

EVOLUTIONARY DYNAMICS IN HETEROGENOUS OLIGOPOLIES FOR THE EXPLOITATION OF RENEWABLE NATURAL RESOURCES

Gian Italo Bischi*, Lucia Sbragia**

ABSTRACT

In this paper we provide an overview of some dynamic oligopoly models proposed in the literature to describe the exploitation of a common pool renewable natural resource (e.g. the fisheries) when agents can switch between different harvesting strategies. We suggest a switching mechanism which is an alternative version of the discrete-time replicator dynamics based on expected profits and myopic imitation of the better performing behaviors, often denoted as exponential replicator. We finally provide a general framework for modeling situations where authorities allow for different alternative strategies to exploit a natural resource and agents can choose among such possibilities by updating their decisions at discrete times on the basis of profitability considerations.

Keywords: Bioeconomic Models, Oligopoly Games, Resource Exploitation, Replicator Dynamics.

JEL Classification: C60, C73, L13, Q22, Q57.

ISSN: 0035-676X (print); 1827-7918 (digital)

DOI: 10.26350/000518_000109

1. INTRODUCTION

Evolutionary game theory studies the behavior of large populations of agents who repeatedly engage in pairwise strategic interactions. At each moment in time, a single individual in a population can choose among a set of alternative behaviors, and his/her payoff depends also on the choices of the other players. The basic idea is that the fraction of the population adopting the best performing behavior in the current situation will increase in the next period. In other words, the distribution of behaviors in a population evolves over time, and the fraction of individuals playing the more successful strategies increases in size. In a biological context, this can be interpreted by saying that more skilled individuals in acquiring food, space or other limiting factors, live longer, are stronger and consequently reproduce more, thus giving rise to newborn individuals that inherit the same kind of behavior. In a social context, this can be

*Gian Italo Bischi, Departments of Economics, Society, Politics (DESP) - University of Urbino - Italy. Email: ✉ gian.bischi@uniurb.it.

**Lucia Sbragia, Department of Economics - Durham University Business School - UK. Email: lucia.sbragia@durham.ac.uk.

We thank Professor Gerd Weinrich for the supervision of Sbragia's PhD dissertation that has led to the production of two papers cited in this work. We also thank the participants of the conference "Mathematical Economics. A workshop in honor of Gerd Weinrich" for their useful suggestions. This work has been developed in the framework of the research project on "Models of behavioral economics for sustainable development" financed by DESP-University of Urbino.

interpreted by saying that the kind of behavior that obtains the largest payoff will be imitated by agents gaining less, so that the more successful behavior will prevail. The study of models incorporating these kinds of dynamics can provide useful information about which behaviors will become extinct and which ones will survive over time, whether the system will approach an equilibrium situation where only one strategy prevails or several of them will coexist in a steady-state or an oscillatory time pattern will emerge.

It is now well known that environmental problems, such as the Climate Change or the exploitation of a natural resource, just to mention a few, require an interdisciplinary approach for their solution, as it occurs in the study of all complex systems. From this point of view, the history of evolutionary games is quite emblematic. The classical theory of games, mainly created in social, political and economic contexts as a mathematical representation of human rational choices in the presence of strategic interactions with other rational players, has been adapted to represent the evolution of biological populations. Indeed, in the Seventies some biologists (see e.g. Maynard Smith, 1982, and references therein) realized that the theory of games, already quite popular after the publication of the seminal book *Theory of games and economic behavior* by von Neumann and Morgenstern (1944), could also be used to represent the evolution of animals' behaviors inside ecological communities. However, the first book about evolutionary games and biology titled "Evolution and the Theory of Games" was published by the mathematical biologist John Maynard Smith in 1982, exactly 40 years ago. He adjusted the methods of traditional game theory to the context of biological natural selection, and he introduced some adaptive rules that govern the time evolution of strategies' distributions over a given population of players in the form of differential or difference equations. In this way, the usual concepts of the qualitative theory of dynamic systems could be retained. Moreover, after the book by Dawkins (1976) *The selfish gene* and the paper by Taylor and Jonker (1978) *Evolutionarily stable strategies and game dynamics*, the most common dynamic foundation of evolutionary dynamics is now known as "replicator dynamics". The basic idea behind this concept is that the number of individuals using a given strategy rises if their expected payoff is greater than the average payoff of the whole population, whereas their number decreases if the payoff of their strategy is below the average. In a biological interpretation, a population consists of animals genetically programmed to use some strategy that is inherited by their offsprings. Initially, the population may consist of animals using different strategies (e.g. more or less aggressive, using different methods to get food etc.) and the payoff to an individual adopting a given strategy is called *fitness*. Animals with higher fitness with respect to the average fitness of the population live longer and are stronger, hence leave more offsprings that inherit the same behavior. So, in the next generation the composition of the population will change.

