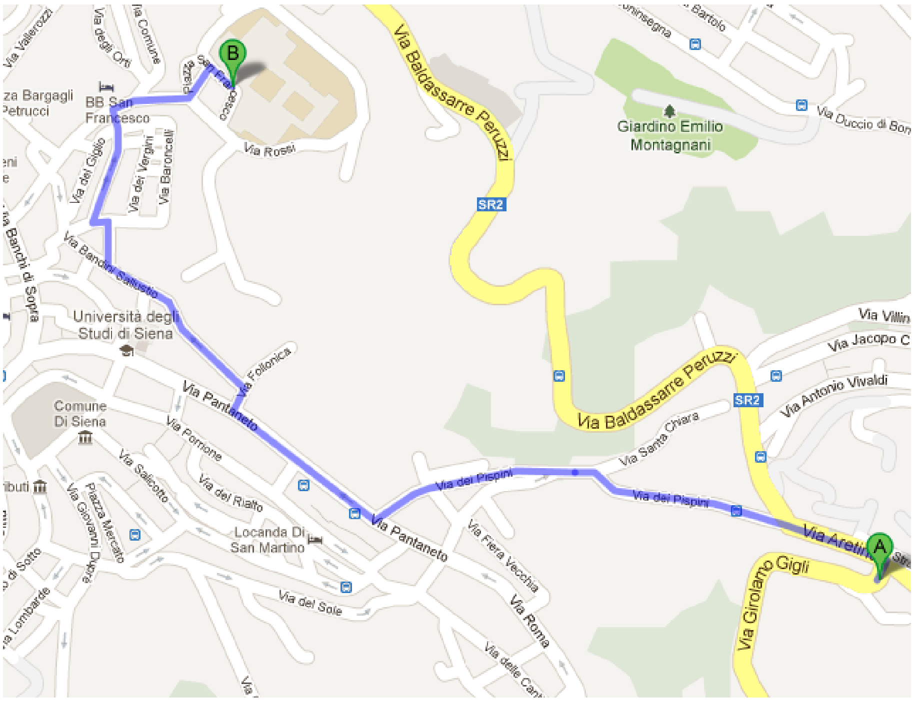
HOTEL ADDRESSES:

- “Hotel Alex”, via G. Gigli, 5; Tel.: +39 0577282338
- “Hotel Athena”, via Paolo Mascagni, 55; Tel.: +39 0577286313
- “Hotel Chiusarelli”, viale Curtatone, 15; Tel.: +39 0577280562
- “Hotel Moderno”, via B. Peruzzi 19; Tel.: +39 0577280034
- “Residenza d’Epoca Locanda San Martino”, via San Martino 14; Tel.: +39 0577271366
- “Residenza d’Epoca Palazzo Fani Mignanelli”, via Banchi di Sopra, 15; Tel.: +39 0577283566

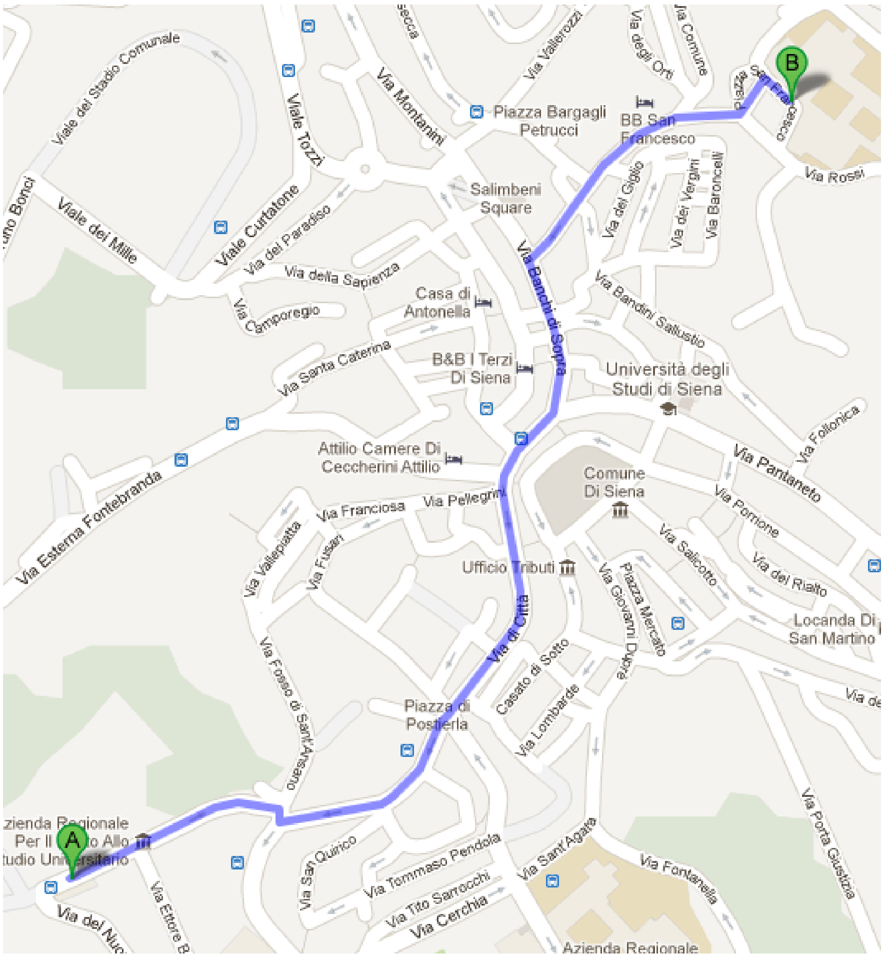
UNIVERSITY CANTEEN “Sallustio Bandini”

Via S. Bandini 47 (just 5 minutes on foot from the faculty)

All participants will be able to gain admission to the University canteen by showing their conference badges. The canteen is self-service and open Mon-Fri between 12:00 and 14:30 at lunch time and between 19:30 and 21:15 in the evening. Prices are 5.50 Euro for a full meal, 4.50 without the second (main) course and 5.00 without the first course.



A, Hotel Alex → B, Piazza San Francesco



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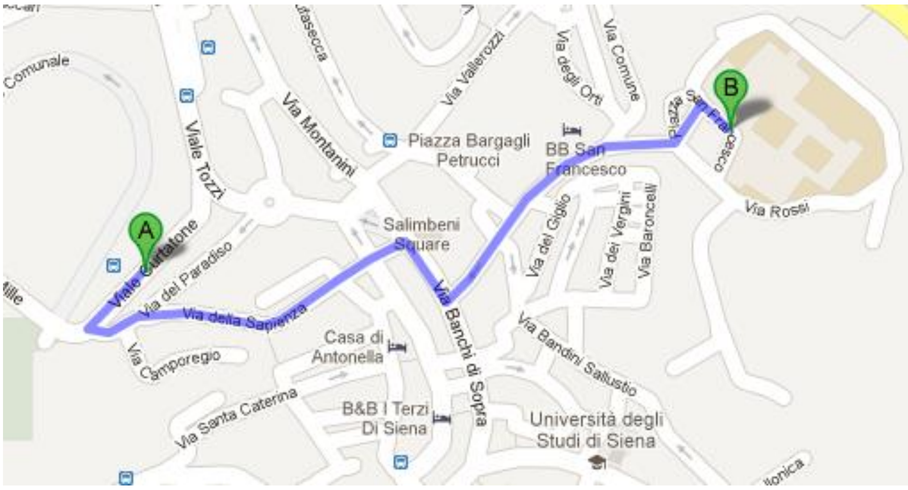

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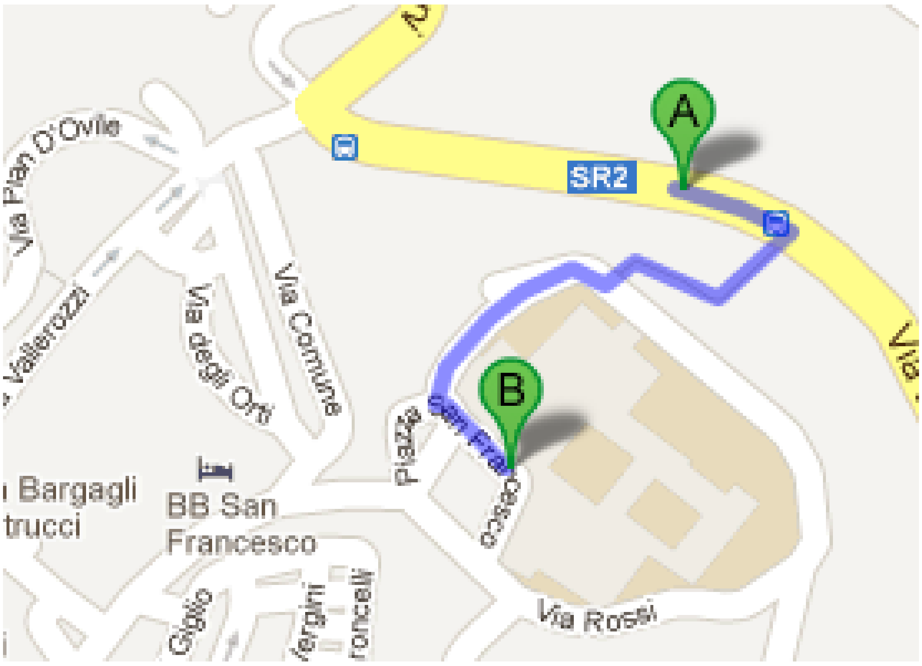

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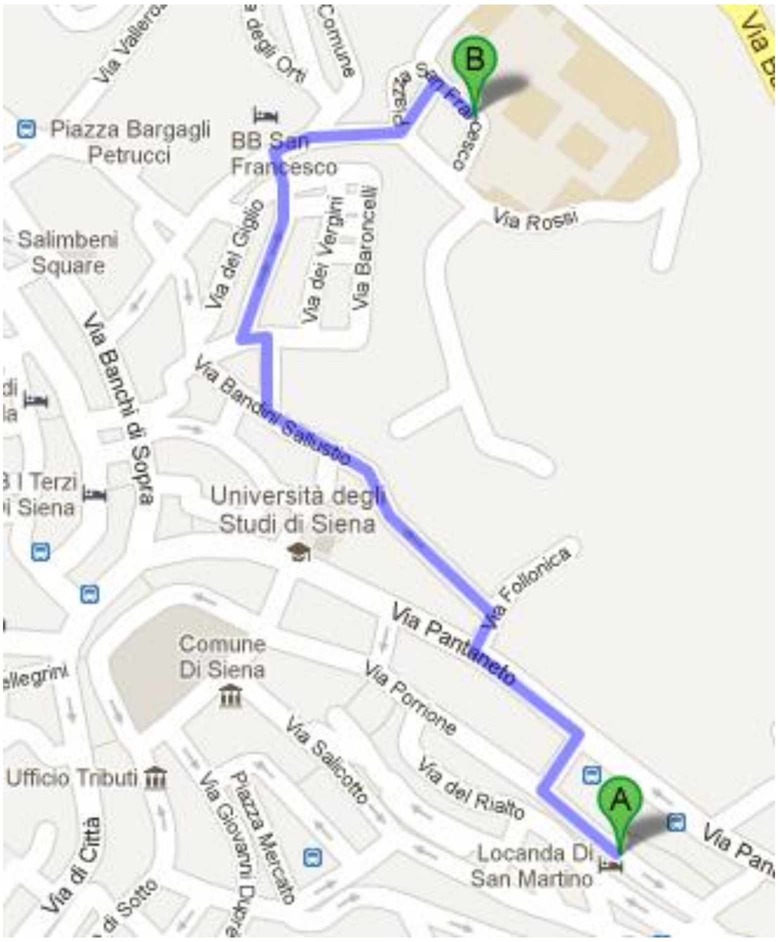

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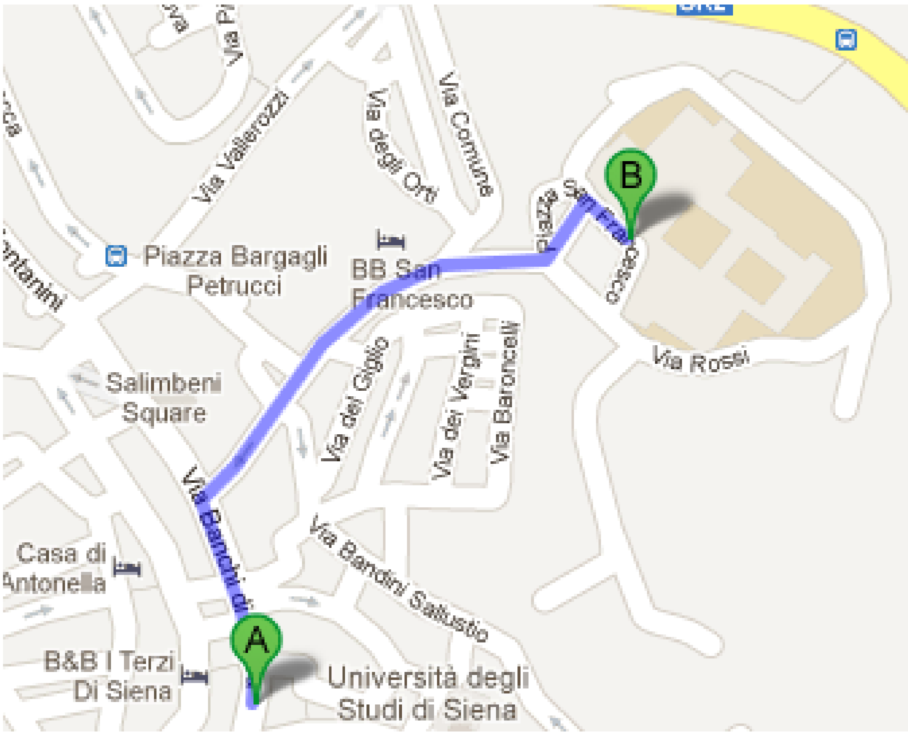
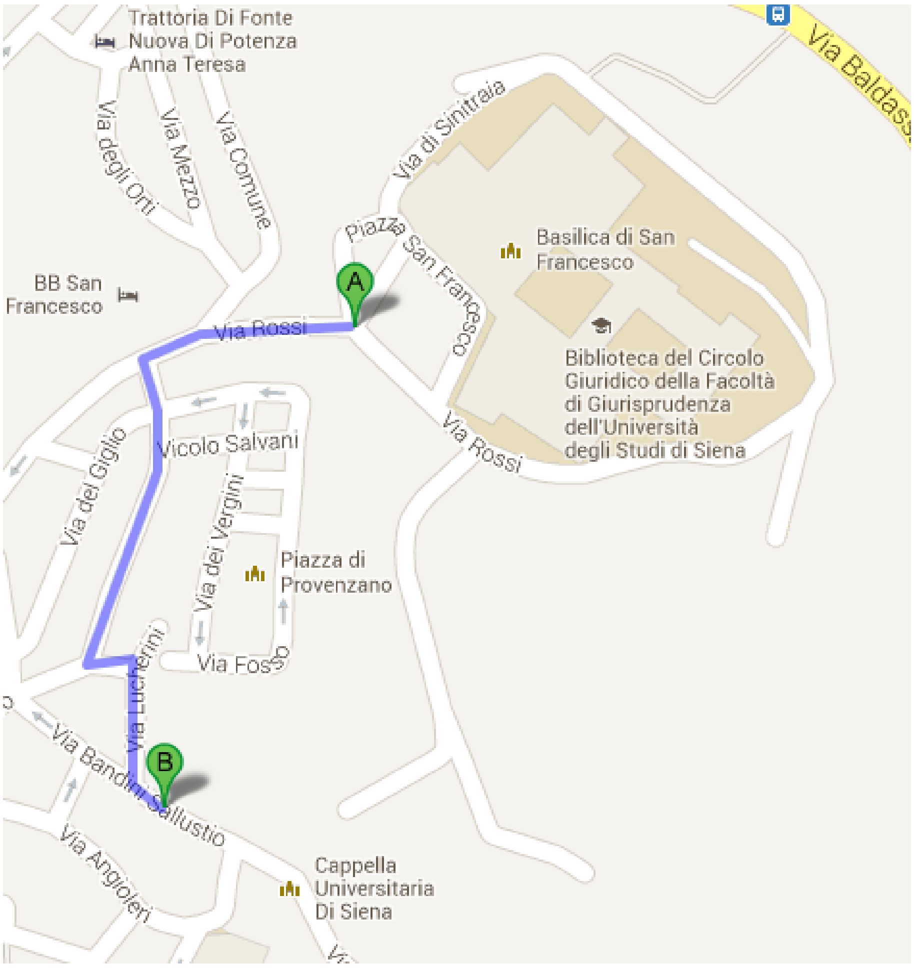


A, Residenza d’Epoca Palazzo Fani Mignanelli → B, Piazza San Francesco



A, Piazza San Francesco $\rightarrow$ B, Mensa Universitaria “Sallustio Bandini” (University Canteen)

PROGRAMME

A9, “Aula 9”, Ground floor
AG, “Aula Goodwin”, II floor
AG2, “Aula Goodwin 2”, I floor

GT, “Goodwinian Themes”
NED, “Nonlinear Economic Dynamics”
WG1, Working Group on “Economic Geography Modeling”
WG2, Working Group on “Institutions and Markets”
WG3, Working Group on “Social and Industrial Interactions”
WG4, Working Group on “Mathematical and Computational Methods”

Programme Overview

Thursday, 4 July

08:30-09:30 Registration **(A9)**
Meeting of the Steering Committee **(AG)**

09:30-09:45 1) Opening Ceremony **(A9)**

09:45-10:30 2) I Keynote speaker: R. ABRAHAM **(A9)**

10:30-11:00 *Coffee break*

11:00-12:40 3) Plenary Session on GT **(A9)**

12:40-15:00 *Lunch break (free)*
14:15-15:00 Meeting of WG3 **(AG)**

15:00-17:05 4) Parallel Session on WG2 **(A9)**
15:00-17:05 5) Parallel Session on WG4 **(AG2)**

17:05-17:30 *Coffee break*

17:30-19:10 6) Plenary Session on NED **(A9**)

Friday, 5 July

08.45-09:30 Meeting of WG2 **(AG)**

09:30-10:15 7) II Keynote speaker: I. KUBIN **(A9)**

10:15-10:40 *Coffee Break*

10:40-12:45 8) Parallel Session on WG1 **(A9)**
10:40-12:45 9) Parallel Session on WG3 **(AG2)**

12:45-15:00 *Lunch break (free)*
14:15-15:00 Meeting of WG1 **(AG)**

15:00-15:45 10) III Keynote speaker: K. Matsuyama **(A9)**

15:45-17:25 11) Plenary Session on GT **(A9)**

17:25 -17:50 *Coffee break*

17:50-19:05 12) Parallel Session on WG4 **(A9)**
17:50-19:05 13) Parallel Session on NED **(AG2)**

Saturday, July 6

09:00-11:05 14) Parallel Session on WG3 **(A9)**
09:00-11:05 15) Parallel Session on WG4 **(AG2)**

11:05-11:30 *Coffee break*

11:30-12:45 16) Plenary Session on COST Action IS1104 **(A9)**

12:45-13:30 Meeting of WG4 **(AG)**

Detailed Programme

Thursday, 4 July	
08:30-09:30 Registration **(A9)**	08:30-09:30 Meeting of the SC **(AG)**
09:30-09:45 1) Opening ceremony **(A9)**: Welcome and introduction to NED2013	
09:45-10:30 2) I KEYNOTE SPEAKER; Chair: L. Gardini **(A9)** R. Abraham, "Chaotic synchronization in economic networks"	
10:30-11:00 *Coffee break*	
11:00-12:40 3) PLENARY SESSION ON GT; Chair: M. Di Matteo **(A9)** 11:00-11:25 L.F. Punzo, "'Zooming into the state': Structure behind dynamics" 11:25-11:50 A. Naimzada, "Chaotic dynamics in alternative discrete-time versions of Goodwin's growth cycle model: A comparison" 11:50-12:15 F. Balibrea, "An non-autonomous extension of the Goodwin model" 12:15-12:40 S. Sordi -A. Vercelli, "The interaction between unemployment, income distribution and financial fragility in a Goodwin-Minsky growth cycle model"	
12:40-15:00 *Lunch break (free)*	14:15-15:00 Meeting of WG3 **(AG)**
15:00-17:05 PARALLEL SESSIONS	
4) WG2 **(A9)**; Chair: A. Kirman 15:00-15:25 F. Purificato, "Financial development and agglomeration" 15:25-15:50 Z. Chen, "Modelling propagation of financial crises" 15:50-16:15 T. Zhi, "Modelling the 'animal spirits' of bank's lending behaviour" 16:15-16:40 A. Negriu, "Financial architecture and technology: A co-evolutionary model" 16:40-17:05 A. Kirman, "An agent-based computational model for sequential Dutch auctions"	5) WG4 **(AG2)**; Chair: M. Matilla 15:00-15:25 R. Cerqueti, "Analysis of the shallow lake model through the method of symbolic images" 15:25-15:50 M. Anufriev, "Heterogeneous beliefs in an asset pricing model with an endogenous fundamental price" 15:50-16:15 V. Mendes, "Learning to play Nash in deterministic uncoupled dynamics" 16:15-16:40 W. Wang, "Economic foundations of technical analysis" 16:40-17:05 M. Matilla, "Detecting the correct temporal lag structure"
17:05-17:30 *Coffee break*	
17:30-19:10 6) PLENARY SESSION ON NED; Chair: B. Rosser **(A9)** 17:30-17:55 T. Asada, "Fiscal and monetary stabilization policies in a Keynesian model of endogenous growth cycle with public debt accumulation" 17:55-18:20 C. Chiarella, "Keynesian DSGD(isequilibrium) modelling: Real-financial market interactions and the term structure of interest rates" 18:20-18:45 H. Dawid , "Strategic location choice under dynamic oligopolistic competition and spillovers" 18:45-19:00 B. Rosser, "Special problems of forests as dynamically complex ecological-economic systems"	

Friday, 5 July

08:45-09:30 Meeting of WG2 (**AG**)

09:30-10:15 7) II KEYNOTE SPEAKER; Chair: P. Commendatore **(A9)**

I. Kubin, "A review of multiregional new economic geography models"

10:15-10:40 *Coffee break*

10:40-12:45 PARALLEL SESSIONS

8) WG1: Economic Geography modeling; Chair: I. Sushko **(A9)**

10:40-11:05 T. Grafeneder-Weissteiner, "Dynamics of outsourcing in a New Economic Geography model"
11:05-11:30 R. Basile, "Semiparametric spatial autoregressive geoadditive models"
11:30-11:55 N.K. Vitanov, "On the nonlinear dynamics of cities growth"
11:55-12:20 C. Proaño, "Economic activity, income distribution and FX-markets in a two-country framework with endogenous speculative gain opinion dynamics"
12:20-12:45 I. Sushko, "Dynamics of a New Economic Geography model in the presence of an outside region"

9) WG3: Social and industrial interactions; Chair: M. Baltov **(AG2)**

10:40-11:05 D. Goldbaum, "Follow the leader: Simulations on a dynamic social network"
11:05-11:30 L. Gori, "Nonlinear dynamics in a duopoly with price competition and isoelastic demand"
11:30-11:55 P. Pin, "Communities and social capital with heterogeneous groups"
11:55-12:20 V. Panchenko, "Asset price dynamics with heterogeneous beliefs and local network interactions"
12:20-12:45 M. Baltov, "On global exponential stability of a class of impulsive discrete neural networks and applications to investment models"

12:45-15:00 *Lunch break (free)*

14:15-15:00 Meeting of WG1 (**AG**)

15:00-15:45 10) III KEYNOTE SPEAKER; Chair: I. Sushko **(A9)**

K. Matsuyama, "Chaos in a credit cycle model with good and bad investments"

15:45-17:25 11) PLENARY SESSION ON GT; Chair: A. Matsumoto **(A9)**

15:45-16:10 P. Flaschel- C. Proaño, "Richard Goodwin's MKS system in a contemporaneous guise"
16:10-16:35 E. Virgillito, "An attempt to overcome Goodwin's growth cycle structural instability in a discrete time framework"
16:35-17:00 G. Feichtinger, "A survey of stable limit cycles in various fields of economics"
17:00-17:25 A. Matsumoto, "Goodwin accelerator model augmented with fixed and distributed time delays"

17:25-17:50 *Coffee break*

17:50-19:05 PARALLEL SESSIONS

12) WG4: Mathematical and Computational Methods; Chair: C. Hommes **(A9)**

17:50-18:15 R. Dieci, "Speculation and real estate cycles: the role of behavioral heterogeneity"
18:15-18:40 J. Tuinstra, "Positive welfare effects of barriers to entry in a dynamic equilibrium model"
18:40-19:05 C. Hommes, "Behavioral rationality"

13) NED; Chair: G. Ricchiuti **(AG2)**

17:50-18:15 E. Ticci, "Investment inflows and welfare reducing structural changes in a natural resource-dependent economy"
18:15-18:40 A. Spelta, "Macroeconomic stability and heterogeneous expectations"
18:40-19:05 M. Ruiz Marin, "Symbolic correlation integral. Getting rid of the epsilon parameter"

Saturday, 6 July

09:00-11:05 PARALLEL SESSIONS

14) WG3: Social and industrial interactions; Chair: F. Lamantia **(A9)**

- 09:00-09:25 H. Yoshida, “Global stability and chaotic oscillations in the Cournot adjustment process”
- 09:25-09:50 M. Muñoz, “On the dynamics of Matsumoto-Nonaka duopoly”
- 09:50-10:15 D. Radi, “Evolutionary Cournot games with different information sets and imitation costs”
- 10:15-10:40 F. Lamantia, “Dynamic mixed oligopoly with socially concerned firms: An evolutionary perspective”

15) WG4: Mathematical and Computational Methods; Chair: F. Tramontana **(AG2)**

- 09:00-09:25 M.P. Martínex-García, “Less than exponential growth with non-constant discounting”
- 09:25-09:50 N. Pecora, “Heuristic selection and heterogeneity”
- 09:50-10:15 M. Sodini, “Local and global dynamics in an OLG growth model with endogenous lifetime and endogenous labour supply”
- 10:15-10:40 M. Bosteranu, “Economics of the earthquake risk mitigation in the urban and constructive structure”
- 10:40-11:05 F. Tramontana, “The bull and bear model of Huang and Day: Some extensions and new results”

11:05-11:30 *Coffee break*

11:30-12:45 16) PLENARY SESSION ON COST Action IS1104; Chair: F. Westerhoff **(A9)**

- 11:30-11:55 A. Panchuk, “Disequilibrium trade and dynamics of stock markets”
- 11:55-12:20 T. He, “Herding, trend chasing and excess volatility”
- 12:20-12:45 F. Westerhoff, “Speculative behavior and the dynamics of interacting stock markets”

12:45-13:30 Meeting of WG4 (**AG**)

ABSTRACTS

1. Ralph ABRAHAM, University of California at Santa Cruz, California, USA

“Chaotic synchronization in economic networks”

Introduction. In his Chaotic Economic Dynamics of 1990, Richard Goodwin repeatedly made use of the Rössler attractor to model economic systems. Since that time, mathematical theory and experiments with coupled Rössler attractors have established robust phenomena of synchronization for such systems. In this talk we will explore the significance of chaotic synchrony for spatial networks of economic systems.

