# Preface

This volume contains the lessons delivered during the "Training School on qualitative theory of dynamical systems, tools and applications" held at the University of Urbino (Italy) from 17 September to 19 September 2015 in the framework of the European COST Action "The EU in the new complex geography of economic systems: models, tools and policy evaluation" (Gecomplexity). Gecomplexity is a European research network, inspired by the New Economic Geography approach, initiated by P. Krugman in the early 1990s, which describes economic systems as multilayered and interconnected spatial structures. At each layer, different types of decisions and interactions are considered: interactions between international or regional trading partners at the macrolevel; the functioning of (financial, labour, goods) markets as social network structures at mesolevel; and finally, the location choices of single firms at the microlevel. Within these structures, spatial inequalities are evolving through time following complex patterns determined by economic, geographical, institutional and social factors. In order to study these structures, the Action aims to build an interdisciplinary approach to develop advanced mathematical and computational methods and tools for analysing complex nonlinear systems, ranging from social networks to game theoretical models, with the formalism of the qualitative theory of dynamical systems and the related concepts of attractors, stability, basins of attraction, local and global bifurcations.

Following the same spirit, this book should provide an introduction to the study of dynamic models in economics and social sciences, both in discrete and in continuous time, by the methods of the qualitative theory of dynamical systems. At the same time, the students should also practice (and, hopefully, appreciate) the interdisciplinary "art of mathematical modelling" of real-world systems and time-evolving processes. Indeed, the set-up of a dynamic model of a real evolving system (physical, biological, social, economic, etc.) starts from a rigorous and critical analysis of the system, its main features and basic principles. Measurable quantities (i.e. quantities that can be expressed by numbers) that characterize its state and its behaviour must be identified in order to describe the system mathematically. This leads to a schematic description of the system, generally a simplified representation, expressed by words, diagrams and symbols. This task is, commonly, carried out by specialists of the real system, such as economists and social scientists. The following stage consists in the translation of the schematic model into a mathematical model, expressed by mathematical symbols and operators. This leads us to the mathematical study of the model by using mathematical tools, theorems, proofs, mathematical expressions and/or numerical methods. Then, these mathematical results must be translated into the natural language and terms typical of the system described, that is economic or biologic or physical terms, in order to obtain laws or statements useful for the application considered. This closes the path of mathematical modelling, but often it is not the end of the modelization process. In fact, if the results obtained are not satisfactory, in the sense that they do not agree with the observations or experimental data, then one needs to re-examine the model, by adding some details or by changing some basic assumptions, and start again the whole procedure. The chapters of this volume are mainly devoted to the mathematical methods for the analysis of dynamical models by using the qualitative theory of dynamical systems, developed through a continuous and fruitful interaction among analytical, geometric and numerical methods. However, several examples of model building are given as well, because this is the most creative stage, leading from reality to its formalization in the form of a mathematical model. This requires competence and fantasy, the reason why we used the expression "art of mathematical modelling".

The simulation of the time evolution of economic systems by using the language and the formalism of dynamical systems (i.e. differential or difference equations according to the assumption of continuous or discrete time) dates back to the early steps of the mathematical formalization of models in economics and social sciences, mainly in the nineteenth century. However, in the last decades, the importance of dynamic modelling increased because of the parallel trends in mathematics on one side and economics and social sciences on the other side. The two developments are not independent, as new issues in mathematics favoured the enhancement of understanding of economic systems, and the needs of more and more complex mathematical models in economics and social studies stimulated the creation of new branches in mathematics and the development of existing ones. Indeed, in recent mathematical research, a flourishing literature in the field of qualitative theory of nonlinear dynamical systems, with the related concepts of attractors, bifurcations, dynamic complexity, deterministic chaos, has attracted the attention of many scholars of different fields, from physics to biology, from chemistry to economics and sociology, etc. These mathematical topics become more and more popular even outside the restricted set of academic specialists. Concepts such as bifurcations (also called catastrophes in the Eighties), fractals and chaos entered and deeply modified several research fields.

On the other side, during the last decades, also economic modelling has been witnessing a paradigm shift in methodology. Indeed, despite its notable achievements, the standard approach based on the paradigm of the rational and representative agent (endowed with unlimited computational ability and perfect information) as well as the underlying assumption of efficient markets failed to explain many important features of economic systems and has been criticized on a number of grounds. At the same time, a growing interest has emerged in alternative approaches to economic agents' decision-making, which allow for factors such as bounded rationality and heterogeneity of agents, social interaction and learning, where agents' behaviour is governed by simpler "rules of thumb" (or "heuristics") or "trial and error" or even "imitations mechanisms". Adaptive system, governed by local (or myopic) decision rules of boundedly rational and heterogeneous agents, may converge in the long run to a rational equilibrium, i.e. the same equilibrium forecasted (and instantaneously reached) under the assumption of full rationality and full information of all economic agents. This may be seen as an evolutionary interpretation of a rational equilibrium, and some authors say that in this case, the boundedly rational agents are able to learn, in the long run, what rational agents already know under very pretentious rationality assumptions. However, it may happen that under different starting conditions, or as a consequence of exogenous perturbations, the same adaptive process leads to non-rational equilibria as well, i.e. equilibrium situations which are different from the ones forecasted under the assumption of full rationality, as well as to dynamic attractors characterized by endless asymptotic fluctuations that never settle to a steady state. The coexistence of several attracting sets, each with its own basin of attraction, gives rise to path dependence, irreversibility, hysteresis and other nonlinear and complex phenomena commonly observed in real systems in economics, finance and social sciences, as well as in laboratory experiments.

