Harvesting dynamics in protected and unprotected areas

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Abstract

We propose a dynamic model for studying the time evolution of fish stocks in an environment divided into two adjacent zones with different fishing policies. We analyze two particular harvesting methods: constant fishing effort and profit maximization. In this case, some agents engage in competition based on maximization of individual profit whereas others cooperate. The asymptotic behavior of the system pinpoints that, even if the presence of a reserve area leads to higher levels of sustainability in exploiting fish stocks, attention should be paid in properly regulating the harvesting activity in order to avoid severe depletion of the resource.
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1. Introduction

The modelling of the commercial exploitations of renewable resources represents a challenging task, as it involves the nonlinear interaction of biological, economic and social components. The sustainability of this exploitation is constrained by the natural growth of the resource, and over-exploitation eventually leads to stock depletion and thus decreasing yields. Different sources of strategic interdependence among competing agents who have access to a public natural resource are present. First, biological externalities must be taken into account, as overexploitation of the

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resource by one agent may have important consequences on the capacity of regeneration of the resource, thus giving a negative externality for the whole community of exploiters. Second, market externalities may exist due to price reduction as a consequence of increasing resource harvesting. Moreover, cost externalities may exist because decreasing stocks of available resources usually imply increasing unitary harvesting costs. In the management of common property renewable resources (such as fisheries, aquifers or rivers, hunting, forestry) harvested by competing individuals, societies or countries, the problem known as "The tragedy of the commons" after Hardin (1968) (see also Gordon, 1954; Clark, 1990; Mesterton-Gibbons, 1993; Conrad, 1999) must be taken into account.

For these reasons many authors suggest that central institutions should impose forms of regulation such as fishery regulations, either by imposing fishing restrictions such as constant fishing efforts, or through taxation (see Clark), or even by stimulating cooperative behavior among exploiters.

However, the complexity of time evolutions of natural populations, due to nonlinear biological growth functions and the ecological complexity of interactions among species, gives rise to several difficulties in implementing suitable regulation policies that are able to combine economically (and socially) efficient exploitation with issues of sustainable exploitation. Indeed, these severe nonlinearities can present themselves at multiple levels and in multiple ways, due to nonlinear interactions between ecological and economic components (see, e.g. Rosser, 2002), and present a serious challenge to policy makers.

In this paper we consider fisheries, and we propose a discrete-time dynamic model for studying the time evolution of fish stock in an aquatic environment divided into two adjacent zones characterized by different fishing policies. Indeed, often fishing legislation divides a fishery into regions such that each region imposes a different harvesting policy. However, fish move among regions so that the stock of one region depends not only on harvesting and biological growth of that region, but also on the stock and catch of the neighboring regions. This gives rise to an interdependence that at each time period can increase the population of the zone where the fish stock is scarcer. Such a situation may occur when different countries harvest in neighboring regions, each in its own region according to different fishing legislation (see, e.g. Datta and Mirman, 1999). One can also imagine that a single country makes a subdivision of a common resource pool into regions where different fishing rules are applied, including the case of regions where fishing is forbidden (reserve areas; see, e.g. Dubey et al., 2003).

We analyze two particular, and in some way complementary, harvesting methods: constant fishing efforts and oligopolistic competition. The first involves a limited number of fishing units, each of them employing a controlled technology and a constrained effort, often imposed by a local regulator, whereas the second harvesting method is based on the free "rational" choice of a given number of profit maximizing agents engaging a Cournot-like competition. In the latter case we assume that the population of agents is split in two fractions, one with individual profit maximizers and the other fraction with group profit maximizers (i.e. agents that cooperate in maximizing the overall profit of a cooperative joint venture). Following Bischi et al. (2004) we compute the Cournot–Nash equilibrium harvesting strategy under the assumption that cooperators and competitors (also denoted as defectors) coexist, the population being subdivided into a fraction of agents that form a cooperative venture and maximize the total profit of the set of cooperators, and the complementary fraction that engage a Cournot competition by maximizing their individual profits. This generalizes the harvesting functions given by Bischi et al. (2005) because the two extreme cases of 100% cooperators or 100% defectors are obtained as limiting cases.

Our model moves from Okuguchi (1998) and its extensions given in Szidarovszki and Okuguchi (1998) where a noncooperative game in continuous time is studied where n players (e.g. countries) harvest from a common pool and each of them sell harvested fish in n markets (home and foreign markets). Szidarovszky and Okuguchi (2000) investigate the case where the players form a coalition and maximize the total profit so that they behave like a sole owner. These two extreme cases (individual and cooperative profit maximization) are compared in a discrete time setting by Bischi et al. (2005), where adaptive expectations are introduced to estimate fish stock.

Our model also generalizes Dubey et al., as they consider the case of two different fishing areas related by fish diffusion and only describe the case of one region with constant efforts and the other one with no fishing activity. The general case of constant effort in one region and oligopolistic competition in the other one will be analyzed in a different paper.

Even if the model proposed can be used to analyze the case of differentiated harvesting policies in the different regions, in this paper we only consider the particular case where one of the two zones is a protected area, that is, where no fishing activity occurs. Indeed, this case may be particularly interesting for real world applications.

The aim of this paper is to study the effects induced by the presence of a marine reserve near a fishery and compare the cases obtained when different harvesting policies are adopted in the fishery. In particular we consider two different, and in some sense complementary, policies: a regulated fishery with imposed constant effort and an oligopoly where profit maximizing agents compete. The main result of the paper, obtained through the study of the different kinds of asymptotic behaviors, reveals that the presence of a protected area favors the sustainability of harvesting, but it alone does not rule out the danger of severe depletion of the resource. With constant effort harvesting we show a sort of "continuity" in the biomass level as effort is varied, whereas with oligopolistic competition it is possible to observe hysteresis effects and irreversibility of the human action. Moreover, the results obtained in each particular model with a reserve area will be compared with the dynamic evolutions obtained for the corresponding one-region model (i.e. without any reserve, represented by a one-dimensional dynamical system). Such elementary one-dimensional models constitute useful benchmarks for understanding, by comparison, the effects of interdependencies between the two regions due to fish diffusion or to market externalities.

The plan of the paper is as follows. In Section 2, the general model with two regions and diffusion is outlined, and the growth function is described. In Section 3, the two different harvesting functions that are considered in this paper are derived: one describing an imposed harvesting policy that forces constant fishing efforts and one characterized by profit maximizing agents that engage a Cournot oligopoly game. In this section the properties of the one-dimensional models that describe the time evolution of fish stocks in the case of a single fishing region are briefly described in order to appreciate better, by comparison, the effects of interdependencies between the two regions due to fish diffusion or to market externalities. In Section 4, two particular models are studied, where one region is a reserve area (i.e. no fishing is allowed in it) and the other one is characterized by one of the two harvesting policies proposed in this paper. This section studies the existence of equilibria and gives some insight about their properties of stability and bifurcations. A summary of the main results as well as a description of possible extensions are given in concluding Section 5.

2. The dynamic model of two-zones fishery

Let us consider two neighboring aquatic environments, labelled as region 1 and region 2, and let $X_i(t)$, $i = 1, 2$, denote, respectively, the biomass density of the (same) fish population at time

period t in the two regions. We denote by G_i and H_i, $i = 1, 2$, the growth functions and the harvesting functions in the two regions, and we assume that the fish population can diffuse across the boundary between the two zones. Under these assumptions the time evolution of fish stock in the two zones can be described by the following two-dimensional map

$$\begin{cases} X_1(t+1) = X_1(t) + G_1(X_1(t)) - H_1 + \sigma[X_2(t) - X_1(t)] \\ X_2(t+1) = X_2(t) + G_2(X_2(t)) - H_2 + \sigma[X_1(t) - X_2(t)] \end{cases} \tag{1}$$

defined in the non-negative orthant $\{X_1, X_2 | X_1 \geqslant 0, X_2 \geqslant 0\}$. In general $G_1 = G_2$ because we are considering the same species of fish in the two regions; however cases with $G_1 \neq G_2$ may be considered to model situations where the two environments have different carrying capacity, for example as a consequence of different reproduction rates (e.g. when the conformation of a zone makes it more suitable for reproduction) or mortality rates (e.g. when a zone undergoes some form of pollution). However, in this paper we focus mainly on the heterogeneity between regions related to different fishing policies (i.e. different harvesting functions H_i, $i = 1, 2$). In particular, we get the cases of a protected area, when $H_i = 0$ in one region (see Dubey et al.). The parameter $\sigma \geqslant 0$ depends on the kind of fish considered and is proportional to the size of the border between the different areas.

