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Strategic interaction and imitation dynamics in patch differentiated exploitation of fisheries

Gian Italo Bischi[a], Fabio Lamantia[b,*], Lucia Sbragia[c]

[a] Department of Economics, University of Urbino, Via A. Saffi 42, Urbino, Italy
[b] Department of Business Sciences, University of Calabria, Via P. Bucci 3C, 87036 Rende (CS), Italy
[c] Department of Economics and Finance, Durham Business School, Durham University, 23/26 Old Elvet, Durham DH1 3HY, UK

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ABSTRACT

In this paper we propose a discrete-time dynamic model for studying the time evolution of fish stocks in an aquatic environment divided into two adjacent patches, one characterized by an imposed constant fishing effort and another one where fishermen are free to decide the quantity to harvest according to oligopolistic competition taking into account economic and biologic externalities. Given the total number of fishermen, we propose an evolutionary switching mechanism, based on imitation dynamics, that endogenously regulates the fraction of fishermen that, at each time period, decide to operate in one of the two regions on the basis of observed past profits. Numerical simulations provide some insights into the role of the main parameters. This model has been motivated by a project for the creation of a marine protected area (MPA) in the Adriatic sea.

1. Introduction

The problem of a *sustainable exploitation* of fisheries involves decisions characterized by several levels of strategic interaction, strongly influenced by biological, social, economic and legislative constraints. "Sustainable" means that fish exploitation should guarantee the preservation of fish stocks for future generations. Any national legislation can only influence fishing activity inside a given distance from national coasts, whereas in the "open sea" only international agreements for species protection can be applied, and open access must be guaranteed to freely competing agents. This often leads to a situation of severe overexploitation and the serious danger of fishery collapse is sometimes faced. Indeed, even in the case of free competition, different economic and biologic externalities exist which may have self-regulatory effects. First, biological externalities must be taken into account, as overexploitation of the resource by one agent may have important consequences on the capacity of regeneration of the resource, thus producing a negative externality for the whole community of exploiters. Second, market externalities may exist, due to price reduction as a consequence of increasing harvesting. Moreover, cost externalities may exist because decreasing stocks of available fish imply increasing unitary harvesting costs. However, in the management of common property renewable resources harvested by competing individuals, societies or countries, the problem known as "The tragedy of the commons" (after Hardin, 1968) must be taken into account (see also Gordon, 1954; Clark, 1976; Mesterton-Gibbons, 1993; Conrad, 1995).

For these reasons, international agreements for the creation of marine protected areas (MPAs) have been recently proposed in the literature, conferences and congresses (see e.g. Bohnsack, 1993; Clark, 1996; Sumaila, 1998; Holland, 2002). This demonstrates increasing interest in such kind of regulation, as results from the proclamation endorsed by 400 prominent marine scientists at the First Symposium on Marine Conservation Biology to "increase the number and effectiveness of marine protected areas so that 20% of Exclusive Economic Zones and High Seas are protected from threats by the year 2020" (Sanchirico and Wilen, 2001), as well as the existence of newsletters like "MPA news, International News and Analysis on Marine Protected Areas" and the special issue of Natural Resource Modeling (2002) devoted to the Proceedings

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* *Corresponding author.* Tel.: +39 0984492253; fax: +39 0984492277.
E-mail addresses: gian.bischi@uniurb.it (G.I. Bischi), lamantia@unical.it (F. Lamantia), lucia.sbragia@durham.ac.uk (L. Sbragia).

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of the International Conference on the Economics of Marine Protected Areas of Vancouver, 2000 (Sumaila and Charles, 2002).

The creation of an MPA in an international sea zone has been recently proposed in the Adriatic Sea, in a region called "Fossa di Pomo" (Antonelli et al., 2005). In order to avoid strong opposition from fishermen, this MPA is not intended as a reserve area (i.e. no fishing zone) but should be a region with carefully regulated fishing activity—for example, fishing with a limited constant effort may be allowed. This may give rise to a possible choice for fishermen as to whether they prefer fishing under an imposed limited effort inside an MPA with abundant fish stocks or fishing without imposed limits (e.g. under oligopolistic competition) outside the MPA, i.e. in a zone with possibly depleted fish stock.

Of course this choice is strongly influenced by the level of imposed fishing effort inside MPA and by the biological and economic externalities mentioned above. This problem is the main motivation for the dynamic model proposed in this paper. In fact, we consider an aquatic environment divided into two adjacent patches, characterized by different, in some way complementary, fishing policies: an imposed *constant fishing effort* in region 1 and *oligopolistic competition* in region 2, where fishermen decide their harvesting on the basis of a boundedly rational "profit incursion" attitude. Moreover, given the total number of fishermen, we propose an adaptive switching mechanism, based on imitation dynamics (Bischi et al., 2003; see also Vega-Redondo, 1996), that endogenously regulates the fraction of fishermen that, at each time period, decide to operate in one of the two regions on the basis of observed profits.

The division of the two different patches is virtual, in the sense that no physical boundaries exist, and fish can move among regions so that the quantity of biomass in one region depends not only on harvesting and biological growth in that region, but also on the stock and catch in the neighboring regions (see e.g. Srinivasu, 2005). The consequence is an interdependence which, at each time period, can increase the population of the zone where the fish density is smaller.

The dynamic model proposed in this paper is a nonlinear four-dimensional discrete-time dynamical system, that constitutes an extension of the model proposed in Lamantia and Sbragia (2005), where the subdivision of fishermen between the two regions is an exogenously given parameter. Bischi and Lamantia (2007) also consider a subdivision of a fishing region into patches, with these two kinds of fishing policies, however the division modelled in that paper is markedly different, because two independent growth functions are considered, in the sense that the fish population of each region is assumed to interact only with individuals of the same region. Instead, following Srinivasu (2005), we assume, like in Lamantia and Sbragia (2005), that the growth function in each patch involves interaction with both subpopulations, so that if we consider the sum of the two dynamic equations, then the growth function of the whole undifferentiated population is obtained, without any influence from the artificial subdivision.

The plan of the paper is as follows. Sections 2 and 3 define, respectively, the biological and economic model. Section 4 introduces the discrete-time dynamic system describing the time evolution of the system under study, whose steady states are analyzed in Section 5. Section 6 provides some insights into dynamics by numerical simulations. Section 7 concludes and suggests some possible improvements and extensions of the model proposed—of course, at the cost of increasing mathematical complexity.

2. The biological model

We assume that, in a given aquatic region, the unharvested fish population evolves according to a logistic growth function with intrinsic growth rate α per year and carrying capacity $K = \alpha/\beta$, i.e.

$$X(t+1) = X(t)G(X(t)) = X(t)(1 + \alpha - \beta X(t))$$

where $X(t)$ denotes the total quantity of fish biomass in the water basin at time period t. We suppose that the water basin is virtually divided in two patches, labelled as patch 1 and patch 2, and we denote by X_1 and X_2 the quantity of biomass in the different regions. Obviously $X = X_1 + X_2$. We can imagine that a central authority draws a virtual line on the water basin so that it can impose a specific harvesting policy for agents operating in one of the two areas, denoted as marine protected area (MPA) in the following. Assuming that fish can migrate between the different patches according to a linear diffusion mechanism, we can model the biological evolution of the resource in each patch by the following two-dimensional dynamical system:

$$\begin{cases} X_1(t+1) = X_1(t)G(X(t)) - \sigma[X_1(t) - X_2(t)] - H_1(t) \\ X_2(t+1) = X_2(t)G(X(t)) + \sigma[X_1(t) - X_2(t)] - H_2(t) \end{cases} \tag{1}$$

where $\sigma > 0$ is a diffusion coefficient, and $H_i(t)$ represents the quantity of fish harvested in time period t in region i. An important feature of this model is that it preserves the logistic growth of the total resource (see Srinivasu, 2005), i.e. summing up the two Eq. (1) we have

$$(X_1 + X_2)(t+1) = (X_1(t) + X_2(t))G(X_1(t) + X_2(t)) - H(t)$$

where $H(t) = H_1(t) + H_2(t)$ is the total harvested quantity.