1. Goodwin’s chaotic attractors. Throughout his book, Goodwin made use of chaotic attractors as models of economic units such as factories or banks. Most frequently, he used the Rössler attractor as his favorite model. A trajectory of this system makes a cycle in the X-Y plane, including a fast spike in the Z direction. Goodwin interpreted X and Y as wages and output (regarded as predator and prey) and Z as control policy.

2. Entrainment of oscillators and chaotic attractors. The entrainment (in frequency and phase) of two oscillators (e.g., pendulum clocks) was observed by Huygens in 1665, and explained with dynamical systems theory by Vassalo-Pereira in 1982. This understanding – generalized by me (1984) to a universal geometric theory based upon the isochron construction of Winfree (1967) – led to my conjecture that a periodic pulsatile force could entrain a chaotic attractor. This conjecture was verified, for the Rössler Attractor, by Emily Stone in 1992. Interpreted in the context of Goodwin's model, this would suggest, for example, the entrainment of a chaotic national business cycle by an endogenous periodic force or global policy.

3. Entrainment in lattices and networks. If we consider now – in place of a pair of oscillators or chaotic attractors – a massive lattice or network of coupled systems, we would expect some emergent cooperative behavior as observed in biological systems such as heart muscle or neural cortices. In the context of complex geospatial economic systems, we would expect regions or cliques of synchrony, bounded by membranes or trade barriers, characterized by competition and conflict.

4. Some recent simulation results. Motivated by such qualitative dynamical speculations as these, the simulation of a 400 by 400 two-dimensional lattice of Rössler attractors has been undertaken by Michael Nivala of UCLA, using software developed for his research on arrhythmias in heart muscle tissue. The emergence of synchronous spatiotemporal oscillations in this simulation is highly suggestive of the evolution of industrial conglomerates and banking networks in our geospatial economic networks.

Conclusion. Further research along this line might seek to mimic real data from networks such as:

** the largest banks in Italy, or the Euro zone, or even the planetary system;
** the global network of international trade or currency exchange;
** data sharing among intelligence services, etc.

2. Mikhail ANUFRIEV, University of Technology, Sidney, Australia

“Heterogeneous beliefs in an asset pricing model with an endogenous fundamental price”

The goal of this paper is to extend the seminal model of financial market with heterogeneous agents introduced by Brock and Hommes (1998, Heterogeneous beliefs and routes to chaos in a simple asset pricing model, *Journal of Economic Dynamics & Control*) by relaxing *at the same time two assumptions*: the assumptions of constant variance and of zero outside supply. By relaxing these assumptions we propose a model where the fundamental price and the market price of an asset co-evolve giving *mutual* feedbacks. By endogenously generating excess volatility and repeated bull and bear markets this model preserves the main features of Brock and Hommes model. In addition, however, this model describes a realistic mechanism leading to market crash and can also be used to study severe consequences of the crashes when not only market price but also the fundamental price falls down in the wake of the high market uncertainty.

Brock and Hommes show that a simple model with fundamentalists and chartists can generate persistent deviations of price from fundamental value resulting in long-lasting bubbles followed by severe crashes. This endogenous dynamics is driven by a so-called Adaptive Belief System, an evolutionary mechanism of switching between fundamentalism and chartism. At each period agents chose their forecasting strategy on the basis of differences in the past performances of fundamentalism and chartism. One parameter, the intensity of choice describing how sensitive the choice of the forecasting strategy to the past performances is, plays the crucial role in the dynamics. Specifically, when the intensity of choice is large, the long-run dynamics of the deterministic system becomes complicated generating quasi-cyclical or even chaotic behavior. Even under constant fundamental price, the market price systematically deviates from the fundamental level, thus generating excess volatility and persistent pattern of bubbles and crashes.

Many subsequent contributions generalized the Brock and Hommes model showing that the model is fairly robust to its assumptions. We contribute to this literature and focus on relaxing *two* assumptions made in the original model: (i) the assumption of constant conditional variance and (ii) the assumption of zero outside supply. Whereas the model has already been studied under relaxation of one of these assumptions, the analysis of the model without both assumptions has never been undertaken to the best of our knowledge. When both assumptions are relaxed we arrive to the model whose unique feature is that the fundamental price depends endogenously on the past price fluctuations. The market crashes can become reinforced in our model by this additional feedback.

This Brock-Hommes model was extended by Gaunersdorfer (2000, Endogenous Fluctuations in a Simple Asset Pricing Model with Heterogeneous Agents, *Journal of Economic Dynamics & Control*), who relaxes the assumption of constant conditional expected variance. Gaunersdorfer assumes that agents estimate the variance using exponentially weighted moving average (EWMA) estimator and use this estimation in their demand schedules. The resulting dynamics is characterized by the same local as well as global properties as original Brock and Hommes model. Both Brock and Hommes and Gaunersdorfer models assume zero outside supply. While generalization on positive supply of the original model has been performed in a number of studies, effect of positive supply in Gaunersdorfer model has never been investigated. Importantly, when the positive supply $z^s > 0$ is assumed, the fundamental price depends on the perceived at time t variance of the price, σ_{t-1}^2, because the investors who hold a positive amount of risky asset in the equilibrium require a risk premium. Under an assumption of i.i.d. dividends the fundamental price is given by

$$p_t^f = \frac{\bar{y} - z^s a \sigma_{t-1}^2}{R-1}$$

We assume that agents condition their behavior on the past prices not only by predicting future returns but also estimating the variance. In particular, when fundamentalists compute the fundamental price they take the EWMA estimator of variance of price into account as above. This introduces an additional feedback loop in the model. Now past prices not only affect the agents' returns expectations, as it was in the original model and its previous generalizations, but they also affect a risk perceived by the agents. We investigate the consequences of this "endogenous fundamental price" on the dynamics of the model, which is as follows

$$Rp_t = n_{1,t-1}(p_t^f + \bar{y}) + n_{2,t-1}(p_t^f + v(p_{t-1} - p_{t-1}^f) + \bar{y}) - z^s a \sigma_{t-1}^2$$

$$n_{1,t-1} = \exp\left[\beta\Big((p_{t-1} + y_{t-1} - Rp_{t-2})\frac{p_{t-1}^f + \bar{y} - Rp_{t-2}}{a\sigma_{t-1}^2} - C\Big)\right]/Z_t$$

$$n_{2,t-1} = \exp\left[\beta\Big((p_{t-1} + y_{t-1} - Rp_{t-2})\frac{p_{t-1}^f + v(p_{t-2} - p_{t-2}^f) + \bar{y} - Rp_{t-2}}{a\sigma_{t-1}^2}\Big)\right]/Z_t$$

$$\mu_t = (1 - w_\mu)R_t + w_\mu \mu_{t-1}$$

$$\sigma_t^2 = (1 - w_\sigma)(R_t - \mu_t)^2 + w_\sigma \sigma_{t-1}^2$$

We study this model both analytically and numerically and find that the intensity of choice parameter plays an important role in the model. When it is low, realized and fundamental prices are quite close to each other with fundamental price being more volatile. This regime is consistent with Efficient Market Hypothesis. Differently from Brock and Hommes model the fundamental price is not constant as its variance depends on the actual price dynamics. On the other hand, when the intensity of choice parameter is high, price disconnects from fundamentals and exhibit significantly higher volatility. Importantly, consequence of crash is now even more severe than in the original model, because perceived fundamental price falls immediately after the crash, driving expectations of both fundamentalists and chartists further down.

3. Toichiro ASADA, Chuo University, Tokyo, Japan (jointly with M. Ouchi)

"Fiscal and monetary stabilization policies in a Keynesian model of endogenous growth cycle with public debt accumulation"

In this paper, we study the impact of the fiscal and monetary policy mix on macroeconomic stability by using a variant of the 'high-dimensional Keynesian macrodynamic model' that was developed by Asada, Chiarella, Flaschel and Franke (2010). In particular, the model in this paper is an integration of the models that were presented in Asada (2010) and Asada and Ouchi (2009).

Asada (2010) studied the effect of the 'Taylor rule' type interest rate monetary policy by using a relatively small scale dynamic model that is described by means of a two-dimensional system of differential equations. In Asada (2010), the dimension of the system could be kept low enough because both of the dynamic effect of capital accumulation and that of public debt accumulation were ignored. On the other hand, Asada and Ouchi (2009) formulated a four-dimensional model of 'Keynesian growth cycle' by introducing Kaldorian type technical progress function and capital accumulation effect. The spirit of this model is 'Keynesian' in the sense that both of the under-employment of labor and under-utilization of capital stock due to the insufficient effective demand are allowed for unlike 'neoclassical' endogenous growth model. In Asada and Ouchi (2009), however, the Taylor rule type interest rate monetary policy was not studied.

In this paper, we present an integrated five-dimensional dynamic model that considers both public debt accumulation effect and capital accumulation effect explicitly, and investigate the dynamic effect of the fiscal and monetary policy mix on the macroeconomic stability, instability and cyclical fluctuations both analytically and numerically. We investigate what is the appropriate policy mix to 'stabilize an unstable economy' in the sense of Minsky.

At first glance, our model is somewhat similar to the now fashionable 'New Keynesian' dynamic model that was presented by Woodford, Galí and others. In fact, core parts of both models are some types of 'IS curve', 'Phillips curve', and Taylor type interest rate monetary policy rule. It is important to note, however, that there are some critical differences between these two models. Typical 'New Keynesian' dynamic model is based on the premise that all economic agents including policy makers can behave perfectly rationally by solving the complicated dynamic optimization problem in the environment of perfect foresight or rational expectations. As Mankiw (2001) correctly pointed out, however, the 'New Keynesian Phillips curve' that is based on such a premise produces empirically implausible paradoxical behavior. Moreover, 'New Keynesian' rational expectations approach treats the variables such as actual and expected rates of inflation and nominal rate of interest as 'jump variables' that transform the unstable equilibrium point (in the traditional sense) to the 'stable' equilibrium point. But, as Mankiw (2001) noted, such a premise contradicts the fact that the effect of monetary policy on inflation is 'delayed and gradual'. On the other hand, our model that is based on high-dimensional dynamic Keynesian model by Asada, Chiarella, Flaschel and Franke (2010) is immune from such paradoxes, because it is based on more traditional approach, so to speak, 'Old Keynesian' approach à la James Tobin.

References

[1] Asada, T. (2010) , "Central banking and deflationary depression : A Japanese perspective". In: Cappello, M., Rizzo, C. (eds.), *Central Banking and Globalization,* Nova Science Publishers, New York, pp. 91-114.
[2] Asada, T., Chiarella, C., Flaschel, P., Franke, R. (2010), *Monetary Macrodynamics.* Routledge, London.
[3] Asada, T., Ouchi, M. (2009), "A Keynesian model of endogenous growth cycle". In : Bailly, R. O. (ed.) *Emerging Topics in Macroeconomics,* Nova Science Publishers, New York, pp. 219-149.
[4] Mankiw, G. (2001), "The inexorable and mysterious tradeoff between inflation and unemployment". *Economic Journal* 111, C45-C61.

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4. Francisco BALIBREA, University of Murcia, Murcia, Spain

"A non-autonomous extension of the Goodwin model"

We present an extension of the standard model on economic cycles introduced in 1967 by R.Goodwin [1]. It is well known that it is one of the first combining cyclical behavior and economic growth. It can be interpreted like a prey-predator model similar to that formulated in the thirties of the former century by Lotka and Volterra in the setting of population dynamics. Goodwin showed that the antagonist relationship between workers and capital owners could lead to economic cycles.

Our model is as follows:

$$v'(t) = v(t) f\left(v(t),u(t)\right) = v(t)\left[r\left(v(t)\right) - u(t)\varphi\left(v(t)\right) - m_1\right]$$
$$u'(t) = u(t) g\left(v(t),u(t)\right) = u(t)[s\left(u(t)\right) + cv(t)\varphi\left(u(t)\right) - m_2]$$

which is a system of differential equations where $v(t)$ denotes the *rate of employment* and $u(t)$ the *share of labour* in national income. The differences with the Goodwin model is the introduction of functions of $v(t)$ denoted by $r(v(t))$ and $s(u(t))$ and the constant rates m_1 and m_2 which transforms the system of differential equations of Goodwin model from autonomous to non-autonomous. It drastically changes the behavior of the system. It is proved that the model has four equilibrium points with different behaviors concerning locally asymptotically stability and instability. One key idea of Goodwin's model which is maintained in the new model is that the economy does not stabilize in a unique equilibrium point but is continuously fluctuating.

1. Indications on the model

- The functions $r(v(t))$ and $s(u(t))$ denote the effect that in $v'(t)$ and u'(t) has the fact that some magnitudes that in the Goodwin model are supposed to stay constant are not, for example changes with time of savings, capital, output, etc. The same for the functions $\varphi(v(t)$ and $\varphi(u(t))$. In order to avoid a too general systems, we have found that the following mathematical conditions are natural to be introduced:
    - for all $v(t) > 0$, is $r(v(t)) > 0$, $r'(v(t)) < 0$, and $\lim_{v(t)\to\infty} r(v(t)) = 0$ and similarly
    - for all $u(t) > 0$, is $s(y(t)) > 0$, $s'(u(t)) < 0$, and $\lim_{u(t)\to\infty} s(u(t)) = 0$

  Nevertheless the net rate of both functions can be stated like $|v(t)r(v(t))|' \geq 0$ and $|u(t)s(u(t))|' \geq 0$.
- The function $\varphi(x)$ is such that

  $$\text{for all } x \geq 0,\ \varphi(x) > 0,\ \varphi'(x) \leq 0, \text{ and } [x\varphi(x)]' \geq 0$$

  where $x\varphi(x)$ is bounded when $x \to \infty$. If we think of $v(t)$ as a prey population and $u(t)$ as a predator one, the former conditions translate the idea that when the prey population increases, the rate of consume of preys per predator increases, but the fraction of the total prey population consumed by predator decreases.

- Parameters m_1 and m_2 denote the total rate of mortality respectively of prey and predators
- The rate per unit of prey and predators are given respectively by

$$f\left(v(t),u(t)\right) = r\left(v(t)\right) - u(t)\varphi\left(v(t)\right) - m_1$$
$$g\left(v(t),u(t)\right) = s\left(u(t)\right) + cu(t)\varphi\left(u(t)\right) - m_2$$

which means that the rate of increasing per unit of every factor decreases with the size of its units. It holds that the auto-regulators terms $f_{v(t)}(v(t),u(t))$ and $g_{u(t)}(v(t),u(t))$ are negative.

2. Equilibria of the system

The former system has four different types of equilibrium points:

- The trivial equilibrium point (0,0) for all values of the parameters
- One equilibrium point of the form $(K,0)$ with $r(K) = m_1$ if and only if $m_1 < r(0)$
- One equilibrium point of the form $(0,M)$ with $s(M) = m_2$ if and only if $m_2 < s(0)$
- One equilibrium point of the form (x^*,y^*) where x^* satisfies the equation

$$cx^*\varphi(x^*) + s\left(\frac{r(x^*) - m_1}{\varphi(x^*)}\right) - m_2 = 0$$

and y^* is given by

$$y^* = \left(\frac{r(x^*) - m_1}{\varphi(x^*)}\right)$$

if and only if is $m_1 < r(0) - M\varphi(0)$ and $m_2 < s(0)$ or $m_1 < r(0)$ and $s(0) < m_2 < s(0) + cK\varphi(K)$

- It can be proved that in the former cases, the equilibrium points can be either locally asymptotically stable or instable. Such results on stability must be interpreted.

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Keywords: Autonomous and non-autonomous systems, Lotka-Volterra, equilibrium points, stability and instability
Mathematics Subject Classification: 91B62, 91B54

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5. Milen BALTOV, Burgas Free University, Burgas, Bulgaria (jointly with I. Stamova and A. Stamov)

"On global exponential stability of a class of impulsive discrete neural networks and applications to investment models"

The investment process is one of the most frequent areas of neural network applications. Some of the most representative problems being solved by neural networks are bankruptcy predictions, risk assessments of mortgage and other loans, stock market predictions (stock, bond, and option prices, capital returns, commodity trade, etc.), financial prognoses (returns on investments) and others.

In most of the applications, neural networks outperformed traditional statistical models, such as discriminant and time series regression analysis. But, most of the existing studies over neural network model training and testing schemes. For example, in Sun et al. (2009) the authors circumvent the difficulty of making a distributional assumption of intra-daily market fluctuations by specifying a neural network approach. With this approach, no distributional assumption regarding the return distribution is required for estimating and forecasting the Value at Risk (VaR) using intra-daily data. Using this approach, they forecast VaR using high-frequency data for the German equity market index. In Skabar & Cloete (2002) the authors describe a methodology by which neural networks can be trained indirectly, using a genetic algorithm based weight optimization procedure, to determine buy and sell points for investment products traded on a stock exchange. The essay of Giacomini (2003) is an overview of numerous applications of neural networks in quantitative finance. Further information on these topics can be found in the books of Shadbolt & Taylor (2002), McNelis (2005), and Trippi & Turban (1996).

One of the important problems in the study of investments neural network models is the stability of the equilibrium states. Given a certain data set a neural network model is only an approximation of the underlying problem. This issue gives rise to the important question of what is the domain of the applicability of a specified neural network model. Could it happen that slight perturbations in the data trigger substantially different outputs, and thus, different conclusions?

If an equilibrium of a neural network is globally asymptotically stable, it means that the domain of attraction of the equilibrium point is the whole space and the convergence is in real time. Similar problems appear in other financial models as Paretian models (see, Rachev & Mitnik, 2000). When a neural network is designed to function as an associative memory, it is required that there exist many stable equilibrium points, whereas in the case of solving optimization procedure, it is necessary that the designed neural network must have a unique equilibrium point that is globally asymptotically stable. Therefore, it is of great interest to establish conditions that ensure the global asymptotic stability of equilibrium points of a neural network.

It is well known that price fluctuations can be caused by time delayed influences. In the case of a regular trader he/she expects that both the drift and the volatility of the stock price process are influenced by certain events that happened before the trading period started. Also, the effect of noise disturbances should be taken into account in studying the stability of delayed investment systems.

In real systems there is often a phenomenon of information launching or the abrupt phenomena of random failures or repairs of the components, sudden environmental changes, altering the subsystem's interconnections. It is recognized that a class of neural networks with Markovian jump parameters is the best system to model them. As a result, neural networks with Markovian jump parameters have gained much attention, and many good results have been presented in the literature, see e.g. Liu et al (2010); Zhand & Yu (2012); Wu et al. (2011); Zhu & Cao (2010) and the references therein.

On the other hand, the state of an investments neural network is often subject to instantaneous perturbations and experiences abrupt changes at certain instants which may be caused by frequency changes or other sudden noise, that is, it exhibits impulsive effects. Economic shocks that shift the price curve are primarily driven by the dynamics of investment, market and technological changes. There are, also, some perturbations in prices, which can be caused by nature and governments. Impulsive neural networks have gained increasing research attention in the past few years, since impulsive control arises naturally in a wide variety of applications. Recently, several good impulsive control approaches for impulsive differential systems with delays have been proposed (Liu& Rohlf, 1998; Liu & Willms, 1996; Li et al. 2007; Stamova & Stamov, 2011; Stamova, Stamov & Simeonova, 2012; Liu, 2004; Sakthivel & Anandhi, 2010; Sun & Zhang, 2003).

However, to the best of our knowledge, there has not been any work so far considering the impulsive control of stochastic discrete investments neural networks with Markovian jumping parameters and mode-dependent delays, which is very important in theories and applications and also is a very challenging problem.

Motivated by the above discussion, in this paper, we will investigate a class of impulsive discrete stochastic neural networks with Markovian jumping parameters and model dependent delays. Applying Lyapunov-Razumikhin method we established some criteria for global exponential stability of the system. The organization of this paper is as follows. In the next section, the problems investigated in this paper are formulated, and some preliminaries are presented. We state and prove our main results in Section 3. Then, an illustrative example of a VaR analysis is given to show the effectiveness of the method in Section 4. Finally, concluding remarks are made in Section 5.

The problem of global exponential stability in mean square of the trivial solution will be investigated. A common technique is Lyapunov direct method. We shall use Lyapunov functions from the class _0. Moreover, the technique of investigation essentially depends on the choice of minimal subsets of a suitable space of piecewise continuous functions, by the elements of which the derivatives of Lyapunov functions are estimated (Razumikhin, 1988). It is well known that Lyapunov-Razumikhin function method have been widely used in the treatment of the stability of impulsive functional differential equations (Stamov, 2012; Stamova, 2009). We assume that training of a neural network has been somehow accomplished, and its weights are fixed during learning process. For training we favor powerful methods based on the extended Kalman filter algorithm and back propagation through time (see, e.g., Barabanov & Prokhorov, 2002), but other methods of training can certainly be used.

By using the Lyapunov-Razumikhin technique, sufficient conditions for global exponential stability in mean square of the zero solution of a class of impulsive neural networks with both Markovian jumping parameters and mode-dependent delays are obtained. Our results improved and generalized some results. We show that by means of appropriate impulsive perturbations we can control the stability behavior of the system. The technique can be extended to study other types of impulsive control delayed systems. An illustrative example in VaR analysis is given to show the effectiveness of our results.

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6. Roberto BASILE, Second University of Naples, Naples, Italy (jointly with R. Mínguez, J.M. Monteroz and J. Mur)

"Semiparametric spatial autoregressive geoadditive models"

Spatial dependence and spatial heterogeneity are two characteristics of spatial data that need to be taken into account when modeling spatial economic dynamics. This is a mantra insistently repeated since the influential textbook of Anselin (1988). Spatial dependence reflects a situation where values observed at one location or region depend on the values of neighbouring observations at nearby locations; spatial heterogeneity points to the lack of spatial stability of the relationships under study (functional forms and parameters vary with location and are not homogeneous throughout the data set).

By far, spatial dependence has been the main attractor and concentrates the most singular results in the literature of spatial econometrics (Lesage and Pace, 2009), although there remain important problems to be solved. Spatial heterogeneity, on the contrary, has attracted more attention in the realm of theoretical spatial economics than in the field of econometric modeling. Krugman (1993) attributes the unevenness of local economics to the impact of "first nature" characteristics of local areas, that is geography, whereas "second nature" characteristics belong to the field of interaction and spatial dependence.

When it comes to the point of building spatial econometric models, it is necessary to consider a specific form of (spatial) heterogeneity which characterises spatial data analysis, that is nonlinearity. This is not a point of consensus in the literature on spatial econometrics, where linearity clearly dominates. The premise is that a linear structure, possibly coupled with some previous functional transformation of the variables warrants enough flexibility to account for the complexity of spatial data formation. However, there is growing evidence, coming from different fields, showing that this is a quite optimistic view. Strong nonlinearities have been detected in regional growth models (Arbia and Paelinck, 2003; Azoumahu et al, 2011; Basile and Gress, 2005; Basile, 2008, 2009; Basile et al., 2012; Ertur and Le Gallo, 2009; Fiaschi et al, 2009; Fotopoulos, 2012), in clustering and spatial agglomeration models (Basile et al, 2013), in environmental studies (Le Gallo and Chasco, 2013), in urban economics (McMillen, 1997; Brueckner et al, 2001), in urban sprawl studies (Brueckner, 2000; Irwin and Bockstael, 2007), in social interaction models (Lee et al., 2010) and in housing models (Bourassa et al., 2010; Kim and Bhattecharya, 2009; Goodman, 2003).

The dominant parametric approach in spatial econometrics is not well equipped to deal simultaneously with the three topics (spatial dependence, spatial heterogeneity and nonlinearities). They have been approached separately and only recently there have been attempts to mix some of them. This is the case of Lambert et al. (2013), which combine spatial dependence and nonlinearity in a STAR model, or the case of the Lotka-Volterra prey-predator model discussed in Griffith and Paelinck (2011). The literature on spatial regimes introduces heterogeneity in models with spatial dependence (Fischer and Stumpner, 2010, and references therein), from which the SALE (Spatial Association Local Estimation) (Pace and LeSage, 2004) and Zoom algorithms (Mur et al., 2010) can be considered limiting cases. To our knowledge, few more references can be added. In fact, the history is very short.

