Mathematical bioeconomic modelling of the interaction between aquaculture and open sea fisheries

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Abstract

We analyze the problem of an integrated management of fisheries by using fish farming as a tool for restocking fish populations depleted by overfishing pressure. We first use a simple heuristic dynamic model, taken from a classical example of mathematical bioeconomics, in order to prove that fish restocking may be an efficient tool for sustainable fishery management in situations where excessive fishing effort is going to cause irreversible stock collapse. Then we propose a two-compartments bieconomic mathematical model, with age structure and age specific harvesting and restocking, to mimic a possible integrated interactions between aquaculture and open sea fisheries, where reared fish are used as substitutes for marine catches. The model proposed in this paper tries to fill a gap between the rich mathematical bioeconomic literature, mainly devoted to the description of open sea fisheries and/or marine protected areas, and the very poor literature on mathematical bioeconomic modelling of interactions between fish farming and fisheries.

1. Introduction

The problem of a sustainable exploitation of renewable resources, in particular fisheries, represents a challenging task, as it involves nonlinear interaction of biological, economic and social components (see Rosser, 2001, and references therein). In the last decades, the mathematical modelling of managed natural populations has given encouraging results, as witnessed by the rapidly increasing amount of literature in the field of so called "Mathematical Bioeconomics" (after the book by Clark, 1976). Even if these models are quite simple with respect to the complex systems they aim to mimic, and they generally give only qualitative and heuristic results, important lessons have been learned from the study of their properties (Clarke, 1976, Getz and Haight, 1989). This has been particularly true in the study of fisheries, for which the effects of several different kinds of sustainable management policies have been modelled and simulated (see e.g. Antonelli et al., 2005a, Bischi et al., 2005). However, many natural fisheries are subject to overexploitation, and this often lead to situations of severe depletion of managed populations. Many authors stress that traditional fishing activities do not seem able to make further increases without irreversibly impoverishing resources (see e.g. Mauracher and Ragazzoni, 2005). An answer to this problem can be given by a more and more intensive fish farming activity. Of course, fish farming will play an increasing role in the reduction of the fishing effort by substituting the latter in catering for the growing demand for fish. Moreover, it may offer interesting opportunities for restocking depleted fish species.

Despite this promising possibility, very few attempts have been made to propose mathematical models that describe the dynamic interplay between open sea fisheries and aquaculture.

Indeed, a rich literature exists to describe the interaction between open fisheries and marine protected areas, such as reserves where no fishing activity is allowed or bounded marine regions where very strong restrictions are imposed on fishing, see e.g. Clark (1996), Lauck et al. (1998), Sumaila, (1998), Pezzey et al. (2000), Holland (2002), Antonelli et al. (2005b). In these papers, marine reserves have been considered as a long term investment and as an insurance to avoid the consequences of unexpected collapses of overexploited fisheries. In fact, even in fisheries where it is possible to tightly control the level of fishing effort there may be a limited ability to manipulate the relative levels of fishing mortality of different species that are caught together. Consequently, any level of fishing effort may be too high for some species.

Indeed, as we shall prove by using a simple bioeconomic model, when a decline of a fish population starts in conditions of strong fishing pressure, it is very difficult to avoid a severe collapse of the fish stock by simply reducing the fishing pressure.

In this case, fish farming can be considered as an integral part of an optimal management system for some fisheries, by using reared fish as substitutes for marine catches, thus acting as an insurance policy against fishery management failures.

In this paper we propose a dynamic model for the simulation of this kind of interaction. Our model is based on a subdivision of the fish stock into age (or size) classes, because overexploitation often implies an increase in catch of older (more valuable) fish, hence a differentiated depletion occurs according to the age classes considered. In particular, some age (or size) portions of intensively fished populations that may be more heavily depleted, such as older (more valuable and also more prolific) fish population, or juvenile portions of fish populations, because of damaged environment or altered intraspecific and interspecific ecological connections

So in open access fisheries many interconnected and growing problems arise, such as overexploitation, stock collapse, loss of biodiversity, truncated or altered age distribution within a fish population. In such situations, export of juvenile and adult biomass from aquaculture may constitute a protection of genetic quality from detrimental effects of fisheries selection, and may constitute a kind of insurance against stock collapse related to overfishing or natural recruitment failure. Such biomass exports may avoid collapse or lead to a more rapid rebuilding of depleted fish stocks.