In the economic interpretation, the population partition among different behaviors (or strategies) changes because (the same) people play the game many times, observe the payoffs gained by other players and consciously switch strategies by imitating the most successful behaviors. So, economists and social scientists realized the value of the evolutionary approach to game theory in modeling social systems, both as a method of providing foundations for the equilibrium concepts of traditional game

theory, and as a selecting tool of an equilibrium in games that admit more than one. Indeed, the two approaches (the biological one and the social sciences one) lead to the same dynamic model denoted as *replicator dynamics* as it can be interpreted not only as a model of natural selection through reproduction (or replication) in biological populations, but also as a model of imitation of successful strategies in social and economic contexts. The main difference is that successful strategies in biology are measured in term of fitness, generally a positive value, whereas in economics profit is used as a proxy of success, a value that may even be negative in some time periods. As we shall see, this may become a problem if the classical replicator dynamics is used. For this reason (and not only this) we prefer to use a different version of the replicator dynamics often denoted as *exponential replicator*, proposed by Cabrales and Sobel (1992) (see also Hofbauer and Sigmund, 2003; Hofbauer and Weibull, 1996; Kopel et al., 2014; as well as Bischi et al., 2015; Bischi and Merlone, 2017; Bischi et al., 2018, for applications in economic and social modeling).

In this paper we revisit three dynamic oligopoly models belonging to the literature of natural resource exploitation, in particular the fisheries, when some practices for a sustainable management are present. In these models, the regulator allows for a set of different exploitation rules leaving to the exploiters the possibility to choose among them according to profit maximization arguments. This naturally leads to model a dynamic switching mechanism which repeatedly selects which strategy is preferred among the allowed ones. In the following examples, in order to model the switching mechanism, we implement the discrete-time exponential replicator, because, as stated above, it allows one to describe situations where payoffs are represented by profits, that can become negative if costs overcome revenues during some time periods.

The exponential replicator dynamic equation is described in Section 2 and the following three sections deal with three different applications without any study of their dynamic properties. This is done to suggest how and why the exponential replicator can be introduced. In Section 6 we develop a general framework where two strategies in an oligopoly context are compared by using the exponential replicator as a device to select the more profitable one. We also suggest possible directions for future analysis and improvements that can be added to the models proposed. Finally, Section 7 concludes the work.

2. STANDARD AND EXPONENTIAL REPLICATOR DYNAMICS

The size of a large population of ex-ante identical individuals at time t is denoted by $N(t)$ and the finite set of available strategies to individuals of the population is denoted by $S = \{s_1, \dots, s_k\}$. Let $N_i(t)$ be the number of agents that play strategy s_i at time t , and let $x_i(t) = \frac{N_i(t)}{N(t)}$ be the corresponding fraction. As any agent is assumed to choose one and only one strategy at each time period, then $\sum_{i=1}^k N_i(t) = N$ and consequently $\sum_{i=1}^k x_i(t) = 1$. Hence, the state of the population can be represented by the vector $\mathbf{x}(t) = (x_1(t), \dots, x_{k-1}(t))$ where $k-1$ is the dimension (degrees of freedom) of the system.

An evolutionary process provides a selecting mechanism that favours some population fractions with respect to others. This mechanism is modelled in terms of differential or difference equations, according to the continuous or discrete time scale considered, so that the usual methods for the study of dynamical systems, with the related concepts of stability, bifurcations and different kinds of attractors, can be applied. The evolutionary process also addresses the issue of mutations i.e. how an invariant set is robust against perturbations of the state \mathbf{x} . This is indirectly taken into account through dynamic stability arguments. Indeed, if a small displacement of the initial condition from an equilibrium is recovered, so that the same equilibrium prevails again in the long run despite the initial perturbation (or mutation), then this can be expressed by saying that the mutants agree on the convention expressed by the original equilibrium.