3. Different harvesting policies

In this section we derive the two different harvesting functions in patch 1 and patch 2, and we compute the profits that fishermen can obtain from them. Let us assume that in patch 1 n_1 fishermen are allowed to harvest with an imposed constant fishing effort, whereas in patch 2 n_2 fishermen are engaged in a Cournot oligopolistic competition. Let $N = n_1 + n_2$ be the total number of fishermen. We denote by $r = n_2/N$ the fraction of fishermen that choose to harvest outside the MPA, choosing the quantity to harvest according to a profit-increasing Cournot oligopoly strategy. Of course $(1 - r)$ represents the fraction of agents fishing in region 1, i.e. the MPA.

If E is the individual fishing effort imposed by a regulator the MPA, denoted as patch 1, then

$$H_1(X_1) = qEN(1 - r)X_1.$$

where q is the "catchability" coefficient, linked to the adopted technology. The idea of a constant fishing effort is very common in the literature on mathematical bioeconomic modelling of the fisheries (see e.g. Gordon, 1954; Clark, 1976, and references therein) and it constitutes the most frequently employed method of control in fisheries.

In patch 2 fishermen engage in a *Cournot oligopolistic competition*, i.e. at each time period they decide their harvesting quantity according to profit-increasing strategies taking into account economic externalities (see Okuguchi, 1998; Szidarovszki and Okuguchi, 1998; Szidarovszky and Okuguchi, 2000; Bischi and Kopel, 2002; Bischi et al., 2004, 2005). Here we assume the existence of a unique market for the harvested resource determined by the linear inverse demand function

$$p(t) = a - bH(t) \tag{2}$$

where a and b are positive parameters representing, respectively, the maximum selling price and the (opposite of the) slope of the inverse demand function. We also assume that the total harvested quantity $H(t)$ is entirely sold in the market.

The profit at time t of a representative fisherman working in zone 1 is given by

$$\begin{aligned}\pi_1(t) &= qEX_1(t)(a - bH(t)) - \delta E \\ &= qEX_1(t)\{a - b[ENq(1 - r(t))X_1(t) + Nr(t)x(t)]\} - \delta E\end{aligned} \tag{3}$$

where δ is the unitary cost of effort, $H_1(t) = N[1 - r(t)]qEX_1(t)$ and $H_2 = Nr(t)x(t)$ represents the total harvesting in patch 1 and 2 respectively, $r(t)$ is the fraction of agents operating outside the MPA in period t, $x(t)$ being the individual catch of an oligopolist, whose determination is given below.

The cost function of a representative fisherman that chooses to fish in zone 2, for harvesting a quantity x when the fish stock is X_2, is given by

$$C(x, X_2) = c + \gamma \frac{x^2}{X_2} \tag{4}$$

where γ is a technological parameter and c is a fixed cost, that includes the effort for solving the harvesting decision problem. The cost function (4) can be derived from a Cobb-Douglas type "production function" with fishing effort (labor) and fish biomass (capital) as production inputs (see Clark, 1976; Szidarovszki and Okuguchi, 1998). It captures the fact that it is easier and less expensive to catch fish if the fish population is large, so that it includes the resource stock externalities (Smith, 1968, 1969).

The quantity of resource harvested by a single oligopolist, $x(t)$, is determined in the following way. Assuming that fishermen are homogeneous, we can write the time t profit of a representative fisherman working in zone 2 for harvesting a quantity $x(t)$ as

$$\pi_2(t) = x(t)(a - bH) - \gamma \frac{x(t)^2}{X_2(t)} - c \tag{5}$$

the MPA, following a Cournot oligopolistic game, or a controlled constant effort exploitation inside the MPA. Since agents do not have a full knowledge of the underlying structure of the game, we assume that the switching mechanism takes place according to an imitative behavior (see Taylor and Jonker, 1978; Weibull, 1995; Vega-Redondo, 1996; Hofbauer and Sigmund, 1998; Noailly et al., 2003). In fact, at each time period any agent is assumed to sample an agent that has chosen a different strategy in the past, and to switch to the other strategy if the profit of the sampled agent reveals to be greater than his own. We assume that the sampling procedure is uniformly distributed so that the probability of meeting an agent adopting a given strategy is proportional to the fraction of those agents. Hence the probability of changing behavior can be modelled according to an imitation dynamic (see Bischi et al., 2003). For this reason we specify the switching probabilities

$$\begin{aligned}&p_{O \to E} = (1 - r)\mathbb{P}(\pi_1 > \pi_2); \quad p_{O \to O} = 1 - p_{O \to E} \\ &p_{E \to O} = r\mathbb{P}(\pi_2 > \pi_1); \quad p_{E \to E} = 1 - p_{E \to O}\end{aligned}$$

where "$p_{O \to E}$" is the probability to change from the "O" strategy ("Oligopoly") to the "E" strategy (constant Effort). This probability is given by the product between the probability to meet an agent involved in the constant effort harvesting, i.e. $(1 - r)$, and the probability that the profit of that agent is greater than the profit for a representative oligopolist, that is $\mathbb{P}(\pi_1 > \pi_2)$; a similar reasoning applies for the other symbols with an obvious meaning of the symbols.

The cumulative distribution (probability) function $\mathbb{P}(\pi_1 > \pi_2) = \Phi(\pi_1 - \pi_2)$ is monotone increasing with $\Phi(-\infty) = 0$ and $\Phi(+\infty) = 1$. Of course, $\mathbb{P}(\pi_2 \geq \pi_1) = 1 - \Phi(\pi_1 - \pi_2)$.

All in all, the dynamic (deterministic mean field) equation describing how the expected fraction of oligopolists evolves is given by

$$\begin{aligned}r(t+1) &= r(t) + [1 - r(t)]\, p_{E \to O} - r(t)\, p_{O \to E} \\ &= r(t) + [1 - r(t)]r(t)[1 - \Phi(\pi_1(t) - \pi_2(t))] - r(t)(1 - r(t))\Phi(\pi_1(t) - \pi_2(t))\end{aligned} \tag{7}$$

Due to the complexity of the underlying model, it could be difficult for an oligopolist to compute the profit maximizing level of harvesting. For this reason we assume that oligopolists act in a "boundedly rational" way, in the sense that they update their harvesting plans in the direction of increasing profits, as specified in the so-called profit-gradient dynamics (see Flam, 1993; Furth, 1986; Bischi and Naimzada, 1999).