In this paper we identify some of the key characteristics and capabilities that are important for modelling the interplay between open sea fisheries and fish farming activities from the point of view of depleted fish restocking.

To do this, we try to extend some bioeconomic models for fisheries to include the presence of fish farming as a device for restoring depleted natural fish populations at the sea and, in particular, for producing juveniles and/or adult population cohorts in order to fill more depleted stages of a given fish population.

In section 2 we propose a very simple aggregated model, in the form of a classroom exercise, in order to show how important a fish restocking is when fishery collapses.

In section 3 we propose a two-compartment model, the two compartments being an aquaculture system and an open fishery (with some traditional form of fishery regulation). The fish population in each compartment is modelled as an age (or size) structured population, by a cohort model with a nonlinear recruitment function and differentiated harvesting for different age (or size) classes. Some final remarks are given in the last section.

2. A simple heuristic dynamic model of irreversible fishery collapse

There is general agreement among scientists, industry, public and politicians that many of the world's marine and freshwater fisheries have been overexploited and that many fish stocks are

depleted and in need of rebuilding. There is also general agreement about the fact that the use of mathematical models can help scientists and policy makers in order to better understand and explain some basic principles and to analyze the effects of fishery management policies, as well as making some qualitative and general forecasts about some possible future scenarios.

In this section we use a simple heuristic dynamic model in order to give a precise meaning to the assessment that the nonlinearity of the interactions between biological growth and human harvesting of a natural renewable population may easily lead to severe and irreversible depletion of the population, and consequently reduced yields. The same model will show that when such dangerous thresholds of fishery exploitations are reached, even a drastic reduction of the fishing effort may be ineffective to restore the originary equilibrium population if it is not associated with population restocking. This implies that the existence of farmed stocks of the husbanded fish species may be very useful, under such circumstances, in order to restock the same species in nature.

2.1 An heuristic dynamic model

Let us denote by $X(t)$ a measure (e.g. total biomass or biomass density) of available fish population in a given environment at a given time period t. The simplest model that mimics the time evolution of the fish stock X is based on the following discrete time dynamical system

$$X(t+1) = F(X(t)) = X(t) + RX(t) - H(t) \tag{1}$$

where t represents time, assumed to be made of discrete units, or periods, that may be measured in terms of days, or months, or seasons according to the features of the system under analysis; R represents the specific growth rate, referred to the time unit and unit of biomass, and is given by the difference between birth and death rate (where death rate includes stock lost due to parasites, predators and senescence); $H(t)$ is the term that represents harvesting, measured as the quantity of fish caught in a time period, determined according to the fishing policy adopted, ranging from free access only limited by profit maximization and market externalities to more severe limitation on fishing effort or maximum allowed quotas imposed by some central authority. So, the function H may be quite complicated as it includes individual strategic decisions as well as the adaptive adjustment of constraints decided on the basis of available data and forecasts about the evaluation of expected fish stock (see Clark, 1976, Getz and Haight, 1989, Antonelli et al., 2005a).

Also the specific growth rate R may be influenced by several factors that may modify recruitment and mortality. In general R is a function of the existing biomass, that is, $R = R(X)$, as fish density influences both death rate, due to competition for food or available space, and fecundity. This implies that the natural growth term in equation (1), given by $X(t)[1 + R(X)]$ is nonlinear. For many natural populations the function $R(X)$ is decreasing, i.e. the growth rate declines as population density increases. This is a form of density-dependent population growth regulation. This is always true for very high levels of population density. However, in fish populations, an opposite effect, called depensation, occurs at low levels of density, i.e. mortality decreases (hence $R(X)$ increases) for increasing X when the population level X is low (see fig. 1a). This phenomenon is commonly observed in fish populations living in schools, such as cod or tuna fish, and can be explained by assuming higher predation rates on smaller school sizes. As we shall see below, depensation in fish population may be a concern, because they can exacerbate irreversible population declines when harvesting with controlled fishing effort is applied.