Given the limitations of the parametric framework, it is important to pay attention to other, more flexible approaches, which offer a more convenient way of addressing simultaneously the three problems: dependence, heterogeneity and nonlinearity. This is the case of the Spatial Autoregressive Semi-parametric Geoadditive Models developed, among other by Basile and Gress (2005), Su and Jin (2010), Su (2012), Basile et al. (2012) and Montero et al. (2012). The objective of this paper is to describe the main methodological contributions produced recently in this field, which helps us to overcome some of the deficiencies encountered in a parametric framework. We also illustrate the discussion with an extended application to the case of housing prices in Lucas County. Spatial housing markets are indeed a very interesting case study because combines the three characteristics previously mentioned (Meen, 1999).

Section 2 discusses the case of a semiparametric geoadditive model. It describes its potential and different technical aspects related to estimation. A control function approach helps to accommodate the technique to the introduction of a spatial autoregressive term among the explicative factors of the equation. Section 3 discusses several orthogonal reparameterizations for this type of model, which introduce more flexibility into the structure of penalizations. This results in the REML models recently developed in the literature. Section 4 includes an extended application to the case of modelling house prices in Lucas County. The application compares parametric and semiparametric model estimates. The differences are clearly in favour of the more flexible semiparametric technique. Section 5 recaps and summarizes main findings in our work.

Keywords: Spatial econometrics, nonlinearities, semiparametric models
JEL Classification: R11, R12, C14

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7. Maria BOSTENARU, Ion Mincu University of Architecture and Urbanism, Bucharest, Romania (jointly with D. Aldea Mendes)

"Economics of the earthquake risk mitigation in the urban and constructive structure"

The aim of this paper is to explore an aspect of disaster management to which little research exist, namely the economic aspects. The priority areas of the COST action touched are the roles of game theory, agent based modeling and to lesser extent networks and urban public policies in designing decision systems for earthquake risk management. In this paper a research encompassing the following will be presented:

1. Review process

First of all a review of literature dealing with the economic value of building restoration, particularly the rehabilitation of historic city centres, was performed, as earthquake retrofit concerns such historic buildings designed before seismic provisions were developed. This is in line with different European platforms dealing with the subject which we try to attract in the COST Action.

Then a review of novel approaches such as game theory, drama theory and conflict based software for multi-criteria decision problems of economic efficiency in the field of climate change and natural disasters was performed. City building games are actually construction management games, based on the same rules as devices. SimCity included in the 2D version simulations of events such as fire after earthquake in San Francisco. We looked at both computer games and hard copy board games, especially at their rules. Drama theory is an alternative way to game theory to look at decision making as the one developed so far by the first author based on utility-value or the one this is compared to in the analytic hierarchy process/balancing method we will approach at decision. Although we look to actors at building scale, the balancing method has been initially developed by Strassert for regional science, which is in the aim of the COST action. So far drama theory has been applied to dealing with climate change (Jason et al, 2009). This project aims to translate the balancing principles for earthquake protection into an actionable management environment. In frame of the drama theory an agreement is looked for, and this can be supported by a software called “confrontation manager” (http://www.ideasciences.com/products/confrontationmanager/), one concurring with other attempts. We had a look at various software applications supporting decision in the context we are looking at. The exploitable foreground out our Marie Curie European Reintegration Grant PIANO was:

1. The idea of ontology for zoning, which can be used for the development of object oriented software dedicated at project management in construction industry, other than based on spaces or building elements as it is now. Ontologies are a way of organising data types for object oriented programming. Right now the software for facility management is based on project management, so this would be a new approach. Additional work includes the definition of taxonomies and the creation of a database to try out the concept. The two exploitable foregrounds are connected by the sense of ontologies/semantics, but differ in the field of application.
2. Relationship between function and sign (semantics, as seen by Umberto Eco, philosophy) applied to the zonification of this housing, to be connected with the ontology. Umberto Eco used ontology and semantics in its philosophic sense. In order to make it an exploitable foreground, the definitions have to be converted to those used in computer science, esp. semantic web.

Along these lines we looked into how Building Information Modelling, in our concrete case the archiCAD software, can be employed in restoration projects, to which building retrofit is a special case. Further, the functional relationships identified in zoning in the project PIANO can be further investigated by means of the so-called “space syntax”, to be investigated with a related software (http://www.casa.ucl.ac.uk/ajax/). The computation of prices in the utility-value method is based on this function.

A second step was looking at the application of game theory for urban simulation of the economic environment: review of architecture and urbanism games and their potential, and translation of the resources from the economic computations used at building scale to the urban scale of games using similar rules to existing board and online games (see details at modeling). We reviewed, in view to our decision approach, the contribution of the European project SIREN Social games for conflIct REsolution based on natural iNteraction (http://sirenproject.eu/). Games can also serve the participative dimension of risk communication and disaster prevention awareness. For example, loss models by Glaister and Pinho (2003) and Borzi (2008) will be used to translate the costs for post-earthquake repair or preventive retrofit from building scale to regional scale. The former method is adequate for reinforced concrete buildings. The way city management games deal with building is strongly related to our computation of preventive retrofit and post-earthquake reparation costs, since it takes into account the resources: materials and people, needed for a certain building element. At the same time it is different from the functional surfaces based computation explained previously. We looked at two kinds of games: cooperative and conflict based games. The conflict based games put face to face restoration and demolition, an example is the Romanian game “Habitat”. Economic aspects play a key role, since demolition and rebuild open the way for speculation and faster wins than in the public participation supported restoration. Cooperative games include such as the ones inspired by the novels of Ken Follett, a different medium to the book and the film we also looked at, and there all players contribute to building the landmarks. The comparison between the digital and non-digital implementation of the game was subject of a previous research. The way building is performed was explained previously. Intervention in case of a disaster involves multiple actors, and this could be the basis of a new game. Aim is to design which components of the real situation can be brought in the abstract (IT) model of a game to support decision making between the actors involved in decision about preventive retrofit compared to post-earthquake repair, the so-called planned conservation. The difference between the mathematical model in collaborative game theory and the architectural/urban approach of designing participative processes through games was investigated. These aspects relate to urban public policies. The study of Markov populations for example can be related to public policies to relate to disasters (Canbolat, 2013), more suitable however for man-made ones. At the same time, the involvement of these multiple actors in the decision system links to agent based modeling.

Going out from a decision tree which we developed between 4 actors: architect, engineer, investor and user, with the later going over to participative models, a review of similar (the group around Caterino, 2009) and alternative decision systems for earthquake resilient planning, ex. agent based modeling of allocation of resources in post-earthquake intervention (Fiedrich, 2006), including the functioning of the street network was performed. The actors have been selected to incorporate a variety of criteria used historically for decision making, including project implementations, theoretical models, and databases of various associations. Several novel methods will be used in developing decision models. These include (i) traditional quantitative decision trees and qualitative balancing methods (Bostenaru, 2004) and (ii) newer methods based on adaptive decision trees. The particularities of the newer methods are based on interdependencies between the various criteria (as supporting or aggravating each other), and developing them into a programming environment (as an ontology).

2. Setting up comparative analyses

First a comparison between agent based modeling (fully computer based choice of resources to be employed in earthquake protection for example) and our approach of decision as well as the drama theory related one was performed. Agent based modeling also involves expert opinion, but not in real time. Instead, plans of reaction in case of a disaster are available for choice after communication between the agents. Our analysis shows that agent based modeling is more suitable for processes of linear action, such as spread of fire and movement along the street network (thus touching a third element relevant for the action), than for modeling the cooperation in pre-disaster retrofit for example, which is directed to specifically chosen buildings or elements of a building, difficult to model also on GIS, for which reason BIM is more adequate. However, we emphasize the role of agent based modeling for the modeling of resource allocation, the process of retrofit and repair being also one of resource allocation (materials and people). Comparison tables are the outcome.

A further development will be the application of a Monte Carlo simulation to numerical simulations at building scale used for the computation of costs using the retrofit elements method developed by the applicant; comparison with real examples of earthquake retrofit and their costs (planned conservation: preventive retrofit versus post-earthquake repair).

3. Modeling

The grid of the planned (re)construction, from case studies to computer games was subject of the modeling. Case study was Lisbon, on which the applicant has worked during an STSM within a previous COST Action (TU0801), and even before that, to model the impact of the earthquake, but without reaching the economic aspects. In particular it can relate to the development of the Baixa quarter and the timber frame typology there. Conclusions can be then drawn for the home city of Bucharest, where several national funded projects on related topics are running. The Baixa quarter has a grid structure typical for the geometric development of Baroque cities when reconstruction was done, and is therefore suitable for this research. We visited several museums in Lisbon which visualise, interactively in multimedia or not this new urban development, having contact with some of the developers of the software to present GIS content on 3D hard copy model and with researchers of the reflection of the postearthquake development today. In continuation of the previous STSM we identified some of the landmarks presented in the Azulejos depiction of pre1755 earthquake Lisbon in the city, with the aim of a 3D city model, from eye level and from above, using Lisbon’s numerous “miradouros” (points of looking from a hill). The 3D city model would assure going over from the building size to the city size, thus modeling the region as required by the action. A special attention was given to Lisbon’s green walls, a feature supported by the particular climate, and to the way green spaces contribute to economic development, such a feature being of potential use in earthquake reconstruction, if happening today.

The modeling of the costs at the level of resources as from device based computation of costs at the single building included the definition of the interface between structural engineering results and construction economics computation, based on either element costs computation – applicable for the rough structure – or on functional space surface computation – including architectural finishing. In a forework to ontology we completed several forms on structural taxonomy of buildings, following World Housing Encyclopedia reports (http://www.world-housing.net/related-projects/share-your-knowledge-of-buildings/building-taxono-my-summary-reports), for which we also analysed to criteria according to the decision tree. It includes employment of the developed ontology for elements and ontology for spaces and how this can react to common architectural design and building management software such as, for example, archiCAD and its new – 2011 – building rehabilitation module. The resources can be translated in the symbols related to materials or building elements in a game, and build thus a basis for collaborative decision in bargaining in an earthquake risk management environment. We envisage further cooperation in developing this with a similar university in Hungary which does research in economic bargain games and in landscape planning, connected to our findings resulting the role of green spaces in reconstruction. The ontology of decision (the IT component of the modeling) must adapt again from another more frequent approach, that of energy modeling (O’Donell et al, 2013, but also present in the archiCAD module) to our topic, of structural earthquake retrofit. The results of the modeling will be integrated into a decision system based on regression between the two scales: building object and urban scale of the quarter.

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8. Roy CERQUETI, University of Macerata, Macerata, Italy (with G. Rotundo)

“Analysis of the shallow lake model through the method of symbolic images”

1 Introduction

This paper aims at exploring the attractors of the shallow lake dynamical system through the method of symbolic images. This method is particularly efficient in our context, in view of the mathematical properties of the evolution map under investigation.

Shallow lake model is an optimal pollution management problem introduced in Brock and Starrett (2003) and Mäler et al. (2003). A wide investigation of the dynamical properties of the shallow lake model has been performed by Wagener (2003) and, more recently, Kiseleva and Wagener (2010). A social planner faces the problem of finding an agreement between the competing objectives of farmers -who have the economic need to indirectly pollute the lake by employing fertilizers that are washed into the lake- and fishermen, tourists and water companies -who pursue the scope to maintaining the lake water as clean as possible. We aim at adding insights in the investigation of dynamical systems using symbolic images. The method transforms a map into a graph through a discretization of the state space, so it opens the way to the usage of graph algorithms and it provides a unified framework for the detection of system features.

2 The ecology of the shallow lake

A lake is said to be shallow when its maximum depth is 3 meters. The shallowness has the relevant consequence, as we will see below, that the stabilization of the sediments plays an important role in the turbidness of the lake water.

The pollution cycle of the lake runs as follows. The fertilizers used by the farmers contain phosphorus, which is washed into the lake due to the rainfalls. As the quantity of phosphorus in the water increases, the population of large aquatic plants grows. This implies the growth of phytoplankton biomass, which in turn leads to water turbidness. Then, a periphyton layer covers the leaves of the water plants. This process has the consequence to reduce the light in the lake water, and the aquatic plants start to die. The impact of the water plants disappearance on this ecological system is dramatic, and consists of two aspects: by one side, the absence of the water plants leads to an easy resuspension of the sediment from the bottom of the lake into the water column; by the other side, the population of zooplankton -which is a predator of phytoplankton-reduces, in that water plants constitute its natural habitat. Hence, the water becomes turbid.

3 The mathematics of the shallow lake

The dynamics of the pollution in the lake is described by the amount of phosphorous at time $t \geq 0$, namely $x(t)$. Phosphorous evolution is affected by the input due to farmers' activities $\{u(t)\}_{t\geq 0} \subseteq [0,+1)$ and by a constant parameter $b > 0$ associated to the rate of loss of phosphorous due to sedimentation. Mäler et al. (2003) and followers adopt a specific dynamics:

$$\begin{cases} x'(t) = u(t) - bx(t) + \dfrac{x(t)^2}{1+x(t)^2}, \quad t > 0 \\ x(0) = x_0 \end{cases} \tag{1}$$

The policymaker maximizes a social utility function which describes the conflicting preferences of two types of agents: farmers – who have the economic need to pollute the lake- and tourists, water companies and fishermen – who have the economic need to maintain the lake water as clean as possible:

$$\begin{cases} x' = u - bx + \dfrac{x^2}{1+x^2} \\ u' = -\left(\rho + b - \dfrac{2x}{\left(1+x^2\right)^2} \right) u + 2cxu^2 \end{cases}$$

4 The method of symbolic images

The method of symbolic images associates a graph to a map through a proper discretization procedure. Such approach has the advantage to offer a unified framework for the analysis of the global structure of vectors. Hence, this opens the way to the study of maps through graph theory analysis. In particular, the investigation of invariant sets of the map corresponds to the detection of strongly connected components of the graph (Avrutin et al., 2006). The structure of the network depends on the discretization. Moreover, the recent development of centrality measures for complex networks allows using other methods for understanding the relevance of network nodes. The set of parameter used for the methods and issues related to the efficiency are going to be shown.

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9. Zhenxi CHEN, Nanyang Technological University, Singapore (jointly with W. Huang)

"Modelling propagation of financial crises"

This paper proposes a heterogeneous agents two-market model. Different agents with fixed compositions are active in both markets and are allowed to invest in other market. A coupling mechanism is proposed for market makers of individual market for price updating such that common factors underlying two markets are captured. The main purpose of this paper is to simulate financial crisis with contagion behaviour within two-market framework. Financial crisis involves a lot of factors and various explanations have attended to justify it, such as over-valuation and shock to market. If a model manages to mimic behaviour of financial crisis, at least it can provide some explanation for it. Investigation is conducted from aspects of endogeneity and exogeneity so that different scenarios could be tested to understand causes of financial crisis.

In terms of endogeneity, we manage to simulate different patterns of financial crisis across two markets endogenously, especially sudden crisis with empirical reference to "Black Monday" of US and UK stock markets in 1987. These simulations imply that financial crisis could occur endogenously without external shocks. As all our simulated financial crises occur at price levels above market fundamental levels, they support hypothesis of market over-valuation causing financial crisis.

In terms of exogeneity, shocks are introduced to change fundamental value of individual market under different scenarios. Relative to study time window, shocks can be categorized as permanent and temporary shocks. For a permanent shock hitting fundamental value of individual market, depending on the magnitude, different patterns of financial crisis can be triggered and spread to two markets, no matter whether the shock is positive or negative. Simulation of temporary shock even shows duration of a shock can produce different results. Exogeneity simulations support hypothesis of financial crisis triggered by shock. In addition, the result that a small shock in one market could cause financial crisis in two markets is analogous to butterfly effect in which wing flapping of a butterfly might cause the formation of hurricane far away. Policy implication is that financial system is fragile. In a world of over-valued and closely linked financial markets, certain policy changes or market shocks in market member(s) might lead to over-adjustment and even disasters to all markets. Policy to remove asset price bubbles must be designed with deliberation. Otherwise, adverse consequence might be caused.

In matching stylized facts, individual market manages to calibrate volatility clustering, fat tails, insignificant autocorrelation of return and significant autocorrelation of absolute return. Moreover, our model also manages to calibrate cross-correlation, which is exclusive to multi-market case. Cross-correlation can match to empirical phenomena of prices co-movement among markets, especially during financial crisis.

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10. Carl CHIARELLA, University of Technology, Sidney, Australia (jointly with P. Flaschel and T. Zhi)

"Keynesian DSGD(isequilibrium) modelling: Real-financial market interactions and the term structure of interest rates"

Financial Crises are an important phenomenon in market economies: are recurrent, can be extremely disruptive and costly, and they raise important issues for theorists and policy makers alike. The ruling paradigm of Dynamic Stochastic General Equilibrium (DSGE) in macroeconomics has done a rather poor job in explaining financial crises and especially the recent global downturn.

This paper proposes a number of departures from DSGE methodology, which can be seen as the building blocks of a new approach in the Keynesian tradition, which we call Dynamic Stochastic General Disequilibrium (DSGD). Following Chiarella and Flaschel (2000), Chiarella *et al* (2005), and Charpe *et al* (2012), we construct an integrated macrodynamic model which incorporates some important feedback channels from the real to the financial sector (and vice versa), and dynamic adjustment process replaces the usual assumption that the economy is always in equilibrium. Further, unlike in much of the macrodynamic literature out of the DSGE approach, we analyse *micro-founded* expectation process on financial markets by incorporating an innovative concept of *animal spirits* developed by Lux (1995) and Franke (2010) instead of the standard rational expectation apparatus.

Three types of assets are traded on financial markets: first, a capital stock asset which is directly owned by households who supply the means of financing to firms. The second asset is a short-term government bond, whose rate of interest is set by the Central Bank which issues the third asset, money M. A portfolio approach based on Tobin (1982) is employed to address disequilibrium adjustment processes on financial markets.

Our main extension to Charpe *et al* (2012) is an incorporation of capital gain expectations for government bonds. We introduce heterogeneous capital gain expectations for the two risky assets (formed by so-called chartists and fundamentalists) and show that the first type of agents tends to destabilise the economy. Global stability can be ensured if opinions favour fundamentalist behaviour far off the steady state. This interaction of expectations and population dynamics is bounding the potentially explosive real-financial market interactions, but can enforce irregular behaviour within these bounds. The size of output and share price fluctuations can be improved by adding suitable policy measures to the dynamics of the private sector.

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11. Herbert DAWID, University of Bielefeld, Bielefeld, Germany (jointly with L. Colombo)

"Strategic location choice under dynamic oligopolistic competition and spillovers"

Location decisions of firms are influenced by a number of factors including availability and costs of input factors of production (in particular skilled labor), proximity to important sales markets, or local regulatory and institutional conditions. In particular, in technologically advanced industries, the consideration of (potential) knowledge flows and the induced technological spillovers play an important role for strategic location decisions. For example, Alcacer and Chung (2007) find in an empirical study based on data from first-time market entrants into the United States between 1985 and 1994 that less technically advanced firms favor locations with high levels of industrial innovative activity, while technically advanced firms avoid economic areas with high industrial activity. They interpret this behavior observing that firms consider both potential incoming and outgoing spillovers when determining their location and that leading firms avoid competitors by steering away from industrial activity. On the other hand, the rich literature on spillovers in clusters (e.g. Jaffe et al. (1993)) suggests that the positive externalities arising in clusters should lead to fast dynamics of knowledge generation, which at least in the long run might also be profitable for technologically advanced firms.

These considerations raise the question under which circumstances a firm interested in maximizing its discounted stream of future profits should choose to locate in a joint cluster with its competitors and under which circumstances it should locate in isolation. In this paper we analyze this issue in a differential game setting, which takes into account both the strategic and the dynamic implications of such location decisions. The existing game theoretic literature on strategic location decisions of firms is entirely static, and therefore our contribution is the first attempt to capture the potential trade-off between short and long term implications of location decisions of firms from a theoretical perspective. In particular, we address the following research questions.

- Under which circumstances is it optimal for technological leaders to locate outside an area with rich R&D activity?
- How do the incentives to locate in a cluster depend on the discount rate, the intensity of spillovers in the core, and the intensity of competition?
- What is the short and long run effect of the leaders' location decision on competitors and consumers?
- Does it make a difference whether technological leaders have temporary or structural knowledge advantages?

To address these questions we characterize the Markov-perfect Equilibria (MPE) of a differential game where n firms producing a homogeneous good compete on a common (world) market by choosing at each point in time *t* an output quantity. The production costs of firms depend negatively on their knowledge stock, where the knowledge stock can be increased by R&D investment. There is a single industrial cluster in the industry and firms located in this cluster receive spillovers proportional to the total size of the knowledge stock of all cluster firms. Each firm is either located in this cluster or in isolation, where firms in isolation neither receive spillovers nor produce outgoing spillovers to any of their competitors. As the first step of the analysis, we characterize the feedback strategies of the firms in the cluster and in isolation with respect to their R&D investments in a Markov perfect equilibrium of the game for an arbitrary location pattern of firms. As the second step, we consider a setup where one firm is more technologically advanced than its competitors and study the incentive of this firm to locate in the cluster, where this incentive is given by the difference between the value functions of the technological leader under the MPE of the differential game for the two corresponding location patterns.

It is shown that regardless whether the technological leader has structural or temporary knowledge advantages it should locate in isolation only if the advantage is larger than some threshold. However, the way this threshold changes with the model parameters and the way the location decision affects the dynamics of instantaneous profits and knowledge stocks differ qualitatively between the scenarios of structural and temporary technological advantages of the leader.

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12. Roberto DIECI, University of Bologna, Bologna, Italy

“Speculation and real estate cycles: the role of behavioral heterogeneity”

A large amount of theoretical and empirical literature on real estate and urban economics has pointed out the tendency of house markets to exhibit large and long-lasting deviations of prices from the so-called ‘fundamentals’, as well as recurrent bubbles and crashes (see, e.g. Cho 1996, Malpezzi and Wachter 2005, Schiller 2008). This view is confirmed by recent dramatic worldwide house price movements. In general, such disequilibrium phenomena in housing markets - as well as in other asset markets - cannot be satisfactorily explained within the Efficient Markets / Rational Expectations theoretical framework, which views asset market movements as 'optimal' adjustments to exogenous shocks to market fundamentals. Rather, such phenomena may be at least partly due to some kind of bounded rational behavior and heterogeneous beliefs of house market investors, namely, optimism and pessimism, imitation and herding, extrapolative and regressive expectations (Schiller 2005, Piazzesi and Schneider 2009). However, compared with financial markets, housing markets have a number of peculiar structural features, such as asset durability and depreciation, supply response, development lags, and the interconnections between the capital (or property) market and the rental market. As pointed out by the literature, such factors have the potential to bring about endogenous price fluctuations, even assuming that house market investors behave ‘rationally’ and adopt trading decisions based on fundamentals (see, e.g. Wheaton 1999, Edelstein and Tsang 2007). Therefore, it is particularly important to understand the specific impact of behavioral factors on house price cycles, in combination with the possible effects of development lags, the downward rigidity of housing stock and other factors that can naturally generate feedback relationships.

This paper seeks to contribute to this discussion by developing and investigating a dynamic heterogeneous agent housing market model in discrete time, which builds on the stylized framework proposed by Wheaton (1999). The basic setup consists of a standard accumulation equation for the stock of housing (or ‘space’), due to depreciation and new constructions, and an asset pricing equation. Due to investment lags, the development of new space (new constructions) is based on the price at the time of decision, and new space is delivered *n* periods after it is begun. The house price at time *t*, P_t, satisfies the standard asset market equilibrium condition:

$$P_t = \frac{P^e_{t+1} + R_t}{1+r}$$

where r is the discount rate, R_t is the rent in the time interval from t to t+1 (inversely related to the stock of housing at time t), and P^e_{t+1} represents the expected end-of-period price from house market investors. In a world with heterogeneous expectations $P^e_{i,t+1}$, i=1, 2, ..., N, with market proportions $w_{i,t}$, the aggregate market expectation of the price is represented as:

$$P^e_{t+1} = \sum_{i=1}^{N} w_{i,t} P^e_{i,t+1}$$

The paper first examines the model behavior and the steady state properties under homogeneous 'fundamental' price expectations, based on a time average $\overline{R}_t$ of observed rents:

$$P^e_{t+1} = P^f_t := \frac{\overline{R}_t}{r}$$

Afterwards, a 'momentum' or extrapolative component is introduced, the impact of which can vary endogenously, according to various switching mechanisms proposed in the heterogeneous-agent literature. In particular, it is assumed that the market impact of the extrapolative component depends negatively on the perceived overvaluation (or undervaluation) of house prices, based on observed price-rent ratio. Analytical results on the steady state stability and bifurcations are possible for low values of the construction lag n and for simple expectations rules. Numerical simulation is adopted for longer lags. Although the model predicts that endogenous price and stock fluctuations are possible even under homogeneous and fundamental-based price expectations, as an effect of the lags and other structural factors, such factors exhibit relatively modest impact for realistic values of the parameters. In contrast, the extrapolative component of investors' expectations, along with the possible endogenous changes of its market impact, appears to be an important source of instability and large swings of house prices. Moreover, the introduction of such behavioral factors makes the dynamics of house prices extremely sensitive to the parameters representing the real-side of the housing market, such as demand and supply elasticities. Results provide theoretical support to the idea that some kind of 'bounded rationality' is a crucial element to understand the repeated cyclic behavior of housing markets, and that house market bubbles and crashes are likely to be strongly affected by the complex interaction between real and behavioral factors.

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Keywords: Housing markets; Speculation; Price-rent ratio; Heterogeneous beliefs; Endogenous dynamics

JEL Classification: D84; R21; R31

13. Gustav FEICHTINGER, Vienna University of Technology, Vienna, Austria

"A survey of stable limit cycles in various fields of economics"

The purpose of this paper is to identify economic mechanisms implying stable limit cycles. In particular, it is shown how the Hopf bifurcation theorem can be used to establish the existence of persistent oscillations in dynamic economic models. In most cases numerical methods have to be used to determine optimal cycles. While we start with a descriptive model, the main part deals with intertemporal optimization models. Several applications in operations research are dealt with. Examples in advertising, production, inventory, employment, R&D, and pollution control are presented.