We are interested in the study of the time evolution of fish stocks in each region, particularly how these are influenced by different harvesting policies, diffusion and market externalities.

To simplify the analysis, in the following we assume that the unharvested resource follows a logistic growth in each zone, with $G_i(X_i) = X_i(\alpha_i - \beta_i X_i)$, $i = 1, 2$, where α_i and β_i, $i = 1, 2$, are *biological parameters* that characterize the fish population we are considering and the environment where it lives: α_i is the *intrinsic growth rate* and $K = \alpha_i/\beta_i$ the *carrying capacity*. However, the results obtained also hold for more general growth functions that satisfy the standard assumptions (i) $G_i(0) = 0$, (ii) $G_i'(0) > 0$, (iii) $G_i''(X) \leqslant 0$ for $0 < X < K$ and (iv) $G_i(K) = 0$.

For some fish, population growth with depensation is observed, characterized by unimodal growth functions that are convex for low values of X and concave for higher values (i.e. assumption (iii) is replaced by $G_i''(0) > 0$ and $G_i''(K) < 0$). Such kinds of growth functions are typical of fish species that form schools. We shall not consider depensation in this paper.

3. Two harvesting functions

In this section we describe the two different harvesting functions considered in the following: imposed constant efforts, in region 1, and oligopolistic competition, in region 2. In the first case a limited number of fishing units is present, each using a controlled technology and a constrained effort, whereas in the second case agents choose quantity to harvest as the result of free "rational" choices (i.e. they try to maximize some profit function engaging a Cournot oligopoly game). For each of these harvesting functions we shall briefly recall the main properties of the corresponding one-dimensional dynamic models

$$X_i(t+1) = F(X_i(t)) = X_i(t) + G_i(X_i(t)) - H_i(X_i(t)) \tag{2}$$

that describe the time evolution of fish stock X_i when only region i exists, where the fishery exploitation occurs according to the harvesting function H_i, $i = 1, 2$. The one-dimensional models (2) constitute useful benchmark cases to better appreciate, by comparison, the effects of interdependencies between the two regions due to fish diffusion or to market externalities.

3.1. Constant effort harvesting

The literature on mathematical bioeconomic modelling of fisheries with constant efforts is really huge, starting with Gordon (1954) and Shaefer (1954, 1957); see Clark and references therein.

Here we assume that in region 1 a central authority imposes a constant total effort in fishing activity. If we denote by E is the individual effort, n_1 the number of agents (fleets of vessels) and q is the catchability rate (linked to the adopted technology), the harvesting policy that imposes a *constant total effort En_1q* is represented by the linear harvesting function

$$H_1(X_1) = En_1qX_1. \tag{3}$$

This is a very common form of controlled management of harvesting activity and very well known in the mathematical bioeconomics literature. We just recall here some properties of the one-dimensional discrete-time dynamic model (2) with logistic growth and constant effort:

$$X(t + 1) = X(t)(1 + \alpha - En_1q - \beta X(t)). \tag{4}$$

The map (4) is conjugate to the standard logistic map $z' = f(z) = \mu z(1 - z)$ with parameter $\mu = (1 + \alpha - En_1q)$ through the linear transformation $X = (1 + \alpha - En_1q)z/\beta$ (see May, 1987; Devaney, 1987; Conrad and Clark, 1987; Clark, 1990). For any $\alpha > En_1q$, the equilibria are

$$X^0 = 0 \quad \text{and} \quad X^* = \frac{\alpha - En_1q}{\beta}$$

that represent, respectively, the extinction of the species and the carrying capacity with harvesting. The extinction equilibrium $X^0 = 0$ is unstable for each $\alpha > En_1q$. However, the effect of increasing the parameters q, n_1 or E (i.e. increasing the level of adopted technology, the number of fishermen and/or the individual effort) is to decrease the value of X^*, and at the value $\alpha = En_1q$ a transcritical bifurcation occurs, after which the origin becomes a stable equilibrium. For $E > \alpha/(n_1q)$ the positive equilibrium no longer exists, and the only long run evolution is towards extinctions of the fish stock.

For $En_1q < \alpha < 2 + En_1q$, the equilibrium X^* is stable and becomes unstable for $2 + En_1q < \alpha < 3 + En_1q$, where a bounded positive attractor around it exists, with periodic or chaotic dynamic. For values of the parameters such that $En_1q < \alpha < 3 + En_1q$, the positive attractor (the fixed point or a periodic or chaotic attractor) has a basin of attraction

$$\mathcal{B} = \left(0, \frac{1 + \alpha - En_1q}{\beta}\right).$$

Every trajectory starting out of $\mathcal{B}$ leads to negative values of X, the extinction of the fish population in finite time (see Clark).

3.2. Competition and cooperation in a common property Cournot oligopoly

Many models in the literature on fisheries deal with Cournot competition among agents; see for instance Levhari and Mirman (1980), Mesterton-Gibbons (1993), Sethi and Somanathan (1996) and Okuguchi (1998).

We now consider the harvesting function on region 2, where harvesting is not regulated by a central management authority, and n_2 agents, allowed to exploit the fishery, are free to decide their harvesting activity by solving a profit maximization problem (i.e. they behave as Cournot oligopolists). In this case the market generates an interdependence among the agents. In fact, increasing harvesting leads to lower prices through a demand function. Moreover, resource depletion leads to increasing unitary harvesting costs. The profit maximization problem solved by the agents is assumed to be myopic, that is, in choosing their current harvesting, agents do not take into account the impact of the current harvest on the future biomass trajectory. Such myopic behavior often characterizes traditional artesanal fisheries, especially in developing countries. As stressed in Sandal and Steinshamn (2004), several reasons for such myopic behavior can be given: first, as long as more than one agent is present, there is always an element of "the tragedy of the commons", that is, "whatever I do not harvest may be harvested by others and therefore I do not have any incentives to save fish for tomorrow". Second, agents may simply not have, or not believe in, information about population dynamics. As a matter of fact, the biology of many fish stocks around the world is poorly understood and many have not even been investigated yet.

As stressed by many authors, myopic and individual profit maximization often leads to over-exploitation, and consequently individual profits may become less and less in the long run, due to severe resource depletion. Of course, since agents may be aware of this, they could prefer to form cooperative ventures and maximize the overall profit of the coalition instead of their individual profit (see Szidarovszky and Okuguchi, 2000). In the following, starting from Bischi et al. (2004) we assume that the population of fishermen is subdivided into two fractions: one fraction formed by agents forming a cooperative venture and the complementary fraction formed by agents preferring an individual competition. Let us assume that n_2 agents harvest fish from the common property fish stock available in region 2 and sell the harvested fish in a market at a price determined by a linear inverse demand function

$$p = a - bH \tag{5}$$

where a and b are positive parameters and H is the total amount of the harvested resource that is sold in the market. The harvesting cost for player k to catch a quantity x_k, when the fish stock is X, is given by $C_k(x_k, X) = c_k + \gamma_k(x_k^2/X)$, where c_k is a fixed cost and $\gamma_k > 0$ represents a technological parameter. This cost function can be derived from a Cobb–Douglas type "production function" with fishing effort (labor) and fish biomass (capital) as production inputs (see Clark, 1990; Szidarovszki and Okuguchi, 1998) and captures the idea that the larger the fish population is, the easier and cheaper it becomes to catch fish.

Moreover, even if each agent decides the quantity to harvest by solving a profit maximization problem, we assume that a fraction s of players, $0 \leq s \leq 1$, act as "cooperators", forming a cooperative venture and consequently trying to maximize the overall profit of the coalition, whereas agents in the remaining fraction, $(1 - s)$, behave as "selfish" profit maximizers and are referred to as "defectors".

These assumptions generalize those given in Bischi et al. (2005), because cooperators and defectors are assumed to coexist and the two limiting cases of "all cooperators" and "all defectors" considered in that paper are here obtained as limiting cases, given by $s = 1$ and $s = 0$, respectively (see also Sethi and Somanathan).