Starting from a given initial fish stock $X(0)$, the dynamic equation $X(t+1) = F(X(t))$ allows one to compute, inductively, the stock levels at any successive time period $t = 1,2, …$, that is, the time evolution of the fish stock. Such equation is based on a simple mass balance consideration: the total fish biomass in a given period is given by the one existing in the previous period increased by the natural growth R and decreased by the fishing activity. It is easy to see that a steady state (or equilibrium) is characterized by $X(t+1) = X(t)$, i.e. $RX(t) = H(t)$, an equation that

states the trivial condition of sustainable harvesting expressed by equality between net natural population growth in a time interval and harvested quantity in the same time unit. Of course, if at a time period t the inequality $RX(t) > H(t)$ holds, then the population stock will increase in the next period, i.e. $X(t+1) > X(t)$, whereas if $RX(t) < H(t)$ then population will decrease in the next period, $X(t+1) < X(t)$.
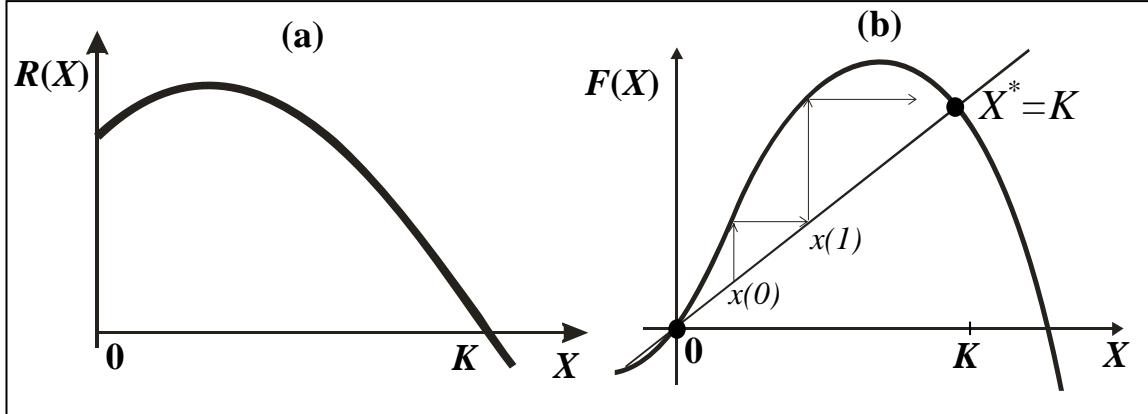

Fig. 1

2.2 Dynamics of unharvested fish population

If we consider an unharvested population, that is $H(t) = 0$, equation (1) describes the natural dynamics of the fish population at hands, only influenced by its biological properties, the natural environment and interactions with other fish populations, through competition, predation etc. If the function $R(X)$ has a shape like the one shown in fig. 1a, i.e. growth with depensation, then the function $F(X) = X(t)[1 + R(X)]$ has a graph like the one shown in fig. 1b, characterized by a unimodal shape (i.e. only one maximum) and an inflexion point, being the function convex for small values of X and concave for higher values. We can notice that there are two equilibrium points: $X^* = 0$, that we shall call "equilibrium of extinction" and $X^* = K$, usually called carrying capacity. From the definition of equilibrium it follows that if, at a given time period, the system is in one of the two steady states X^*, then it would stay there in subsequent periods as well. However, the two equilibriums behave differently if the value of the population is slightly altered with respect to the equilibrium value. In fact, at $X^* = 0$ a small increase in the biomass will be amplified by the model dynamics, and consequently the successive values $X(t)$ obtained by the repeated application (or iteration) of the map F will definitely move the system away from the extinction equilibrium. Instead, in the case of the viable equilibrium $X^* = K$ the endogenous forces of the system will tend to diminish any small displacement from the equilibrium value, thus bringing the system back to the original equilibrium. In fact if X is decreased, i.e. moved to the left of $X^* = K$, there $F(X) > X$ hence X will increase in the following period, whereas if X is increased, i.e. moved to the right of $X^* = K$, then it will enter a region where $F(X) < X$, so that it will decrease in the next period. This is expressed by saying that $X^* = K$ is a stable equilibrium and $X^* = 0$ is unstable (see Bischi et al., 2004, for an elementary expositions of the concepts of dynamic evolution and stability in discrete time). An easily deduced consequence is that if, for any reason, the fish stock is strongly reduced with respect to the natural equilibrium $X^* = K$, then the endogenous dynamic forces in the system are capable to recover the perturbation and restore, in the long run, the original natural equilibrium. However, as we shall see in the next subsection, the situation becomes quite different if a fishing activity is introduced, even if it is a regulated one.