14. Peter FLASCHEL, Bielefeld University, Bielefeld, Germany (jointly with C. Proaño)

"Richard Goodwin's MKS system in a contemporaneous guise"

In this paper we set up a baseline, but nevertheless advanced and complete model representing detailed Keynesian goods market dynamics, segmented labor markets as in Marx's Capital, Vol. I and their impact on income distribution, in this respect dual and cross-dual wage-price adjustment processes, and finally Schumpeter-Marx type endogenous technical change. The cyclical movements of output generates, through Okun's law, employment variations in the heterogeneous labor market. The core of the resulting MKS macrodynamics is however given by credit-financed investment behavior and loanrate setting by credit suppliers. The framework is constructed in such way that simplified, lower dimensional versions of the model can be obtained by setting parameters describing specific feedback effects from one sector to another equal to zero. Starting from such low dimensional sub-dynamics, as for example the conflict about income distribution, we show the local stability of the full 7D model through a 'cascade of stable matrices' approach if the feedback chains are sufficiently tranquil in their transmission mechanisms. However, local stability is only the point of departure for the numerical investigation of local explosiveness and the forces that can bound such a behavior.

Keywords: Macroeconomic (In-)Stability, Segmented Labor Markets, Business Cycles, Uzawa-Lucas-Leontief production functions.
JEL Classification: E12, E24, E31, E52

15. David GOLDBAUM, University of Technology, Sidney, Australia

"Follow the leader: Simulations on a dynamic social network"

In the developed model and simulations, the desire to be seen as a trendsetter motivates adaptive behavior from agents in a dynamic social network setting. In a repeated choice environment, individuals choose from a new set of options each round. Faced with a discrete choice about which option to choose, agents are rewarded for adopting early an option that subsequently emerges as popular within the population.

Agents adopt one of the following actions each round. They can *lead*, adopting one of the new options immediately without waiting to see what others adopt. Alternatively, they can *follow* by imitating the

adoption decision of someone else in the population. The environment demands coordination as everyone benefits by the emergence of a single popular option in each round. The environment invites competition in that a leader benefits more than her followers since followers must wait to find out what there leader has done before making a choice.

In a companion working paper, Goldbaum (2012), considers a static version of this dynamic choice environment and establishes that Nash Equilibrium play produces a hierarchical social structure consisting of a single leader heading a population of followers.

The dynamic model developed for this analysis offers a process by which the equilibrium can emerge from an initially unorganized social structure and identifies factors in determining who is likely to emerge in the leadership position. Agents are modeled as choosing from the action set probabilistically, updating the probabilities using the Experience Weighted Attractor (EWA) of Camerer and Ho (1999). Adjustment is driven by how well the action taken fairs given the behavior of the remainder of the population and by the estimated counter factual payoffs that alternate actions would have generated. The reward structure pays players a fixed rate x for each agent in the population who adopts the same option as the player and an additional y for agents adopting the same choice subsequent to their own adoption.

Simulations explore the influence of the environment and individual behavior on the social organization that emerges. Consistent with the static model equilibrium, the EWA driven dynamic system is capable of producing a single leader with a population of followers. For x/y sufficiently large, the entire population organizes into a hierarchy under the leader. For x/y below a threshold value, those otherwise at the bottom of the hierarchy are better off choosing independently, though they are without any followers. With this strategy, the possibility of independently choosing the same option as the leader, thereby earning the same payoff as the leader, compensates for the forgone smaller but certain payoff earned by being among the last to adopt the popular choice.

Emergence of the equilibrium structure is fairly robust as a consequence of the backwardslooking adaptive behavior of the EWA. The performance measures generated by the EWA identify the most promising action given the current social connections. Adjusting probabilities to favor better performing actions codifies early transient events into permanent advantage in the emerging social structure. As a consequence, luck is one factor in determining who emerges as the leader. Persistence is another as consistently leading is generally a prerequisite to emerging as a leader with followers. This is because late adopting followers will have difficulty recovering to attract followers in subsequent rounds.

The EWA also reinforces advantages in the existing social structure. With the population initially dispersed over an underlying network of potential links, agents with a relatively high number of incoming potential links possess an advantage in attracting and retaining followers. This social advantage is thus a third factor in determining who emerges as leader.

Behavior that fails to reinforce transient events undermines the process that generates a leader and hierarchy. Certain parameters for the EWA will alter the relative importance of the different factors in determining the leader and can slow or prevent the emergence of a leader and follower hierarchy. For example, when agents down-weight the importance of the counterfactual estimated payoffs, transient good fortune can go unnoticed so that attracting a following becomes difficult. Similarly, the random component of the players' decision in the EWA can make it difficult for a player to consistently produce standout performance and thus difficult to attract followers.

Players holding positions vulnerable to off-equilibrium action of other players are those most harmed by small random elements in the individual decisions. For example, a player dependent on a long chain of imitation links to the leader faces disruptions in the flow of information about what to adopt. This can lead to the emergence of non-equilibrium formation of numerous smaller hierarchies under independent leaders as agents look for alternative structures to accommodate the unreliable flow of information from a distant leader.

This paper establishes the feasibility that an initially unstructured population of independent agents can, through a process of self-interested adjustment, organize into the equilibrium social structure despite the high degree of coordination demanded and the possibility that competitive competition might undermine coordination.

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16. Luca GORI, University of Genoa, Genoa, Italy (jointly with M. Sodini)

"Nonlinear dynamics in a duopoly with price competition and isoelastic demand"

This paper analyses nonlinear dynamic phenomena in a Bertrand duopoly with isoelastic demand (with price elasticity different from one) and horizontal product differentiation. The model is characterised by the hypothesis of homogeneous players: firms have the same cost function and the decision of fixing the price in the future by both firms is taken by considering the steepest (local) slope of the profit function at the current state of production as in Bischi, Stefanini and Gardini (1998) and Bischi, Gallegati and Naimzada (1999).

The papers most closely related to ours are Fanti, Gori and Sodini (2013) and Fanti, Gori, Mammana and Michetti (2013). The former paper analyses a nonlinear Cournot duopoly with isoelastic demand by showing that a price elasticity of demand different from one is responsible for the existence of several nonlinear phenomena. The latter one studies a Bertrand duopoly with linear demand and horizontal product differentiation and shows that varying the degree of product differentiation causes onoff intermittency, synchronisation and several global dynamic events. Unlike their works, in this paper we explore a symmetric two-dimensional map describing and observe phenomena such as coexistence of attractors, coordination failures and complex structures of the basins of attraction.

In addition, from an economic point of view, by assuming an isoelastic demand function makes it relevant to implement policies on the demand elasticity that can have effects on the stability of the fixed point.

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Keywords: Bertrand; Isoelastic demand; Nonlinear dynamics
JEL Classification: C62; D43; L13

17. Theresa GRAFENEDER-WEISSTEINER, University of Vienna, Vienna, Austria (jointly with I. Kubin)

"Dynamics of outsourcing in a New Economic Geography model"

This paper analyzes outsourcing in a new economic geography framework. We set up a model of two trading regions with three sectors (agriculture, manufacturing and an investment sector) and two factors of production (knowledge capital and labour). Trade in manufactured goods involves iceberg transport costs. Since one unit of capital is required for each manufacturing firm, the total number of firms/varieties is equal to the capital stock.

The two regions differ by their productivities in agriculture and their possibilities to outsource production in the manufacturing sector. In particular, the North has an absolute productivity advantage and thus a higher wage rate than the South. Moreover, a Northern manufacturer can decide between producing at home or abroad (outsourcing) while a Southern firm is restricted to home manufacturing production. Shifting manufacturing production to the South has the advantage of a lower wage rate but also implies some costs associated with outsourcing.

We characterize the short-run equilibrium for any given regional allocation of manufacturing production. This results in equilibrium operating profits for a Southern firm producing in the South, a Northern firm producing in the North as well as for a Northern firm outsourcing to the South. The ratio of the two last-mentioned profit rates is the decisive variable with respect to the outsourcing decision.

As standard in the New Economic Geography we use the replicator dynamics to model the outsourcing dynamics, in our case in discrete time. The resulting map is one-dimensional and highly non-linear. We first analyze the equilibrium outsourcing pattern for varying trade costs. Interestingly, increasing trade freeness might first reduce the amount of outsourcing before leading to full relocation of manufacturing production to one region. This non-monotonous pattern results from interplay of two forces. A productivity advantage of the North translates into higher wages. At the one hand this increases the market size of the North and fosters capital location in the North, on the other hand this constitutes a cost advantage of the South which tends to foster capital location in the South.

In addition, we show analytically that the interior equilibrium loses stability via a Flip bifurcation at sufficiently high values for the outsourcing speed. With respect to the global dynamics, first simulations indicate endogenous emergence of asymmetric outsourcing patterns, border collision bifurcations and coexisting attractors with complex basin boundaries.

Keywords: Outsourcing, Firm location, Footloose capital, New economic geography
JEL Classification: F12, F20, H5, R12

18. Tony HE, University of Technology, Sydney, Australia (jointly with C. Di Guilmi and K. Li)

"Herding, trend chasing and excess volatility"

Through the master equation, this paper extends the continuous-time asset price model with heterogeneous beliefs and adaptive switching in He and Li (2012) (JEDC2012) to study the effect of herding on financial markets. There are two types of agents, the fundamentalists who trade on the fundamental value and the trend followers who extrapolate the market price trend based on weight moving average over a finite time horizon. The herding behaviour among the agents is characterized by the master equation with transition rates depending on performance-based switching and herding. The market price is determined by a market maker who adjusts the market price to the excess demand from the fundamentalists and trend followers, together with a market noise. The resulting model is characterized by a six dimensional system of stochastic delay differential equations. It provides a uniform framework to examine the joint impact of time horizon, switching, and herding in a continuous-time model.

By combining an analytical analysis on the stability of the underlying deterministic model and numerical simulations, we obtain four main results. (i) The herding mechanism does not affect the local stability of the fundamental steady state of the underlying deterministic model, comparing to the adaptive switching model. (ii) Herding leads to different market dominance for different time horizons. With short (long) time horizons, herding leads to the dominance of the trend followers (the fundamentalists). However, herding always increases the fluctuations in market prices and market fractions of the two types of agents, generates excess price volatility, and contributes to market bubbles and crashes. An increase in the time horizon increases the market price deviation from the fundamental price. (iii) Herding amplifies market price volatility when agents increasingly switch to better performed strategy, although herding alone leads to excess volatility. (iv) With the herding, market noise plays more important role than the fundamental noise in generating insignificant autocorrelations (ACs) for the return, but significantly decayed AC patterns for the absolute and squared returns, characterized by the long-range dependence in volatility.

In summary, the model shows that herding and trend chasing contribute significantly to the market excess volatility and stylized facts observed in financial markets.

Keywords: Heterogeneous beliefs, herding behaviour, stability, stochastic delay differential equations.
JEL Classification: C62, D53, D84, G12

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19. Cars HOMMES, University of Amsterdam, Amsterdam, The Netherlands

"Behavioral Rationality"

We discuss a theory of *behavioral rationality* and its empirical and experimental relevance, focusing on expectation formation. Expectations are behaviorally rational if agents use simple and intuitive forecasting rules (heuristics) and these individual expectations self-organize into almost self-fulfilling aggregate behavior. Expectations are not model-consistent and not perfectly self-fulfilling, but the forecasting mistakes are hard to detect and improve upon from observed market realizations. We discuss three cases of behavioral rationality.

The first example concerns an asset pricing model driven by exogenous autocorrelated dividends. Agents use a simple AR(1) forecasting rule to predict prices, but optimize the parameters such that the sample average and first order autocorrelation become self-fulfilling. This simple misspecification leads to multiple behavioral learning equilibria, and the system may switch between phases of low persistence and low volatility and phases of high persistence and high volatility, consistent with financial time series.

The second example concerns an asset pricing model with heterogeneous expectations –mean reversion versus trend extrapolation. Agents switch between the two strategies based upon recent realized profits. Asset prices are characterized by boom and bust cycles, with bubbles strongly amplified by trend extrapolation. Estimation of this heterogeneous expectations model to quarterly S&P500 data shows strong amplification of the bubble by trend-followers before the financial crisis and rapid decline back towards the fundamental reinforced by the mean reversion strategy.

Finally, we discuss behavioral rationality in learning-to-forecast laboratory experiments, where the market price depends upon the average forecasts of individuals. In these experiments individual forecasting behavior is characterized by simple heuristics such as adaptive expectations, AR(1) forecasting rules, trend following and contrarian rules. Individual and aggregate behavior depend crucially on the type of expectations feedback. For negative expectations feedback, coordination of expectations is relatively slow but individuals coordinate on the rational expectations (RE) equilibrium price. In contrast, for positive expectations coordination of individual expectations is very quick, but agents quickly coordinate on a non-RE price and the market fluctuates with persistent deviations from the RE equilibrium price. For both negative and positive feedback, individual expectations self-organize into behavioral rationality.

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20. Alan KIRMAN, University of Aix-Marseille, Marseille, France (jointly with E. Guerci)

"An agent-based computational model for sequential Dutch auctions"

This paper investigates Sequential Dutch/descending auctions and uses an agent-based computational model in order to understand the emergence of certain well-known "stylized facts" such as the declining price paradox. This paper follows on from the strand of research on declining price paradox with a specific focus on the nature of sequential descending auction in the context of fish markets. Fish markets have particularly "good" properties for studying the economic phenomenon of price formation. They are perishable good markets, therefore there is no possibility of agents having an inter-temporal strategy involving the postponement of the sale of the good (fresh fish cannot be stocked for more than one day and still be sold as "fresh"). Hence the whole amount of the produced (fished) good must be traded or thrown away. This makes the economic analysis much simpler and makes it easier to get to conclusions which answer the basic questions raised here.

The standard agent-based modeling approach is to exploit a well-established behavioral model which identifies minimal behavioral requirements in agents' decision making processes which are capable of replicating "styled facts" at the aggregate level, such as the declining price paradox. Such models deliberately aim at carefully addressing context-dependent modeling issues which may provide a rationale for departing from standard learning models. It is common to adopt similar algorithms for different economic contexts where the only things that vary are, for example, the level of information available or where the repeated nature of the interaction among the same market actors is neglected. On the contrary, in this paper, we aim to identify a "rich" decision-making model which incorporates behavioral assumptions which are highly plausible in the economic context under consideration. The plausibility stems from the fact that the authors had the opportunity to actually visit the market and discuss with market actors. We aim to give an account which is consistent with what we observed as to the available level of information available to market operators about the market mechanism and conditions, the conjectures that individual buyers and sellers made about opponent's behavior and the inter-temporal arbitrage opportunities inherent to the daily sequential descending auctions. Thus, we propose to implement a non-standard computational learning algorithm in the attempt to identify specific behavioral aspects that need to be taken into account in reproducing the emergence of certain well-known "stylized facts". One thing that comes out of this research is thus to show how behavioral aspects which are commonly neglected can generate the economic phenomena that we observed.

The goods, which are assumed to be indivisible units of a perishable and homogenous good, are traded daily in the early morning wholesale market and after in retail markets. The former market is modeled as a sequential Dutch/descending auction market comprising T identical rounds each trading only one unit of the good. The latter is represented by assuming that each buyer of the wholesale market is a monopolistic seller in a different retail market facing a linear demand. Every period/day d the same sequential trading procedure comprising T rounds is repeated. Furthermore, we assume, realistically, that each market operator meets all other operators every day, the same population of agents is kept alive in the computational market model. Indeed, wholesale buyers meet repeatedly among themselves on a daily basis. The market actors are one auctioneer, several sellers and buyers. In each round/auction the auctioneer is responsible for establishing the starting price P. The starting price is seldom varied and is often set low, by the auctioneer. This behaviour is motivated by the attempt to avoid buyers using strategic reasoning. A fish market auctioneer mentioning an analogy with the game of bridge said that auctioneers do not want buyers to implement the strategy equivalent to counting and memorizing the cards which have been played in order to calculate which cards remain in the hands of their opponents. They achieve this by speeding up the process. Several sellers or equivalently a representative seller are modeled and the daily supply of the good is assumed to be exogenous. Sellers are characterized by a reservation price $\underline{P}$ corresponding to an opportunity cost related to a sale to sellers outside the market in question. In our model, both sellers and the auctioneer are "zero-intelligence" actors, the rationale is that, as in reality, buyers exhibit the most interesting and relevant behavior among market actors with respect to price formation. N learning buyers represents the daily demand. Both demand and supply may vary on a daily basis.

The model focuses, in particular, on the behavior of buyers, and proposes a rich computational model implementing inter-temporal profit maximization, conjectures on opponents' behavior and fictitious learning. Buyers learn to behave and in so doing move towards maximizing their long-term profits by means of an original learning algorithm that we define qEWA-learning model which comprises several behavioral assumptions. In particular, we adopt an action-state representation (condition-action rule), a reinforcement learning rule (explore and exploit mechanisms), the use of a EWA-based counterfactual reasoning, and finally an intertemporal decision-making based on temporal difference learning. In our market model, there is no room for belief-based reasoning (belief-based models assume that agents explicitly model opponents' behavior) because of the high number of competing buyers and the fast decision-making process. In reality, as mentioned, the auctioneer speeds up the trade by diminishing occasionally the starting price.

Nonetheless, some behavioral aspects have been neglected. We implicitly neglect the emergence of loyalty among buyers and sellers. In reality, auctioneers try to mitigate what might be thought of as inefficient market outcome by randomly ordering the presentation of the goods of the sellers throughout the auction rounds. Obviously, this action is not sufficient, because buyers may wait, taking some risk, when they know the identity of the current seller/vessel. However, this aspect turns out not to be very important for explaining the declining price phenomenon, the random sorting of the vessels does not provide a rationale for a decline in price as auction rounds go by. Buyers, expectations formation about daily supply has been left out. In reality, form expectations about the daily overall market supply. Buyers also anticipate daily supply and bid accordingly, thus they can condition their bidding behaviour on some priors. In fact, our behavioural model has been trained over different scenarios which have been kept stable throughout each independent simulation (number of buyers, number of auction rounds/unit of goods sold).

Our agent-based computational model successfully replicates a price declining phenomenon in particular by isolating three potential sources, that is, diminishing marginal profits (linear retail demand model), time pressure (perfectly inelastic retail demand model) and excess of supply. All these aspects in a more complex and realistic market setting can jointly contribute to amplify such price formation phenomenon. Finally, it is worth remembering that all simulated scenarios have considered a homogeneous population of learning buyers. The heterogeneity among buyers (in particular in terms of marginal profits) is an aspect which has not been addressed in order to make our simulation outcomes easier to interpret. Obviously, the latter aspect may further contribute to "exogenously" reinforce the declining price effect on market prices.

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21. Ingrid KUBIN, WU-Vienna University of Economics and Business, Vienna, Austria

"A review of multiregional new economic geography models"

In this lecture, it will be presented and overview of the existing multi-regional New Economic Geography (NEG) models. The NEG approach aims to explain how industrial economic activity tends to spread or agglomerate across space. Originating from Krugman's Core-Periphery model (1991, JPE), several NEG model variants have been proposed in the literature. The main ingredients are increasing returns and goods differentiation in the manufacturing production; transport costs which determine geographical distance / separation between two regions; and, finally, factor mobility between regions driven by an economic incentive. In the C-P model, the migration decisions of workers depend on real wage differentials. Workers enter in the production process both as a fixed and as a variable factor. In the footloose entrepreneurs (FE) model, proposed by Forslid and Ottaviano (2003, JEG), the mobility hypothesis involves entrepreneurs / human capital. This model is much more suitable to describe the integration processes taking place in Europe. NEG analyses highlight that in the long run several regional distributions of economic activity are possible. Notwithstanding the fruitfulness of the NEG approach, a clear shortcoming is a lack of realism of some of its assumptions (Fujita, Fujita & Thisse, RSUE, 2009). In particular, the analysis is often limited to the two-region case. As stated recently by Fujita and Thisse (2009), the existence of more than two regions may involve effects (i.e. spatial spillover effects, hub effects) that cannot emerge in a two-region context. Moreover, in a multi-country and multi region context agglomeration / dispersion processes may emerge that cannot be explained in a simple two-region setting. Due to the complicated structure of NEG models, there is still a scant, even though, expanding literature on multi-regional / multi-country models. This literature will be reviewed highlighting the effects on the spatial distribution of the economic activity of economic integration and entrepreneurial mobility in a multiregional setting. We notice that the long run spatial distribution of the economic activity in these models depends on the counterbalancing of agglomeration and dispersion forces. The final outcome depends on parameter settings and on the authors' modeling choice (Brühlart, 2011).

22. Fabio LAMANTIA, University of Calabria, Cosenza, Italy (jointly with M. Kopel and F. Szidarovszky)

"Dynamic mixed oligopoly with socially concerned firms: An evolutionary perspective"

Corporate Social Responsibility (henceforth CSR) plays an increasingly important role for firms in today's globalized market environment. Investors and consumers alike pressure companies to consider social and environmental issues. Documenting the importance of this trend, the consulting firm KPMG reports that nearly 95% of the largest 250 companies worldwide issued CSR reports, up from 80% in 2008 and 50% in 2005 (KPMG International Survey of Corporate Social Responsibility Reporting 2011, 2008, 2005). Hence, it is safe to conclude that CSR has become mainstream in the corporate world.

Empirical and theoretical work in economics, industrial organization, and management research is still divided about the question if pursuing a social strategy can serve shareholder value. Empirical studies investigating the link between Corporate Social Performance and Corporate Financial Performance do not show clear-cut results (McWilliams et al. 2006, McWilliams and Siegel 2000). Theoretical work starts recognizing the long-term investment characteristic of CSR activities, but primarily uses static models with only a few exceptions (e.g. Wirl et al. 2013). The goal of our paper is to gain insights into the drivers for the recent trend in corporate social responsibility and to shed some light on the potential dynamics of an industry involving firms with profit and non-profit motives.

We consider an evolutionary setting based on a mixed N-firm oligopoly model, where a group of socially concerned firms (SCFs) competes with a group of profit-maximizing firms (PMFs). In contrast to a PMF, a SCF maximizes its profit plus a fraction of consumer surplus (e.g. Kopel and Brand 2012 and the literature cited there). We assume that the products offered by the two groups are horizontally and vertically differentiated. First, we characterize the Cournot-Nash outcome in the general N-firms setting. For the evolutionary model, we then focus on the duopoly case. We discuss the equilibria arising in a two-stage model, where in the first stage of the game the owners of the firms choose simultaneously whether to be SCF or PMF and in the second stage they choose the quantity to produce in order to maximize their expected payoff. Here, the share of consumer surplus included in the SCF's objective function as well as the degree of product differentiation are assumed to be exogenously given. Depending on the combination of these parameters, the model is either dominance solvable, with both firms playing PMF or both playing SCF, or it is an anti-coordination game, with a mixed-strategy Nash equilibrium in which both strategies, PMF and SCF, are played with positive probability. This demonstrates that depending on the product characteristics and the level of social concern the industry evolves towards a homogeneous or a heterogeneous population of firms.

Following Droste et al. (2002), we then address the issue of robustness of CSR behaviour under evolutionary pressure in the duopolistic setting. Differently from their paper, in our setting agents have in common the same information and are endowed with the same degree of rationality, but they can make different choices when selecting their objective function. Thus, firms are randomly matched to play the mixed Cournot duopoly game with differentiated products. The fraction of PMFs and SCFs evolves over time according to the realized profits in the quantity game through an asynchronous updating mechanism (see Hommes, 2009).

With respect to firms' behavioural rules, we develop the evolutionary game in the cases of competition between "Nash" players and between "best reply" players (see Droste et al. 2002 and Hommes et al., 2013). When players are "Nash", they are assumed to select the Cournot-Nash outcome prescribed by the chosen strategy (PMF or SCF) in the quantity stage. In this case, the dynamic model assumes the form of a unidimensional map for the fraction of SCFs. Through the stability analysis of the map, we show that when the game is dominance solvable the pure strategy Nash equilibria are always stable attractors, regardless of firms' intensity of choice. However, in the case of anti-coordination game, the mixed Nash equilibrium always loses stability through a flip bifurcation for high enough agents' intensity of choice.

We then reformulate the problem with best-reply players. In this case, both quantity and evolutionary dynamics are taken into account. The stability analysis of the equilibria is studied as well as a comparison between the dynamics with Nash players and best reply players. Also in the case of best reply players there exists a specific threshold level for the intensity of choice after which the mixed Nash equilibrium loses stability.

In the final part of the paper, a three-stage version of the game is studied, including also the optimal choice of the share of consumer surplus that each SCF firm include in its objective function (e.g. Königstein and Müller 2001). The analysis here captures the firms' owners choice of an optimal social strategy. Some discussions on possible extension of the basic model conclude the paper.

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23. María Pilar MARTÍNEZ-GARCÍA, University of Murcia, Murcia, Spain (jointly with F. Cabo and G. Martín-Herrán)

"Less than exponential growth with non-constant discounting"

1 Introduction

Most of the basic models in economic growth theory rely on the assumption that households have a constant rate of time preference, ρ. However, the rationale for this assumption is unclear. Recent literature argues that individuals are highly impatient about consuming between today and tomorrow, but are much more patient about choices advanced further in the future, for example, between 365 and 366 days from now. In this paper we study the economic growth for a neoclassical growth model with consumers discounting at a declining rate during an infinite time horizon. The evolution of the economy is described by a differential equation system for consumption, the rate of return on assets and λ (a time-varying weighted average of the instantaneous rates of time preference). We prove that consumption grows at a declining rate converging to a nonhyperbolic steady-state. We study the associated center manifold and prove that the convergence to the equilibrium is slower than $O(1)$. Then we can guarantee an unlimited amount of consumption despite of the diminishing returns to scale in the production function. That is what is called *less than exponential unbounded growth.*

2 The model

We start by considering the standard Ramsey economy with a constant population size. Each household is the single decision-maker who decides consumption per capita in order to maximize the discounted flow of utility through an infinite time horizon subject to the budget constraint. A utility function with constant intertemporal elasticity of substitution, σ, and a Cobb-Douglas technology are considered. It is also assumed that the discount rate declines through time, that is, at each date t the t-agent solves the following problem

$$\begin{aligned} &\max_{c_t(h)} \int_t^{\infty} u\left[c(h)\right]\theta(h-t)\,dh \\ &s.t.:\ \dot{k}_t(h) = f\left(k(h)\right) - c_t(h),\ \ k(t) = k_t \end{aligned} \tag{1}$$

where t is the current date and $\theta(j) \geq 0$ is the discount function which measures the time preference. By definition, the discount function, $\theta(j)$ is a decreasing function, initially equal to one. Moreover, defining the instantaneous discount rate as $\rho(j) = -\dot{\theta}(j)/\theta(j) > 0$, we consider that $\rho(j)$ decreases through time, and it tends to zero at infinity: $\theta(0) = 1$, $\dot{\theta}(j) < 0$, $\dot{\rho}(j) < 0$, $\lim_{j\to+\infty} \rho(j) = 0$.