Let h_i^c and h_k^d represent the quantities harvested and sold on the market, respectively, by cooperator i, $i = 1, \ldots, n_2 s$, and defector k, $k = 1, \ldots, n_2(1 - s)$. Then assuming $H = H_2$ (i.e.

the total supply in region 2 is harvested by the n_2 agents[1]), we have that[2]

$$H_2 = \sum_{i=1}^{n_2 s} h_i^c + \sum_{i=1}^{n_2(1-s)} h_i^d.$$

Therefore, the expected profit of ith cooperator is

$$\pi_i^c = h_i^c[a - bH_2] - \gamma_i \frac{(h_i^c)^2}{X_2} - c_i = \tag{6}$$

$$= h_i^c \left[a - b \left(\sum_{i=1}^{n_2 s} h_i^c + \sum_{i=1}^{n_2(1-s)} h_i^d \right) \right] - \gamma_i \frac{(h_i^c)^2}{X_2} - c_i \tag{7}$$

where X_2 is the estimation on future fish stock in the second region.[3] Similarly, the profit of the ith defector is

$$\pi_i^d = h_i^d(a - bH_2) - \gamma_i \frac{(h_i^d)^2}{X_2} - c_i = \tag{8}$$

$$= h_i^d \left[a - b \left(\sum_{i=1}^{n_2 s} h_i^c + \sum_{i=1}^{n_2(1-s)} h_i^d \right) \right] - \gamma_i \frac{(h_i^d)^2}{X_2} - c_i. \tag{9}$$

The defectors solve the optimization problem $\max_{x_i^d} \pi_d^i$, which leads, assuming interior optimum, to the condition

$$\frac{\partial \pi_i^d}{\partial h_i^d} = a - b \left(\sum_{i=1}^{n_2 s} h_i^c + \sum_{i=1}^{n_2(1-s)} h_i^d \right) - bh_i^d - 2\frac{\gamma_i}{X_2}h_i^d = 0. \tag{10}$$

Instead, each cooperator determines h_c^i by solving the optimization problem $\max_{h_i^c} \pi^V = \max_{h_i^c} \sum_{i=1}^{n_2 s} \pi_i^c$, where π^V, which is a concave function in the variables h_i^c, denotes the total profit of the cooperative venture. Assuming interior optimum also in this optimization problem, the first order conditions are

$$\frac{\partial \pi^V}{\partial h_i^c} = a - 2b \sum_{i=1}^{n_2 s} h_i^c - b \sum_{i=1}^{n_2(1-s)} h_i^d - 2\frac{\gamma_i}{X_2}h_i^c = 0. \tag{11}$$

Moreover, we show below that both profit functions are always positive at the interior optimum. Since each cooperator (defector) solves the same optimization problem, we denote by $h^c = h_i^c$ and $h^d = h_i^d$ the optimal harvesting of a representative cooperator and defector, respectively. Moreover, to stress the heterogeneity between the two groups, we assume that, in general, a representative cooperator and a representative defector adopt different fishing technology, represented,

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[1] We assume this since we are here concerned with the presence of a protected area. The case $H = \delta H_1 + H_2$, with $\delta \in (0, 1]$, will be discussed briefly in Section 5 and analyzed in a different paper.

[2] Since n_2 is finite, admissible values of s should be discrete (i.e. $s = k/n_2$ with $k = 0, 1, \ldots, n_2$). However, as usual in population dynamics, we abstract from this and we allow s to be a real number in the interval [0,1], even if we shall consider $n_2 s$ and $n_2(1 - s)$ integers that sum to n_2, by assuming some approximation of $n_2 s$ to the nearest integer.

[3] In the following we shall assume that agents have perfect foresight on next period amount of fish stock available (for a more realistic assumption, see Bischi and Kopel, 2002).

respectively, by the parameters γ_c and γ_d. This assumption can be motivated as a consequence of different attitudes of the two groups toward the environmental sustainability of harvesting: cooperators for instance could decide to harvest according to a "precautionary approach" (e.g. adopting particular gear restrictions) whereas defectors could be unconcerned with it (e.g. fishing with trawls).

All in all Eqs. (10) and (11) reduce to a system of two linear equations:

$$\begin{cases} bn_2sh^c + h^d\left[b\left(n_2(1-s)+1\right)+\dfrac{2\gamma_d}{X_2}\right] = a \\ bn_2(1-s)h^d + 2h^c\left[bn_2s+\dfrac{\gamma_c}{X_2}\right] = a. \end{cases}$$

We can easily derive the following linear *reaction functions*,

$$h_c = f_c(h_d) = \frac{aX_2}{2(bsn_2X_2+\gamma_c)} - \frac{b(1-s)n_2X_2}{2(bsn_2X_2+\gamma_c)}h_d$$

$$h_d = f_d(h_c) = \frac{aX_2}{b(1+(1-s)n_2)X_2+2\gamma_d} - \frac{bsn_2X_2}{b(1+(1-s)n_2)X_2+2\gamma_d}h_c,$$

and compute the unique positive Nash equilibrium (h_c^*, h_d^*), located at the intersection of the reaction functions, given by

$$h_d^*(X_2) = \frac{X_2(bsn_2X_2+2\gamma_c)a}{D(X_2)} \tag{12}$$

and

$$h_c^*(X_2) = \frac{X_2(bX_2+2\gamma_d)a}{D(X_2)} \tag{13}$$

where

$$D(X) = (bX(bn_2(2+n_2(1-s))sX+2\gamma_c(1+n_2(1-s)))+4\gamma_d(bn_2sX+\gamma_c)). \tag{14}$$

The total harvesting *at the Nash equilibrium* is given by

$$H_2^*(X_2) = n_2[sh_c^*(X_2)+(1-s)h_d^*(X_2)]. \tag{15}$$

The function $H_2^*(X_2)$ in (15) is an increasing and concave function such that $H_2^*(0) = 0$. For the main properties of $h_d^*(X_2^*)$ and $h_c^*(X_2^*)$ we refer to Bischi et al. (2004). By plugging expressions (12) and (13) in (6) and (8), we have that the profit functions of a representative defector and competitors, respectively, are always positive at the equilibrium:

$$\pi^c(h_c^*, h_d^*) = \frac{a^2X_2(bn_2sX_2+\gamma_c)(bX_2+2\gamma_d)^2}{[D(X_2)]^2} > 0$$

and

$$\pi^d(h_c^*, h_d^*) = \frac{a^2X_2(bn_2sX_2+2\gamma_c)^2(bX_2+\gamma_d)}{[D(X_2)]^2} > 0.$$

It is interesting to note that $H_2^*(X_2)$ approaches the horizontal asymptote,

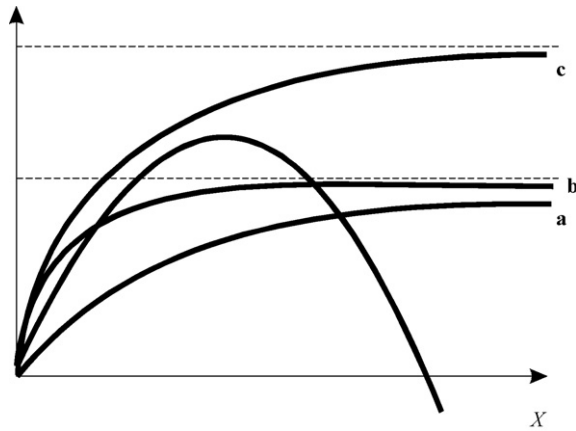$$H_2^*(+\infty) = \frac{a}{b}\left(1-\frac{1}{2+n_2(1-s)}\right). \tag{16}$$

Fig. 1. Equilibria in the one-dimensional model of Cournotian oligopolistic competition with cooperators and noncooperators. (a) A single positive equilibrium exists. (b) Two equilibrium exist. (c) No positive equilibrium exists.

3.2.1. Existence of equilibria

We consider now the main properties of the dynamic Eq. (2) with harvesting function (15). In this way, we can describe the time evolution of the fish stock X in a model with a single region, where harvesting is assumed to occur according to the Nash equilibrium of the Cournot game described above. The steady states are the non-negative solutions of the equation

$$X(\alpha - \beta X) = H_2^*(X) \tag{17}$$

So, for any given value of s, the equilibria are located at the non-negative intersections between a parabola and the concave function (15) with $X_1 = 0$. The equilibrium $X^0 = 0$ (extinction of the resource) always exists. Furthermore, depending on the values of the parameters, one, two, or no positive equilibria may exist (see in Fig. 1 curves a, b and c, respectively). Indeed, we can distinguish three different situations according to the existence of positive steady states.[4]

(a) *A unique positive equilibrium exists, say* X^1 *with* $0 < X^1 < \alpha/\beta$.
(b) *Two positive equilibria exist, say* X^2 *and* X^1, *such that* $0 < X^2 < \alpha/2\beta < X^1 < \alpha/\beta$.
(c) *No positive equilibria exist.*

3.2.2. Stability analysis of equilibria

Concerning the stability of the equilibria, from the first derivative $DF(X)$ of the map in (2) we can deduce that in case (a), if Eq. (29) in Appendix in Supplementary data holds, then $DF(0) > 1$, so the fixed point $X^0 = 0$ is unstable. This means that if Eq. (29) in Appendix in Supplementary data is satisfied (i.e. the number of fishermen is not too high, or prices are not too high or cost parameters are not too small), then even if the resource stock is reduced at an arbitrary small positive value by some exogenous shock, the endogenous dynamics of the system is such that it spontaneously evolves to viable levels of the resource stock, close to X^1. The unique positive equilibrium X^1 may be stable (i.e. $|DF(X^1)| < 1$) or unstable (with $DF(X^1) < -1$). In the latter