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2.3. Harvesting with constant effort and hysteresis effects

The harvesting function $H(t)$ may assume several different forms, according to fishing policies imposed or chosen by fishermen organizations. The simplest kind of regulation is given by the imposition of a constant *fishing effort*, a parameter that expresses a measure of intensity of fishing activity, in terms of number of fishing boats and time dedicated to fishing activity. Under this assumption, the quantity harvested at each time period is proportional to the currently available fish stock

$$H(t) = q \, E \, X(t), \tag{2}$$

where E is the fishing effort and the parameter q, called *catchability coefficient*, is a measure of the ease with which fish are captured. It depends, among other things, on the technologies used by fishermen.

With harvesting (2) the dynamic model (1) becomes

$$X(t+1) = F(X(t)) = X(t) \, (1 + R(X) - qE) \tag{3}$$

The extra-mortality term causes a downward shift of the graph of the function F with respect to the one that gives the dynamics of unharvested populations, shown in fig. 1, see fig. 2a where the function F defined in (3) is represented by a solid line, compared with the one without harvesting, i.e. $E=0$, represented by the dashed curve. As it can be seen in fig. 2a, for sufficiently low values of the effort E the qualitative properties of the dynamics induced by the iteration of the function $F(X)$ are essentially the same, the only effect being a slight decrease if the viable equilibrium $X^*=K_E$, whose value decreases for increasing values of the parameter E. Instead, as the effort E is increased over a threshold value, say E_1, we can observe the creation of a new positive equilibrium point, say X_E, located between the extinction equilibrium and the viable equilibrium K_E, i.e. $0 < X_E < K_E$ (fig. 2b). At the bifurcation value $E=E_1$, the extinction equilibrium becomes stable and the new equilibrium X_E is unstable. It acts as a watershed between the initial conditions that generate trajectories converging to the extinction equilibrium (if $X(0) < X_E$) and those generating stock paths evolving towards the positive stable equilibrium K_E (if $X(0) > X_E$, see the arrows in fig. 2b). Accordingly, the unstable equilibrium X_E is called *survival threshold.*

If the fishing effort E is further increased, the two stable equilibria X_E and K_E get closer (fig. 2c). This has two effects: first, the fish stock at the stable equilibrium K_E decreases (a quite intuitive effect due to the increased harvesting); second, the survival threshold X_E increases, and this implies that the system becomes more vulnerable, because a smaller exogenous fluctuation is sufficient to move the population level below the survival threshold. If E is further increased, then X_E continues to increase and K_E to decreases, until the two equilibria merge and disappear (fig. 2d). This event marks another bifurcation, occurring at a bifurcation value denoted by E_2 in fig. 2. After this bifurcation, the only possible evolution of the system is towards stock collapse. It is important to notice that if, just after this occurrence, i.e. when the population begins to decline but it is still viable, an authority imposes to decrease the effort, so that a value E such that $E_1 < E < E_2$ is restored, it may happen that fish stock continues to decline. In fact, even if the equilibria are restored, if the population level is below the threshold, i.e. $X < X_E$, then the decline will continue. In other words, the bifurcation at $E = E_2$ marks an irreversible departure from the stable equilibrium, even if the effort is reduced at a value $E < E_2$.
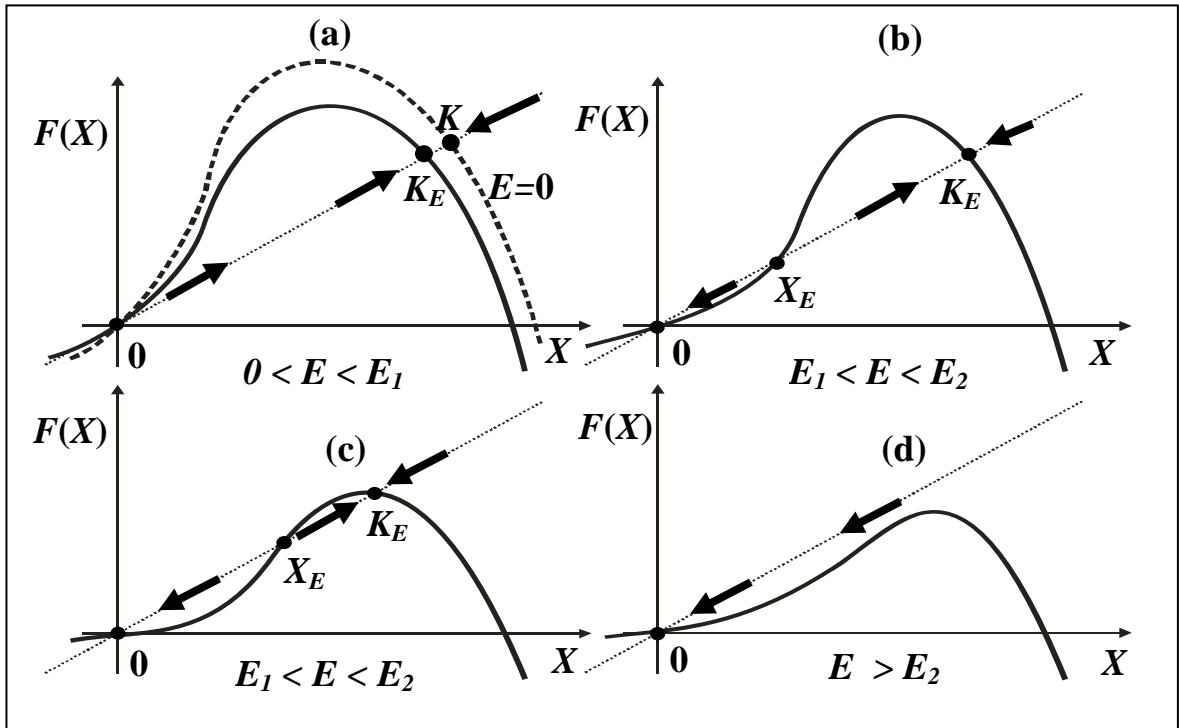
Fig. 2