Thus the quantity $x(t+1)$ of a representative oligopolist as a function of $r(t), x(t), X_1(t)$ and $X_2(t)$ can be written as

$$\begin{aligned}x(t+1) &= x(t) + k\frac{\partial \pi_2(t)}{\partial x(t)} = \\ &= x(t) + k\left\{a - bN[2r(t)x(t) + Eq(1 - r(t))X_1(t)] - \frac{2x(t)\gamma}{X_2(t)}\right\}\end{aligned} \tag{6}$$

where $k \in (0, 1]$ is the speed of adjustment to the direction of the profit gradient.

4. The dynamic model

Agents can decide at any time period which patch to fish in, and consequently the harvesting strategy, i.e. whether to harvest the resource according to a free competition outside

$$= r(t)[1 - (1 - r(t))M(\pi_1(t) - \pi_2(t))] \tag{8}$$

where similarly to Bischi et al. (2003), we define $M(.) = 2\Phi(.) - 1$. Eq. (7) can be interpreted as a balance equation, saying that the fraction of agents that at time $t+1$ operates outside the MPA, $r(t+1)$, is increased by the expected fraction of agents that decide to exit the MPA, $[1 - r(t)]\, p_{E \to O}$, and decreased by agents entering the MPA, i.e. $r(t)\, p_{O \to E}$.

The main properties that the function M inherits from the commonly used cumulative distributions Φ are: (1) $M(0) = 0$; (2) $\lim_{z \to -\infty} M(z) = -1$; (3) $\lim_{z \to +\infty} M(z) = 1$; (4) M is an odd function and (5) M is convex in $(-\infty, 0)$ and concave in $(0, +\infty)$.

Here we explicitly assume the functional form

$$M(z) = \frac{2}{\pi} \arctan\left(\frac{\lambda \pi}{2} z\right)$$

where the parameter $\lambda = M'(0) = (1/2)\Phi'(0) > 0$ represents the agents' propensity to strategy switching, inversely proportional to the dispersion of the probability distribution of $\pi_1 - \pi_2$, i.e. greater values of λ represent more peaked probability density around 0 (see Fig. 1).

The dynamical system considering the natural resources evolution in Eq. (1) with $H_1(t) = N[1 - r(t)]qEX_1(t)$ and $H_2(t) =$

$Nr(t)x(t)$, individual oligopolistic harvesting in (6), and expected fraction of oligopolistic competitors (7) can be described by the following map in $\mathbb{R}^4$ in the dynamic variable X_1, X_2, x and r:

$$T:\begin{cases} X_1(t+1) = X_1t(t)G(X_1(t)+X_2(t)) - \sigma[X_1(t)-X_2(t)] - N[1-r(t)]qEX_1(t) \\ X_2(t+1) = X_2(t)G(X_1(t)+X_2(t)) + \sigma[X_1(t)-X_2(t)] - Nr(t)x(t) \\ x(t+1) = \max\left[0, x(t) + k\left\{a - bN[2r(t)x(t) + qE(1-r(t))X_1(t)] - \dfrac{2x(t)\gamma}{X_2(t)}\right\}\right] \\ r(t+1) = r(t)\left\{1 - (1-r(t))\dfrac{2}{\pi}\arctan\left[\dfrac{\lambda\pi}{2}(\pi_1(t)-\pi_2(t))\right]\right\} \end{cases} \tag{9}$$

It is important to note that the max operator prevents oligopolists from choosing negative quantities of harvest. The sets $r = 0$ and $r = 1$ are dynamic invariant, in the sense that if $r(t) = 0$ (1) then also $r(t+1) = 0$ (1). This corresponds to the obvious statement that when a strategy is not performed at a given time it does not occur anymore. In fact, imitation is impossible in such limiting case. When $r = 0$, i.e. all agents exert constant effort, the model reduces to a bidimensional map, as the equations for x and r in (9) are redundant. This case corresponds to the one in Lamantia and Sbragia (2005), where analysis of existence and stability of equilibria as well as numerical investigations of the global dynamics are carried out.

5. Steady states of the model

A steady state of the dynamical system (9) is a point $S = (X_1^*, X_2^*, x^*, r^*)$ satisfying the following nonlinear system of equations

$$\begin{cases} X_1[\alpha - \beta(X_1+X_2)] - \sigma(X_1 - X_2) - N(1-r)qEX_1 = 0 & (1) \\ X_2[\alpha - \beta(X_1+X_2)] + \sigma[X_1 - X_2] - Nrx = 0 & (2) \\ aX_2 - bNX_2[2rx + qE(1-r)X_1] - 2x\gamma = 0 & (3) \\ r(1-r)(\pi_1 - \pi_2) = 0 & (4) \end{cases} \tag{10}$$

where π_1 and π_2 are given in (3) and (5) respectively. When no biomass is present in each patch, i.e. $X_1 = X_2 = 0$, from (10), (2) we get $x = 0$ and (10), (4) reduces to $r(1-r)(c - \delta E) = 0$. In this case equilibria are given by $S_1^0 = (0,0,0,0)$ and $S_2^0 = (0,0,0,1)$. When $c = \delta E$ any level $r^* \in [0,1]$ guarantees an equilibrium of the type $S_3^0 = (0,0,0,r^*)$. All these cases represent extinction equilibria as they involve no biomass in each patch and can be economically interpreted in a straightforward way. About the stability of these equilibria, the following result holds:

Proposition 1. *Equilibria S_1^0, S_2^0, S_3^0 are unstable for all parameters values.*

A proof can be given by standard equilibrium analysis and it is outlined in Appendix A. As these equilibria are never stable, we can deduce that the system does not tend spontaneously to extinction.

Let us now consider the case $r = 0$, i.e. all agents harvesting inside the MPA. Obviously Eq. (10, 4) is satisfied and from (10, 3) we find that the harvesting equilibrium level of each representative agent is given by $x_0^* = ((a - bNqEX_1)X_2/2\gamma)$. Note that as all agents harvest inside the MPA, the equilibrium level x^* has no practical relevance whatsoever in this case.

Hence system (10) reduces now to

$$\begin{cases} X_1[\alpha - \beta(X_1+X_2)] - \sigma(X_1 - X_2) - NqEX_1 = 0 \\ X_2[\alpha - \beta(X_1+X_2)] + \sigma[X_1 - X_2] = 0 \end{cases}$$

From the second equation, for $X_2 \neq (\sigma/\beta)$, we obtain $X_1 = g(X_2) = (X_2(\alpha - \sigma - \beta X_2))/(\beta X_2 - \sigma) > 0$ which, substituted back in the first equation, lets study the equilibrium level in the variable X_2 as a zero of the single variable function

$$f(X_2) = \frac{X_2[(2X_2\beta - \alpha)(\alpha - 2\sigma)\sigma + ENq(\sigma - X_2\beta)(\alpha - \sigma - X_2\beta)]}{(\sigma - X_2\beta)^2}$$

Clearly it results $f(0) = 0$, $\lim_{X_2 \to +\infty} f(X_2) = +\infty$, and f has a vertical asymptote at $X_2 = (\sigma/\beta)$, as $\lim_{X_2 \to (\sigma/\beta)} f(X_2) = -\infty$. By continuity of f in the interval $((\sigma/\beta), +\infty)$ we have that at least one $X_2^* > (\sigma/\beta)$ such that $f(X_2^*) = 0$, hence defining the equilibrium point $S_0^* = (g(X_2^*), X_2^*, x^*, 0)$. Note that condition $f'(0) > 0$, equivalent to $(\alpha/\sigma) < 1 + (1/(1+ENq))$, implies the existence of another equilibrium point such that $X_2^* \in (0, (\sigma/\beta))$.