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[4] See the Appendix in Supplementary data.

case, which occurs with high values of the parameter α, a more complex bounded attractor, which may be periodic or chaotic, may exist around X^1. The bounded attracting set is confined inside the trapping set $I = [c_1, c]$ where c is the maximum value of the function F and $c_1 = F(c)$. In any case, the basin of the bounded attractor is bounded by the unstable fixed point $X^0 = 0$ and its rank one preimage X^0_{-1}, that is, $\mathcal{B} = (0, X^0_{-1})$, where X^0_{-1} is the positive solution of the equation

$$1 + \alpha - \beta X = \frac{H_2^*(X)}{X}. \tag{18}$$

It is straightforward to see that under assumption (29) in Appendix in Supplementary data, Eq. (18) has a unique positive solution X^0_{-1} such that $X^1 < X^0_{-1} < (1 + \alpha)/\beta$. An initial condition with $X(0) > X^0_{-1}$ is mapped by the iterated function F to a negative value, so we consider as *unfeasible* such a trajectory.[5]

It is worth noticing that the equilibrium value X^1 is influenced by the value of s. In fact, if s is increased, then the asymptote of H_2^* moves downwards, and this implies that X^1 increases with s (i.e. $X^1(s)$ is an increasing function). The intuition behind this is clear: more cooperators imply an higher resource stock at the long run equilibrium, due to a more conservative (or sustainable) resource exploitation.

If the aggregate parameter at the left hand side of Eq. (29) in Appendix in Supplementary data is increased (i.e. the number of fishermen and/or prices become higher compared with intrinsic growth of the fish species in the environment considered, and/or the fishing costs are lowered by using more sophisticated technologies), for

$$n_2 a \left(\frac{s}{\gamma_c} + \frac{1-s}{\gamma_d} \right) = 2\alpha \tag{19}$$

we have $DF(0) = 1$, and if $n_2 a(s/\gamma_c + (1-s)/\gamma_d)$ is further increased (or α decreased), then a transcritical (or stability exchange) bifurcation occurs after which the equilibrium $X^0 = 0$ becomes stable (i.e. $-1 < DF(0) < 1$), and a second fixed point X^2 enters the positive orthant, thus giving the situation (b). The new positive equilibrium X^2 is unstable, being $DF(X^2) > 1$, and it belongs to the boundary that separates the basin of the stable equilibrium $X^0 = 0$ from the basin of the positive attractor. In this situation X^2 constitutes a threshold population level such that if the current population $X(t)$ falls below X^2 then the resource stock will spontaneously evolve towards extinction. It is worth noticing that if s is increased, so that the asymptote of H_2^* moves downwards, then threshold value X^2 moves to the left (i.e. it is a decreasing function of s). This means that more cooperators imply not only a higher resource stock at the long run equilibrium X^1, but also an enlargement of its basin of attraction.

As usual with noninvertible maps, all the rank one preimages of X^2 belong to basin boundaries, so the basin of the positive attractor is now given by

$$\mathcal{B} = (X^2, X^2_{-1}), \tag{20}$$

X^2_{-1} being the rightmost solution of the equation $F(X) = X^2$.

When two positive equilibria exist, let us consider given values of the biological parameters α and β, so that the parabola in Fig. 1 is fixed. If the other parameters are varied with the consequence that the asymptote of H_2^* moves upwards, the two positive equilibria become closer and closer,

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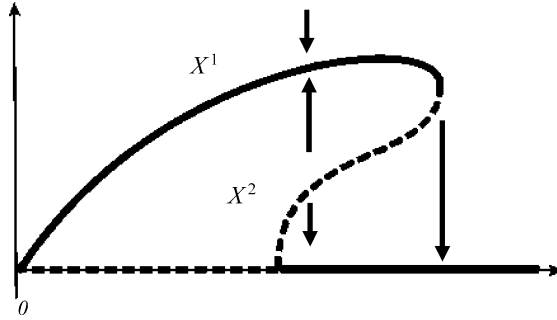[5] This may be interpreted as a resource extinction due to overcrowding effects.

Fig. 2. Cournotian oligopolistic competition with cooperators and noncooperators. Bifurcation diagram as a/b is increased and/or s decreased. The thick lines represent stable equilibria, the dashed ones unstable equilibria.

so that the basin of X^0 enlarges and, therefore, the basin of the viable equilibrium X^1 shrinks. This can be obtained, for example, by increasing prices (i.e. increasing a/b) or decreasing values of s.

Finally, the situation (c), where the extinction equilibrium $X^0 = 0$ is the unique steady state, may be obtained as the final effect of increasing $H_2^*(+\infty)$. The transition from a dynamic scenario characterized by two positive steady states to one with no positive steady states occurs via a fold (or tangent) bifurcation, due to a progressive decrease of X^1 and increase of X^2 (so that the basin $\mathcal{B}$ becomes smaller and smaller) until they merge with $DF(X_1) = DF(X_2) = 1$, and then they disappear.

These different dynamic scenarios, obtained by a gradual shift of the horizontal asymptote $H_2^*(+\infty)$ (see (16) with $X_1 = 0$) are summarized in the bifurcation diagram in Fig. 2, where in the horizontal axis a parameter that moves upwards the horizontal asymptote (16) is reported (such as increasing values of a/b or decreasing values of s). The thick lines represent stable equilibria, the dashed ones unstable equilibria. If a is increased, or s decreased, so that the two positive equilibria merge and disappear, the system will evolve toward extinction, and a successive opposite variation (i.e. increasing a or increasing s) will not be sufficient to move back to the previous positive equilibrium. So, this bifurcation diagram shows the occurrence of a certain hysteresis effect, similar to the sudden and irreversible jumps familiar in catastrophe theory (see, e.g. Rosser, 2000).

4. The coexistence of protected and unprotected areas

The dynamic model with two regions (1) characterized by the two different harvesting functions described in the previous section becomes

$$\begin{cases} X_1(t+1) = X_1(t) + G_1(X_1(t)) - En_1qX_1(t) + \sigma[X_2(t) - X_1(t)] \\ X_2(t+1) = X_2(t) + G_2(X_2(t)) - H_2^*(X_2(t)) + \sigma[X_1(t) - X_2(t)]. \end{cases} \tag{21}$$

However, in this paper we only focus on two particular cases (and meaningful per se) in which one region is a reserve area (i.e. no fishing is allowed in it).

For sake of simplicity we shall consider the case $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$, that is when the same growth functions in each region is given by $G(X) = X(\alpha - \beta X)$, so that the only difference between the neighboring zones is due to the presence of harvesting.

4.1. Constant effort and a protected area

This model can be obtained from (21) by assuming $n_2 = 0$ (hence $H_2^* = 0$). In this case the map (21) assumes the form of a linearly coupled quadratic map, given by

$$\begin{cases} X_1(t+1) = X_1(t)(1 + \alpha - \sigma - En_1q - \beta X_1(t)) + \sigma X_2(t) \\ X_2(t+1) = X_2(t)(1 + \alpha - \sigma - \beta X_2(t)) + \sigma X_1(t), \end{cases} \tag{22}$$

the linear coupling being caused by the diffusion terms. A rich literature exists about this kind of discrete dynamical systems (see, e.g. Schult et al., 1987; Gardini et al., 1994; Inque and Nishi, 1996; Maistrenko et al., 1998; Dobrynskiy, 1999; Bischi and Kopel, 2001, just to cite a few).

4.1.1. Existence of equilibria

The equilibria in the bidimensional model are the non-negative solutions of the fourth degree algebraic system

$$\begin{cases} X_1(\alpha - \sigma - En_1q - \beta X_1) + \sigma X_2 = 0 \\ X_2(\alpha - \sigma - \beta X_2) + \sigma X_1 = 0 \end{cases} \tag{23}$$

located at the intersections of the two convex parabolas

$$X_1 = f_1(X_2) = \frac{1}{\sigma} X_2(\beta X_2 + \sigma - \alpha) \quad \text{and}$$

$$X_2 = f_2(X_1) = \frac{1}{\sigma} X_1(\beta X_1 + En_1q + \sigma - \alpha). \tag{24}$$

The origin $X^0 = (0, 0)$ is always a steady state of the model (22) and the following result holds, as proved in Appendix in Supplementary data (see also Fig. 3):

Proposition 1. *The model* (22), *with constant effort* $En_1q \in [0, +\infty)$ *and* $\sigma < \alpha$, *has a unique positive equilibrium* (X_1^*, X_2^*) *with* $X_2^* \in (\frac{\alpha - \sigma}{\beta}, \frac{\alpha}{\beta}]$ *and* $X_1^* = \max(0, f_1(X_2)) \in (\max(0, \frac{\alpha - \sigma - En_1q}{\beta}), \frac{\alpha}{\beta})$.