As it has been proved in the literature, in the absence of any commitment, the usual Ramsey rule for the growth rate of consumption would be modified to $\gamma_c(t) = \dot{c}(t)/c(t) = \left(r(t) - \lambda(t)\right)/\sigma$, where $r = f'(k) = \alpha A k^{\alpha-1}$ in the case of a Cobb-Douglas production function, and

$$\lambda(t) = \frac{\int_0^{\infty} \rho(j) w(t,j)\,dj}{\int_0^{\infty} w(t,j)\,dj} > 0, \qquad w(t,j) = \theta(j) e^{(1-\sigma)\int_t^{t+j} \gamma_c(\tau)\,d\tau} \tag{2}$$

Choosing $\theta(j)$ such that $\dot{\rho}(j) - \rho^2(j) = -\phi\rho(j),\ \phi > 0$, the system dynamics is given by:

$$\dot{s} = -(1-s)\left[\left(\frac{1}{\sigma} - s\right)r - \frac{\lambda}{\sigma}\right], \tag{3}$$

$$\dot{r} = -\frac{1-\alpha}{\alpha} s r^2, \tag{4}$$

$$\dot{B} = \frac{1-\sigma}{\sigma}(r-\lambda)B + B(B-\lambda), \tag{5}$$

$$\dot{\lambda} = -\rho_0 B - \lambda(\lambda - B) + \phi\lambda \tag{6}$$

where s is the saving rate, $B(t) = 1/\int_0^{\infty} w(t,j)\,dj$, and $\rho(0) = \rho_0 \in (0, 1)$. Equations (3)-(6) constitute an autonomous four differential-equations system with (1,0,0,0) as steady state. This type of steady-state is a non-hyperbolic equilibrium. The flow in the center manifold is approximated by

$$\dot{u} = -u\left\{\frac{\left[(\sigma-1)u + \sigma\phi\right](u-r)}{\sigma^2\phi} + r - u\frac{\phi}{\rho_0}\right\}, \tag{7}$$

$$\dot{r} = -\frac{(1-\alpha)r^2}{\alpha}(1+x), \tag{8}$$

$$\dot{x} = -x\left\{\frac{\left[(\sigma-1)u+\sigma\phi^2\right](u-r)}{\sigma^2\phi} + \frac{(1-\rho_0)\sigma(1-\phi)u^2}{\rho_0\sigma^2\phi^2} + r(1+x)\right\}. \tag{9}$$

where $u = \rho_0 B/\phi$ and $x = s - 1$. From this system of equations, it follows that the rate of return converges towards zero slowly, at most, of order one. Then we can conclude that the consumption grows at a positive growth rate indefinitely. Furthermore, we numerically prove that the speed of convergence is lower than $O(1)$, which guarantees an unlimited amount of consumption as time goes to infinity.

24. Mariano MATILLA-GARCIA, University of Madrid, Madrid, Spain (jointly with R. Ojeda and M. Ruiz)

"Detecting the correct temporal lag structure"

This paper suggests a new nonparametric statistical test and procedures for selecting relevant lags in the model description of a general linear or non- linear stationary time series. All the techniques are based on correlation integrals, a concept that has played a relevant role in the literature on non-linear dynamical systems. The tests seem to be robust to the selection of correlation integral usual parameters. We also show that the test can be used as a diagnostic tool.

1. Introduction

The selection of the lags that should be included in a general model description of a time series is certainly a crucial first step for model selection, which is essential in forecasting and economic model building. Traditionally, autocorrelation and partial autocorrelation coefficients have been utilized in specifying the appropriate lags. Nevertheless, it is well established (Granger and Weiss, 1983) that processes with zero autocorrelation could still exhibit higher order dependence or nonlinear dependence. This is the case for some bilinear processes and even for pure deterministic chaotic models, among others. In general, autocorrelation-based procedures may be misleading for non-linear models, and so might fail to detect important nonlinear relationships present in the data, being hence of limited utility in detecting appropriate time delays (lags), especially in those scenarios where nonlinear phenomena are more the rule than the exception. That major economic variables are of nonlinear nature and that nonlinear relationships abound among them are facts generally accepted.

From an econometric and a statistical point of view, this situation has motivated the development of tests for serial independence (see Tjostheim, 1996 for a review) with power against alternatives exhibiting general types of dependence, being the vast majority of these tests of nonparametric nature, hence trying to avoid restrictive assumptions on the marginal distributions of the model. However these tests, by construction, are not design for selecting relevant lags, and this partially explains the scarcity of nonparametric techniques for investigating lag dependence, regardless the linear or nonlinear nature of the process, which is an aspect that is unknown in most of the practical cases. Some notable exceptions to this relative scarcity are Granger and Lin, 1994, Tjostheim and Auestad, 1994, Granger et al., 2003, and Matilla-García and Ruiz Marin, 2009. A common characteristic of most (but not all) of these techniques is the use of entropy-based measures to identify the correct lag.

In this paper we take a different way and we rely on the well-known correlation integral to develop a new nonparametric statistical procedure for identifying what lags to use when building models of unknown nature (linear or nonlinear). In addition, we design a bootstrap based statistical test for selecting among potential relevant lags. This later contribution makes an interesting difference with other available procedures, which do not provide a hypothesis-testing framework.

Correlation integral, introduced by Grassberger and Procaccia, 1983, and initially designed for measuring the fractal dimension of chaotic data, currently constitutes an important tool in physics and natural sciences for analyzing some time series' properties (see for example Figueiredo et al., 2011, and Mathew and Picu, 2011, for the latest contributions). Correlation integral has been widely used in finance and macroeconomics basically for testing serial independence. The most relevant contribution is the well-known BDS test (Brock et al., 1996), which is precisely based on the fact that correlation integral factorizes when the elements of a time series are i.i.d. (independent and identically distributed). Interestingly, correlation integral has also been used (Hiemstra and Jones, 1994) to detect potential nonlinear causal relationships between time series.

There have been several critical comments to the BDS test (Tjostheim, 1996, De Lima Pedro, 1996, Barnett et al., 1997, among others) regarding a larger over-estimation of the right level of the test, hence tending to report a high false alarm rate. This can be explained, in part, because of the non-trivial impact of several ex ante decisions about several decisive parameters that are required to be taken in order to use the test. Unfortunately there is very limited statistical guidance (most of them based on massive Monte Carlo experiments) about its selection, being likely to choose inappropriate values. As we show in this paper, our test, despite the fact it is based on correlation integral is robust to those ex ante decisions, particularly minimizing the role played by them when selecting a proper lag for model selection. Closely related, another especially relevant property is that our procedure is not affected by the "curse of dimensionality" problem, as we do not rely on a nonparametric estimation of different moments. Rather, we rely only on the correlation integral, which originally (as presented by Grassberger and Procaccia) requires incorporating a delay-time parameter to fully and better describe the basic dynamics properties of the process. This characteristic, which was not used in BDS-type tests since delay time was fixed at 1, allows considering scenarios where dependence is not necessarily contained consecutive-formed classical m-histories, since now we can properly delay such vectors and therefore searching through several lags. Another contribution of the paper is that the proposed technique can be used to examine lag structure in model residuals, and therefore it seems to be useful as a potential diagnostic tool. This two last attributes makes the approach to be certainly general either for macroeconometrics and financial analyzers.

The paper is organized as follows. In the second section we introduce the basic notation and mains concepts that are used throughout the rest of the paper. We show the original concept of correlation integral, which uses time delay as a parameter, and its relation with Takens' theorem, which accounts for the general dynamical properties of the underlying process. In the third section we present a result that informs about the behavior of correlation integrals when evaluated at true (correct) lags. This result is robust regarding the embedding dimension and distance. The fourth section firstly introduces a bootstrap based test that helps in the identification of relevant lags, given a dependent stationary time series. Secondly, the behavior of the test for finite sample sizes is studied for 10 possible data generating processes of linear and nonlinear nature. And thirdly, an automatic procedure for detecting potential lags is proposed and evaluated. Section 5 deals with using the test as a diagnostic test for residuals.

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25. Akio MATSUMOTO, Chuo University, Tokyo, Japan

"Goodwin accelerator model augmented with fixed and distributed time delays"

A Goodwin 1951 accelerator business cycle model is developed for local and global dynamics of national income. The main aim is to show analytically and numerically that the delay matters in macro dynamics. In the literature of time delay, there are two different modeling ways in continuous-time scale, fixed time delay and continuously distributed time delay. Goodwin adopted fixed time delay in his seminal work of the accelerator model. In this study, introducing distributed time delay into Goodwin's original fixed delay mode, we study the effect of a change of the length of the delay upon the solution of the model when the steady state loses stability. Dynamics is described by a mixture of delay differential and integro-differential equations. Conditions for the local stability are derived. Global dynamics is examined by applying Hopf bifurcation theorem.

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26. Kiminori MATSUYAMA, Northwestern University, Evanston, Illinois, USA (jointly with I. Suhsko and L. Gardini)

"Chaos in a credit cycle model with good and bad investments"

The idea that market mechanisms are inherently dynamically unstable can be traced back at least to [1]. Recent events have also renewed interest in the hypothesis that financial frictions are responsible not only for amplifying the effects of exogenous shocks but also for causing macroeconomic instability [2,4]. Although a vast majority of research continues to study amplification effects of financial frictions within a setting that ensures the existence of a stable steady state toward which the economy would gravitate in the absence of recurring exogenous shocks, there exist several micro-founded, dynamic general equilibrium models of financial frictions, in which the steady state is unstable and persistent fluctuations occur without exogenous shocks.

Our paper builds on one such model developed by [3], which generates endogenous fluctuations of borrower net worth and aggregate investment. This model considers an overlapping-generation economy in which entrepreneurs arrive sequentially. Upon arrival, they first sell labor and other endowments of inputs, which are used to produce the consumption good, in order to acquire some net worth that could later be used to finance their own projects or to lend to finance the projects run by others. There are two types of projects, *the Good* and *the Bad*. The Good projects generate capital, which produces the consumption good using inputs supplied by next generations of entrepreneurs who could undertake projects in the future. By competing for these inputs, more Good projects drive up the price of these inputs, thereby improving the net worth of next generations of entrepreneurs. In contrast, the Bad projects are independently profitable as they directly generate the consumption good. Without generating demand for any inputs, these projects do not improve the net worth of next generations of entrepreneurs. Furthermore, the Bad projects are subject to borrowing constraints due to the limited pledgeability of their revenue so that the entrepreneurs need to have enough net worth of their own to finance them. The unique equilibrium path of this economy, characterized by a one-dimensional nonlinear piece-wise smooth map, may fluctuate persistently for almost all initial conditions. With a low net worth, all the credit flows to finance the Good, even when the Bad projects are more profitable than the Good projects. This overinvestment to the Good creates a boom, which generates demand spillovers to the next generation of the entrepreneurs by improving their net worth. During a boom, with their improved net worth, the entrepreneurs are now able to finance the Bad projects. Credit flows are now redirected from the Good to the Bad. This change in the composition of credit flows at the peak of the boom causes a deterioration of borrower net worth. The whole process repeats itself. The equilibrium path oscillates, as the Good breed the Bad and the Bad destroy the Good. Such instability and persistent fluctuations occur whenever the Bad projects are sufficiently profitable but come with an intermediate degree of pledgeability.

The contribution of our paper is twofold. First, it shows that the same nonlinear, piecewise smooth map can be derived under a much simpler set of assumptions. Such a streamlined presentation of the model helps to highlight the key underlying economic mechanisms that cause instability and persistent fluctuations. Second, we offer a much detailed analysis of the nature of fluctuations under the additional assumption that the production function of the consumption good is Cobb-Douglas. With this assumption, the map has four-parameters, the share of capital in the Cobb-Douglas production function (α), the profitability of the Bad projects (B); the pledgeability of the Bad projects (μ); and the fixed investment size of the Bad projects (m). We characterize the bifurcation structure of the map in terms of these four parameters. For example, it is shown that the flip bifurcation of the fixed point is supercritical for $\alpha > ½$, degenerate for $\alpha = ½$, and subcritical for (the empirically more relevant case of) $\alpha < ½$. Before such a subcritical flip bifurcation, the stable fixed point and a stable period-2 cycle co-exist, along with an unstable period-2, which separates their basins of attraction. This implies *corridor stability*, i.e., the steady state of the economy is stable against small shocks but unstable against large shocks. Furthermore, when the steady state loses its stability as a parameter change causes such a subcritical flip bifurcation, the effect is catastrophic and irreversible in that restoring the stability of the steady state by reversing the parameter change is not enough for the economy to return to the steady state. Recent advances in the theory of piecewise smooth map (e.g., border collision bifurcations, the skew tent map as a normal form) also allow us to show that the bifurcation structure of our map has many features that are quite distinct from those of smooth maps. They include, among others, an immediate transition to chaos after one bifurcation; robust chaotic attractors (i.e., existing for an open set of parameter values); merging and expansion bifurcation of such chaotic attractors.

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Keywords: borrower net worth, composition of credit flows, financial instability, piecewise smooth nonlinear map, subcritical flip bifurcation, corridor stability, bordercollision bifurcation, skew tent map, robust chaos

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“Learning to play Nash in deterministic uncoupled dynamics”

This paper is concerned with the following problem. In a bounded rational game where players cannot be as super-rational as in Kalai and Leher (1993), are there simple adaptive heuristics or rules that can be used in order to secure convergence to Nash equilibria, or convergence only to a larger set designated by correlated equilibria? Are there games with uncoupled deterministic dynamics in discrete time that converge to Nash equilibrium or not? Young (2008) argues that if an adaptive learning rule follows three conditions – (i) it is uncoupled, (ii) each player's choice of action depends solely on the frequency distribution of past play, and (iii) each player's choice of action, conditional on the state, is deterministic – no such rule leads the players' behavior to converge to Nash equilibrium. In this paper we present a counterexample, showing that there are simple adaptive rules that secure convergence, in fact fast convergence, in a fully deterministic and uncoupled game. We used the Cournot model with nonlinear costs and incomplete information for this purpose and also illustrate that this convergence can be achieved with or without any coordination of the players actions.

1 Introduction

The particular problem of "learning" in game theory – or the dynamic process through which the players. actions may (or may not) converge to the equilibria of the game – has a long history in economics; in fact, we can say that the former is almost as long as the very history of game theory itself. Already in the early 1950s we find seminal contributions to the subject. Brown (1951) presented a dynamic adjustment now widely known as "fictitious play" to be taken as a learning process for computing the equilibria of games, while Hannan (1957) and Blackwell (1956) put forward specific proposals to evaluate the success of convergence of various learning rules to the equilibria.

However, despite such a long history, it seems no exaggeration at all to argue that it was essentially over the last ten years or so that the learning problem took over as one of the crucial elements of modern game theory. The literature has grown and flourished so much that it is totally impossible to acknowledge much of its large volume in a necessarily short space, which includes a signi.cant number of advanced textbooks – see e.g. Fudenberg and Levine (1998), Rubinstein (1998), Young (2004), Sandholm (2008), Camerer (2003) and Cesa-Bianchi and Lugosi (2006) – and more than a hundred papers, see two recent excellent surveys by Hart (2005) and Sandholm (2007).

This paper is concerned with the following problem. In a bounded rational game where players cannot be as super-rational as in Kalai and Leher (1993) – where they were found to be unbounded in what they can remember, compute, or anticipate – are there simple adaptive heuristics or rules that can be used in order to secure convergence to Nash equilibria, or convergence only to a larger set designated by correlated equilibria? Do games with uncoupled deterministic dynamics in discrete time converge to Nash equilibrium or not? Does the dynamics in fact converge to, or does it just come close to the Nash equilibria? By uncoupled dynamics, following Hart and Mas-Colell (2003), we mean a game in which the strategy may depend on the actions of the other players but not on their preferences.

This issue has been extensively discussed since the late 1990s. Foster and Vohra (1997,1998), Fudenberg and Levine (1998), Hart and Mas-Colell (2000, 2003) proved that by constructing a calibrated procedure for forecasting opponents' play convergence of time-averaged behavior to the set of correlated equilibria can be achieved, independently of the game considered. Notice that we are mentioning correlated equilibria, not the set of Nash equilibria which is generally speaking a strict subset of the set of the former equilibria. It was Foster and Young (2003) who tackled the problem of convergence to the set of Nash equilibria and showed that the dynamics are most of the time close to but are not Nash-convergent. The state of the problem, as we currently have it, can be clearly highlighted by a quotation from a very recent paper by Young (2008):

> "We have repeatedly said that interactive trial and error learning cause behaviors to come close to Nash equilibrium a high proportion of the time. Why not just say that behaviors converge to Nash equilibrium? Because typically they do not converge. In fact, there are very severe limits to what can be achieved if one insists on convergence to Nash equilibrium. To be specific, suppose that a learning rule has the following properties: (i) it is uncoupled, (ii) each player's choice of action depends solely on the frequency distribution of past play (as in fictitious play), and (iii) each player's choice of action, conditional on the state, is deterministic. Hart and Mas-Colell (2003) show that for a large class of games, no such rule causes the players' period-by-period behavior to converge to Nash equilibrium."(p.7)

Despite the problem being spelled out above with such strong clarity and conviction, from a mere intuitive point of view one may raise some doubts about such generality of the no convergence result to Nash equilibria if certain conditions are considered: if the game has a stationary structure, if one accepts that the game is allowed to be played for a long period of time, and, finally, if players are allowed to learn from the past experience, even if only in a bounded fashion. In fact, the problem of possible no convergence to the Nash equilibrium was already acknowledged by Shapley (1964), but there is a feeling that if the game has an internal dynamic structure that is ergodic (either fully deterministic or stochastically ergodic), it must be subject to some level of prediction or control when the players can use information from past outcomes in order to decide what strategy should be followed. For example, this was exactly what happened with the recent paper by Germano and Lugosi (2007), who showed that the no convergence result of Foster and Young (2003) could be easily reversed if the players were allowed to add experimentation to their learning procedures. Notice that in their approach a rationale of bounded rational players is still adopted, keeping the game far away from the strong rationality hypothesis of Kalai and Lehrer (1993), but the crucial point in their paper is that there is a simple and adaptive process – bounded rational learning – that does in fact deliver convergence to the Nash equilibrium.

But the counterpoint presented by Germano and Lugosi can also be found in many other recent papers. For example, it is well known in game theory that the strategy pair sequence produced by following a gradient ascent algorithm may never converge, see Owen (1995). However Singh, Kearns and Mansour (2000) showed that in general stochastic games if both players follow an Infinitesimal Gradient Ascent (IGA) learning process, then their strategies will converge to a Nash equilibrium or the average payoffs over time will converge in the limit to the expected payoffs of a Nash equilibrium. Their first theorem is extremely interesting because it states one of the first convergence results for a rational multiagent learning algorithm, although the convergence was still somewhat week. This weakness was quickly overcome by Bowling and Veloso (2002), who showed that if the learning process followed a WoLF principle ("Win or Learn Fast") we will obtain a stronger notion of convergence, i.e., players will always converge to a Nash equilibrium. Successful convergence to the Nash equilibrium can also be found in the learning approach proposed by Zinkevich (2003), under the name of Generalized Infinitesimal Gradient Ascent (GIGA), and one finds in Leslie and Collins (2006) an explanation of why bounded rational players might learn to play Nash equilibrium strategies without having any knowledge of the game, or even that they are playing a game.

In this paper we continue on this route of bounded rational learning and the convergence to the Nash equilibrium. In particular, we take the three conditions presented in the above quotation by Young as a delimitation criteria for whether the Nash equilibrium can be learned or not, and put one of the most simple games that has been used in game theory (the Cournot model) to the test. We provide a counterexample that clearly violates those three general conditions above. We take a standard Cournot model in strategic form, with pretty conventional convex cost curves that can be found in any undergraduate microeconomics textbook, and we add bounded learning in order to overcome the extremely high computational requirements needed to achieve the Nash equilibrium of the game. The game is fully deterministic and clearly satis.es the three conditions above: (i) it is uncoupled, because the strategies depend only upon the other player's actions (not upon the opponents profit function); (ii) each player's choice of action depends solely on the past play, and (iii) each player's choice of action is entirely deterministic.

We will show that, under these conditions, a simple adaptive learning rule going back in time as far as t–1; concerning information on the actions taken (output in this case), will deliver very fast convergence to the Nash equilibrium, even in a case where we have multiple Nash equilibria. That is, if the game starts to be played relatively near any one of the various Nash equilibria, and players are bounded rational – so that they are not able to compute straight away the level of output correspondent to any of those equilibria – but take decisions for the next period by using information on the output produced at t and t–1 by both firms, the dynamics converge very fast to that particular Nash equilibrium. This occurs if both players adopt similar learning procedures, but can also be achieved if just one of the players corrects his mistakes by incorporating in his strategy past information on his own actions and the actions of his rival.

Another interesting point in this bounded rational game consists in the fact that even very simple rules of thumb can be very powerful rules to lead to optimal decision making, because they may render optimal decisions to be achievable by trial and error, in a situation where without them such optimal decisions were hardly feasible, unless a player is equipped with super-rational powers. Such a point was firstly highlighted by Baumol and Quandt (1964) in one of the first papers applying the concepts of bounded rationality to economics:

> "It is easy to jump to the conclusion that the widespread use of rules of thumb is good evidence of sloppy workmanship on the part of business management. We shall argue [...] on the contrary, rules of thumb are among the more efficient pieces of equipment of optimal decision making." (p. 23).

The paper is organized as follows. In section 2 the Cournot game with nonlinear costs and incomplete information is presented. Section 3 introduces the bounded rational process adopted by players in order to overcome the extreme complex computations which are required to play Nash in a one shot game. Section 4 analyses with some rigor the local and global dynamics associated with the bounded rational game. Section 5 discusses the simple time-delayed adaptive process to secure convergence to the Nash equilibrium, and the final section presents some concluding remarks.

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"On the dynamics of Matsumoto-Nonaka duopoly"

In [2] a two-market model consisting in two firms which produce differentiated goods is introduced. The first firm produces goods x in the first market and the second firm produces goods y in the second market. It is assumed that externalities of different sign exist. Although in [2] a more general model is introduced, we are interested in the following particular case. Inverse demands functions are given by

$$\begin{aligned} p_1(x, y) &= (\alpha-1)^2 - \frac{x}{2} + (\alpha y)^2, \\ p_2(x, y) &= 1 - \frac{y}{2} + (\beta x)^2, \end{aligned}$$

where p_1 and p_2 are the market prices of goods x and y, respectively, and $\alpha \in [1,2] \subset \mathbb{R}$ and $\beta \in [0,2] \subset \mathbb{R}$. Each firm decides future production depending on the other firm's choice and production externalities. It is assumed that each firm tends to maximize its profit. The function

$$R_{\alpha,\beta}(x,y) = (f_\alpha(y), g_\beta(x)) = ((\alpha y - \alpha + 1)^2, (\beta x - 1)^2),$$

is the reaction function for the outcome (x,y). The iterations are given by

$$\begin{aligned} x_{t+1} &= f_\alpha(y_t) = (\alpha y_t - \alpha + 1)^2, \\ y_{t+1} &= g_\beta(y_t) = (\beta x_t - 1)^2, \end{aligned}$$

that is,

$$(x_{t+1}, y_{t+1}) = R_{\alpha,\beta}(x_t, y_t) = (f_\alpha(y_t), g_\beta(y_t)),$$

$\alpha \in [1,2] \subset \mathbb{R}$ and $\beta \in [0,2] \subset \mathbb{R}$. The parametric region, $[1,2] \times [0; 2] \subset \mathbb{R}^2$, is such that the reaction map $R_{\alpha,\beta}$ is a map from $[0,1] \times [0,1]$ into $[0,1] \times [0,1]$. Thus,

$$R^2_{\alpha,\beta} = f_\alpha \circ g_\beta \times g_\beta \circ f_\alpha$$

and therefore, one might expect that the dynamics of the whole system can be derived from the one dimensional maps $F_{\alpha,\beta} : [0,1] \to [0,1]$ and $G_{\alpha,\beta} : [0,1] \to [0,1]$ given by

$$\begin{aligned} F_{\alpha,\beta}(x) &= f_\alpha \circ g_\beta(x) = (1 + \alpha\beta x(\beta x - 2))^2, \\ G_{\alpha,\beta}(x) &= g_\beta \circ f_\alpha(y) = (\beta(\alpha y - \alpha + 1)^2 - 1)^2 \end{aligned}$$

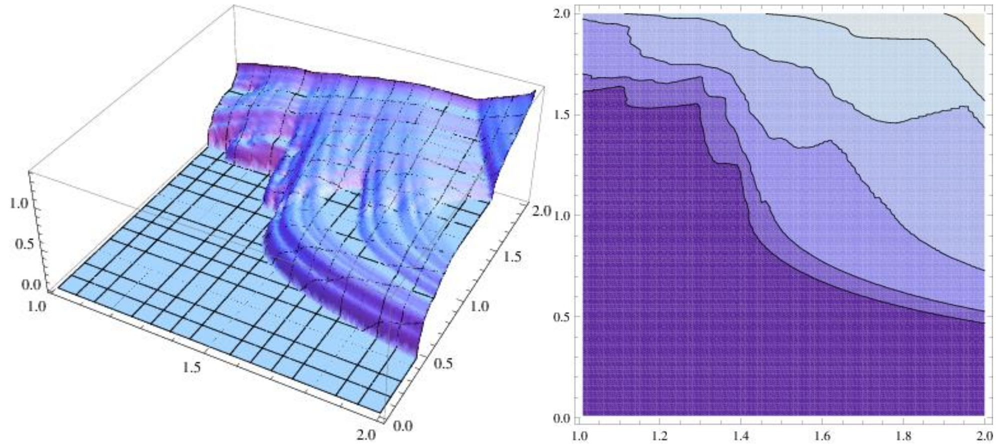

Figure 1: Topological entropy with accuracy 10^{-3} for each $(\alpha,\beta) \in [1,2] \times [0,2]$, 3D view (left) and projection (right).