The analysis of the case $r = 1$, i.e. all agents harvesting outside the MPA, leads to a similar exercise. Again Eq. (10, 4) is satisfied and the harvesting equilibrium level of each representative agent as $x_1^* = (aX_2)/2(bNX) > 0$. The existence of equilibria can be studied similarly to the previous case. Under the likely assumption that $\alpha > 2\sigma$ it is possible to prove analytically that no equilibrium exists with $X_1^* < (\sigma/\beta)$ and at least one equilibrium $S_1^* = (X_1^*, g(X_1^*), x_1^*, 1)$ always exists with $X_1^* > (\sigma/\beta)$. Moreover, it is easy to verify that condition $a < (4b\gamma\sigma(\alpha - 2))/\beta$ ensures uniqueness of equilibrium S_1^* under the case $r = 1$. The details are left to the reader.

Now we tackle the problem of existence of inner equilibria with $r \in (0,1)$ and with $X_1, X_2 > 0$. As before, we can find equilibria in terms of a two-dimensional system of equations, whose derivation is carried out in Appendix A. This system can be easily solved numerically. An easy situation is depicted in Fig. 2a, with parameters $\alpha = 2$; $\beta = 0.2$; $\sigma = 0.6$; $E = 1$; $N = 80$; $q = 0.1$; $a = 2$; $b = 0.3$; $\gamma = 0.5$; $\delta = 0.5$; $c = 0.5$, where the unique positive

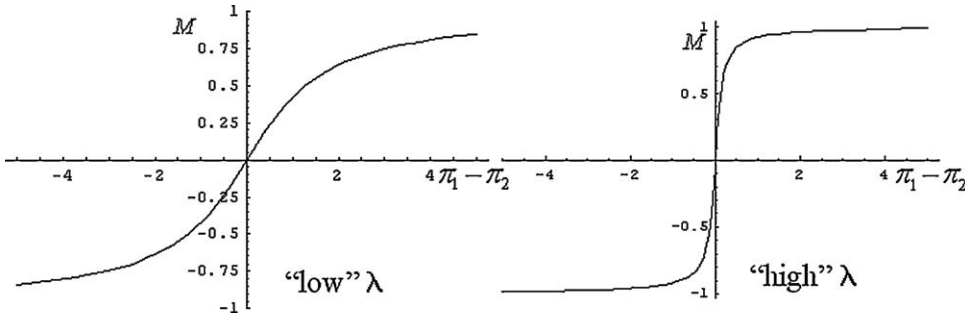

Fig. 1. Sketches of $M(z)$ for (a) “low” and (b) “high” propensity to strategy switching λ.

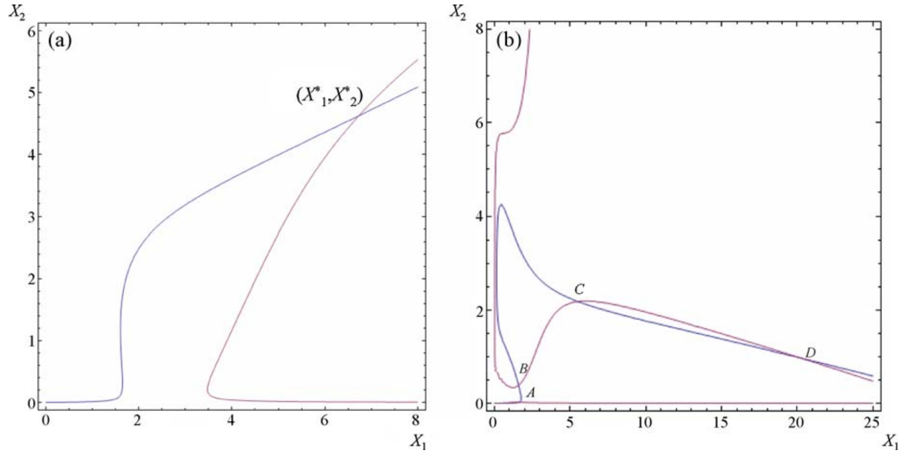

Fig. 2. Existence of inner equilibria. (a) Uniqueness of an inner equilibrium. (b) Multiple equilibria.

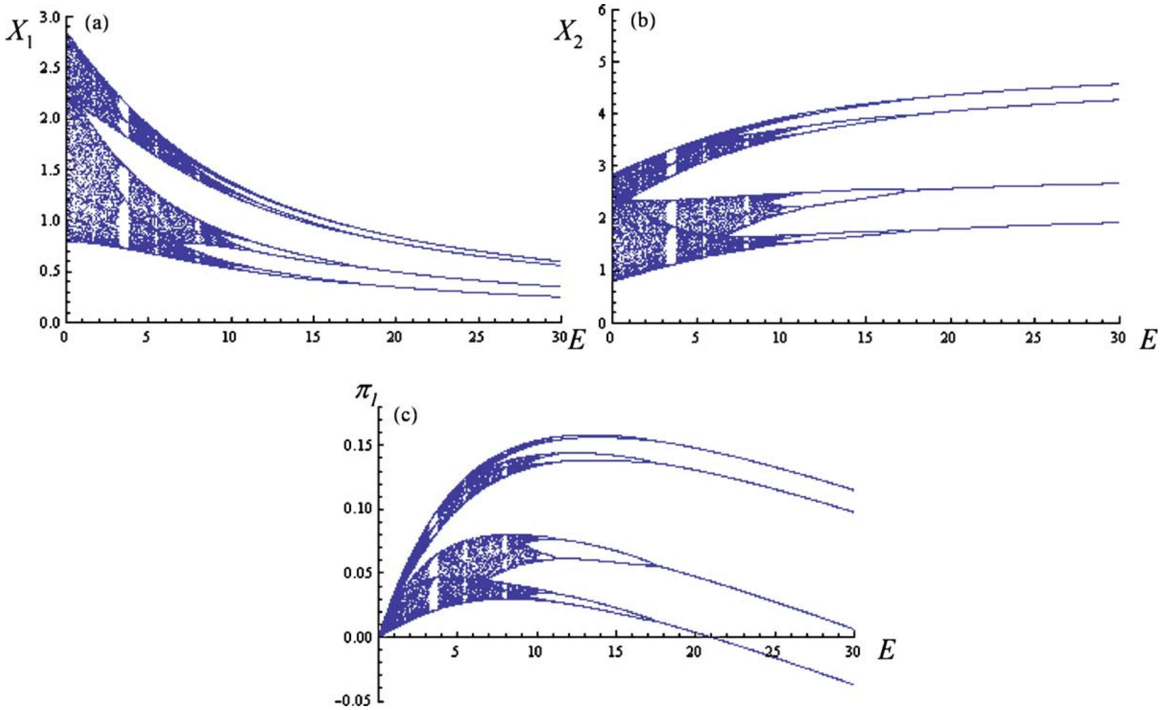

Fig. 3. Bifurcation diagrams for $E \in [0, 30]$. (a) and (b) show in the vertical axis the asymptotic values of biomass in patch 1 and 2; the corresponding individual profits for agents operating in patch 1 are depicted in (c).

equilibrium with $(X_1^*, X_2^*) \simeq (6.7341, 4.6296)$ is represented as the intersection point of two curves defined by the previous system. It can be easily checked that the corresponding equilibrium level of r and x are also feasible (i.e. $0 \leq r \leq 1$ and $x \geq 0$). More involved equilibrium curves are depicted in Fig. 2b, where parameters are set as $\alpha = 3$; $\beta = 0.4$; $\sigma = 0.3$; $E = 2$; $N = 80$; $q = 0.1$; $a = 2$; $b = 0.3$; $\gamma = 0.5$; $\delta = 0.5$; $c = 0.5$, where four equilibria A, B, C, D are obtained (also in these cases the corresponding equilibrium level of r and x are also feasible).