It is easy to compute the equilibrium in the two limiting cases of no fishing effort and infinite fishing effort. When $En_1q = 0$, $X_1^* = X_2^* = \alpha/\beta$ (the two carrying capacities) whereas $\lim_{n_1qE \to +\infty} X_2^* = (\alpha - \sigma)/\beta$ and $\lim_{n_1qE \to +\infty} X_1^* = 0^+$.

We remark that the result given in Proposition 1 contrasts with the classical results obtained for the model of harvesting with constant effort without a neighboring reserve area, recalled in Section 3.1, where for $En_1q > \alpha$ no positive equilibrium exists. Instead, thanks to the presence of the neighboring reserve area a positive equilibrium always exists for the model (22).

4.1.2. Stability analysis of equilibria

Since Proposition 1 only concerns the existence of a unique positive equilibrium of the model (22), it is not sufficient to rule out the extinction of the fish stock. In fact, in order to make a meaningful comparison with the corresponding one-region model with constant effort harvesting, it is necessary to investigate the stability of the two equilibria.

The Jacobian matrix of (22) is symmetric

$$DT(X_1, X_2) = \begin{pmatrix} 1 + \alpha - 2\beta X_1 - \sigma - En_1q & \sigma \\ \sigma & 1 + \alpha - 2\beta X_2 - \sigma \end{pmatrix}; \tag{25}$$
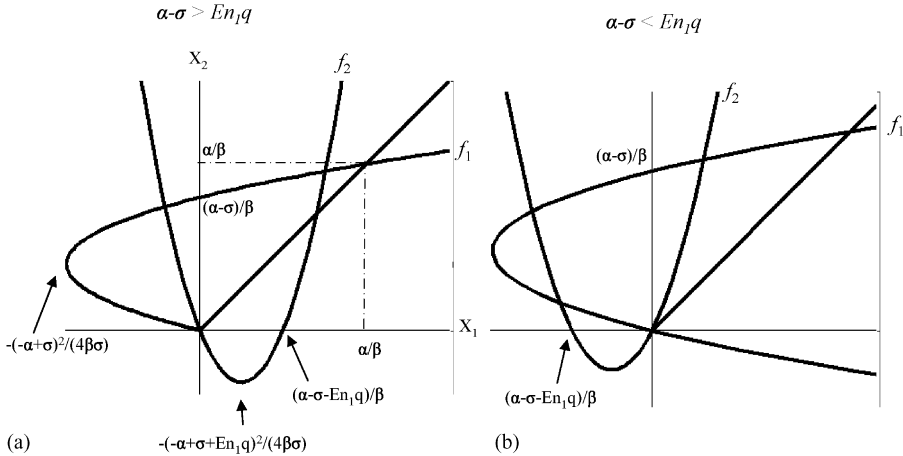
Fig. 3. Existence and uniqueness of the positive equilibrium with constant effort and a protected area. (a) $(\alpha - \sigma - En_1q)/\beta > 0$ and (b) $(\alpha - \sigma - En_1q)/\beta < 0$.

hence its eigenvalues λ_1, λ_2 (given in Appendix in Supplementary data) are always real. When no harvesting takes place (i.e. $En_1q = 0$), it is easy to show that the equilibrium with no fish stock X^0 is unstable for each value of α and σ (in fact $\lambda_2 = 1 + \alpha > 1$). Moreover, if $0 < \sigma < \alpha$ the equilibrium with no fish stock is always unstable, in contrast with the case of one region where the equilibrium with no fish stock becomes stable for $En_1q \leqslant \alpha$ (details are provided in Appendix in Supplementary data). However, this result does not rule out extinction in finite time (i.e. trajectories that involve negative values of X_1), as we shall show by numerical simulations.

Now we turn to the positive equilibrium.

Fold and pitchfork bifurcations, characterized by one eigenvalue $\lambda_i = 1$, cannot be present in this case, because of the uniqueness of the positive equilibrium. Moreover, Neimark–Hopf bifurcations cannot occur because the eigenvalues are real, being Jacobian matrix symmetric. So, the positive equilibrium may lose stability only through a flip (or period doubling) bifurcation. For instance, if condition $2 < \alpha < 2 + \sigma$ is met, starting from a situation where the equilibrium is an unstable node (i.e. $\lambda_1, \lambda_2 < -1$), there exists a level of En_1q above which $\lambda_2 > -1$ so that the equilibrium becomes a saddle point or a stable node, depending on λ_1. Analytical details on the stability of the positive equilibrium are again given in Appendix in Supplementary data.

Let us first consider the case of no fishing (i.e. $En_1q = 0$). In this case the only positive equilibrium $(X_1^*, X_2^*) = (\alpha/\beta, \alpha/\beta)$ is stable if and only if condition $\alpha + 2\sigma < 2$ is satisfied (i.e. overcrowding effects, due to natural growth or migration, are sufficiently low). At $\alpha + 2\sigma = 2$ the point $(\frac{\alpha}{\beta}, \frac{\alpha}{\beta})$ looses stability through a flip bifurcation. If $(2 - 2\sigma) < \alpha < 2$ then (X_1^*, X_2^*) is a saddle point, being $\lambda_1 < -1$ and $-1 < \lambda_2 < 1$, whereas for $\alpha > 2$ it is an unstable node, being $\lambda_2 < -1$ as well.

Moreover, from the analysis of the eigenvalue λ_1 and being $\lambda_1 < \lambda_2 < 1$, we can deduce that when $\alpha + 2\sigma < 2$ there exists always a bifurcation value $b^* \in (0, 4)$ such that the positive equilibrium (X_1^*, X_2^*) looses stability through a flip bifurcation at $En_1q = b^*$. Through the same reasoning it is possible to prove that in the case $\alpha + 2\sigma > 2$ so that the equilibrium is unstable at $En_1q = 0$, a backward flip bifurcation is associated with any flip bifurcation; that is, increasing
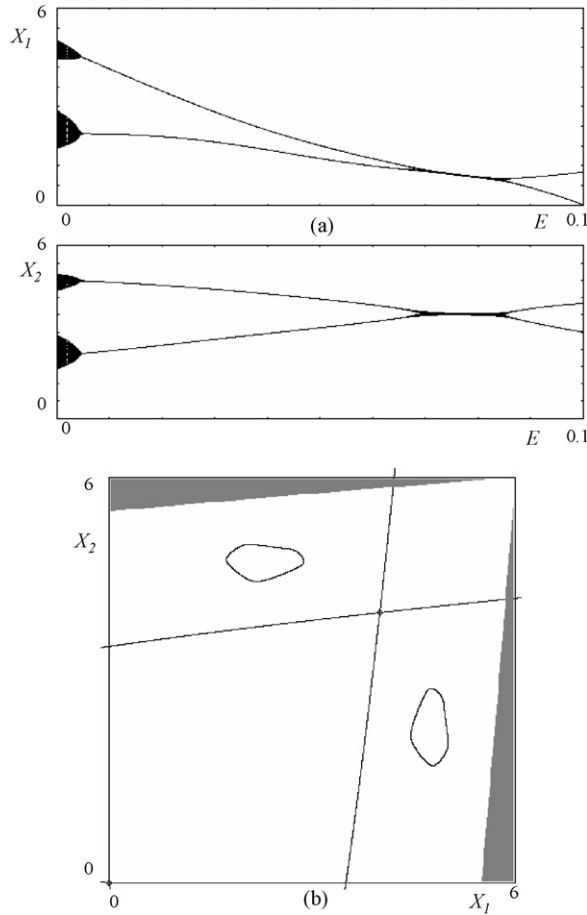
Fig. 4. Constant effort and a protected area with parameters $n_1 = 30$, $q = 1$, $\alpha = 2$, $\beta = 0.5$, $\sigma = 0.25$. Being $\alpha + 2\sigma > 2$, the equilibrium is unstable for $E = 0$. (a) Bifurcation diagram with E ranging in the interval [0,0.1]. An interval of E where stabilization of the equilibrium is present is clearly visible. (b) $E = 0$. The asymptotic dynamic is a two-cyclic quasi-periodic attractor.

values of $E n_1 q$ can lead to stability through a backward flip (i.e. period halving) bifurcation and again to instability through a regular flip bifurcation.[6]

This results are relevant not only from a mathematical point of view, but also from a regulatory point of view since we could find an interval for the parameters $E n_1 q$ inside which both eigenvalues have modulus less than one so that stability is achieved, and consequently preservation of the fish stock in both areas is favored.