To better explain this point, and in order to consider possible actions to avoid such an irreversible collapse, we show a bifurcation diagram, where the steady states are represented as functions of the bifurcation parameter E (fig. 3).

In fig. 3 the solid lines represent stable equilibria, the dashed lines unstable ones. For $E_1 < E < E_2$ there are two stable equilibrium curves, one positive at $X = K_E$ one vanishing, at $X = 0$, separated by an unstable equilibrium curve, at $X = X_E$, that acts as a watershed between the two stable ones (see the arrows in fig. 3). As E increases, the basin of attraction of the stable equilibrium K_E shrinks, until it disappears at $E = E_2$ and the only global attractor remains the extinction equilibrium $X = 0$. So, for $E > E_2$ the fish stock can only evolve towards extinction.

Now, let us assume that when the effort-population values are at the point denoted by A in fig.3 the fishermen, or some central authority, realize that it is urgent to decrease the effort, and impose an effort decrease ΔE. As this is not sufficient to cross the threshold level to reach the basin of attraction of K_E, the fish stock will continue to decrease, and the position B in the diagram of fig. 3 will be reached, where the situation is even worst. Instead, the reduction of effort may be more effective if it is associated to some form of exogenous fish restocking, i.e. a method to partially rebuild the depleted population. This is equivalent to move the system to position C in the diagram of fig. 3, thus crossing the survival threshold and entering the basin of K_E. This means that, without any further exogenous action, the endogenous forces of the system will lead again the stock to the originally stable equilibrium K_E.

Without any restocking action the population level will decrease more and more, because the stock value $X(t)$ will remain below the survival threshold, i.e. it is trapped inside the basin of attraction of the extinction equilibrium. In order to exit from this trap, the effort must be reduced below the first bifurcation value E_1.

This sequence of events, characterized by irreversibility, i.e. the difficulty to restore the conditions existing before the bifurcation at $E = E_2$, unless a strong reduction of the bifurcation parameter is applied, is called hysteresis, a typical occurrence studied in the framework of theory of catastrophes (see e.g. Rosser, 2000).