A well known dynamic evolutionary selection process is given by the so called “replicator dynamics”. It is based on the assumption that individuals of a large population under scrutiny are randomly pairwise matched to play a two-person game with given payoffs. The probability of being matched with an agent playing a given strategy s_i is proportional to the fraction x_i . This implies that in all the computations, especially of the payoffs, the fractions $x_i(t)$ can be interpreted as probabilities, like in a game with mixed strategies.

In the following we shall focus on a discrete time scale based on the idea that human decisions, in social and economic contexts, generally occur at discrete times, due to adjustment costs and system inertia.

The derivation of the discrete time replicator dynamic equation is straightforward. Let r be the intrinsic growth rate of the population, independent of the game played inside the population. Moreover, assume that the growth rate of the portion of population playing strategy s_i is proportional to the average (or expected) payoff π_i obtained by a representative player i of that population class when randomly pairwise matched with other individuals of the same population, so that:

$$N_i(t+1) = [r + \pi_i(\mathbf{x}(t))] N_i(t)$$

Summing up over classes $i = 1, \dots, k$ we have

$$N(t+1) = [r + \bar{\pi}(\mathbf{x})] N(t)$$

as $N_i(t) = N(t)x_i(t)$ and $\sum x_i(t)\pi_i(\mathbf{x}(t)) = \bar{\pi}(\mathbf{x})$ is the average payoff in the whole population. Dividing each side of the first equation by the corresponding side of the second one, we get

$$x_i(t+1) = \frac{r + \pi_i(\mathbf{x}(t))}{r + \bar{\pi}(\mathbf{x})} x_i(t) \quad (1)$$

provided that $r + \pi_i(\mathbf{x}(t)) > 0$ for each $i = 1, \dots, k$ and for each t . Equation (1) describes the discrete-time replicator dynamics, which essentially states that the fraction x_i of the population playing strategy s_i grows whenever it obtains a payoff greater than the average payoff. However, the assumption of positive payoffs may be quite critical in applications, as negative payoffs may occur (consider, for example,

the case of payoffs given by profits, as it is often assumed in economic applications where $r + \pi_i$ may be negative for some periods even if it is positive in the long run). This gives values of $x_i(t)$ outside the interval $[0,1]$, which are meaningless, and even if such unfeasible values are obtained just in one time period then all the successive states are meaningless as well. So, an alternative to (1) is obtained by considering a monotone transformation of payoffs given by $u(\pi_i) = \exp(\beta\pi_i)$, with $\beta \geq 0$, and consequently $\bar{u} = \sum_{i=1}^k x_i \exp(\beta\pi_i)$.

The equation that describes the growth of the portion of the population playing strategy s_i then becomes

$$N_i(t+1) = \exp[r + \beta\pi_i(\mathbf{x}(t))] N_i(t) = e^r e^{\beta\pi_i(\mathbf{x}(t))} N_i(t).$$

Summing up over classes $i = 1, \dots, k$, we have

$$N(t+1) = \sum_{i=1}^k N_i(t) = e^r \sum_{i=1}^k e^{\beta\pi_i(\mathbf{x}(t))} N_i(t) = e^r N(t) \sum_{i=1}^k e^{\beta\pi_i(\mathbf{x}(t))} x_i(t)$$

and, again, dividing each side of the first equation by the corresponding side of the second one, we get

$$x_i(t+1) = \frac{e^{\beta\pi_i(\mathbf{x}(t))}}{\sum_{i=1}^k x_i e^{\beta\pi_i(\mathbf{x}(t))}} x_i(t) \quad (2)$$

which guarantees that $x_i(t) \in [0,1]$ for each t without further conditions. This (exponential) replicator equation, based on the monotone selection dynamics proposed in Cabrales and Sobel (1992), is a simple and useful way to avoid the problem of negative payoffs, see e.g. Hofbauer and Weibull (1996) and Hofbauer and Sigmund (2003). The parameter $\beta \geq 0$ is referred to as the *intensity of choice*, which measures how fast the players are at selecting profit-increasing behavioral rules. The extreme case $\beta = 0$ gives a model with fixed fractions, being $x(t+1) = x(t) = x(0)$. The other extreme case $\beta = \infty$ corresponds to a situation where all firms immediately switch to the behavioral rule showing a (even negligible) better performance, i.e. $x_i(t) \rightarrow 1$ if $\pi_i > \pi_j$ for each $j \neq i$. This evolving adjustment mechanism has been recently used in Bischi and Merlone (2017) and in Bischi et al. (2018) to model evolutionary binary games, as well as in Bischi et al. (2015) and Bischi et al. (2018) to study the time evolution of different boundedly rational behaviors in oligopoly models with the possibility of switching from one to another. Of course, even if many evolutionary models are based on the replicator dynamics, this is not the only evolutionary selection mechanism, as many others have been proposed in the literature, see e.g. Bischi et al. (2003); Brock and Hommes (1997); Droste et al. (2002); Hommes (2013), for different switching mechanisms and their applications.