Both these bifurcations, occurring as the fishing effort E increases, can be seen in the bifurcation diagram shown in Fig. 4a, obtained with $n_1 = 30$, $q = 1$, $\alpha = 2$, $\beta = 0.5$, $\sigma = 0.25$ and E ranging in the interval [0,0.1]. In this case $\alpha + 2\sigma > 2$, so the equilibrium is unstable for $E = 0$ and the

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[6] In this case whenever a backward flip bifurcation takes place a flip bifurcation occurs, but it is possible to show by counterexamples that for given parameters' values, flip never occurs.
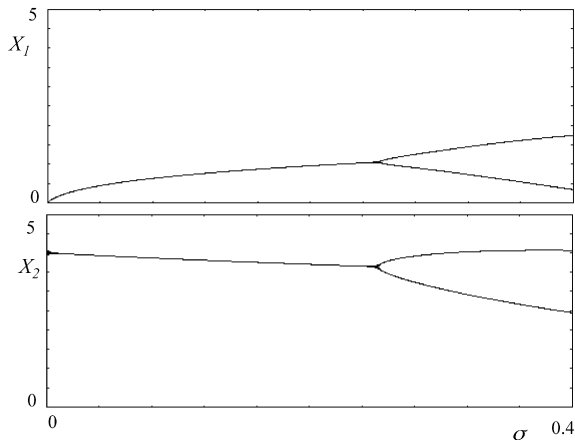
Fig. 5. Constant effort and a protected area with parameters $n_1 = 30$, $q = 1$, $\alpha = 2$, $\beta = 0.5$, $E = 0.07$ and σ ranging in the interval [0,0.4]. The positive equilibrium flip bifurcates for increasing values of σ.

asymptotic dynamics are characterized by a two-cyclic quasi-periodic attractor (see Fig. 4b). As E is increased a cycle of period two is obtained and then the positive equilibrium become stable via a backward flip (period halving) bifurcation. A further increase of E causes a loss of stability of the positive equilibrium that flip bifurcates giving rise to a stable two-cycle again. Of course, the fish stock of region 1 decreases as E is increased, and we have extinction in finite time for $E = 0.1$, as the oscillations involve negative values of X_1.

Instead, for a fixed value of E an increment of the diffusion coefficient σ increases the fish stock in region 1. This can be seen in the bifurcation diagram in Fig. 5, obtained with parameters $n_1 = 30$, $q = 1$, $\alpha = 2$, $\beta = 0.5$, $E = 0.07$ and σ ranging in the interval [0,0.4]. In this case, the positive equilibrium flip bifurcates for increasing values of σ.

More complex attractors can be obtained by considering higher values of the growth rate α, as shown in Fig. 6. The structure of these chaotic attractors that assume the shape of folded veils, suggests us that the map (22) is a noninvertible map. Indeed, as typically occurs for coupled quadratic maps, the locus of vanishing Jacobian that coincides with the set of merging preimages LC_{-1} (see Mira et al., 1996) is given by an equilateral hyperbola, and the map is a noninvertible map of type $Z_4 - Z_2 - Z_0$. The Riemann foliation is very similar to the one described in Bischi and Kopel (2001) or Bischi and Naimzada (2000).

As is now well known, the property of noninvertibility of an iterated map may give rise to basins of attraction with a complex topological structure. This also happens in this case. In fact, as usual with coupled logistic maps, bounded and positive trajectories cannot be obtained if the initial conditions are taken far from the origin. So, a region of negative and unbounded trajectories exists, sometimes called basin of unfeasible trajectories (see, for example, the grey region in Fig. 6b). If a contact between the boundary of this region and a critical curve $LC = T(LC_{-1})$ occurs such that a portion of the basin of unfeasible trajectories enters a zone Z_k characterized by an higher number of preimages, then non-connected portions of such basin may appear, nested inside the basin of bounded attractors As an example see Fig. 7, obtained with $n_1 = 30$, $q = 1$, $\alpha = 2.4$, $\beta = 0.7$, $E = 0.001$ and $\sigma = 0.25$. In this case, the positive attractor is a cycle of period 2, but its basin (white color) is quite intermingled with the one of unfeasible trajectories (grey region). In this case unfeasible trajectories lead to extinction in finite time of the fish stock in both regions.
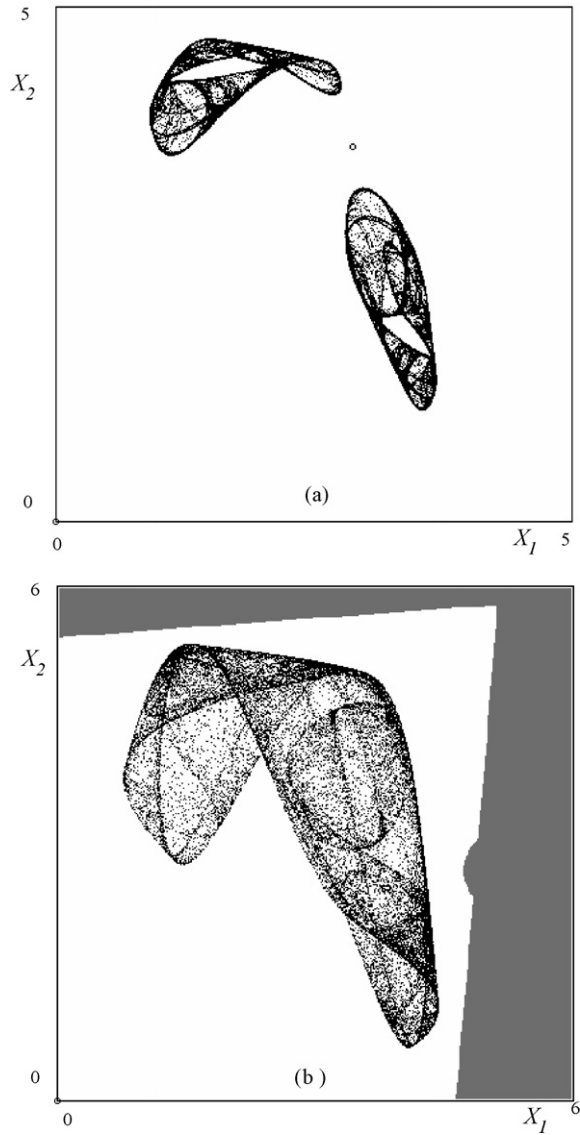
Fig. 6. Constant effort and a protected area. (a) A two pieces chaotic attractor is obtained with parameters $n_1 = 30$, $q = 1$, $\alpha = 2.26$, $\beta = 0.6$, $E = 0.02$ and $\sigma = 0.3$. (b) A chaotic attractor with its basin of attraction (white region) is obtained with parameters $n_1 = 30$, $q = 1$, $\alpha = 2.4$, $\beta = 0.7$, $E = 0.015$ and $\sigma = 0.25$. The grey region represents the basin of attraction of trajectories diverging to minus infinity, leading to extinction in finite time.

4.2. A common property region with oligopolistic competition and a protected area

This model can be derived from (21) by assuming $n_1 = 0$ or $E = 0$. The harvesting function of region 2 is given in (15).[7]

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[7] In this section we denote the harvesting function $H_2^*(\cdot) = H^*(\cdot)$ to simplify the notation.
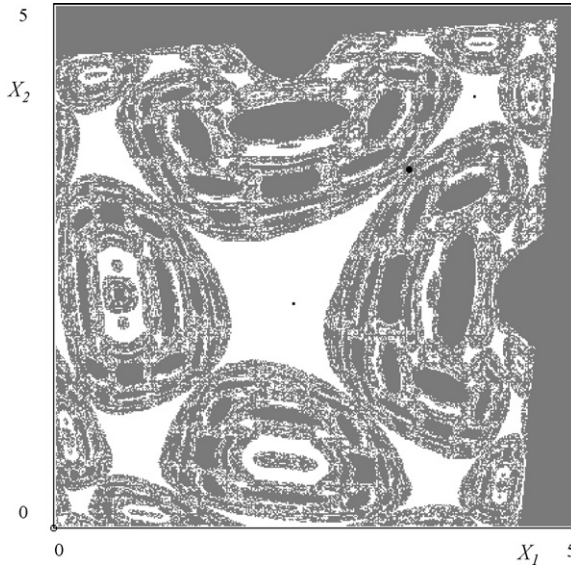
Fig. 7. Constant effort and a protected area. Two-cycle attractor with basin of attraction (white region) intermingled with the basin of unfeasible trajectories (grey region), obtained with parameters $n_1 = 30$, $q = 1$, $\alpha = 2.4$, $\beta = 0.7$, $E = 0.001$ and $\sigma = 0.25$.