Firstly, we will give an analytical proof of the existence of chaos (in the sense of Li-Yorke) in the model by computations of topological entropy with prescribed accuracy, see Figure 1. On the other hand, we study the number and the type of attractors. Our approach is different from that made in [1]. The attractors of the reaction map $R_{\alpha,\beta}$ can be studied through the attractors of the maps $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$. The attractors of a one-dimensional map $f : I \to I$ can be: a) a periodic orbit; b) a solenoidal attractor, which is basically a Cantor set in which the dynamics is quasi periodic; c) a union of periodic intervals $J_1, \ldots, J_k$, such that $f^k(J_i) = J_i$ and $f^k(J_i) = J_j$, $1 \le i < j \le k$, and such that f^k is topologically mixing. Topologically mixing property implies the existence of dense orbits on each periodic interval (under the iteration of f^k). Moreover, if f has an attractor of type b) and c), then they must contain the orbit of a turning point, and therefore its number is bounded by the turning points. In the case of our model, we can strength the above result by noticing that $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$ have negative Schwarzian derivative. Hence, we can deduce that the number of attractors of $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$ can be at most 2. Using numerical simulations allow us to find parameters in which the number of attractors of these maps is 2. The existence of chaos does not imply that chaos can be observed. We also analyze this situation.

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29. Ahmad NAIMZADA, University Bicocca, Milan, Italy (jointly with S. Sordi)

"Chaotic dynamics in alternative discrete-time versions of Goodwin's growth cycle model: A comparison"

In a paper published in 1981, Pohjola [1] considers a 1-D, discrete-time version of Goodwin's growth cycle model [2], originally framed in terms of the Lokta-Volterra 2-D differential equations system. In contrast to the original model, the share of wages is assumed to depend linearly on the rate of employment. The final model results in a logistic map representing the evolution of the employment rate. The emergence of chaotic motions is shown to depend positively on the ouput-capital ratio, on the bargaining power of the capitalist and negatively on the natural rate of growth, which is exogenously given. In Sordi [3] the structure of Pohjola's model is slightly modified in another way, that is, by the use of a non-linear bargaining relation between the wages share and the employment rate. The main result of Sordi's exercise is that the consideration of a non-linear bargaining equation does not increase the likelihood of chaotic solutions in Pohjola's model. In the same paper Sordi proposes a second exercise, increasing the dimensionality of the model by considering Goodwin's original non-linear version, in discrete time, of the bargaining equation; in this case, discretisation is obtained by relating the rate of variation of wages to the past level of employment. Her conclusion is that such a discretisation of the continuous original model is not theoretically satisfactory: the origin is a saddle point and the non-trivial steady-state is an unstable focus; trajectories starting near the unstable focus fluctuate with increasing amplitude, eventually entering in the basin of attraction of the saddle point.

In this paper we develop two different discrete versions of Goodwin's model. In the first case, we depart from Pohjola's 1-D model by using a sigmoid bargaining relation between the share of wages and the employment rate, one that considers historical and institutional factors such as the real subsistence wage of workers, and the capitalists' minimum profit rate. The model results in a bimodal map with economically meaningful behaviours. What we find is an increasing likelihood of chaotic solutions.

In the second exercise we consider a 2-D map where, in contrast to Sordi's discretisation, the rate of variation of wages is related to the current level of employment. This last model admits a rich set economically meaningful behaviours ranging from periodic to chaotic solutions and multistability phenomena.

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Keywords: Growth cycles; Nonlinearity; Stability; Bifurcations; Multistability.

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30. Anghel NEGRIU, University of Amsterdam, Amsterdam, Netherland

"Financial architecture and technology: A co-evolutionary model"

Empirical evidence points to a relation between financial architecture and industrial technology: market-based financial systems support the development of industries where innovation is typically radical whereas incremental innovation thrives in association with bank-based finance. I set-up a model where firms choose between either radical or incremental innovation and access external finance either from markets or from a profit-maximizing monopolist bank. In a static environment where all economic agents make optimal decisions, it is the distribution of the firms' heterogeneous ability for radical innovation and other model parameters that uniquely determine the choices made by the firm population and the banker. I run extensive simulations of a dynamic agent-based version of this model where agents adapt their behavior through reinforcement learning and bankruptcy provides a selection mechanism. I find that this stylized economy is prone to a lock-in phenomenon that can lead the system into different ecologies of technology and finance for the same starting parameters. I conclude that the observed relation between financial institutions and technology can be thus explained by co-evolution, an important insight for institutional design and reform.

The analysis begins with a stylized static model of imperfect information and optimizing agents inspired by the imperfect markets literature[1] where a population of heterogeneous firms have to access external funding in order to invest in one of two available projects. In relative terms, the incremental project is assumed to be safer - higher probability of success - but less profitable than the radical one - lower returns in case of success. I consider a firm population with heterogeneous ability for radical innovation and model this

[1] See, for instance, Rajan and Zingales (2001) for a survey of this literature.

ability as the return of a successful radical project. When the innovation project fails, the firm may be incapable of repaying its loan. Depending on whether it has borrowed from the bank or from the market, the firm may have to liquidate fixed assets at a fixed cost in order to repay the financier. Consistent with empirical evidence and previous models of entrepreneurial financing, I assume that the bank is more flexible, showing the firm leniency in case of innovative failure, but in exchange it can extract rents when the firm is successful. This is modelled as a state-contingent contract offered by the bank, with a low repayment in case of failure, R_L and a high repayment in case of success, R_H. The market offers a contract with a fixed rate, assumed to be exogenous throughout the paper.

Analysis of this model shows that there is a threshold in the heterogeneous ability for radical innovation that splits the firm population into bank-financed incremental innovators and market-financed radical innovators. Additionally, if the fixed cost of liquidation is sufficiently high, the bank can extract increasingly higher rents on top of the market rate.

The static model is then cast into a dynamic agent-based framework where banks and firms are no longer fully rational but instead use reinforcement learning to adapt to their changing environment. Firms choose between the 4 finance-innovation strategies available: bank-incremental, bank-radical, market-incremental, and market-radical. Similarly, the monopolist bank generates a set of contracts (combinations of R_L and R_H) over which it uses reinforcement learning to determine which one would be more profitable to offer the firms. Additionally, I introduce an accumulation process for firm wealth by which successfully innovative firms grow their stock of fixed assets. It is out of this stock that firms pay liquidation costs when they have insufficient cash to cover current debt and. When the stock of fixed assets becomes negative, firms become bankrupt and exit the market to be replaced by newborn entrants. Finally, I introduce learning-by-doing for the radical technology in the form of an increase in the future radical ability for successful radical innovators.

This more realistic, dynamic, boundedly rational, agent-based model behaves much like the static model: in the long run, the firm population again splits into bank-financed incremental innovators and market-financed radical innovators. However, the processes of accumulation described above play an important role in determining the relative dominance of the two strategies in the long-run. In fact, the financing-innovation choices of the firms co-evolve with the contracting behavior of the bank and eventually lock-into different economic ecologies that correspond to the systemic variations of techno-institutional complexes observed in today's capitalist economies.

Given its theoretical framework and the results obtained, the model is one of very few formal responses to the recent calls for an integrated analysis of institutions and technological change by leading scholars in both fields, North and Wallis (1994) and Nelson (2002). Such models are needed not only for a better understanding of the dynamic forces that underline the co-evolution of institutional structures of a society and its technology, but can also help to guide institutional and industrial policy by identifying both inertial forces and unexpected transitions.

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31. Valentyn PANCHENKO, University of New South Wales Sydney, Australia (jointly with S. Gerasymchuk and O. V. Pavlov)

"Asset price dynamics with heterogeneous beliefs and local network interactions"

This paper investigates the effects of network structures on asset price dynamics. We introduce network communication structures into a simple present value discounted asset pricing model with heterogeneous expectations. Every period the agents choose a predictor of the future price based on the past performance of their own and alternative predictors if they are observed and determine their demand for a risky asset. The information about the performance of alternative predictors is available only through locally connected agents. We model communication structure using four types of commonly considered networks: a fully connected network, a regular lattice, a small world, and a random network. The results show that the network structure influences asset price dynamics in terms of the regions of stability, amplitudes of fluctuations and their statistical properties.

Keywords: asset pricing, local interactions, networks, small world, heterogeneous beliefs, price dynamics.
JEL Classification: C45, C62, C63, D84, G12.

32. Anastasiia PANCHUK, Institute of Mathematics, NASU, Kiev, Ukraine (jointly with T. Puu)

"Disequilibrium trade and dynamics of stock markets"

In the present work we focus on pricing and trade dynamics for stock commodity markets. Dealing with flow markets one may define unique demand and supply functions, as well as their equilibria, as the commodities disappear in each period and then re-emerge. On the contrary, a durable commodity (a stock) remains on the market to the next period and may just change owner through exchange. However, such a redistribution modifies demand and supply functions, and hence the possible equilibrium state to which a dynamic process may be heading. Thus, such processes are provided with memory of the actual exchange history, and are entirely different from the familiar flow market dynamics. As a result, in stock markets a so-called disequilibrium trade may also occur which should be taken into account.

As an example, one may think about the housing markets, being subjected to certain instabilities recently, which is usually explained in terms of speculative bubbles. However, unlike stock exchange, the housing market seems to be dominated by people who use houses and apartments just as habitations. No doubt there exists some speculation through the intervention of real estate dealers, but, there are other issues at work which may explain at least some of the phenomena without reference to speculation. For instance, constant changes in the housing market due to demographical factors, including (im)migration, and trade in disequilibrium states.

Using a simplified case with only two traders of two stock commodities, and focusing on pure trade, it is possible to specify the exact conditions for making a deal in each step of the dynamic process. As a result, the trajectory may end up in one of the map equilibria (which are infinitely many), or trade can stick in some disequilibrium point while complex, even chaotic, price dynamics goes on.

The exchange process is built as follows. First, for each of the players the desired optimum subjected to the budget constraints is defined. Namely, for the first player it is

$$x_1 = a\left(X + pY\right), \quad y_1 = \left(1-a\right)\left(\frac{X}{p} + Y\right) \tag{1}$$

while for the second one

$$x_2 = 1-\beta\big(1-X+p(1-Y)\big),\quad y_2 = 1-(1-\beta)\left(\frac{1-X}{p}+1-Y\right) \tag{2}$$

Then, the two agents are trying to make a deal, and the trade is limited to the smallest change that any of them wants to make, because the agent wanting to change more would then also benefit from moving part of the way towards his optimum, whereas he has no means to force the other agent to move further than he wants. Thus, in case when the two optima x_1 and x_2 are located on different sides of the current value X, if the distance from X to x_1 is smaller than the distance to x_2 then the new distributions of goods $(X', Y') = (x_1, y_1)$, and it is $(X', Y') = (x_2, y_2)$ otherwise. In case when x_1 and x_2 are located on the same side of X, it means that both agents would like to sell/buy the same good, and therefore the deal is not possible implying $(X', Y') = (X, Y)$. We call such a situation a *disequilibrium trade*.

Finally, the new price is obtained as

$$p' = p\cdot \mathrm{e}^{\delta(x_2-x_1)} \tag{3}$$

Combining (1), (2), and (3) we get

$$\left(X',Y'\right) = \begin{cases}(x_1,y_1) & \text{if}\quad (x_1-x_2)(x_1-X)\le 0,\\ (x_2,y_2) & \text{if}\quad (x_2-x_2)(x_2-X)\le 0,\\ (X,Y) & \text{if}\quad (X-x_1)(X-x_2)<0,\end{cases} \qquad p' = p\cdot\mathrm{e}^{\delta(x_2-x_1)} \tag{4}$$

The final map (4) is three-dimensional and piecewise smooth. However, one may notice, that the second variable Y is somehow dependent on the first variable X. As a result, the map trajectories are substantially two-dimensional, and the variable Y is excessive. To get rid of it we use a certain variable transformation.

Let us consider the sequences $\{(x_1^{(n)}, y_1^{(n)})\}_{n=0}^{\infty}$ and $\{(x_2^{(n)}, y_2^{(n)})\}_{n=0}^{\infty}$ of the successive optima for the first and the second agents, respectively. We notice that

$$y_1^{(n)} = \frac{1-\alpha}{\alpha}\frac{x_1^{(n)}}{p_{n-1}},\quad y_2^{(n)} = 1-\frac{1-\beta}{\beta}\frac{1-x_2^{(n)}}{p_{n-1}} \tag{5}$$

where $\{p_n\}_{n=0}^{\infty}$ is the sequence of the corresponding price values. Substituting (5) to (1) and (2) and then to (4), we get

$$x_1^{(n+1)} = x_1^{(n)}\left(\alpha+(1-\alpha)\frac{p_n}{p_{n-1}}\right),\quad x_2^{(n+1)} = 1-\left(1-x_2^{(n)}\right)\left(\beta+(1-\beta)\frac{p_n}{p_{n-1}}\right). \tag{6}$$

By introducing a new variable $q_n = p_n/p_{n-1}$ the map (4) is reduced to a two-dimensional map:

$$X' = \begin{cases} x_1 = X(\alpha+(1-\alpha)q) & \text{if}\quad (x_1-x_2)(x_1-X)\le 0,\\ x_2 = 1-(1-X)(\beta+(1-\beta)q) & \text{if}\quad (x_2-x_2)(x_2-X)\le 0,\\ X \quad \text{if}\quad (X-x_1)(X-x_2)<0, \end{cases} \qquad q' = \mathrm{e}^{\delta(x_2-x_1)}. \tag{7}$$

33. Nicolò PECORA, Catholic University, Piacenza, Italy (jointly with A. Agliari and C.H. Hommes)

"Heuristic selection and heterogeneity"

In the last decades economics and finance have been moving inside an important paradigm change, from a representative and rational agent point of view to a behavioral, agent-based approach, in which economic environment is populated with boundedly rational, heterogeneous agents using rule of thumb strategies. The approach of neoclassical economics, the rational approach, assumes that the decision maker is able to select the alternative that maximizes the utility or profit, given his beliefs of other economic factors; moreover it is assumed that decision maker's original beliefs are self-fulfilling, in the sense that he is able to predict exogenous as well as endogenous variables.

The bounded rationality viewpoint (see [8]) relies on different requirements because, from this standpoint, what the rational approach requires is too demanding: generally speaking, a boundedly rational agent is modeled as able to choose what he perceives as the best for himself, but doesn't own a perfect knowledge of the environment structure. This behavioral approach fits much better with agent based-simulation models and challenges the traditional, representative, rational agent framework.

Furthermore the evolution of economic variables, such as stock prices, interest rates, exchange rates and inflation rates, is affected by expectations of agents operating in the financial and real markets: for this reason it's possible to think to the market as an expectations feedback system: market history shapes individual expectations which, in turn, determine current aggregate market variables and so on.

Laboratory experiments with human subjects have shown that individuals do not behave in a full rational way but follow simple heuristics which can account for persistent biases in taking decisions. This occurrence explains why prices may persistently deviate from fundamentals in laboratory markets, similarly to what can be observed in real stock markets. Moreover heterogeneity is crucial to the aim of expounding a number of evocative findings of the recent learning to forecast experiments.

In the present paper we develop a multi-agent model which presents evidence that evolutionary selection among different forecasting heterogeneous heuristics can explain coordination on individual behavior. In particular the model presented here is an attempt to generalize the idea of adapted belief system (ABS) introduced by Brock and Hommes in [3] and [4]) and then developed by Anufriev and Hommes in [1] and [2]. We shall show that the model is able to exhibit either convergence to an equilibrium price or persistent deviations from that, with the appearance of strange dynamics, similar to what it is possible to observe in reality: indeed asset price fluctuations are characterized by high volatility with large price changes irregularly interchanged by episodes of low volatility with small price changes.

In this multi-agent model, endogenous fluctuations are caused by a generic phenomenon, that is coexistence of two attractors (a steady state and periodic or quasi-periodic trajectories). The economic intuition behind these different outcomes (persistent oscillations and convergence) could be explained by the interaction and the evolutionary switching between trend extrapolation and stabilizing fundamental analysis that may lead to coexistence of locally stable fundamental steady state and an attracting closed curve or cycle far from the steady state.

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34. Paolo PIN, University of Siena, Siena, Italy (jointly with A. Dalmazzo and D. Scalise)

"Communities and social capital with heterogeneous groups"

Different, even diverging, notions clash together when the concept of "social capital" is considered in the economic literature. Some authors see social capital as the individual's accumulation of place-specific relationships, like group membership or political activism, possibly alternative to human capital accumulation. Guiso, Sapienza and Zingales (2010) criticize such a relation-building idea of social capital arguing that this notion is fully compatible with socially undesirable activities like, to the extreme, affiliation to criminal gangs. More in general, when members of society are partitioned into competing groups, or "clans", they may invest in membership activities that, in the aggregate, reduce social welfare.

Indeed, in general, the individual production of actions that are in principle pro-social (such as political participation, voluntary activity in charitable associations, etc.) is most often framed into communities' membership, such as associations, parties, clubs, ethnic groups, and families/tribes. On the one side, group membership can generate mechanisms that reinforce individual incentives to undertake more intense actions. On the other side, however, the activities of other groups in society can rival with own group's activity, producing negative externalities.

First of all, we motivate this community-based distinction with empirical evidence from the World Value Survey (WVS) extending the empirical results of Alesina and La Ferrara (2000) to non-US countries. Then, we analyze a one-shot contribution game between agents in a continuum interval, with both positive and negative externalities, due to an exogenous partition of the agents in *groups*. Agents are the elements of the continuum interval Ω, with mass 1, that we call the *society*. Agents are exogenously partitioned into a finite number of groups Θ, so that each agent $i \in \Omega$ is member of one and only one group $G_i \in \Theta$, $G_i \subseteq \Omega$. We denote as q_I the size of group I in the society.

The strategy of each agent i is an effort $x_i \geq 0$ that she can put in social activities (e.g. showing and promoting the identity of her group, provide public goods that are available only to the group and may be detrimental for those outside, participate to social, religious or political activities that are strictly and exclusively related to the group, or affiliate to associations promoting those activities). We denote with the function $\mathbf{x} : \Omega \to \mathbb{R}_+$ the aggregate strategy profile, and with $\overline{x}_I$ its average value. We denote with $\overline{x}_I$ the average effort of all the members of group I, and with $\overline{x}_{\neg I}$ the average effort of all the agents that are not members of group I (so that they are members of other groups).

Given the aggregate strategy profile $\mathbf{x}$ of the whole society, the payoff of an agent i, belonging to group I is given by

$$U_i = ax_i - \frac{k}{2}x_i^2 + bx_i\overline{x}_I q_I - cx_i\overline{x}_{\neg I}\left(1-q_I\right) \quad (1)$$

The payoff of the typical individual embeds positive externalities and strategic complementarity from fellow members' actions and, potentially, negative net externalities from actions of members in "rival" groups. Under this setup we are also able to consider explicitly the issue of "discipline" in groups as the ability of coordinating mandatory actions: we call the equilibrium with group-coordination the cooperative equilibrium, in contrast with the non-cooperative one where each atomic individual maximizes independently her own action. This feature allows us to consider the role of "obedience", seen by Tabellini (2010).

From (1), we get that the optimal best response for agent i, as an individual, is

$$x_i^* = \frac{a + b\bar{x}_I q_I - c\bar{x}_{\neg I}(1 - q_I)}{k} \qquad (2)$$

Analyzing this linear form of best reply we obtain the following non trivial results (that rely on many pages of analysis and intermediary results).

Proposition 1 *In the symmetric case, there is a non-linear condition between the parameters of the model such that the non-cooperative equilibrium leads to a strictly greater welfare than the group centralized equilibrium.*

This tells us that aggregate welfare in the society may be higher without coordination inside the groups, when the groups are of equal size. This is generalized in the following for the special case of two groups.

Proposition 2 *When N = 2, if*

$$\frac{(b - 2c)}{(c - b)^2} < \frac{1}{k},$$

Then there are h,l ∈ [0,1/2], *such that*:

- *if* $q_1 \in (1/2 - l, 1/2 + l)$ *then* $U_\Omega^* > U_\Omega^{I*}$ (*i.e. the non-cooperative equilibrium leads to a higher aggregate welfare than the group-centralized equilibrium*);
- *if* $q_1 \in (1/2 - h, 1/2 + h)$ *then* $U_1^* > U_1^{I*}$ *and* $U_2^* > U_2^{I*}$ *(i.e. the non-cooperative equilibrium is Pareto dominant with respect to the group-centralized equilibrium).*

This second result tells us that, for the case of two groups, even when the groups are not symmetric, group-coordination can be harmful for the whole members of the society, not even on aggregate but also individually for each of them.

The mechanism underlying the results in the Propositions goes as follows. As shown in the paper, when a group planner coordinates members she will choose an action which is *greater* that the action that would have been individually chosen, as in the absence of coordination. In the aggregate, thus, the presence of negative externalities ($c > 0$) across groups makes group-level mandatory actions socially inefficient. The level of inefficiency is paramount when society is polarized into two groups of equal size. Indeed, the propositions also highlights the role of "heterogeneity" in society, as measured by the probability of "meeting" a member of the other group. With $N = 2$, maximum heterogeneity is reached for $q_1 \to$ ½, making group discipline most costly in terms of welfare.

Furthermore, the conditions underlying Proposition 2 require that ($b - 2c$) be sufficiently low. That is, the benefit *b* from actions of co-members is low, relative to the cost *c* imposed by actions of other group's members. Thus, given *b*, compliance to group discipline imposes greater welfare losses, the greater the cost *c* imposed on other groups.

Alesina and La Ferrara (2000) have argued that "social capital" can be proxied by participation in "associations", such as religious groups, sports clubs, and voluntary organizations of different kinds. Our framework, instead, puts emphasis on the character of such organizations. A society fragmented into organizations, or family clans which have antagonistic and "particularistic" values, will suffer from strong negative externalities from groups interaction (high *c*). On the opposite, organizations such as voluntary medical associations or ecological groups, which share "universalistic" values, may well be associated with positive externalities. In this case then $c < 0$, and the conditions required for Proposition 2 may fail to hold. Finally, our propositions suggest that individual group members in polarized societies would have a net benefit from collapse in group-level "obedience".

Finally, in the paper, we empirically test some predictions of the model by using the panel data from WVS.

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35. Christian PROAÑO, The New School for Social Research, New York, USA

"Economic activity, income distribution and FX-markets in a two-country framework with endogenous speculative gain opinion dynamics"

Heterogeneous expectation formation is introduced into the Rational Expectations Turnovsky (1986) framework of (symmetric) two-country interactions within the IS-LM-PC framework. We are therefore deviating from the RE assumption and are allowing for endogenous opinion dynamics as financial-market oriented drivers of business fluctuations. The number of behavioral traders in the FX market changes endogenously due to a switching mechanism proposed by Franke (2011). We also change the Turnovsky approach by making use of the dynamic multiplier and an interest rate rule in place of his IS-LM structure. Moreover, and more importantly, we use wage-price dynamics in place of a single price Phillips curve and give income distribution a role to play aggregate demand which allow for the distinctions between wage-/profit-led and labor-market vs. goods-market led in the interaction of wages, prices and aggregate goods demand. We show that world-wide averages are converging under very mild conditions. Numerical simulations of country differences show that the occurrence of chartist herding might temporarily cause situations of high volatility, though the overall dynamics of differences remain globally bounded due an increasing existence of fundamentalists under such circumstances. This result is in sharp contrast to the empirically implausible strict convergence outcomes which were generally the only possibility under Rational Expectations.

Keywords: 2-country model, Inflation, FX-dynamics, Opinion dynamics, Persistent Fluctuations
JEL Classification: E12, E24, E31, E52.

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36. Lionello F. PUNZO, University of Siena, Siena, Italy

"'Zooming into the state': Structure behind dynamics"

All his life Richard Goodwin worked on the project of combining a fine description of a system structure with the aggregate picture of its dynamics. The latter (the nonlinear dynamics as taught by Le Corbellier) is most probably the better known of the two, but the former (the input output description he derived from Leontief's work) was no less important: in his (and R. Frisch's) view dynamics is the manifestation of inner structure. Linearity here, aggregation there were prices to be paid to mathematical difficulty. 'Local' analysis was the realm of the structural approach; loss of 'the fine view' the shortcoming of the nonlinear approach. Very early on, Goodwin found the nexus between the two in spectral analysis and diagonalization. Later, he came back to it in his work on complex dynamics.