3. COOPERATORS AND DEFECTORS IN FISHERIES

In this section we first illustrate the dynamic model proposed in Bischi et al. (2004) which describes an open access fishery exploited by a population of interacting

agents that sell the harvested resource in the same market, and then we replace the standard replicator dynamics (1) with the exponential one given by (2).

The situation addressed in the model concerns the classical problem in the exploitation of common property renewable resources known as “the tragedy of the commons” (Hardin, 1968, see also Clark, 1990). The essence of this “tragedy” resides in the nature of the resource: as a good that is not excludible and rivalrous in its consumption, any savings coming from an individual conservative management of the resource do not end up in the hands of the saver, giving rise to a use of the resource of the type first-come, first served.

This problem can be modelled as a prisoner’s dilemma (see e.g. Mesterton-Gibbons, 1993): if harvesting agents play non-cooperatively by maximizing their own individual payoff they all choose their dominant strategy and, in the short-run (disregarding competitors’ profits), a severe depletion of the resource is obtained with low profits for all in the long run. On the other hand, if fishers cooperate to maximize the aggregate profit, then a sustainable exploitation of the resource is more likely to be obtained, which implies higher profits for all in the long run. However a cooperative behavior is not robust to unilateral defections, i.e. the decision of an agent to harvest intensively while the other players harvest moderately may lead to higher profits for the defector, and consequently to profit loss for the cooperators.

The model developed in Bischi et al. (2004) uses a Cournot oligopoly setting to describe the exploitation of the natural resource, as in Levhari and Mirman (1982) or Szidarovszky and Okuguchi (1998), Szidarovszky and Okuguchi (2000). In these papers, it is assumed that the strategic interactions among players occur both via the selling price, determined by the total harvested quantity for a given demand function, and via a cost externality, since resource stock reductions, as consequence of players’ harvest, lead to higher unitary fishing costs (see also Bischi and Kopel, 2002). In Szidarovszky and Okuguchi (1998), every player decides his/her harvesting activity by solving his/her profit maximization problem (non-cooperative approach). By contrast, in Szidarovszky and Okuguchi (2000) it is assumed that fishermen form a grand-coalition (i.e. a cooperative venture) and each player determines his/her harvesting activity such that the joint profit of all players is maximized. In both cases, the solution of the optimization problem leads to harvesting functions that depend on the fish stock, whose dynamics is governed by a biological growth function with an extra mortality term due to the harvesting activity.

In Szidarovszky and Okuguchi (1998), the time setting is assumed to be discrete. Even if populations’ growth functions in dynamic ecological models have been traditionally formulated in continuous-time, discrete-time population dynamics have received increasing attention not only for the complex and intriguing time evolutions that they can produce even in the simplest nonlinear systems, but also for relevant usefulness in ecological modelling. Indeed, several authors stress that discrete-time population models should be used whenever reproduction happens at given breeding seasons, as several animal species successfully mate only during certain times of the year, thus giving non-overlapping generations (see e.g. Geritz and Kisdi, 2004; May, 1975). As a consequence, an increasing number of discrete-time population models have been proposed in the literature (see e.g. Cushing et al., 2004; Getz and Haight, 1989). A comparison between continuous-time

and discrete-time dynamics in imperfectly competitive international commercial fisheries is given in Eriaee and Okuguchi (2006).

In terms of the biological model, the fish stock at time period t , denoted by $X(t)$, is assumed to be regulated by the logistic growth function

$$X(t+1) = F(X(t)) = X(t)(1 + g - mX(t)) - H(X(t)) \quad (3)$$

where $g > 0$ represents the natural growth rate of the population, $m > 0$ is an extra-mortality term due to overcrowding (lack of food or space) and H is the total harvest to which the fishery is subjected.