As stability analysis for equilibria of the type S_0^* and S_1^* as well as for inner equilibria with $r \in (0, 1)$ is not an easy task, we only perform numerical analysis for these cases in the next section.

6. Numerical simulations

In the following we present some numerical simulations that give an insight on the richness of possible dynamic scenarios arising from (9). We present two main cases. In the first one, only constant effort is exerted, so that a part of the region can be regarded as a no-take area. In this case we show that the model suggests that an intermediate level of effort should prevail which maximizes profits in a sustainable way. This is very similar to the notion of maximum sustainable yield presented in Gordon (1954). The second example shows some dynamic scenarios arising in the complete model, when harvesting both inside and outside the MPA is allowed. A useful tool for numerical investigation is the bifurcation diagram, showing the possible long-term values (equilibria/fixed points, periodic or chaotic orbits) of the system as a function of a bifurcation parameter. Even if the cases presented in this section are based on fixed parameter values, we found similar situations for a broader set of parameters, as briefly outlined below. Before discussing these cases in detail, however, we observe that often the system (9) diverges when $r(0) > 0$: if we let even one agent harvest outside the MPA, then many fishermen will imitate him and so the resource will be depleted so that it becomes extinct in finite time. We find this point very interesting,

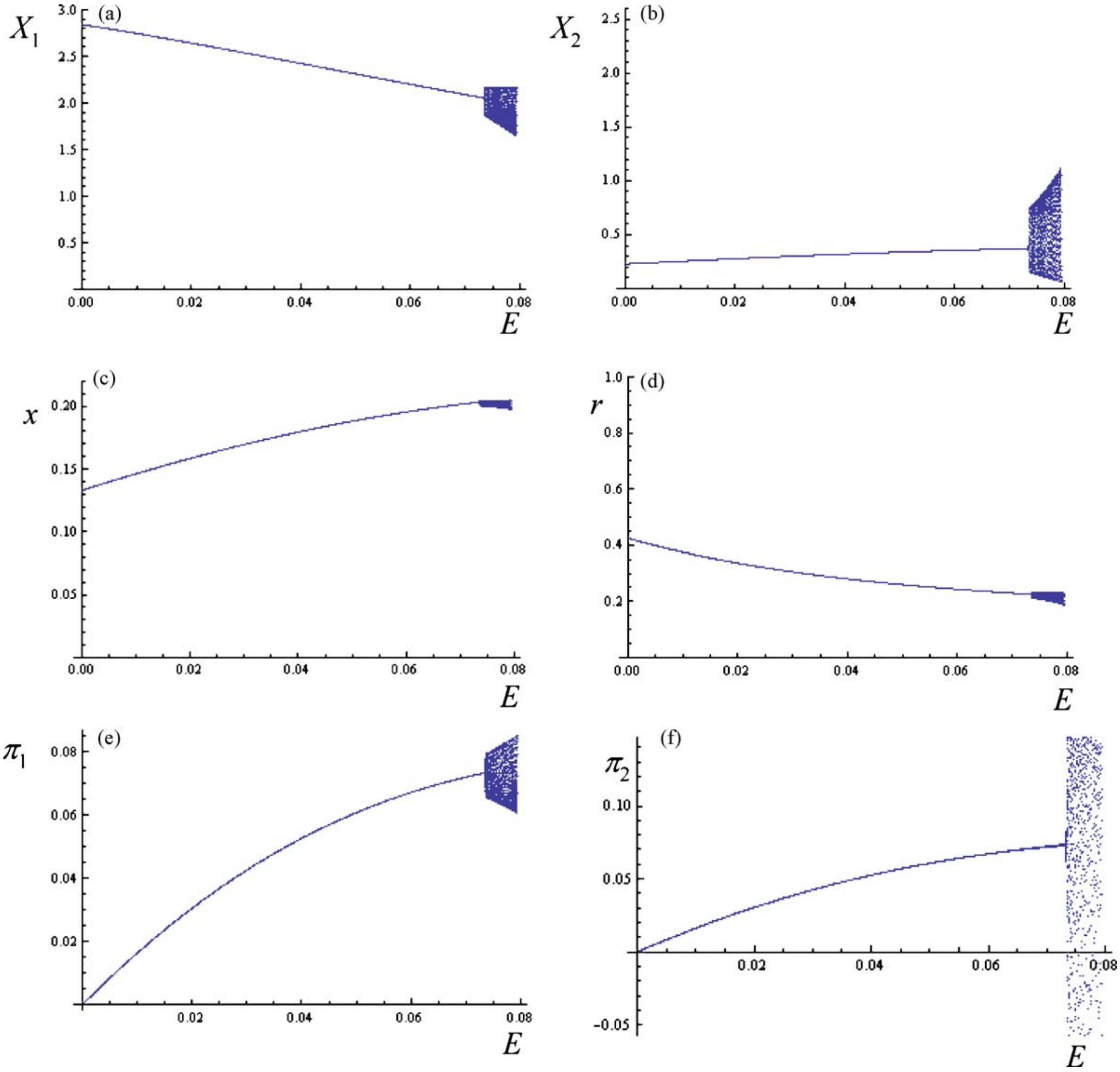

Fig. 4. Bifurcation diagrams for $E \in [0, 0.1]$ with a "low" propensity to strategy switching λ. (a), (b), (c), (d) in the vertical axis the asymptotic values for state variables X_1, X_2, x, r are reported respectively; (e), (f) corresponding individual profits π_1 and π_2 for agents operating in patch 1 and 2 respectively.

as it resembles the "tragedy of the commons" recalled in the introduction.

We start by considering a case with all agents inside the MPA, i.e. $r = 0$. If $r(0) = 0$ then the last equation in (9) reduces to the identity $r(t + 1) = r(t) = 0$ for each t, and so also the third equation in (9) loses practical relevance, as no oligopolist exists. So the system is described by the iteration of a two-dimensional discrete dynamical system for the state variables $X_1(t)$ and $X_2(t)$.

Inside the MPA, harvesting is regulated by the authority by setting a constant fishing effort level E. Of course the authority is interested in setting this parameter conveniently, as low effort levels reduce agents' profits whereas high levels can deplete the resource. Therefore we mainly focus on the impact of parameter E on biomass and agents' profits.