To complete the heuristic exercise proposed in this section, we suggest a simple analytic expression of the specific growth function with depensation , so that a numerical example can be attached to the qualitative analysis described above. Let us consider $R(X)=3+2X-X^2$. This is a typical specific growth function with depensation, i.e. a graph similar to the one shown in fig. 1a, with $R(0)=3$, maximum at $X=1$, where $R(1)=4$, and carrying capacity $K=3$. If $E=0$ the function $F(X)$ that characterizes the unharvested dynamics has a graph like the one shown in fig. 1b, with maximum at $X=2$ and inflection point at $X=1/3$. The corresponding function $F(X) = X(1+R(X))- qEX = (4-qE)X+2X^2-X^3$, besides the equilibrium at $X=0$, has a unique positive equilibrium if $qE<3$, two positive equilibria for $3<qE<4$, no positive equilibria for $qE>4$.
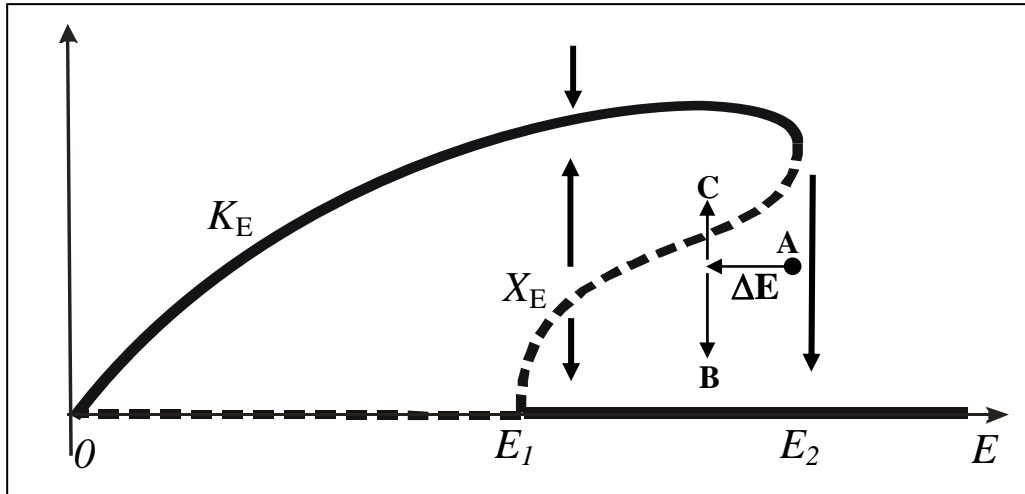

Fig. 3

3. A two compartments model with age structure and cohort flows.

The simple model described in the previous section suggests that it may be very useful to insert farmed fish into depleted fish populations when fishing effort exceeded the maximum allowed levels. Of course, this is only possible if the aquatic organisms reared are very similar, or identical, to their counterparts in nature, and the aquaculture considered is ecologically compatible.

However, the description given by the heuristic model presented in the previous section is too simple, because such aggregated model describes an homogenous, i.e. undifferentiated, fish population, and also the fishing pressure is considered as homogeneous on the whole population. Instead, overexploitation often implies an increase in catch of older (and generally more valuable) fish, and this implies an important change in the age structure of the population with respect to the natural age distribution. Moreover, being the older individuals the more prolific, the depletion of higher age classes may be even more dangerous from the point of view of the survival of the whole population. Also in this case, a proper restocking of older cohorts of a population, by using mature individuals coming from an ecologically compatible aquaculture, may be a suitable method to fill exhausted overfished stocks.

Moreover, selective fishing harvesting should be considered, such that it operates differentiated fishing pressure on different age classes. This may have good effects if properly programmed, such as those harvesting policies that aim to avoid any fishing pressure on juvenile cohorts. However, in many cases the increased harvesting of mature age classes may indirectly cause the depletion of juvenile cohorts due to the induced decrease of population fertility. Moreover, alterations of intraspecific and interspecific competition of predation connections induced by fishing activity may cause a disequilibrium in the biodiversity composition of an aquatic

environment, for example due to modifications of predation and competition.

On the basis of these arguments, we introduce a model with two compartments, given by a population in open sea and a population of the same fish species in an aquaculture, and each of these populations is subdivided into age (or stage) classes, such that each sufficiently mature class contributes to the juvenile age class by a proper stock-recruitment function, and the total fish harvesting can be subdivided into specific harvesting functions referred to any age class. Of course, proper survival coefficients (or functions) determine how the individuals of the populations grow and move from one class to the successive one. Finally, transfers are allowed from the compartment of fish farming to the one of open sea population whenever depletion of a given age class is detected.