Following the usual setup of evolutionary games, the population of n fishermen is subdivided into a fraction $0 \leq s \leq 1$ of cooperators, that form a cooperative venture where each one decides the harvesting quota x_c^i by maximizing the profit of the coalition. Members of the complementary fraction $(1 - s)$ are called competitors (or “defectors”) and each of them decides his/her harvesting x_d^i by maximizing his/her own profit. The total harvest is so given by

$$H = \sum_{i=1}^{ns} x_c^i + \sum_{i=1}^{n(1-s)} x_d^i,$$

and the selling price p is determined by the total harvested quantity H according to the given linear demand function

$$p = a - bH. \quad (4)$$

Player’s i cost function for harvesting a quantity x_i when the fish stock level is X is given by (see e.g. Clark, 1990; Szidarovszky and Okuguchi, 1998)

$$C(x_i, X) = \gamma_i \frac{x_i^2}{X} \quad (5)$$

with $\gamma_i \geq 0$ which satisfies the common assumptions that costs are decreasing in the fish stock and increasing in harvest¹.

Following Sethi and Somanathan (1996), an extra-cost is added to the profit functions due to the presence of social norms that are intended to punish fishers that behave as defectors, i.e. self-interested profit maximizers without any care of social optimum. Like in Sethi and Somanathan (1996), cooperators are entrusted to sanction defectors by applying some sort of punishment. This may be done directly by expressing social disapproval or by physically damaging the person and/or her equipment as observed in less developed societies; punishment can be also accomplished indirectly by alerting authorities so that they can impose sanctions according to the laws in force. Such punishment is costly for the defectors, with the cost being given by $ns\xi$, where ξ is the amount of the sanction and ns represents the probability that a

¹ This type of cost function is derived from a “production function” of the Cobb-Douglas-type with fishing effort and biomass as the two inputs.

defector is notified by a cooperator. However, in general, this kind of punishment is also costly for the cooperators, the cost being proportional to the number of defectors. Let $n(1-s)\psi$ be this extra cost for cooperators (of course $\xi > \psi$, and we shall often consider $\psi = 0$ in the following). All in all, the profit of the i -th cooperator is

$$\pi_c^i = x_c^i (a - bH) - \gamma_c \frac{(x_c^i)^2}{X} - n(1-s)\psi \quad (6)$$

where γ_c refers to the fishing technology adopted by all cooperators and $n(1-s)\psi$ represents the cost that a single cooperator has to face in order to punish all the defectors. The profit of the i -th defector is

$$\pi_d^i = x_d^i (a - bH) - \gamma_d \frac{(x_d^i)^2}{X} - ns\xi \quad (7)$$

where γ_d refers to the (business-as-usual) fishing technology adopted by the defectors² and $ns\xi$ represents the punishment that a single defector bears for causing the negative externality in the community.

Each cooperator determines x_c^i by solving the optimization problem given by

$$\max_{x_c^i} \pi^V = \max_{x_c^i} \sum_{i=1}^{ns} \pi_c^i \quad (8)$$

where π^V , which is a concave function in the variables x_c^i , denotes the total profit of the cooperative venture.

Assuming an interior optimum, the first order conditions give a system of linear equations in the unknowns x_c^i

$$\frac{\partial \pi^V}{\partial x_c^i} = a - 2b \sum_{k=1}^{ns} x_c^k - b \sum_{k=1}^{n(1-s)} x_d^k - \frac{2\gamma_c}{X} x_c^i = 0 \quad (9)$$

Each defector determines x_d^i by solving the optimization problem represented by

$$\max_{x_d^i} \pi_d^i \quad (10)$$

Assuming, again, an interior optimum, the first order conditions give a system of linear equations in the unknowns x_d^i

$$\frac{\partial \pi_d^i}{\partial x_d^i} = a - b \sum_{k=1}^{sn} x_c^k - b \sum_{k=1}^{(1-s)n} x_d^k - bx_d^i - \frac{2\gamma_d}{X} x_d^i = 0 \quad (11)$$

Note that the optimization problems (8) and (10) are intended as static optimizations, solved at a given time at which the available biomass is assumed to be constant. Equa-

² At this stage we do not make any specific assumption on the relationship between γ_c and γ_d .