As in the previous case, the only coupling between the two regions is given by the diffusion term, being

$$\begin{cases} X_1(t+1) = X_1(t)(1 + \alpha - \sigma - \beta X_1(t)) + \sigma X_2(t) \\ X_2(t+1) = X_2(t)(1 + \alpha - \sigma - \beta X_2(t) - H^*(X_2(t))) + \sigma X_1(t) \end{cases} \tag{26}$$

where H^* is a rational function given by the ratio between two quadratic polynomials, according to (12) and (13).

4.2.1. Existence and stability of equilibria

The equilibria, defined by the two steady state conditions $X_1(t+1) = X_1(t)$, $X_2(t+1) = X_2(t)$, are the non-negative solutions of the system

$$\begin{cases} X_1(t)(\alpha - \sigma - \beta X_1(t)) + \sigma X_2(t) = 0 \\ X_2(t)(\alpha - \sigma - \beta X_2(t) - H^*(X_2(t)) + \sigma X_1(t) = 0. \end{cases}$$

and for $\sigma \neq 0$ they are located at the intersections, in the non-negative orthant, between the two curves defined by the functions

$$\begin{cases} X_2 = f_2(X_1) = \dfrac{1}{\sigma} X_1(t)(\sigma - \alpha + \beta X_1(t)) \\ X_1 = f_1(X_2) = \dfrac{1}{\sigma}[X_2(t)(\sigma - \alpha + \beta X_2(t)) + H^*(X_2)] \end{cases} \tag{27}$$

where $X_i = f_i(X_j)$ represents the locus of points such that X_j does not change under the application of the map. The curves $X_1 = f_1(X_2)$ and $X_2 = f_2(X_1)$ are a sigmoid curve and a convex parabola, respectively (some situations are shown in Fig. 8). The two curves cross at the origin of
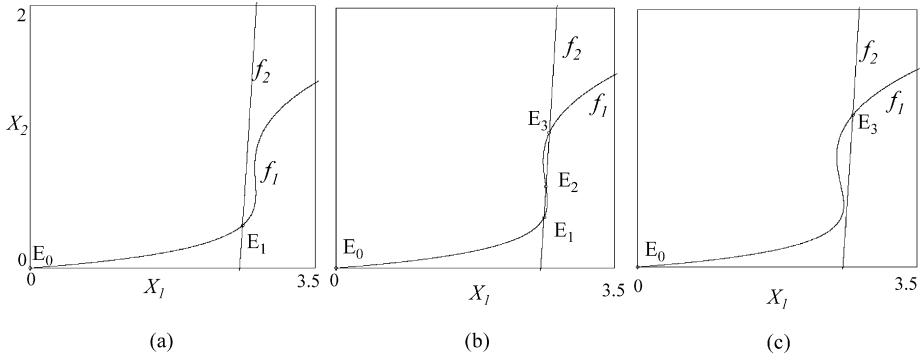
Fig. 8. Oligopolistic competition and a protected area. (a) A unique (attracting) equilibrium point E_1 is obtained as intersection of two curves, with parameters $\alpha = 2$, $\beta = 0.7$, $n_2 = 30$, $\gamma_c = \gamma_d = 2.5$, $a = 1$, $b = 0.3$, $\sigma = 0.2$ and $s = 0.7$. (b) Two positive equilibria, E_2 and E_3, are created by a saddle node bifurcation increasing the parameter s. The parameters are as in (a) but with $s = 0.78$. (c) Only the (attracting) equilibrium E_3 remains, after the merging of E_1 and E_2 through a new saddle-node bifurcation ($s = 0.8$).

the coordinate axes; hence the point $(0,0)$ is always an equilibrium, and there may be up to three intersections in the positive orthant.

As before, nontrivial equilibria can be obtained by (27). In fact, $f_2(X_1)$ is the same as $f_1(X_2)$ in the previous section. Instead, the function $f_1(X_2)$ is a sigmoid, proportional to the sum between a convex parabola and the concave function $H^*(X_2)$ given in (15). Some useful properties of $f_1(\cdot)$ can be deducted by the previous analysis. In fact we have that $f_1(\cdot)$ is differentiable, with $f_1(0) = 0$, definitely strictly increasing (at least for $X_2 > (\alpha - \sigma)/2\beta$) and with $\lim_{X_2 \to +\infty} f_1(X_2) = +\infty$. A first important consequence is

Proposition 2. *At least one positive equilibrium exists for all parameters values, with* $(X_1, X_2) \in (\frac{\alpha-\sigma}{\beta}, \frac{\alpha}{\beta}] \times (0, \frac{\alpha}{\beta}]$.

Moreover, f_1 is strictly increasing at the origin if $\alpha - \sigma < (na/2)((s/\gamma_c) + ((1-s)/\gamma_d))$ holds.[8] When condition $H^{*\prime\prime}(0) < -2\beta$ holds, then $f_1(X_2)$ is concave in a right neighborhood of the origin and convex for sufficiently high values of X_2, (recall that $\lim_{X_2 \to +\infty} f_1^{\prime\prime}(X_2) = 2\beta/\sigma > 0$) and at least one inflection point exists. In this case the uniqueness of the positive equilibrium is not guaranteed. In fact the positive equilibrium can bifurcate via saddle-node, with the consequent creation of two more positive equilibria. A necessary condition for the existence of three equilibria in the set $(\frac{\alpha-\sigma}{\beta}, \frac{\alpha}{\beta}] \times (0, \frac{\alpha}{\beta}]$ is that $f_1(X_2)$ is a bimodal map. These equilibria can be numerically computed as the zeroes of a one-dimensional function. In fact, substituting the second equation in the first one, it is easy to show that the equilibria can be obtained as the positive fixed points of the composition between f_1 and f_2: $f_1 \circ f_2(X_1) = X_1$, such that $X_2 = f_2(X_1) \geqslant 0$ (or, equivalently, the positive fixed points of $f_2 \circ f_1(X_2)$ such that $X_1 = f_1(X_2) \geqslant 0$).

The equilibria can also be obtained as the non-negative zeroes of the function

$$Z(X_1) = \frac{1}{\sigma}[G_1(X_1) + G_2(f_2(X_1)) - H^*(f_2(X_1))] = 0 \tag{28}$$

such that $X_2 = f_2(X_1) \geqslant 0$.

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[8] Being $\frac{\partial f_1}{\partial X_2}|_{X_2=0} = \frac{-\alpha+\sigma+(na/2)((s/\gamma_c)+((1-s)/\gamma_d))}{\sigma}$.
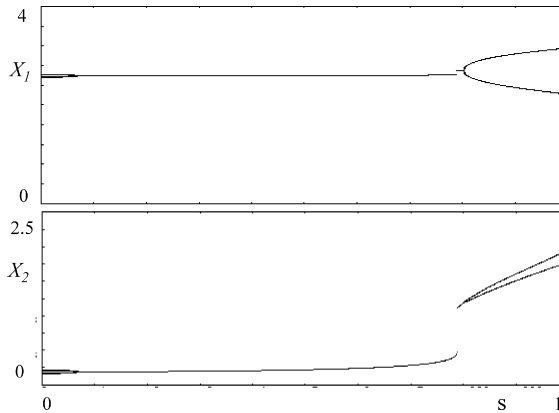
Fig. 9. Oligopolistic competition and a protected area. Bifurcation diagram with $\alpha = 2$, $\beta = 0.7$, $n_2 = 30$, $\gamma_c = \gamma_d = 2.5$, $a = 1$, $b = 0.3$, $\sigma = 0.2$ and s ranging in the interval [0,1]. The discontinuity of the bifurcation diagram depends on the chosen initial condition (i.c.) $(X_1, X_2) = (0.5, 1)$ and on the two successive saddle-node bifurcations through the intermediate bistability.