In this section we try to represent these factors in the form of a discrete time dynamical system, that may be seen as a generalization of the lumped model described in the previous section. Let $X(t)$ and $Y(t)$ denote the fish biomass of a given fish species in open sea and in fish farming respectively, and let $x_1, ..., x_n$, the age (or stage) classes of the fish species in open sea, $y_1, ..., y_n$ the age (or stage) classes of the fish species in the fish farm. Of course, the relations

$$X(t) = \sum_{i=1}^{n} x_i(t) \text{ and } Y(t) = \sum_{i=1}^{n} y_i(t) \tag{4}$$

hold at each time period t. In this case the discrete time periods considered in the model counts down the discrete times of transfer from one age group to the next one. The dynamic equations for the cohorts of the fish population in the sea are

$$x_1(t+1) = x_1(t)(1-s_1) + \sum_{i=1}^{n} b_i x_i(t) R_i(X(t)) - H_1(t) + \phi_1$$

$$x_k(t+1) = x_k(t)(1-s_k) + s_{k-1} x_{k-1}(t) - H_k(t) + \phi_k \qquad k = 2,...,n-1 \tag{5}$$

$$x_n(t+1) = x_n(t) + s_{n-1} x_{n-1}(t) - H_n(t) + \phi_n$$

where s_i, $0 \le s_i \le 1$, $i=1,...n$, denote the survival coefficient for age (or stage) class i, defined as the proportion of individuals alive in class i that survives to class $i+1$, of course; b_i, $i=1,...n$, denotes a natality coefficient (we may have $b_i=0$ for younger age classes up to $b_i=1$ for the more prolific age classes); H_i represent the harvesting of individuals in age class i; R_i, $i=1,...n$, represents the specific growth functions for age class i, ϕ_i represent the fish transfers from aquaculture to the open sea, subdivided into age classes. A comparison with the lumped model of the previous section is possible by defining

$$H(t) = \sum_{i=1}^{n} H_i(t) \text{ and } \Phi = \sum_{i=1}^{n} \phi_i. \tag{6}$$

Analogously, for the cohorts in the fish farming population we have:

$$y_1(t+1) = y_1(t)(1-s_1^y) + \sum_{i=1}^{n} b_i^y y_i(t) R_i^y(Y(t)) - H_1^y(t) - \phi_1$$

$$y_k(t+1) = y_k(t)(1-s_k^y) + s_{k-1}^y y_{k-1}(t) - H_k^y(t) - \phi_k \qquad k = 2,...,n-1 \tag{7}$$

$$y_n(t+1) = y_n(t) + s_{n-1}^y y_{n-1}(t) - H_n^y(t) - \phi_n$$

where the superscripts "y" in the survival coefficients, specific rates and harvesting functions mean that these quantities are generally different with respect to the ones in the equations that describe open sea cohorts.

The transfer parameters ϕ_i may be transformed into functions of x_i, such that any single cohort flow ϕ_i is activated when x_i is smaller than a given threshold value $\overline{x}_i$, i.e.

$$\phi_i(x_i) = \begin{cases} y_i \Phi(\overline{x}_i - x_i) & \text{if } x_i < \overline{x}_i \\ 0 & \text{if } x_i > \overline{x}_i \end{cases} \tag{8}$$

where Φ is a bounded increasing function with values in the range $[0,1)$, such as $\Phi(z) = \dfrac{2}{\pi}\arctan(z)$ or $\Phi(z) = \tanh(z)$ or $\Phi(z) = \dfrac{z}{1+z}$, i.e. a function whose graph is like the one shown in fig. 4. It is plain that the *transfer functions* ϕ_i are bounded above because of the limited availability of farmed fish y_i in each age class i, see fig. 4.
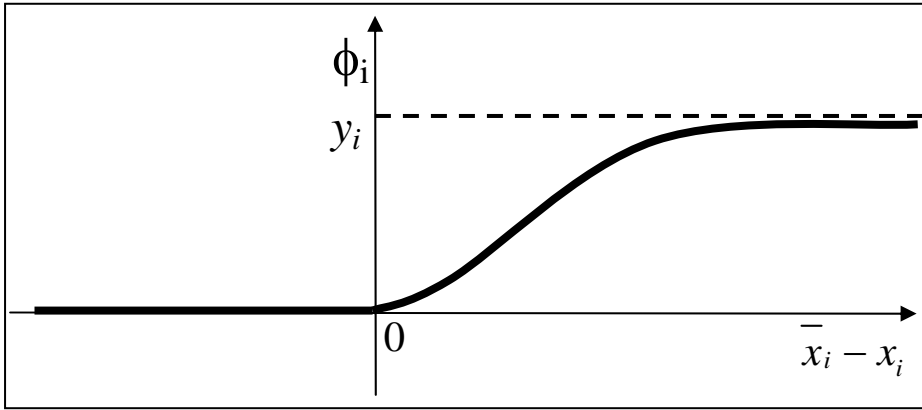