tions (9) and (11) constitute a linear system of n equations with n unknowns. However, it is straightforward to see that any cooperator faces the same optimization problem, and analogously for defectors. In fact, if we denote by $x_c^{TOT} = \sum_{k=1}^{sn} x_c^k$ the total harvest of cooperators and by $x_d^{TOT} = \sum_{k=1}^{(1-s)n} x_d^k$ the total harvest of defectors, from (9) we get

$$x_c^i = \frac{X}{2\gamma_c} (a - 2bx_c^{TOT} - bx_d^{TOT}) \quad \forall i = 1, \dots, sn$$

and from (11) we get

$$x_d^i = \frac{X}{bX + 2\gamma_d} (a - b(x_c^{TOT} + x_d^{TOT})) \quad \forall i = 1, \dots, (1-s)n$$

Thus, denoting the optimal harvesting decision of each cooperator by x_c and the optimal harvesting decision of each defector by x_d , these quantities can be calculated by solving the two linear equations

$$\begin{cases} a - 2\left(bsn + \frac{\gamma_c}{X}\right)x_c - b(1-s)nx_d = 0 \\ a - bsnx_c - \left[b(1+(1-s)n) + \frac{2\gamma_d}{X}\right]x_d = 0 \end{cases}$$

from which the two linear *reaction functions* are obtained as

$$\begin{aligned} x_c = h_c(x_d) &= \frac{aX}{2(bsnX + \gamma_c)} - \frac{b(1-s)nX}{2(bsnX + \gamma_c)}x_d \\ x_d = h_d(x_c) &= \frac{aX}{b(1+(1-s)n)X + 2\gamma_d} - \frac{bsnX}{b(1+(1-s)n)X + 2\gamma_d}x_c \end{aligned} \quad (12)$$

These reaction functions allow one to compute, respectively, the optimal harvesting decision of a “representative cooperator”, given the harvesting decision of a representative defector, and the optimal harvesting decision of a “representative defector”, given the harvesting decision of a representative cooperator. These two reaction functions always intersect in a unique point (x_d^*, x_c^*) , whose coordinates are given by

$$x_d^*(X, s) = aX \frac{bsnX + 2\gamma_c}{b^2sn(n(1-s) + 2)X^2 + 2b(2ns\gamma_d + \gamma_c + \gamma_cn(1-s))X + 4\gamma_c\gamma_d} \quad (13)$$

and

$$x_c^*(X, s) = \frac{X}{2} \frac{a - b(1-s)nx_d^*(X, s)}{bsnX + \gamma_c} \quad (14)$$

For any $s \in [0, 1]$ both x_d^* and x_c^* are increasing functions of X and they vanish at $X = 0$, where their slopes are $\frac{\partial x_d^*(X, s)}{\partial X} \Big|_{X=0} = \frac{a}{2\gamma_d}$ and $\frac{\partial x_c^*(X, s)}{\partial X} \Big|_{X=0} = \frac{a}{2\gamma_c}$, respectively.

Moreover, both x_d^* and x_c^* tend to saturate as $X \rightarrow +\infty$, at the values

$$x_d^*(+\infty, s) = \frac{a}{b[(1-s)n+2]} \quad \text{and} \quad x_c^*(+\infty, s) = \frac{a}{bsn[(1-s)n+2]}$$

respectively. The gap between x_d^* and x_c^* is always positive and, for large values of the fish stock, it increases with increasing prices and with the number ns of cooperators, being

$$x_d^*(+\infty, s) - x_c^*(+\infty, s) = \frac{a}{b[(1-s)n+2]} \left(1 - \frac{1}{sn}\right)$$

The total harvest at the Nash equilibrium by the two groups, given by

$$H^*(X, s) = n[sx_c^*(X, s) + (1-s)x_d^*(X, s)] \quad (15)$$

is an increasing and concave function with respect to X , such that $H^*(0, s) = 0$,

$$\frac{\partial H^*(X, s)}{\partial X} \Big|_{X=0} = \frac{na}{2} \left(\frac{s}{\gamma_c} + \frac{1-s}{\gamma_d} \right)$$

and for $X \rightarrow \infty$ it saturates at the value

$$H^*(+\infty, s) = \frac{a}{b} \left(1 - \frac{1}{2 + (1-s)n}\right)$$

Notice also that $H^* < a/b$ for each $X > 0$, so that prices are always positive. Moreover, if s decreases, i.e. the number of defectors increases, then $H^*(+\infty, s)$ increases. In other words, as expected, in the presence of abundant resources the total harvest is greater if the number of defectors increases. In the limiting case $s = 0$ (all defectors) we have $H^*(+\infty, 0) = \frac{a}{b} \left(1 - \frac{1}{2+n}\right)$, and in the opposite limiting case $s = 1$ (all cooperators, equivalent to the case of a sole owner) we get $H^*(+\infty, 1) = a/2b$, the monopolist optimal harvesting.