Let us consider the bifurcation diagrams shown in Fig. 3, where parameters are given as[1] $\alpha = 2.7, \beta = 0.6, \sigma = 0.2, N = 50, q = .001, a = 15, b = 0.3, \gamma = 0.1, \delta = 0.005, c = 3, k = 0.02, \lambda = 0.1$, and initial condition (i.c.) $X_1(0) = 0.3, X_2(0) = 0.2, x(0) = 0.07$ and $r(0) = 0$. The bifurcation parameter E ranges in the interval [0,30]. For low values of E, harvesting is kept at low levels and so biomass in both patches varies according to natural fluctuations, driven by logistic growth. However the shrinking of the chaotic attractor for exploited biomass X_1 as E is increased is evident (see Fig. 3a). Further increasing the level of imposed effort E causes the chaotic attractor to vanish through a sequence of period halving bifurcations and a stable four cycle appears. In this case, the initial condition $r(0) = 0$ forces every agent to stay inside the first region (MPA), as there is no oligopolist to imitate. Consequently, biomass in the second region, X_2, is left unharvested and its value grows on average, as is clearly visible in Fig. 3b. The corresponding values of individual profits as constant effort is increased are shown in Fig. 3c. It is interesting that for low levels of efforts, profits are positive and agents are better off when the authority imposes an average level of effort E. High levels of effort (and so of resource exploitation), not only lead to resource depletion in the region, but they can also cause losses to fishermen. It is interesting to compare the situation depicted in Fig. 3c with the one described in the seminal paper by Gordon (1954), in particular with the notion of maximum sustainable yield. For parameters as in Fig. 3, if we consider an initial condition with $r(0) > 0$, then in numerical simulations we always observe that the system diverges, i.e. the resource becomes extinct in finite time.

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[1] As for parameter α, expressed in day^{-1}, we considered in the numerical simulations values over 2.5, following Conrad and Clark (1987, p. 98).

Now we turn our attention to the complete case where both patches are exploited and agents can switch strategies according to the imitation dynamic mechanism previously described. In this case the initial condition of the system (9) is such that $r \in (0, 1)$ and any steady state of the dynamical system is characterized by constant levels of biomass and harvesting in both patches, so that agents are indifferent to harvesting in patch one or in patch two, as profits are equals.

We begin this analysis by comparing two situations obtained with, respectively, a low and a high λ, that regulates agents' propensity to change strategy. In the bifurcation diagrams of Fig. 4, parameters are given by $\alpha = 2.7, \beta = 0.7, \sigma = 0.6, N = 30, q = 0.3, a = 3, b = .3, \gamma = 1.7, \delta = 0.4, c = 0.2, k = 0.0003, \lambda = 0.1$, imposed efforts $E \in [0, 0.1]$ and i.c. $X_1(0) = 0.3, X_2(0) = 0.2, x(0) = 0.07$ and $r(0) = 0.3$.

For low levels of efforts, agents split themselves almost equally between the two regions ($r \to 0.43$), but this fraction decreases as more harvesting is allowed in patch one (see Fig. 4d). For low levels of efforts, however, the long run behavior of the system stabilizes to a fixed point. As effort in the MPA is increased, competition rises and so agents outside the MPA are forced to increase unitary catches to keep up with competitors (see Fig. 4c). At $E = \tilde{E}_1 \approx 0.07261$ the steady state loses its stability through a Neimark-Sacker (NS) bifurcation, i.e. it loses stability and a stable closed curve emerges, where (quasi)periodic motion takes place. Also in this case it is interesting to consider agents' profits in both regions. As E is further increased, inside the MPA profits increase as a consequence of more intensive resource exploitation and profits outside the MPA also increase due to greater individual harvesting x, as noticed before. However, the NS bifurcation leads to high biomass variability, which has a deep impact on profits, especially for oligopolists. Consequently, they can gain huge profits or suffer big losses as time goes on (see Fig. 4e and f).

It is interesting to compare this result with the situation depicted in Fig. 5, where all parameters are as in Fig. 4 but $\lambda = 0.8$. Again a Neimark-Sacker bifurcation causes the loss of stability of the fixed point, but two main differences with respect to the previous case are evident: firstly the fixed point has a very small region of stability, as the bifurcation appears for a lower value of effort, namely at $\tilde{E}_2 \approx 0.0011$. Secondly, and more interestingly, after the NS bifurcation takes place, larger oscillations in the state variables with quasi-periodic motion appear. A larger effort inside the MPA in general reduce the biomass in the MPA and causes an increase of the variability of the biomass outside the MPA. Again

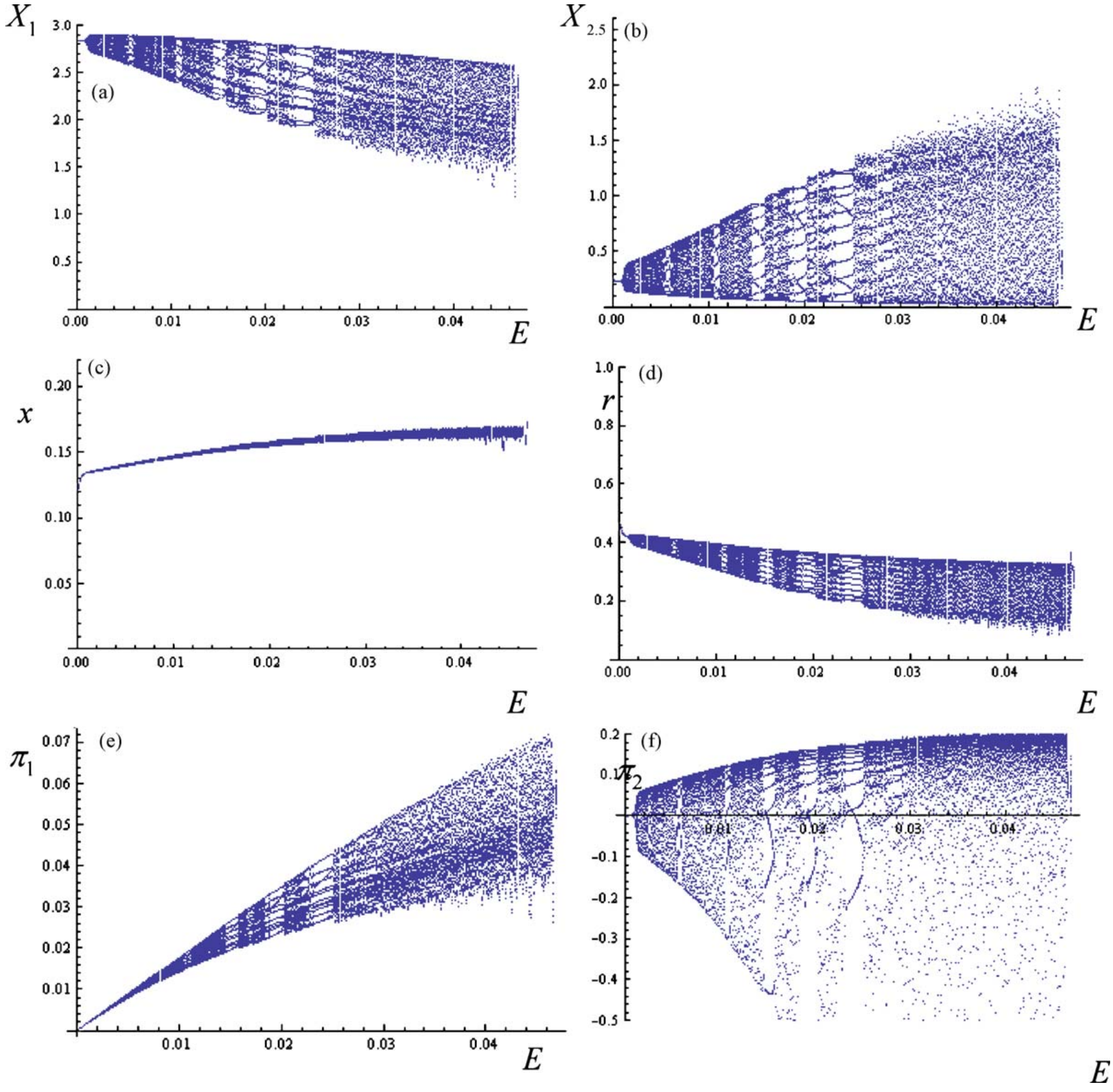

Fig. 5. Bifurcation diagrams for $E \in [0, 0.1]$ with a "high" propensity to strategy switching λ. (a), (b), (c), (d) in the vertical axis the asymptotic values for state variables X_1, X_2, x, r are reported respectively; (e), (f) corresponding individual profits π_1 and π_2 for agents operating in patch 1 and 2 respectively.