Fig. 4

The dynamic model proposed may be used to simulate different scenarios according to the choices of the growth functions $R_i(X)$, the harvesting functions $H_i(X)$, the natality coefficients b_i. For example, following the arguments given above, we may realistically assume that the natality coefficients b_i are higher for higher age classes, being mature fishes more prolific, and also the harvesting functions have greater values for higher age classes, as mature fishes have a greater size so they generally are more valuable and are easier to be captured, i.e.

$$H_k = q_k E x_k. \tag{9}$$

have higher catchability coefficients q_k for higher values of k. This implies that higher age classes of the open sea fish populations, i.e. x_k with k close to n, will be more depleted, and will need to be restocked with sufficiently high transfers ϕ_k. This means that we should try to increase the number of farmed fish of the corresponding age classes y_k, for example by setting $H_k^y = 0$ for the higher age classes in the aquaculture.

These simulations may help us to obtain an optimal regulation of the controllable parameters, like the effort E and the catches H_k^y in the fish farming in order to minimize the probability of fish stock collapses.

Of course other situations can be simulated, by properly tuning the parameters q_k. In fact, by setting $q_k = 0$ for small values of k it is possible to simulate fisheries where small size fishes are not captured, for example by using proper mesh selection, i,e, sufficiently large fishing nets so that fish below a given size can pass through them. It may be interesting how this parameters' choice will influence the dynamics of the whole integrated system.

This model may be improved by properly modifying the harvesting functions H_k in order to include some other traditional policies for the controlled management of fisheries, such as controlled quotas. In other words, a mix of policy instruments can be simulated.

4. Conclusions

In this paper we have proposed a two-compartments dynamic model for the description of two age structured fish populations, one in open sea and one in fish farming, in order to simulate an integrated fishery management that includes the possibility of differentiated harvesting and fish restocking.

The model has not been studied here, as its study is still at its initial stage. However, it is proposed in order to fill a gap in the literature, as bioeconomic mathematical models has not been applied to describe this kind of integrated management of fisheries, despite the fact that this kind of integration between open sea fisheries and aquaculture has been indicated by several authors as a promising method for sustainable management of fishing activity.

Models of aquatic systems characterized by several compartments (or patches) have been proposed for the description of interactions between open access fisheries and marine protected areas, see e.g. Sumaila and Charles (2002), and references therein, Bischi and Lamantia, 2005. However, the modeling of spatial and age structure for fish farming is quite different to the marine protected areas one, so ad hoc models are necessary.

The model proposed in this paper is only a first step towards a more complete modelling of interactions between aquaculture and open sea fisheries. In fact, many different assumptions can be made on specific growth functions in open sea and aquaculture systems, and different harvesting policies may be described for the open sea compartment, ranging from the open access only regulated by profit maximization and competition (see Bischi and Kopel, 2002, Bischi et al., 2005) to fish catches under severe constraints imposed by restrictive laws. In addition, multispecies models may be considered, characterized by nonlinear specific interactions of competition and predation.

Moreover, the mathematical modelling of the economics of fisheries in the presence of aquaculture, may also take into account other externalities: economic externalities, due to the increased supply of cultivated fish, that must be considered in the demand function, and ecological externalities, related to competition between fishes coming from aquaculture and their sea analogues. The modelling of these factors may represents an important advancement in the understanding of trade off between aquaculture and sustainable open sea fisheries.

However, it is well known that simpler models may often lead to insights not gained from very detailed models (see e.g. Clark, 1976, Getz and Haight, 1989, for interesting discussions on this point).

Finally, it is worth to stress that, as remarked by several authors (see e.g. Frankic and Hershner, 2003) the introduction of exogenous fishes in natural environment may have some negative side effects, due to a substantial alteration of the benthic community. This may be particularly critical if the aquaculture does not follow some ecological standards. This is one of the reasons that testify the necessity of bioeconomic models that allow the operators to simulate the behavior of complex ecological systems before taking decisions. So, the use of mathematical modelling as a decision support system for institutions that operate in the field of resource management is one of the challenges of next years.

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