The above results allow us to compute, at the Nash equilibrium, the profits of both a representative defector and of a representative cooperator, given by

$$\pi_d^* = x_d^*(a - bH^*) - \gamma_d \frac{(x_d^*)^2}{X} - ns\xi \quad \text{and} \quad \pi_c^* = x_c^*(a - bH^*) - \gamma_c \frac{(x_c^*)^2}{X} - n(1-s)\psi \quad (16)$$

respectively. Plugging the expressions of x_c^* , x_d^* and H^* into (16), π_d^* and π_c^* can be rewritten as:

$$\pi_d^* = \frac{a^2 X (bnsX + 2\gamma_c)^2 (bX + \gamma_d)}{[bX (bn(-n(1-s) - 2))sX - 2bX(n(1-s) + 1)\gamma_c - 4(bnsX + \gamma_c)\gamma_d]^2} - ns\xi \quad (17)$$

and

$$\pi_c^* = \frac{a^2 X (bnsX + \gamma_c) (bX + 2\gamma_d)^2}{[bX (bn(-n(1-s) - 2)) sX - 2bX (n(1-s) + 1) \gamma_c - 4(bnsX + \gamma_c) \gamma_d]^2} - (1-s)n\psi \quad (18)$$

respectively. These expressions show that, at the Nash equilibrium, the profits of defectors (cooperators) are positive provided that the applied sanctions (the costs to apply sanctions) are not too heavy. For example, under the assumption $\psi = 0$ and $\xi > 0$, we have x_c^* always positive and x_d^* positive or negative depending on the size of the sanctions applied and on the number of cooperators.

For this reason, when an evolutionary mechanism in discrete time is introduced to describe how, at each time period, the dynamics of $s(t)$ is updated, we propose the exponential replicator (2) with payoffs $x_c^*(t)$ and $x_d^*(t)$, whereas in Bischi et al. (2004) the standard replicator dynamics (1) is used after suitable assumptions to maintain profits always positive. In this new version of the model, such assumptions are no longer necessary, and the evolution of the bioeconomic system is then governed by the iteration of the two-dimensional map $T : (X(t), s(t)) \rightarrow (X(t+1), s(t+1))$ given by

$$T : \begin{cases} X(t+1) = X(t) (1 + g - mX(t)) - H^*(X(t), s(t)) \\ s(t+1) = s(t) \frac{e^{\beta\pi_c^*(X(t), s(t))}}{s(t)e^{\beta\pi_c^*(X(t), s(t))} + (1-s(t))e^{\beta\pi_d^*(X(t), s(t))}} = \frac{s(t)}{s(t) + (1-s(t))e^{\beta\Delta\pi^*}} \end{cases} \quad (19)$$

where

$$\Delta\pi^*(X, s) = \pi_d^*(X, s) - \pi_c^*(X, s)$$

and $H^*(X, s)$ is given by (15), with π_d^* and π_c^* defined in (13) and (14) respectively, π_d^* , π_c^* , given in (17) and (18) respectively.

Furthermore, when (1) is replaced by (2) a new parameter $\beta \geq 0$ is introduced. This parameter is important because it can be tuned to allow for different degrees of fishermen reactivity (or speed of adjustment) in their decisions to switch from one strategy to the other so giving rise to new dynamic patterns.

It is straightforward to see that $s(t) \in [0, 1]$ implies $s(t+1) \in (0, 1)$ despite the sign of π_d^* and π_c^* (whereas in the classical formulation (1) used in Bischi et al. (2004) this is true only if profits are positive at each time t). As usual, if $s(t) = 0$ then $s(t+1) = 0$ for each $t \geq 0$, and if $s(t) = 1$ then $s(t+1) = 1$ for each $t \geq 0$, i.e. the two boundary lines $s = 0$ and $s = 1$ are trapping lines, on which the dynamics are governed by the one-dimensional unimodal maps, given by the restrictions of the two-dimensional map (19) to them. These two cases correspond to particular benchmark cases, i.e. when agents are only defectors and when agents are all cooperators respectively, see Szidarovszky and Okuguchi (1998), Szidarovszky and Okuguchi (2000) and Bischi et al. (2005) on this point. Some general properties of these one-di-