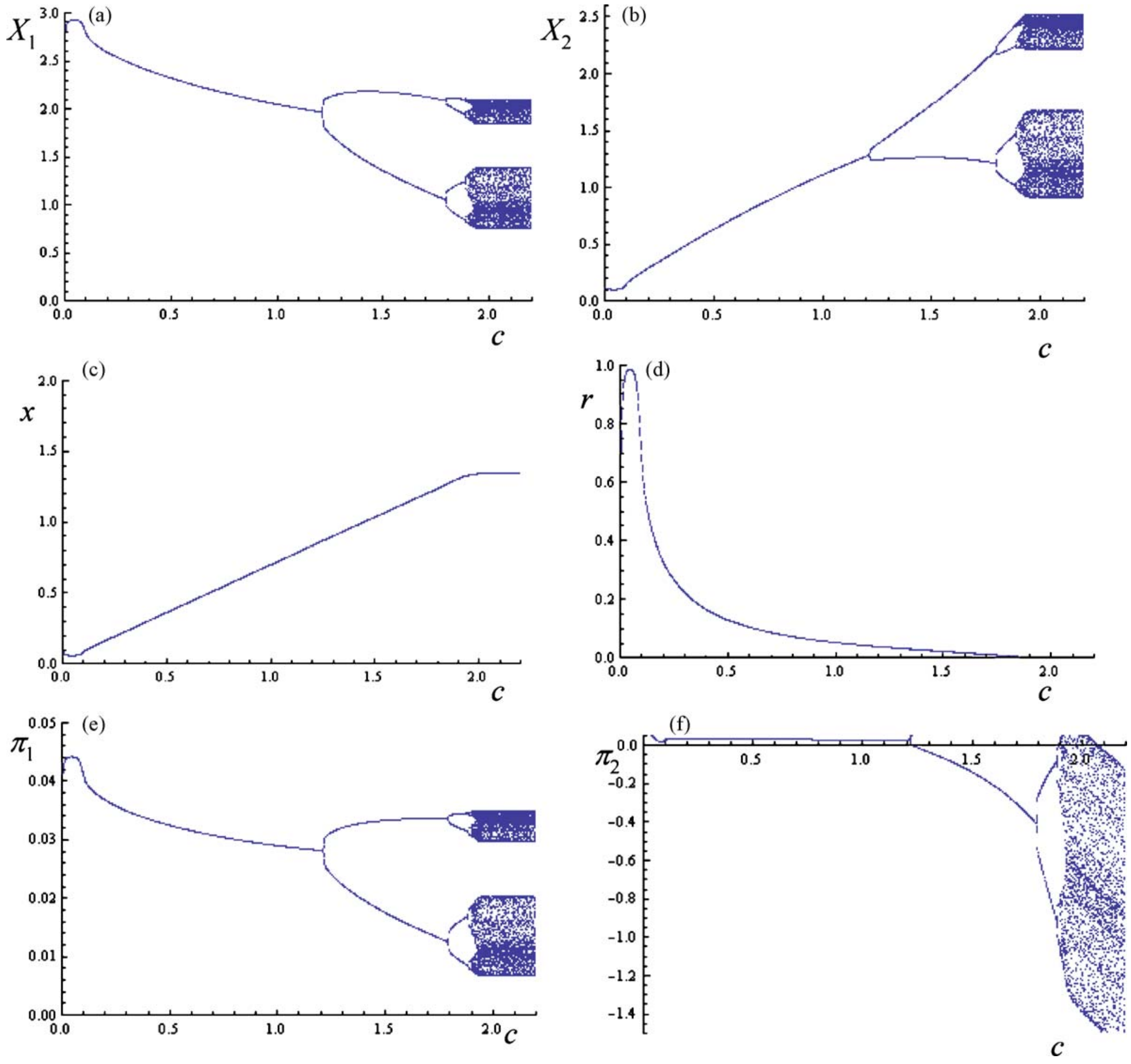

Fig. 6. Bifurcation diagrams for $c \in [0, 2.2]$. (a), (b), (c), (d) in the vertical axis the asymptotic values for state variables X_1, X_2, x, r are reported respectively; (e), (f) corresponding individual profits π_1 and π_2 for agents operating in patch 1 and 2 respectively.

profits oscillate, but the high propensity to change behavior amplifies strongly their variability.

From an economical point of view we can summarize these last results as follows. When the regulator is able to enforce low effort levels in the first patch, then the system is characterized by stability in all relevant quantities: biomass in each patch, harvesting and fraction of oligopolists. This is a consequence of imposing low harvesting levels in the first patch so that many agents are better off by operating in patch two. Indeed patch one serves as a restocking area, since harvesting levels are kept at a minimum. However, if the regulator lets agents increase their catches in patch one, then a larger fraction of fishermen tends to switch to the first patch. As a consequence, also individual harvesting in patch two is increased (see Fig. 4c), in order to compensate higher harvesting (and profit) from patch one (see Fig. 4e). This mechanism is more complicated after the NS bifurcation occurs, as endogenous oscillations in state variables (biomasses, harvesting, and fraction of oligopolists) appear. These effects are more evident with higher agents' propensity to switch strategies, with consequently more persistent oscillations in biomass levels and fractions of oligopolists. Similar results can be obtained, for a given level of imposed efforts, by varying the cost of effort δ. In particular we observe that the higher the cost of effort is, the more stable the system appears. In particular the oscillatory behavior encountered with low costs of effort disappears as δ is increased over a given value.

Now we consider the effect of changing fixed costs c of a representative oligopolist, see Eq. (5). This fixed cost can also be interpreted as a fixed tax imposed on each oligopolist at each time period. In this case, as before, there is a range of fixed costs ensuring stability of the steady state but now loss of stability of the steady state can happen through flip (period doubling) bifurcations. Let us consider again parameters as in Fig. 4 but with $E = 0.025$ and $c \in [0, 2.2]$. In this case we can notice that the fraction of oligopolists decreases as fixed cost c increases over a small threshold $\widehat{c}$ (see Fig. 6d). Since oligopolists prefer to change behavior and exert constant effort as a consequence of taxation c, biomass in patch two tends to increase and, consequently, patch one is heavier exploited. At a level $\tilde{c_1} \approx 1,21$, a flip bifurcation takes place, and biomass in both regions starts oscillating between a low and an high level (see Fig. 6a and b). When c is above a given level, exploiting patch two gives rise to sure losses (see Fig. 6f) so that almost all agents operate in patch one (see Fig. 6d). Hence patch two tends to become a no-take area. In this scenario we have that $r \rightarrow 0$ as time passes and the third equation in (9), representing harvesting of a representative oligopolist, does not influence biomass levels in practice: with heavy taxation on oligopolists, the model reduces to the one proposed in Lamantia and Sbragia (2005), that can be regarded as a benchmark case of (9) when oligopolistic behavior is ruled out.