From the description given above, it is evident that the analysis of adaptive systems can be formulated in the framework of the theory of dynamical systems, i.e. systems of ordinary differential equations (continuous time) or difference equations (discrete time); the qualitative theory of nonlinear dynamical systems, with the related concepts of stability, bifurcations, attractors and basins of attraction, is a major tool for the analysis of their long-run (or asymptotic) properties. Not only in economics and social sciences, but also in physics, biology and chemical sciences, such models are a privileged instrument for the description of systems that change over time, often described as "nonlinear evolving systems", and their long-run aggregate outcomes can be interpreted as "emerging properties", sometimes difficult to be forecasted on the basis of the local (or step by step) laws of motion. As we will see in this book, a very important role in this theory is played by graphical analysis, and a fruitful trade-off between analytic, geometric and numerical methods. However, these methods built up a solid mathematical theory based on general theorems that can be found in the textbooks indicated in the references.

Chapter 1, by Gian Italo Bischi, Fabio Lamantia and Davide Radi, is the largest one, as it contains the basic lessons delivered during the Training School. It introduces some general concepts, notations and a minimal vocabulary about the mathematical theory of dynamical systems both in continuous time and in discrete time, as well as optimal control.

Chapter 2, by Anastasiia Panchuk, points out several aspects related to global analysis of discrete time dynamical systems, covering homoclinic bifurcations as well as inner and boundary crises of attracting sets.

Chapter 3, by Anna Agliari, Nicolò Pecora and Alina Szuz, describes some properties of the nonlinear dynamics emerging from two oligopoly models in discrete time. The target of this chapter is the investigation of some local and global bifurcations which are responsible for the changes in the qualitative behaviours of the trajectories of discrete dynamical systems. Two different kinds of oligopoly models are considered: the first one deals with the presence of differentiated goods and gradient adjustment mechanism, while the second considers the demand function of the producers to be dependent on advertising expenditures and adaptive adjustment of the moves. In both models, the standard local stability analysis of the Cournot-Nash equilibrium points is performed, as well as the global bifurcations of both attractors and (their) basins of attraction are investigated.

Chapter 4, by Ingrid Kubin, Pasquale Commendatore and Iryna Sushko, acquaints the reader with the use of dynamic models in regional economics. The focus is on the New Economic Geography (NEG) approach. This chapter briefly compares NEG with other economic approaches to investigation of regional inequalities. The analytic structure of a general multiregional model is described, and some simple examples are presented where the number of regions assumed to be small to obtain more easily analytic and numerical results. Tools from the mathematical theory of dynamical systems are drawn to study the qualitative properties of such multiregional model.

In Chap. 5, Fabio Lamantia, Davide Radi and Lucia Sbragia review some fundamental models related to the exploitation of a renewable resource, an important topic when dealing with regional economics. The chapter starts by considering the growth models of an unexploited population and then introduces commercial harvesting. Still maintaining a dynamic perspective, an analysis of equilibrium situations is proposed for a natural resource under various market structures (monopoly, oligopoly and open access). The essential dynamic properties of these models are explained, as well as their main economic insights. Moreover, some key assumptions and tools of intertemporal optimal harvesting are recalled, thus providing an interesting application of the theory of optimal growth.

In Chap. 6, Fabio Tramontana considers the qualitative theory of discrete time dynamical systems to describe the time evolution of financial markets populated by heterogeneous and boundedly rational traders. By using these assumptions, he is able to show some well-known stylized facts observed in financial markets that can be replicated even by using small-scale models.

Finally, in Chap. 7, Ugo Merlone and Paul van Geert consider some dynamical systems which are quite important in psychological research. They show how to implement a dynamical model of proximal development using a spreadsheet, statistical software such as R or programming languages such as C++. They discuss strengths and weaknesses of each tool. Using a spreadsheet or a subject-oriented statistical software is rather easy to start, hence being likely palatable for people with background in both economics and psychology. On the other hand, employing C++ provides better efficiency at the cost of requiring some more competencies. All the approaches proposed in this chapter use free and open-source software.

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