37. Francesco PURIFICATO, University of Naples Federico II; Naples, Italy (jointly with P. Commendatore and E. Michetti)

"Financial development and agglomeration"

1. Paper Overview

1.1 Motivation

The New Economic Geography (NEG) literature has paid little attention to the role of banking industry in affecting agglomeration phenomena of the economic activity. The paper aims to fill this gap by analysing how the level of regional financial development affects locational choices of firms. In a context where financial markets are becoming increasingly integrated this issue could seem not appealing; if a rise in the international capital mobility or legal order homogeneity promotes a rise in the uniformity of credit conditions for firms among regions, then the regional financial development would not bound locational choices of firms. Regardless of these arguments, empirical evidence supports the opposite view.[1]

1.2 Model outline

The paper modifies the standard version of the Footloose Entrepreneur (FE) model by introducing a banking sector; the model framework is the following. The economy consists of two trading regions, three sectors (Agriculture, Manufacturing, and Banking industry) and three production factors (unskilled workers, entrepreneurs, and banks). Entrepreneurs are mobile across regions, whereas unskilled workers are inter-regionally immobile and freely mobile across sectors. The banking industry is characterised by the next assumptions. First, banks cannot move from one region to another and cannot lend to firms located in other regions, that is, we do not allow for inter-regional/international bank mobility and for inter-regional/international capital mobility.[2] Second, banks provide the working capital to firms because there is a friction in the payment system of the economy: workers cannot buy goods before to receive wages, while firms cannot pay wages before to sell goods; so that entrepreneurs borrow financial resources from banks to anticipate wage payments, that is, the demand for loans in each region is equal to variable labour costs, but they have to borne interest payments.[3] Third, banks operate in an oligopolistic market; particularly, we implement the standard model of oligopolistic competition among banks developed by Klein (1971) and Monti (1972) as reported in Frexias and Rochet (1999). This model gets the result that interest rates charged on loans are negatively related to demand elasticities and positively to the concentration degree of banking sector.[4]

The two trading regions are characterised by the same number of unskilled workers, the same production technology, and the same consumer preferences; while they could differ in terms of the number of banks. The solution of the bank's optimization problem allows to get a closed solution for the interest rate, which comes to be a decreasing function in the number of banks operating in each region; therefore, the last variable defines the concentration degree of the banking industry.

1.3 Results

The competition level of the banking industry affects the firm locational choice. When we consider the symmetric case of regions with the same concentration degree of the banking industry, an increase in the interest rate drives entrepreneurs to rise the price level and so we observe an increase in living cost and bank

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[1] Guiso et al. (2004) shows that the individual's odds of starting a business or the ratio between new firms and population is positively affected by a variable denoting local financial development.

[2] For the empirical evidence supporting this assumption, see Felici and Pagnini (2008), Agarwal and Hauswald (2010), Alessandrini et al. (2009a; 2009b) and Bofondi and Gobbi (2006).

[3] Within a neoclassical model of an open economics, Neumeyer and Perri (2005) makes a similar assumption to justify the working capital need for firms.

[4] The empirical evidence appears to show that banks in more concentrated markets apply higher loan rates or lower deposit rates (Berger and Hannan, 1989; Neuberger and Zimmermann, 1990; Berger and Hannan, 1992; Corvoisier and Gropp, 2002; for an opposite view, see Jackson, 1992). Moreover, the empirical evidence recorded in Corvoisier and Gropp (2002) allows to observe that the concentration degree of credit markets is statistically different across euro area countries and that this concentration degree positively affects interest margins of banks for short-term loans.

profits. In this context, the agglomeration forces, that is, the market access effect and the living cost effect, tend to become stronger than the dispersion force, that is, the local competition effect; therefore, an asymmetric long-run equilibrium with full agglomeration of manufacturing activities in one region could emerge as the most likely stable equilibrium.

When we consider the case of regions with a different degree of financial development, an asymmetric interior equilibrium emerges and the aforementioned results are reinforced; so that manufacturing firms tend to move in the region where the concentration degree in the banking industry and the interest rate are lower. Moreover, the model also shows a hysteresis or a path dependency of locational choice of firms. For example, a temporary shock that reduces the competition level of the banking industry in one region implies a migration of manufacturing firms towards this region and leads the economy from a symmetric equilibrium to another stable equilibrium characterised by full agglomeration; nevertheless, the removal of the shock could not restore the symmetric equilibrium.

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38. Davide RADI, University of Bergamo, Bergamo (jointly with G.I. Bischi and F. Lamantia)

"Evolutionary Cournot games with different information sets and imitation costs"

In this paper we analyze the dynamics of an evolutionary Cournot game with heterogeneous agents. We assume an industry with N firms operating with a decreasing return to scale technology and a market characterized by constant price elasticity. Moreover, we assume the presence of firms with different information sets. According to their information set the firms determine the current production as a function of past productions.

Let us assume the presence of two behavioural rules a and b, and let $r_t \in [0,1]$ indicates the fraction of firms using strategy a at time t and $1 - r_t$ the fraction of firms using strategy b at time t. Moreover, let $q_{a,t}$ be the production of a single firm using strategy a at time t and let $q_{b,t}$ be the production of a single firm using strategy b.

Behavioral rule a requires information about the market where they operate which is not common knowledge and requires costly market researches, so we associate a fixed cost to this strategy equal to k. On the contrary behavioral rule b (imitators) only requires knowledge about the current production of firms using strategy a. We assume that, the larger is the fraction of firms using strategy a the easier and the less expensive is to imitate them. In particular, the cost associated with behavioral strategy b is a function, $w(r_t)$, of the fraction of firms using strategy a, such that $w(r_t)$ is a monotone non-increasing function.

In the paper we define the output dynamics of the Cournot game coupled with a population dynamics describing the share of the two behavioral rules among the firms. The time evolution of the system is modelled by a three-dimensional map

$$T:\begin{cases} q_{a,t+1} = R^a\left(q_{a,t}, q^e_{b,t+1}, r^e_{t+1}\right) \\ q_{b,t+1} = q_{a,t} \\ r_{t+1} = G_\beta\left(q_{a,t}, q_{b,t}, r_t\right) \end{cases}$$

where $R^a(\cdot)$ is the reaction function of firms using behavioral strategy a, that depends on the information set of the type a firm, and $G_\beta(\cdot)$ is the population dynamic. As a switching rule, we use the so-called *logit dynamic* where β is the intensity of choice parameter. The quantities $q^e_{b,t+1}$ and r^e_{t+1} are the expectations of the firms using strategy a on the level production of the imitators (firms using strategy b) and on the share of firms using strategy a respectively.

The aim of the paper is to investigate the effects on production output of different behavioral rules based on different information sets. At this aim, we propose the three-dimensional map T in four different settings according to the information set of the firms in group (a), while firms in group (b) are always assumed to be imitators.

Let us assume different information sets for the firms in group (a) as follows:

1. Nash firms: they know the demand function, the fractions of firms using a specific behavioral strategy and *production level of the firms in group (b) for the next period*, $R^{a,NF}(q_{a,t},q_{b,t+1},r_{t+1})$;
2. Best Reply firms: *they know the demand function and production level of the firms in group (b) for the current period, i.e. best reply with naive expectations*, $R^{a,BR}(q_{a,t},q_{b,t},r_t)$;
3. Local Monopolistic Approximation (LMA) firms: *they only know the slope of the demand function at the current production levels*, $R^{a,LMA}(q_{a,t},q_{b,t},r_t)$;
4. Gradient method firms: *they decide the production level on the basis of the estimated marginal profit and they know the production level of the firms in group (b) for the current period*, $R^{a,GM}(q_{a,t},q_{b,t},r_t)$.

We prove that in all of these four cases the Nash-Cournot equilibrium is a fixed point for the map T, (q^*_a, q^*_b, r^*) characterized by the same cost for both strategy a and b, i.e. $k = w(r^*)$. Moreover, we provide an analytical and numerical study of the map T and we compare the stability properties of the Nash-Cournot equilibrium for all of these four cases.

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39. J. Barkley ROSSER, James Madison University, Harrisonburg, Virginia, USA

"Special problems of forests as dynamically complex ecological-economic systems"

Ecologic-economic systems tend to exhibit greater complexity than systems that are purely ecological or economic. The interactions between the two types often generate nonlinear relations that lead to various kinds of complex dynamics that complicate management and decision-making regarding them. Of these, forests have characteristics that lead them to have special problems not usually encountered in the management of such systems. A central one is the long time periods involved managing forests compared to most other such systems. This means that the issues regarding determination of discount rates for valuing future outcomes are more important for forestry management than for many other systems. Also, forests generate a wider range of externalities than do most other ecologic-economic systems, with implications for various hierarchical levels of management. This paper considers the array of these problems as they appear for a variety of forestry management issues.

40. Manuel RUIZ MARÍN, Technical University of Cartagena, Cartagena, Spain (jointly with M. V. Caballero and M. Matilla)

"Symbolic correlation integral. Getting rid of the ε parameter"

The BDS test is built upon the concept of correlation integral, as a measure of the frequency with which temporal patterns are repeated in the data.

For a time series $\{X_t\}_{t\in I}$ define the correlation integral as

$$C_m(\varepsilon) = \frac{1}{T(T-1)}\sum_{t=1}^{T-1}\sum_{k=t+1}^{T} I_\varepsilon(X_t(m), X_k(m)) \text{ if } m>1 \quad (1)$$

where $X_t(m) = (X_t, X_{t+1}, \dots, X_{t+m-1})$ is an m-history and

$$I_\varepsilon(X_s(m), X_t(m)) = \begin{cases} 1 & \text{if } \| X_s(m) - X_t(m) \| \le \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

$\|\cdot\|$ is the supremum norm, and ε refers to the distance between $X_s(m)$ and $X_t(m)$.

Then, under the null of independence, the BDS statistic is:

$$BDS = \sqrt{T}\,\frac{C_m(\varepsilon) - C_1(\varepsilon)^m}{\sigma_{m,R}(\varepsilon)}$$

is asymptotically $N(0,1)$ distributed where

$$\sigma_m(\varepsilon) = 4\left[K^m + 2\sum_{j=1}^{m-1} K^{m-j}(C)^{2j} + (m-1)^2 (C)^{2m} - m^2 K (C)^{2m-2} \right]. \qquad (2)$$

$C = C(\varepsilon)$ in the last equation can be consistently estimated by $C_1(\varepsilon)$; moreover a consistent estimate for $K = K(\varepsilon)$ is:

$$K(\varepsilon) = \sum_{t<s<r} h_\varepsilon(X_t(m), X_s(m), X_r(m)) \left[\frac{6}{T(T-1)(T-2)} \right] \qquad (3)$$

where $h_\varepsilon(i,j,k) = \frac{I_\varepsilon(i,j)I_\varepsilon(j,k) + I_\varepsilon(i,k)I_\varepsilon(k,j) + I_\varepsilon(j,i)I_\varepsilon(i,k)}{3}$.

This test depends on the proximity (ε) and the embedding dimension (m) parameters both of which are chosen by the researcher. The behavior of the test is really sensitive to different values of these parameters. Although different studies have been carried out to provide an adequate selection of the proximity parameter ε, there is no an optimal choice for it. A bad choice of the proximity parameter may introduce a severe bias when testing for independence.

In this paper we overcome the problem of the selection of the epsilon parameter by defining the symbolic correlation integral $SC(m)$ for a fixed embedding dimension m. In order to define $SC(m)$ some notation has to be introduced.

Denote by S_m the symmetric group of order $m!$, that is the group formed by all the permutations of length m. Let $\pi_i = (i_1, i_2, \dots, i_m) \in S_m$. We will call an element π_i in the symmetric group S_m a symbol.

Now we define an ordinal pattern for a symbol $\pi = (i_1, i_2, \dots, i_m) \in S_m$ at a given time $t \in I$. We say that t is of $\pi-$type if and only if $\pi = (i_1, i_2, \dots, i_m)$ is the unique symbol in the group S_m satisfying the two following conditions:

$$(a) \quad X_{t+i_1} \le X_{t+i_2} \le \dots \le X_{t+i_m}, \text{ and}$$

$$(b) \quad i_{s-1} < i_s \text{ if } X_{t+i_{s-1}} = X_{t+i_s}$$

Condition (b) guaranties uniqueness of the symbol π. This is justified if the values of X_t have a continuous distribution so that equal values are very uncommon, with a theoretical probability of occurrence of 0.

Notice that for all t such that t is of π_i-type the m-history $X_m(t)$ is converted into a unique symbol π_i. This symbol π_i describes how the order of the dates $t+i_1 < t+i_2 < ... < t+i_m$ determined the order of the values in the time series.

Then the symbolic correlation integral is defined as:

$$SC(m) = \frac{2}{T(T-1)} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} I_{ij} \tag{4}$$

where

$$I_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are both of the same symbol } \pi-\text{type} \\ 0 & \text{otherwise} \end{cases}$$

Under the null of i.i.d. for $\{X_t\}_{t\in I}$, it follows that the expected value of $SC(m)$ is $E(SC(m)) = \frac{1}{m!}$ and the variance $\sigma^2_{SC(m)}$ can be estimated. Under this setting we prove the following result.

Theorem 1. Let $\{X_t\}_{t\in I}$ be a stationary i.i.d. time series. Let $m>1$ be a positive integer. Then the following statistics

$$\frac{SC(m) - \frac{1}{m!}}{2\sigma_{SC(m)}} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

asymptotically follows a standard normal distribution $N(0,1)$.

Finally with a Monte Carlo simulation we show the finite sample performance of the statistic in terms of size and power, for linear and nonlinear data generating processes.

Moreover, symbolic correlation integral is shown to be an useful tool to test for determinism in time series.

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41. Mauro SODINI, University of Pisa, Pisa, Italy (jointly with L. Gori)

"Local and global dynamics in an OLG growth model with endogenous lifetime and endogenous labour supply"

The present study introduces endogenous labour/leisure decisions in an overlapping generations model à la Diamond (1965) with endogenous longevity as in Bhattacharya and Qiao (2007). Life of the typical agent is divided between working period (youth) and retirement period (old age). An agent can increase the probability of surviving at the end of youth by incurring private investments in health. However, complementary tax-financed public health programmes also exist. While Bhattacharya and Qiao (2007) study a model whose dynamics is described by a one-dimensional map and show that there exists a unique fixed point that can be stable or unstable depending on the relative size of the health tax rate, our model is two-dimensional and may show multiple fixed points. This may cause local and global indeterminacy and other complex phenomena that cannot be observed in the one-dimensional model by Bhattacharya and Qiao (2007).

The main message of this paper is to show that the link between private expenditure on health and public one causes dramatically different phenomena than when individual labour decisions are exogenous.

The paper is strongly related to the burgeoning literature with endogenous lifetime in models with overlapping generations (Chakraborty, 2004; Fanti and Gori, 2013), and with indeterminacy (Grandmont et al., 1998; Cazzavillan, 2001).

Keywords: Bertrand; Isoelastic demand; Nonlinear dynamics
JEL Classification: C62; D43; L13

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42. Alessandro SPELTA, University of Pavia, Pavia, Italy (jointly with N. Pecora)

"Macroeconomic stability and heterogeneous expectations"

The recent macroeconomic literature has been stressing the role of heterogeneous expectations in the formulation of monetary policy and laboratory experiments provided more evidence about this phenomenon. We use a simple model made up by the standard aggregate demand function, the New Keynesian Phillips curve and a Taylor rule to deal with different issues, such as the stabilizing effect of different monetary policies in a system populated by heterogeneous agents. In particular we investigate whether the policy makers can sharpen macroeconomic stability in the presence of heterogeneous expectations about future inflation and output gap and how this framework is able to reduce volatility and distortion in the whole system.

We consider a parsimonious model with simple heuristics which is able to generate endogenous waves of optimism and pessimism (animal spirits) introducing rationality in the model through a selection mechanism in which agents evaluate the performance of the forecasting they are following and decide to change their strategy depending on how well it performs relative to other ones, [3].

We consider a scenario in which agents can choose among different symmetric forecasting rules, where positive and negative biases are exactly balanced around the Rational Expectations Equilibrium (REE). In this environment agents roughly know the fundamental steady state of the economy whereas they disagree about the correct value of the fundamental output and inflation steady state. Considering the simplest scenario in which agents can choose among three different forecasting rules, we provide an analysis of the dynamics which depends on intensity of choice parameter, the RE predictor cost and the Taylor inflation coefficient. Assuming no cost and a small reaction coefficient to inflation when the intensity of choice is relatively low, there exists only one steady state, the RE equilibrium. In this case the realized inflation remains close to the fundamental steady state. As the intensity of choice increases, two new steady states are created along with the RE equilibrium, which becomes a saddle. Moreover, as the rationality further increases, the RE equilibrium coexists with two steady states and two saddles.

We perform the same exercise assuming a moderate monetary policy (Taylor principle is satisfied) and zero cost for the fundamental predictor. The difference between this scenario and the previous is that now the RE equilibrium is still locally stable.

By numerical investigation, we found that, fixing the inflation reaction coefficient of the Taylor rule equal to c_1^* = 4.425, the REE is unique and globally stable. We define the monetary policy as "aggressive" if $c_1 > c_1^*$. The Central Bank has to react aggressively to an inflation deviation from its RE level in order to send correct signals for the evolutionary selection of the strategies, generating stable dynamics that settle down to the RE equilibrium.

Differently from [1], our 2D model gives different policy intuitions. Employing a standard 3-equations New-Keynesian model, we conclude that the monetary authority has to react more aggressively in order to guarantee the uniqueness (and global stability) of the RE equilibrium. Therefore an aggressive monetary policy can influence heterogeneous expectations by sending correct signals to agents that correct their expectations in order to break down the reinforcing process that arises between inflation and output.

But "What happens when the number of constant forecasting rules increases and approaches to infinity?". In order to study the dynamics of the system as long as the number of predictors become large, we apply the concept of Large Type Limit (LTL) developed in [2]. We found that the space in which the RE equilibrium is unstable implies the intensity of choice to be high and the Taylor principle not satisfied. On the other hand, the RE equilibrium is locally stable if the reaction coefficient for inflation in the Taylor rule is bigger than one. The stability conditions can be achieved only if the variances for the predictors of both output and inflation are not large, meaning that the agents can choose among a continuum of forecast that are not too much distant from the RE predictor.

Finally we adopt an agent-based approach employing our model as an artificial laboratories to carry out computer experiments to improve our insights into the working of certain regulatory mechanisms. We follow Westerhoff [4] introducing two indexes, the first measures the mean of the deviation of the relevant variable from its steady state, the second denotes the rate of change of the time series.

We run the model for 1000 quarters with different values of the Taylor rule parameters, the inflation reaction and the output reaction coefficient. We perform Monte Carlo simulations varying also the intensity of choice coefficient.

If the Central Bank is keen in inflation targeting with a moderate monetary policy, it is possible to reduce both output and inflation variability. The relation is non-linear and, with a too high inflation stabilization parameter, there exist a trade-off where lower inflation variability is obtained at the cost of increased output variability. Even with a Taylor rule coefficient that could lead to a multiplicity of equilibria, as we have previously shown, the variability of the two variables can be lowered at the same time. Moreover some output stabilization is good because it reduces both output and inflation variability by preventing too large switches in forecasting behavior.

Depending on the target of the monetary authority, inflation volatility and distortion can be minimized but also output stabilization can be taken into account. Indeed, if the central Bank shifted its target from inflation to output, results suggest that there exists a trade-off between inflation and output distortion but, a strong reaction to output is more likely to stabilize the economy.

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43. Irina SUSHKO, National Academy of Sciences of Ukraine, Kiev, Ukraine (jointly with P. Commendatore, I. Kubin and C. Petraglia)

"Dynamics of a New Economic Geography model in the presence of an outside region"

The New Economic Geography (NEG) models originate from Krugman's seminal contribution [1], where he put forward the well-known Core-Periphery (CP) model. This model describes an economy composed of two symmetric/identical regions and two productive sectors: agriculture and manufacturing. The footloose entrepreneurs (FE) model [2] is a variant of the CP model, and both of them were originally framed in continuous time. In [3,4] discrete time versions of these models are presented, which preserve many of the most interesting properties of their continuous time counterparts, and in the meantime possess additional features such as chaotic dynamics, multiple attractors, etc.

One of possible directions of further research in frame of NEG approach is related to the fact that typically it is not accounted the existence of outside regions in addition to the ones involved in the integration process. This possibility is explored in [5] by introducing a third region to a FE model under the simple assumption that the manufacturing sector can be localized at most in two regions, while the traditional or agricultural sector is located in all three regions. The model is described by a one-dimensional *piecewise smooth map* Λ: [0, 1] → [0, 1] defined as

$$\Lambda:\lambda \to \Lambda(\lambda)=\begin{cases}0, & M(\lambda)<0,\\ M(\lambda), & 0\le M(\lambda)\le 1,\\ 1, & M(\lambda)>1,\end{cases}$$

where

$$M(\lambda)=\lambda\left(1+\gamma(1-\lambda\frac{T(\lambda)}{1+\lambda T(\lambda)}\right);$$

$$T(\lambda)=\frac{(\sigma-\mu)\left(l\left(\frac{\Delta_1}{\Delta_2}+\phi\right)+2(1-l)\Delta_1\right)+2\mu\phi}{(\sigma-\mu)\left(l\left(\frac{\Delta_2}{\Delta_2}+\phi\right)+2(1-l)\Delta_2\right)+2\mu\phi}\left(\frac{\Delta_2}{\Delta_1}\right)^{1+\frac{\mu}{\sigma-1}}-1;$$

$$\Delta_1=1+\lambda(\phi-1);\;\;\Delta_2=\phi-\lambda(\phi-1).$$

Here the variable λ (resp., $1-\lambda$) corresponds to the share of entrepreneurs located in the region 1 (resp., 2), $0 \le \lambda \le 1$; the parameter l is a share of unskilled labour equally distributed between the manufacturing regions 1 and 2, and the rest is located in region 3, $0 \le l \le 1$; the parameter μ is the exponent in the utility function indicating the invariant share of disposable income devoted to the manufactured goods, $0 < \mu < 1$; the parameter ϕ is so-called "trade freeness" parameter, $0 < \phi < 1$; the parameter γ represents the migration speed, $\gamma > 0$; and σ is the constant elasticity of substitution in a composite of manufactured goods, $\sigma > 1$.

In [5] preliminary analysis of the bifurcation structure of the parameter space of the map Λ is presented. Purpose of the present paper is to describe the dynamics of Λ in more details, in particular, to investigate dependence of the observed bifurcation scenarios on the parameter l, and give economic interpretations of the obtained results.

44.Elisa TICCI, University of Siena, Siena, Italy (jointly with A. Antoci, S. Borghesi and P. Russu)

"Investment inflows and welfare reducing structural changes in a natural resource-dependent economy"

This article discusses the impact of external investment inflows on the development of local rural economies with a focus on the role of environmental externalities and of capital market segmentation. These are two recurrent features of many developing countries where local borrowing and investment capacity is often more limited than in other regions or countries, while external investors enjoy better access to capital markets. Moreover, unlike new incoming activities, local production is usually profoundly dependent on environmental dynamics. The purpose is to analyze the role that free access natural resources play in the relationship between growth, poverty and distribution in contexts (as in several developing countries) characterized by asset concentration, presence of relevant environmental externalities and significant dependence of economic activities on natural capital.

In particular, we analyze a two-sector model describing the dynamics of a small open economy with three factors of production: labor, a renewable natural resource and physical capital. In this economy, agents belong to two different communities: "External Investors" (I-agents) and "Local Agents" (L-agents). I-agents invest their capital in the economy as long as the return on capital generated is higher than in other economies. L-agents use their time endowment partly working as employees for external investors and partly in the local sector, where they directly exploit the natural resource.

Given that L- and I- agents' investments in physical capital follow different mechanisms and rules, we assume that the capital market is completely segmented and it is accessible only by the External Investors, while Local Agents can invest only their savings.

We assume that the production function of the representative L-agent is given by:

$$Y_L = K_L^{\alpha} E^{\beta} L^{1-\alpha-\beta}$$

where E is the stock of a free access environmental resource; L is the amount of time the representative L-agent spends on local sector production; K_L is the physical capital accumulated by the representative L-agent; $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$ hold.

The L-agent's total amount of time is normalized to 1 and leisure is excluded, thus $1 - L$ represents the L-agent's labor employed by the representative I-agent as wage work. The production function of the representative External Investor is represented by:

$$Y_I = K_I^{\gamma}\ (1-L)^{1-\gamma}$$

where K_I denotes the stock of physical capital invested by the representative I-agent in the economy. The representative I-agent chooses her labor demand $1-L$ and the stock of physical capital K_I which she invests in the economy in order to maximize her profits:

$$K_I^{\gamma}(1-L)^{1-\gamma} - rK_I - w(1-L)$$

where w and r are, respectively, the wage rate and the rental rate of K_I (which can be interpreted as an opportunity cost), considered as exogenously determined by the representative I-agent. However, the wage w is endogenously set in the economy by the labor market equilibrium condition (we exclude the import of labor from other economies), while r is an exogenous parameter. We assume that K_I inflow is potentially unlimited. Therefore the time evolution of K_I is not linked to I-agents' savings but only to productivity of K_I (which, in turn, depends on L and K_I).

The representative L-agent, in each instant of time t, chooses L in order to maximize her revenues:

$$K_L^{\alpha} E^{\beta} L^{1-\alpha-\beta} + w(1-L)$$

taking as given the wage rate w, while the accumulation process of K_L is assumed to be described by the following behavioral mechanism:

$$\dot{K}_L = s\left[K_L^{\alpha} E^{\beta} L^{1-\alpha-\beta} + w\left(1-L\right)\right] - \delta K_L$$

where $\dot{K}_L$ is the time derivative dK_L/dt of the variable K_L, the positive parameter δ represents the depreciation rate of K_L and the parameter $s \in (0,1)$ represents the saving rate.

The dynamics of E are described by a logistic function modified by taking into account of the impact of economic activities:

$$\dot{E} = E\left(\overline{E} - E\right) - \varepsilon \overline{Y}_L - \eta \overline{Y}_I$$

where $\overline{Y}_L$ and $\overline{Y}_I$ are the aggregate values of Y_L and Y_I, respectively, and ε and η are positive parameters measuring the environmental impact caused, respectively, by the aggregate production of L and I-agents. The positive parameter E represents the carrying capacity of the environmental resource.