7. Conclusions

In this paper we have proposed a dynamic nonlinear fishery model to simulate the time evolution of fish stock in an aquatic environment which is divided into two adjacent patches characterized by different exploitation methods: harvesting in one patch is limited by a constant fishing effort rule imposed by a regulator, whereas in the other patch the individual catch is freely determined by fishermen according to a decision rule based on the knowledge of economic (demand and cost) externalities and on profit-increasing attitude (profit gradient dynamics). The available fish stock in each time period in each patch is determined by reproduction, by the fishermen's harvesting decisions and by migration of fish from one patch to the other according to differences between fish densities. It is also assumed that fishermen can switch harvesting decision rules according to an imitation dynamic, based on comparison of profits obtained in the previous by fishermen fishing in the two different patches. The patch where oligopolistic competition takes place is intended to represent an open access high sea fishery, whereas the patch with imposed fishing effort is intended to represent a marine protected area (MPA).

Our simulations of models with two different kinds of fishing strategies reveal a much broader array of dynamic behavior with respect to those obtained in the study of system with a homogeneous fishing environment: besides steady state behavior, periodic, quasi-periodic and chaotic dynamics emerge, and qualitative changes (or bifurcations) are detected as some parameters of the model are varied.

The main conclusions are related to the sustainability of exploitation in the whole region. In fact, sustainability is favored by reducing the harvesting levels in one of the two patches, which hence serves as a restocking area. Among the tools to disincentive harvesting in one region, we considered the simplest ones, such as reduction of individual efforts, taxation on oligopolists or on effort. All in all we believe that these results can provide useful insights into the complicated matter of management of renewable resource in the presence of regions where different fishing policies are adopted and strategic interactions occur.

In particular, the important role of marine protected areas can be deduced from the results shown in this paper. In this paper we have focused our attention on the level of effort to impose within the MPA. In fact we found that low levels of effort tend to stabilize the system around equilibrium levels, whereas large variability of state variables are present when effort is over some threshold values. This effect is more evident as agents' propensity to switch strategy is increased. The stability of state variables is important as it also implies stability in profits. In any case, as effort is increased, we observed more variability in profits for oligopolists than for constant effort exploiters; hence oligopolistic behavior can be very risky when the MPA is not properly enforced. This point suggests keeping regulated effort at low levels not only to prevent overfishing inside the MPA, but also to reduce profit variability to oligopolists. Moreover the regulator can influence the fishing level outside the MPA by properly tuning the parameter c, which may be interpreted as a taxation on oligopolists.

A peculiar feature deduced from the numerical results shown in this paper is the tendency to produce oscillatory time patterns, with periodic, quasi-periodic or chaotic oscillations. In a real system this property may create some difficulties in the analysis of empirical data when policy makers try to understand the trend of available fish stock after the creation of a MPA. In other words, the benefits of the creation of a MPA can be properly evaluated only if they are observed for a long period after the MPA exists.

Of course, the model proposed in this paper gives a very simplified and stylized representation of the real system described. In fact, the presence of many important factors have been neglected, such as the presence of different species of fish as well as the existence of different age (or size) classes. This fact is particularly important in the particular case that motivated this paper, the project of a MPA in the region known as Fossa di Pomo in the central Adriatic Sea. In fact, the importance of this region is related to the presence of young (hence small) fishes (it is denoted as a "nursery"). So, the limitation of fishing activity in that region has the important effect of increasing the number of big size fishes around it. This effect, as well as other boundary effects around a MPA, has been neglected in the model proposed. Moreover, the effects of the mobility of fishes different from linear diffusion due to density differences have also been neglected. In other words, the model proposed in this paper constitutes a first modest step towards more realistic models that can be obtained by improving it through inclusion of these more realistic assumptions. Of course, the inclusion of more realistic assumption has the consequence of giving the model a more complicated mathematical structure, and the usual trade-off between mathematical simplicity and a realistic description of real systems always constitutes a difficult and challenging problem.

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Appendix A

Proof of Proposition 1. Stability of extinction equilibria S_1^0, S_2^0, S_3^0 can be shown by standard linearization analysis. For instance for equilibrium S_1^0, eigenvalues of the Jacobian matrix for dynamical system (9) read

$$\lambda_{1,2} = 1 + \alpha - \sigma - \frac{1}{2}\left(ENq \pm \sqrt{ENq^2 + 4\sigma^2}\right)$$
$$\lambda_3 = 1 - \frac{2k\gamma}{X_2}$$
$$\lambda_4 = 1 - \frac{2}{\pi}\arctan\left[\frac{\pi}{2}\lambda(c - E\delta)\right]$$

It is straightforward that, assuming $\alpha > \sigma$, eigenvalue $\lambda_1 = 1 + \alpha - \sigma - (1/2)(ENq - \sqrt{ENq^2 + 4\sigma^2})$ is never inside the stability interval, being $\lambda_1 > 1$ for all parameter values. Moreover we observe that eigenvalues λ_3 approaches $-\infty$ as $X_2 \to 0^+$ and that it is $\lambda_4 \in (-1, 1)$ as long as $\delta E < c$. Therefore S_1^0 cannot be stable for all possible parameter values. The details for S_2^0, S_3^0 are similar and are left to the reader.

Inner equilibria with $r \in (0,1) X_1, X_2 > 0$. From (10, 1) we can write $r(X_1, X_2) = (ENq - \alpha + \beta(X_1 + X_2) - \sigma(X_1 - X_2)/ENq)$. By adding together From (10, 1) and From (10, 2) and substituting back the expressions for r and $1 - r$, we obtain an expression for $x(X_1, X_2) = ([(\alpha - \beta(X_1 + X_2))X_2 + \sigma(X_1 - X_2)]EqX_1/X_1[ENq - \alpha + \beta(X_1 + X_2)] - \sigma(X_1 - X_2))$. By employing the obtained expressions for

$r(X_1, X_2)$ and $x(X_1, X_2)$ in (10, 3) and (10, 4) we obtain the following system of equations only in the variables X_1 and X_2 that characterize inner equilibria with $r \in (0,1)$ and $X_1, X_2 > 0$

$$\begin{cases} a - bNqE(1-r)X_1 - 2\left(bNr + \dfrac{\gamma}{X_2}\right)x = 0 \\ qEX_1[a - b(qENX_1(1-r))] - \delta E + c = x\left[a + bNqEX_1(2r-1) - \left(bNr + \dfrac{\gamma}{X_2}\right)x\right] \end{cases}$$

where the last equation can be referred to as the locus of points where profits inside and outside the MPA are equal. Note that any solution (X_1^*, X_2^*) of the previous system only define a feasible equilibrium point when $r(X_1^*, X_2^*) \in [0,1]$ and $x(X_1^*, X_2^*) > 0$.

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