Interesting situations are obtained if some parameters of the model are varied so that the fold bifurcation that creates two positive equilibria occurs. For example, in Fig. 8a–c three situations are shown with three different values of the parameter s, the fraction of cooperators. With fixed values of the parameters $\alpha = 2$, $\beta = 0.7$, $n_2 = 30$, $\gamma_c = \gamma_d = 2.5$, $a = 1$, $b = 0.3$, $\sigma = 0.2$, the two equilibrium curves are represented in Fig. 8a for $s = 0.7$. In this case a unique positive equilibrium exists, denoted by E_1, which is stable. If the fraction of cooperators is slightly increased to $s = 0.78$, two new positive equilibria can be seen, created by a saddle node bifurcation, denoted by E_2 (a saddle point) and E_3 (a stable node). A situation of bistability is obtained, and the two stable equilibria E_1 and E_3 are characterized by quite different values of equilibrium fish stock in region 2. The basins of attraction of these two stable equilibria are represented in Fig. 10 by the light grey and white regions, respectively. The boundary that separates these two basins is given by the stable set of the saddle E_2 whereas the grey region represents the basin of unfeasible trajectories.

If s is further increased, the two equilibria E_1 and E_2 merge and disappear via another saddle-node bifurcation, and the upper equilibrium E_3 remains the unique attractor to which the generic feasible trajectory converges.

To sum up, an increase in the number of cooperators in region 2 causes a transition from a situation of a low equilibrium fish stock to one in which the sustainable fish stock is much higher. This can be also seen from the bifurcation diagram depicted in Fig. 9, obtained with the same set of parameters as in the other figures of this section and s ranging in the interval [0,1]. The sudden jump to an upper sustainable level of fish biomass is quite evident. Of course, in the range of s such that coexistence of two stable equilibria occurs, the exact value of s at which the jump occurs depends on the initial condition chosen to get the bifurcation diagram. In other words, the two successive saddle-node bifurcations that mark the transition from a lower to an upper equilibrium, through an intermediate situation of bistability, creates a typical hysteresis effect that resembles the fast transitions of the models discussed in the framework of the catastrophe theory (see, e.g. Rosser, 2000; Arnold, 1992).

The bifurcation diagram also shows that if s is further increased, then a flip bifurcation of E_3 occurs, leading to a stable cycle of period 2 as unique attractor of the feasible trajectories.
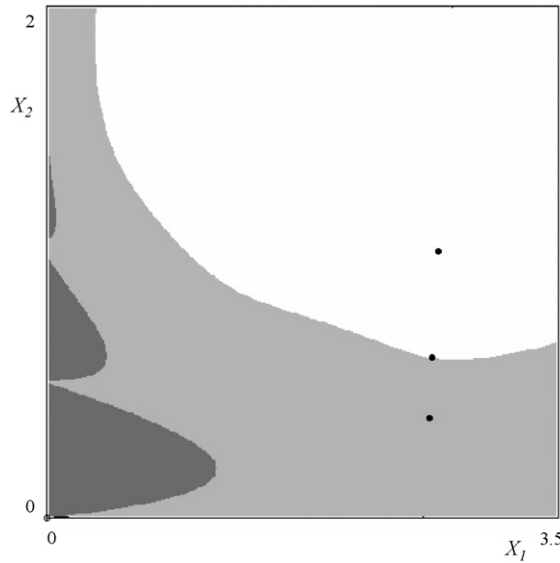
Fig. 10. Oligopolistic competition and a protected area. Basins of attraction of the stable equilibria E_1 and E_3 (light grey and white regions, respectively), in the case of bistability in Fig. 8b. The stable set of the saddle E_2 is the boundary separating the two basins. The grey region is the basin of diverging trajectories.

The stability analysis for the equilibria is analogous to the case with constant effort. The Jacobian matrix is

$$DT(X_1, X_2) = \begin{pmatrix} 1 + \alpha - 2X_1\beta - \sigma & \sigma \\ \sigma & 1 + \alpha - 2X_2\beta - \sigma - H^{*\prime}(X_2) \end{pmatrix}$$

with eigenvalues

$$\lambda_1 = 1 - \frac{H^{*\prime}(X_2)}{2} + \alpha - \sigma - (X_1 + X_2)\beta - \frac{1}{2}\sqrt{[H^{*\prime}(X_2) + 2\beta(X_2 - X_1)]^2 + 4\sigma^2}$$

$$\lambda_2 = 1 - \frac{H^{*\prime}(X_2)}{2} + \alpha - \sigma - (X_1 + X_2)\beta + \frac{1}{2}\sqrt{[H^{*\prime}(X_2) + 2\beta(X_2 - X_1)]^2 + 4\sigma^2}.$$

The condition $\alpha > \beta + \sigma$, under the reasonable assumption $\gamma_c = \gamma_d = \gamma$, in sufficient to ensure that $\lambda_2 > 1$ and hence that the origin is unstable.[9] Using the same argument as before and recalling that $H^{*\prime}(X_2) > 0$, it possible to show that it is always $\lambda_1 < 1$ and that Neimark–Hopf bifurcations are ruled out by the symmetry of the Jacobian. Hence it is the eigenvalue $\lambda_2 = 1$ that causes the saddle-node bifurcations and the consequent creation/destruction of two (positive) equilibria.

We remark that also the map (26) is noninvertible, because the system to extract $X_1(t)$, $X_2(t)$ from (26) in terms of $X_1(t + 1)$, $X_2(t + 1)$ is an algebraic system of degree eight, so up to eight rank one preimages of a point of the plane may be obtained; consequently for this model quite complicated structures of the basins can be observed.

Concluding this section we can state that the presence of an exploitable zone where cooperative and noncooperative behavior takes place introduces in the model a kind of complexity that is not

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[9] It easily follows from $\lambda_2(0,0) = 1 + \alpha - \beta - \frac{an_2}{4\gamma} - \sigma + \frac{1}{4}\sqrt{\frac{a^2 n_2^2}{\gamma^2} + 16\sigma^2}$.

present when a level of constant effort is imposed. Firstly uniqueness of the equilibrium is not guaranteed for all parameters values, but could be replaced by coexistence of multiple equilibria with possible bistability and hence path-dependence (depending on current fish stocks in both regions) of the asymptotic steady state, as shown in Fig. 10. Secondly the creation mechanism of multiple equilibria itself through two successive saddle-node bifurcations introduces typical hysteresis patterns, clearly visible in Fig. 9.

5. Conclusions

In this paper a general framework has been introduced for the study of discrete-time dynamic models that represent a system of two adjacent zones where different harvesting policies are adopted, and we focused on the important case where a given harvesting policy is adopted on one side and no harvesting is allowed on the other side. The main results have been compared with those obtained with more traditional models where no subdivision is described. In the region where harvesting occurs two different kinds of fishing policies have been considered: first the case of regulated fishery according to constant effort regulation, and second the case of an oligopoly where profit maximizing agents compete. In any case the presence of a protected area leads to higher levels of sustainability, since it helps to avoid the danger of biomass decreases. However, this risk persists when intensive and competitive fishing occurs (such as in the oligopoly case with low degrees of cooperation). In the case of constant effort we showed that stability of the fish stock is obtained by imposing intermediate values of effort. In particular we showed that there exists a range of effort inside which stability of the biomass can be achieved. Moreover, the loss of stability in this model takes place through flip bifurcations (i.e. oscillations with higher and lower levels of biomass occur after the loss of stability). This issue is relevant from the point of view of a central planner (i.e. when implementing regulatory policies based on forecasting of fish stock): if the oscillations are not properly understood, it is possible to systematically underestimate or overestimate the fish stock, with severe impact on the sustainability of the exploitation.

In the case of oligopolistic competition, we showed that even though a protected area exists, it is possible to observe hysteresis phenomena, leading to irreversibility of the human activity. In these cases it is even more important to adopt a precautionary principle toward fishery: if stocks are depleted below certain points, it could become practically impossible to restore the previous levels of biomass. In order to avoid these kinds of difficulties we showed that, in addition to the institution of a protected area, other tools favor the sustainability of harvesting. Within our model, we identified as such a tool every action aimed at diminishing the maximum selling price and/or the efficiency of the adopted technology or at increasing the level of agents' cooperation.

As a concluding remark we note that this framework can be extended in many directions. The first obvious extension of the analysis presented in this paper is obtained by considering different harvesting policies in the two regions, for instance constant effort in a region and oligopolistic competition in the other one. In doing so, it is possible to derive the main equations of the model, namely the harvesting function in the region where oligopolistic competition occurs, by making different assumptions on the structure of market. For example one may assume separate markets in the two regions (i.e. each agent sells the harvested fish only in the home market) or that, more realistically, a global market may be considered, so that each agent sells fish both in the home and the foreign market. Of course intermediate cases may be considered, where each agent mainly sells fish in the home market and only partially (i.e. with some restrictions) sells fish in the foreign market.

Other possible extensions can be obtained by weakening the assumption of agents' perfect foresight on the available fish stock, for example by introducing adaptive expectations as in Bischi and Kopel (2002) or Bischi et al. (2005). These extensions will be the considered in a different paper.

Acknowledgments

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2005.06.012.

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