In the above-described economy, external investors can defend themselves against a reduction in capital returns in the local economy by moving their capital towards other economies. On the other hand, local agents have fewer defensive strategies and are vulnerable to environmental degradation. They can react to a reduction in labor productivity in the local sector due to depletion of the natural resource by increasing their labor input in the external sector. However, if the external sector produces a high impact on the environmental resource, such strategy fuels a self-enforcing growth process of external investments associated to a reduction in the welfare of the local agents. However, we find that this result does not imply that full specialization in the local sector always ensures the highest level of welfare for local agents. Under strict conditions, spelt out in the paper, the arrival of external capital inflows and the convergence to a stationary state with the coexistence of both sectors leads to a higher welfare level among local agents than the absence of external investments.

Our results suggest that environmental preservation and protection should be considered as a complementary rather than a supplementary measure to the openness to inflows of external investment in impacting sectors even in an economy where barriers to labor movement do not represent an obstacle towards full specialization in the incoming activities which are not dependent on environmental resources. Environmental defense, in the form of support to the initial endowment of natural capital and to the environmental carrying capacity, and limitation to pollution impact of external investment are prerequisites for a welfare improving coexistence between the two sectors in the economy we have analyzed.

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45. Fabio TRAMONTANA, University of Pavia, Pavia, Italy (jointly with F. Westerhoff and L. Gardini)

"The bull and bear model of Huang and Day: Some extensions and new results"

The development and analysis of financial market models with heterogeneous interacting agents began with the seminal paper by Day and Huang (1990). Based on a few stylized institutional and behavioral facts, Day and Huang proposed a simple financial market model with three types of agent: a market maker who adjusts prices with respect to excess demand; chartists who believe in the persistence of bull and bear markets; and fundamentalists who bet on mean reversion. In their model, market participants' transactions may cause apparently unpredictable price dynamics with randomly alternating periods of generally rising or generally falling prices, so-called bull and bear market dynamics. Since then, literally hundreds of follow-up papers have been presented, deepening our understanding of how financial markets function.

The one-dimensional map studied in Day and Huang (1990) is nonlinear since fundamentalists become increasingly aggressive as the price runs away from its fundamental value. Huang and Day (1993) reformulated their original model such that it corresponds to a one-dimensional continuous piecewise-linear map by assuming that fundamentalists only start trading if the distance between the price and its fundamental value exceeds a critical threshold value. Otherwise, speculators' trading strategies are linear and thus the model is represented by three connected linear branches. Of course, simplifying assumptions must be made to derive such a piecewise-linear map. Due to the model's piecewise-linearity, however, Huang and Day (1993) were able to study how certain model parameters, e.g. chartists' and fundamentalists' reaction parameters, affect the distribution of prices. They found, for instance, that an increase in fundamentalists' aggressiveness tends to fatten the tails of the distribution of prices. Obviously, such insights are quite important for our understanding of financial markets.

In addition, the simplified model of Huang and Day (1993) initiated a substantial number of studies proving that simple piecewise-linear models enable us to derive new analytical insights into how financial markets function.

These models are able to explain some important stylized facts concerning financial markets such as bubbles and crashes, excess volatility, and long-memory effects. Given the repeated emergence of severe financial crises and the herewith associated risk to the real economy, we believe that more work should be undertaken in this exciting research direction.

In this paper, we propose a financial market model that may be regarded as a generalization of the model of Huang and Day (1993). Let us briefly recall why their one-dimensional piecewise-linear map is continuous. Close to the fundamental value, only chartists are active in the market. Hence, the slope of the map's inner branch is larger than one. If the price deviates too far from the fundamental value, additional fundamentalists enter the market. Since their demand is zero when they enter the market, all three branches of the map are connected, and the slope of the two outer branches is lower than the slope of the inner branch. Instead, we assume that a number of chartists and fundamentalists are always active in the market, that additional chartists and additional fundamentalists may enter the market when the distance between the price and its fundamental value exceeds a critical level, and that new traders.demand may be non-zero at the market entry level. As a result, the dynamics of our model is due to a quite flexible one-dimensional discontinuous piecewise linear map. In particular, the three branches are typically disconnected, and there are no restrictions to the values their slopes may assume.

Nevertheless, we are able to provide a comprehensive analysis of the model dynamics. For instance, we are able to determine the frontiers, in parameter space, that separate bounded dynamics from divergent dynamics. This analysis demonstrates that both chartists and fundamentalists can contribute to or prevent market stability, making the regulation of financial markets a delicate issue. Moreover, we find that quite different scenarios can lead to intricate bull and bear market dynamics. As in Huang and Day (1993), we observe repeated price rallies and subsequent market crashes if the slope of the two outer branches is negative, due to fundamentalists' aggressive trading behavior. However, we also observe such – and even more intricate – boom-bust dynamics if the slope of the two outer branches is positive, due to chartists' aggressive trading behavior. Overall, the emergence of endogenous bull and bear market dynamics may thus be regarded as a robust and characteristic feature of speculative markets.

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46. Jan TUINSTRA, University of Amsterdam, Amsterdam, The Netherlands (jointly with F. Westerhoff)

"Positive welfare effects of barriers to entry in a dynamic equilibrium model"

In this paper we consider a simple equilibrium model with two, *A* and *B*. In each region the same commodity is produced and consumed. Firms from region *A*, which produce more efficiently than those from region *B*, may also supply their output in region *B* but face barriers to entry. These barriers to entry take the form of small but positive import tariffs. We consider a cobweb-type model where firms switch between supplying in a particular region on the basis of expected profitability of those regions. We study the effect of import tariffs on stability of the equilibrium and the resulting welfare distribution. First, we find that the autarkic equilibrium, where import tariffs are prohibitively high and every firm produces only for its home market, is locally stable. Second, in the absence of entry costs the equilibrium induces some welfare-enhancing reallocation of firms over the two regions. However, for many parameter values this 'free-trade' equilibrium is locally unstable leading to endogenous dynamics in prices, consumption and firms profits and typically a decrease in welfare. We study the trade-off between higher welfare due to low barriers to entry and lower welfare due to endogenous dynamics and establish that, in contrast to conventional economic wisdom, there may be an optimal non-zero level of barriers to entry in such a setting.

We extend the model by endogenizing the barriers to entry. In particular we will allow special interest groups (such as consumers or producers from the different regions) to lobby for a particular level of import tariffs. This lobbying effort will be positively related to dissatisfaction with status quo.

47. Alessandro VERCELLI, University of Siena, Siena, Italy (jointly with S. Sordi)

"The interaction between unemployment, income distribution and financial fragility in a Goodwin-Minsky growth cycle model"

Though one of the most attractive features of the Goodwin (1967) model is its iconic simplicity, many economists have attempted to extend the theoretical and empirical scope of the model by adding one or more state variables and independent equations. This paper focuses on one of the extension strategies explored so far in order to contribute to its advancement: the study of the interaction between the real fluctuations modelled by Goodwin in his model and the financial fluctuations as explained by Minsky within his Financial Instability Hypothesis (FIH: 1976, 1982). We emphasize two basic reasons for focusing on this intriguing research path:

(i) it goes in the direction advocated by Goodwin (e.g., 1986) of building models within an M-K-S system of thought, i.e. a system based on a creative synthesis of the visions of Marx, Keynes and Schumpeter. While the 1967 model captures in a simple but powerful way one crucial aspect of the real cycles rooted in a Marxian view of class struggle over the distribution of income, it misses completely any reference to the financial side of the economy that plays a crucial role in both Keynes and Schumpeter, as well as in Marx himself. Minsky suggested a theory of financial fluctuations that builds explicitly on the financial side of Keynes and, at the same time, explains the crucial role of credit and the ensuing indebtedness within a cyclical growth perspective reminiscent of the Schumpeterian view.

(ii) the study of the coupling between Goodwin's model and Minky's model of financial fluctuations promises to increase the explanatory power of Goodwin's original model beyond the real fluctuations triggered by the struggle over income distribution, such as those that were prominent in the 1960s and 1970s that inspired the model, including those of the last three decades growingly affected by financial instability.

The pioneer of this research path is no doubt Steve Keen who published a series of models (e.g., Keen 1995, 1996) aiming at coupling Goodwin's model with insights from Minsky's FIH. We follow him by adopting the basic assumption that started his research programme. The crucial role of credit in a monetary economy, as was well understood by Minsky (and Schumpeter before him), is that of releasing investment, the mainspring of capitalist dynamics, from the rigid constraint of saving. Therefore, in order to consider the role of credit and of the financial side of the economy we have to relax Goodwin's equality between investment and saving by assuming that their difference is accommodated by changes in extant credit. We point out, however, that the formalization suggested by Keen is characterized by a few shortcomings; in particular:

(i) his formulation of an independent investment function in the real ('Goodwinian') part of the model appears somehow *ad hoc*;

(ii) the consequences for the dynamics of the model of the possibility to finance investment by debt are not fully formalized and, as a consequence, the dimension of the 'Goodwinian' part of the dynamic system is unaffected;

(iii) the role of credit is seen only from the point of view of the economic units' liquidity and not of their solvency. We believe that financial instability and financial fluctuations crucially depend on the interaction between the liquidity and financial conditions of the economic units (Vercelli, 2011). This is also a basic assumption of Minsky who always started his many versions of the FIH by his famous classification of the financial units crucially based on the contemporaneous use of an index of liquidity and an index of solvency studying then the modifications of these indexes as drivers of financial fluctuations (ibid.).

In order to get round these shortcomings we suggest a different way of coupling Goodwin's model with Minsky's insights based on our previous contributions (see, in particular, Sordi and Vercelli, 2012), in which a basic role is played by the interaction between liquidity and solvency indexes. Our results suggest that by coupling Goodwin's model with a reduced form of the dynamic system of the latter, we obtain a model that allows an explicit analysis of the system's financial fragility and its dynamics. We conclude that this model may be seen as the first step towards a more general model that studies the interaction of real and financial fluctuations in the spirit of Goodwin and Minsky, by updating and developing their far-reaching insights.

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48. Maria Enrica VIRGILLITO, University of Pisa and Sant'Anna School of Advanced Studies, Pisa, Italy

"An attempt to overcome Goodwin's growth cycle structural instability in a discrete time framework"

In this work we discuss reformulations in discrete time of the class struggle model presented by Goodwin in 1967. We intend to describe the business cycle movement as the combined effect of output and wage growth rate. We present two different discrete time versions: the first one maintains the same topological structure of the original model. Being the Lotka-Volterra system characterized by an inherent structural instability, we end up in explosive oscillations that do not allow discussing economic meaningful results. This outcome confirms how the class-struggle model is not invariant to the adoption of discrete time.

In order to overcome the structural instability we elaborate the second extension that presents a coupled dynamic: output and income growth rate simultaneously depend on output and wage level. The aim of this extension is dual: from the one hand overcoming the structural instability of Goodwin contribution, from the other hand comparing Keynesian and classical roots in leading to output fluctuations. Performing simulations we get, for some parameter ranges, a limit cycle that resembles the invariant closed orbits of the original conservative Goodwinian one, generated by a Neimark-Sacker bifurcation. Additionally, our model is able to reproduce endogenous chaotic dynamics, generated by a Flip bifurcation that evolves into a period doubling bifurcation. This chaotic motion is finally proven by the presence of a Henon-like attractor. The richer dynamic comes from the introduction of a coupled interaction of both variables in determining the output and the wage growth rate. It introduces the relevance of the path dependence in explaining variables movements. This reformulation allows comparing which is the behaviour of the system when investments are demand-led (Keynesian approach) or when investments are profits-led (Classical approach). The question that remains open is how to interpret empirical results on the ground of the economic intuition provided by this model. It is a challenging conjecture that requires corroboration both on the modeling side and on the econometric side.

49. Nikolay K. VITANOV, Bulgarian Academy of Sciences, Sofia, Bulgaria (jointly with Z.I. Dimitrova and M. Ausloos)

"On the nonlinear dynamics of cities growth"

The data for the evolution of the distribution of the size of Bulgarian cities show that the Gibrat's law (that city growth is independent on the city size) does not hold - Fig. 1. From the mathematical point of view there are two possible explanations for this deviation: presence of variable coefficients in the model stochastic differential equation (Variant 1) or randomization of the time of the observation (Variant 2).

The Variant 1 leads to dependence of the exponent of the size distribution from the city sized. The Variant 2 leads to the double Pareto lognormal distribution. The rescaled city size is defined as $S_i(t) = N_i(t)/N(t)$, where $N_i(t)$ is the population of the i-th city and $N(t)$ is the population of all cities in year t. Gibrat's law states that the distribution of the growth rate of an unit (city, firm, etc.) is independent on its size. This kind of growth leads to power law distributions as follows. The model stochastic differential equation is:

$$dS_t \ = \ \nu(S_t)dt + \tau(S_t)dW_t \tag{1}$$

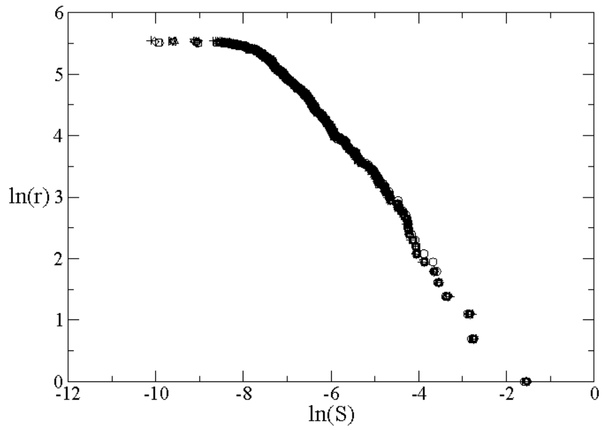

Fig. 1: Rank-size relationship for the rescaled sizes S of the Bulgarian cities from 2004 till 2011. Circles: 2004. Squares: 2005. Diamonds: 2006. Triangles up: 2007. Triangles left: 2008. Triangles down: 2009. Triangles right: 2010. Pluses: 2011. It appears that the rank-size relationships seem to be almost collapsing on a unique relationship.

W is a Wiener process. For the particular case $\nu(S) = gS$ and $\tau(S) = vS$ and in presence of reflecting barrier at $S = S_{\min}$ (which means that the city size can't be smaller than $S_{\min}$), the solution of the Fokker-Planck equation that corresponds to Eq.(1) is a power law (Pareto) distribution:

$$P(S > x) = \left(\frac{x}{S_{\min}} \right)^{-\zeta} \tag{2}$$

The constant value of the exponent ζ from Eq.(2) was not observed in the case of Bulgarian cities (Fig. 1). One possible explanation for this is that g and v are not constants and then deviations from the power law are possible: the exponent ζ can depend on the city size S. In this case the stochastic differential equation is:

$$dS_t = g(S_t)S_t dt + v(S_t)S_t dW_t \tag{3}$$

and the solution of the corresponding Fokker-Planck equation has exponent

$$\zeta(S) = 1 - 2\frac{g(S)}{v^2(S)} + \frac{S}{v^2(S)}\frac{\partial v^2(S)}{\partial S} \tag{4}$$

However there is a second possible explanation of the data. It is based on the observation that the both tails of the distribution of the city sizes are power laws. This feature is exactly the feature of the double Pareto lognormal distribution which can be obtained by means of the stochastic differential equation

$$dS \ = \ gSdt \ + \ vSdW \tag{5}$$

$S(0) = S_0$ is lognormally distributed with mean μ and variance σ^2. Eq (5) has constant coefficients but let now the moment T at which the process is observed be a random variable with p.d.f.

$$f_T(t) = \lambda \exp(-\lambda t) \tag{6}$$

Then the distribution of the variable $S^* = S(T)$ which is the variable S observed at the random moment T is as follows. Let α and $-\beta$ ($\alpha > 0, \beta > 0$) be the roots of the equation

$$\frac{v^2}{2}\kappa^2 + \left(\mu - \frac{v^2}{2}\right)\kappa - \lambda = 0 \tag{7}$$

Then S^* is distributed according to the double Pareto lognormal distribution which p.d.f is:

$$p(x) = \frac{\alpha\beta}{2(\alpha+\beta)}\exp\left(\alpha\mu + \frac{\alpha^2\sigma^2}{2}\right)x^{-\alpha-1}\left[1 + \text{erf}\left(\frac{\ln(x) - \mu - \alpha\sigma^2}{\sigma\sqrt{2}}\right)\right] +$$
$$\frac{\alpha\beta}{2(\alpha+\beta)}\exp\left(-\beta\mu + \frac{\beta^2\sigma^2}{2}\right)x^{\beta-1}\left[1 - \text{erf}\left(\frac{\ln(x) - \mu - \beta\sigma^2}{\sigma\sqrt{2}}\right)\right]$$

In Eq.(8) $x > 0$ and $\alpha, \beta, \mu,$ and σ, are the parameters of the distribution. erf is the error function.

The double Pareto lognormal distribution follows different power laws: $p(x) \approx x^{\beta-1}$ at $x \to 0$, and, $p(x) \approx x^{-\alpha-1}$ at $x \to 1$. This corresponds quantitatively to the distributions we obtained by the data for the Bulgarian city sizes.

In the paper we discuss in detail the application of the Variants 1 and 2 for explanation of the distribution of the sizes of Bulgarian cities. In addition we connect the mathematical models to economic processes that are relevant for the growth of the cities.

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50. Wanying WANG, Nanyang Technological University, Singapore (jointly with W. Huang)

"Economic foundations of technical analysis"

Though technical analysis has gained huge popularity among the practitioners for over two centuries, it is still known as "voodoo finance" to academicians due to lack of solid theoretical supports. Among all the technical indicators, price patterns attract the most disputes due to their subjective identification processes. While many pieces of empirical evidence have been found to support the profitability of technical analysis, showing that those indicators indeed help the speculators to "beat" the market, no one theoretical or statistical model has been developed to replicate all price movements and stylized facts that are documented in financial market. Besides, no existing literature can further justify how the price trends following the chart patterns are pre-determined.

Therefore, the aim of this paper is to go a step further to develop a solid theoretical model to replicate those charting indicators, specifically the price pattern. We approach this goal from the standpoint of technicians that "Price are determined by the demand and supply" and "History repeats itself", by extending the classic deterministic heterogeneous agent model of Day and Huang (1990). One of the most distinguished features of this framework lies in the common saying in technical analysis that "History repeats itself." If we were to accept the potential and profitability of using technical analysis to outperform market, we must then need to realize that these chart patterns are not determined by unknown stochastic factors. Therefore, in contrary to previous studies that include stochastic components, we use a purely deterministic model where not only price dynamics are seen to follow some deterministic movements, but the trend of the patterns is also predetermined. In fact, some studies have already demonstrated that a deterministic process could be the underlying reason that is responsible for the seemingly irregular stock movements.

Day and Huang (1990) are among the pioneers to make use of a deterministic HAM to explain the chaotic asset price fluctuations and the switch between bull and bear market. Huang et al. (2010) also document the possibility of replicating three different types of crisis, namely the sudden crisis, the smooth crisis and the disturbing crisis, using the same deterministic model. Based on this backdrop, we are motivated to explore further potential of HAM in simulating various chart patterns found in technical analysis and therefore provide a solid theoretical model that can be used to explain the underlying internal working mechanisms of such chart patterns used in technical analysis. By this deterministic HAM, we show that (1) the seemingly chaotic fluctuations in price and price-volume relations in the real market can be simulated with high compatibility; (2) most of the commonly-seen chart patterns and their following predictive powers can be easily replicated, (3) popular stylized facts, including fat tails, unit root process and the volatility clustering, that documented by extensive literature can be captured.

This study also goes further to provide economic arguments to the rationale of technical analysis. It is demonstrated that supporting zones and resisting zones can be endogenously developed and once price falls into these zones, the previous trend will be reversed and a reversal point occurs. In this way, since we applied a purely deterministic framework, we can safely conclude that those perpetually important points in the chart patterns can be used to help speculators to detect these supporting zones and resisting zones and predict the trends.

51. Frank WESTERHOFF, University of Bamberg, Bamberg, Germany (jointly with N. Schmitt)

"Speculative behavior and the dynamics of interacting stock markets"

We develop a simple agent-based financial market model in which speculators, who apply technical and fundamental analysis to determine their orders, can trade in two different stock markets. In each time step, a speculator selects a strategy/market combination and this decision depends on predisposition effects, herding behavior and market circumstances. Simulations reveal that our model is able to replicate some important stylized facts of stock markets. We observe, for instance, bubbles and crashes, fat-tailed return distributions and volatility clustering. Most notably, the speculators' behavior creates also realistic stock market interactions, including coevolving stock prices and cross-correlated volatilities.

Keywords: Stock markets; stylized facts; technical and fundamental analysis; agent-based modeling; bounded rationality; simulation analysis.
JEL Classification: C63; D84; G12.

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52. Hiroyuki YOSHIDA, Nihon University, Nihon, Japan

"Global stability and chaotic oscillations in the Cournot adjustment process"

Goodwin is one of the most remarkable pioneers in the field of nonlinear economic dynamics. He developed a set of nonlinear mathematical equations describing both micro- and macro-economic phenomena. For instance, Goodwin (1947, Econometrica) emphasized the importance of interconnectedness of different markets and constructed dynamical coupling models, which exhibited irregular oscillations.

In this paper we examine the effects of interconnectedness of economic agents in a particular market. To put it another way, we investigate the Cournot oligopoly model, which captures the strategic interactions among a small number of firms in a market for a single homogeneous commodity. As usual, we assume that each firm tries to maximize its profit by taking the output choices of the other firms as given. Needless to say, this economic framework is essentially equivalent to a non-cooperative game.

The purpose of the present paper is twofold. First, we attempt to formulate a basic model of the Cournot adjustment process and establish the global stability of the Cournot-Nash equilibrium by using a system of ordinary differential equations. We deal with the following n-firm oligopoly model:

$$\dot{x}_i = \alpha_i \left[R_i \left(\sum_{j \neq i} x_j \right) - x_i \right] x_i, \quad i = 1, 2, \cdots, n \tag{1}$$

where x_i is the output level of firm i, $R_i\,(\Sigma_{j \neq i}\, x_j)$ is a reaction function of firm i and α_i indicates an adjustment speed parameter, which is constant. In this case we show that the Cournot-Nash equilibrium point is globally asymptotically stable by applying the method of the Lyapunov function.

Second, we seek to investigate the possibility of complex dynamics in the Cournot adjustment process, in which time is continuous. To attain our purpose, we modify the basic model by introducing four discrete time lags, which are inherent in the information process when we study the production activities of firms. We carried out numerical simulations of the modified model for certain parameter values. As is evident from Figure 1, we can observe the occurrence of chaotic behavior in the Cournot adjustment process with discrete time lags. This result indicates that the existence of time lags could destroy the global stability of the equilibrium point and further induce chaotic oscillations in the market.

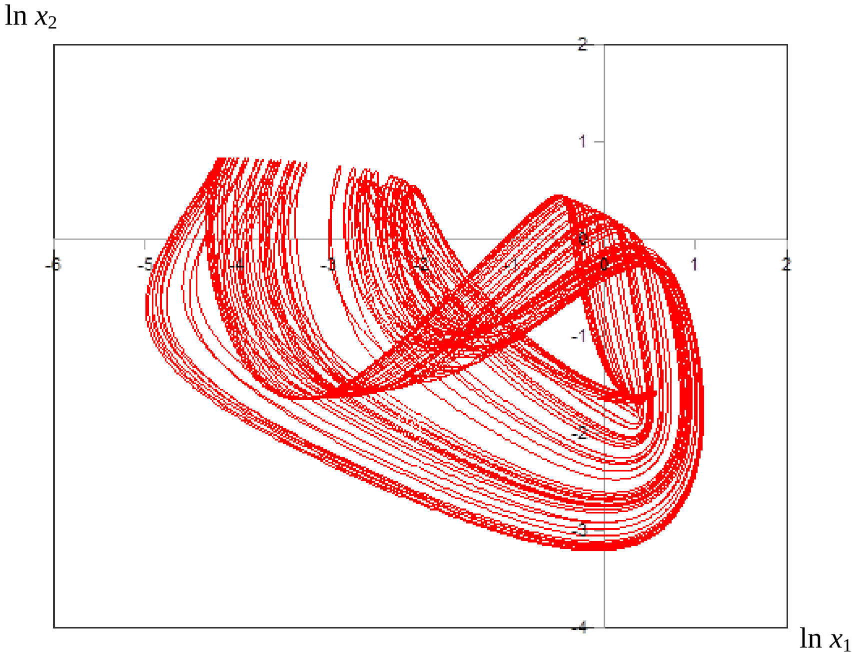

Figure 1: Chaotic attractor

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53. Tianhao ZHI, University of Technology, Sidney, Australia (jointly with C. Chiarella and C. Di Guilmi)

"Modelling the 'animal spirits' of bank's lending behaviour"

The idea that the animal spirit amplifies the business cycle is one of the central insights that Keynes elaborates in his General Theory. In particular, the collective waves of optimism and pessimism of bankers play an important role in the formation of credit cycles. In this paper we propose a model and analyse how the lending attitude of banks affects their behaviour, which ultimately contributes to the boom and bust of credit bubbles and amplifies the business cycle in the real sector. We adopt the Lux (1995) framework to model the opinion contagion of bankers. The model has a particular emphasis on the banks' role in credit creation. It shows that the opinion contagion and heterogeneous expectation amongst banks play an important role in propagating the credit cycle and destabilizing the real economy. The boom phases trigger banks' optimism that collectively lead the banks to lend excessively, thus reinforcing the credit bubble. Eventually the bubbles collapse due to an over-accumulation of debt.

Keywords: animal spirits; contagion; pro-cyclical credit cycle; financial fragility
JEL Classification: E12, E17, E32, G21.

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