mensional dynamical systems can be easily obtained. For example, the dynamics along the invariant edge $s = 0$, when all players are defectors, are governed by the one-dimensional restriction of (19) with $s = 0$ and

$$H^*(X, 0) = nx_d^*(X, 0) = \frac{naX}{b(n+1)X + 2\gamma_d}$$

So the steady states are given by $X = 0$ and the solutions (if any) of the equation

$$bm(n+1)X^2 + (2m\gamma_d - \alpha b(n+1))X + na - 2g\gamma_d = 0 \quad (20)$$

On the other invariant edge $s = 1$, when all players cooperate, the dynamics are governed by the one-dimensional restriction of (19) with $s = 1$ and

$$H^*(X, 1) = nx_c^*(X, 1) = \frac{naX}{2(bnX + \gamma_c)}$$

So the fixed points are given by $X = 0$ and the solutions (if any) of the equation

$$2bmnX^2 + 2(m\gamma_c - \alpha bn)X + na - 2g\gamma_c = 0 \quad (21)$$

The analysis is the same as in Szidarovszky and Okuguchi (2000) or in Bischi et al. (2005).

As usual, the starting point for the qualitative analysis of a nonlinear dynamical system is the localization of the steady states and the study of their local stability. The steady states of the two-dimensional dynamical system (19) are the fixed points of the map T , solutions of the system $T(X, s) = (X, s)$. It is straightforward to see that two *corner equilibria* always exist, given by $E_0 = (0, 0)$ and $E_1 = (0, 1)$, both characterized by the extinction of the resource. Other boundary equilibria may exist along the invariant lines $s = 0$ and $s = 1$, given by the solutions, if any, of equations (20) and (21) respectively. If two equilibria with positive fish stock exist both on the invariant edge $s = 0$ and on the invariant edge $s = 1$, say $X_2(0)$, $X_1(0)$ and $X_2(1)$, $X_1(1)$ respectively, then the following relation must hold: $X_2(1) < X_2(0) < X_1(0) < X_1(1)$. A necessary condition for the existence of two positive equilibria along $s = 0$ is that two positive equilibria exist along $s = 1$. However, it may happen that two positive equilibria exist along $s = 1$ and no positive equilibria exist along $s = 0$. The stability of these equilibria with respect to the one-dimensional dynamics trapped inside the invariant edges can be easily deduced from the discussion on the one-dimensional dynamics.

Another interesting aspect to assess is the stability of the equilibria with respect to perturbations transverse to the invariant edges, i.e. what happens if a few defectors appear starting from a situation with all cooperators, or what happens if a few cooperators appear starting from a situation with all defectors. The question to be answered is about whether such small mutations are eliminated by the evolutionary dynamics, so that the original benchmark case is restored (case of transverse stability) or these behaviors spread out thus causing an irreversible departure from the original benchmark case.

An answer to these questions requires the study of the local stability of the boundary equilibria, i.e. the localization, in the complex plane, of the eigenvalues of the Jacobian matrix of (19) computed at the boundary steady states. This is not difficult in principle, as eigenvalues are always real because the Jacobian matrix of (19) is a triangular matrix along the invariant edges. This implies that we can only have nodes or saddle equilibria on the boundaries. However, the expressions of the eigenvalues are quite involved, and the derived stability conditions are not easy to interpret.

Interior equilibria, i.e. steady states characterized by the coexistence of cooperators and defectors, are obtained solving the nonlinear system

$$\begin{cases} X(g - mX) = H^*(X, s) \\ \pi_c^*(X, s) = \pi_d^*(X, s) \end{cases} \quad (22)$$

with $0 < s < 1$. The set of points of the plane (X, s) that satisfy the first equation represent the locus of points that give one-period stationary resource stock, i.e. $X(t + 1) = X(t)$. This set of points may be formed by two branches, say $X_1(s)$ and $X_2(s)$, with $X_2(s) < X_1(s)$ for each s . Moreover, $X_2(s)$ is a decreasing function and $X_1(s)$ is increasing, so the branch $X_1(s)$ has positive slope and the branch $X_2(s)$ has negative slope in the plane (X, s) . The intersection of the branch $X_1(s)$ ($X_2(s)$) with the invariant edge $s = 1$ gives the X coordinate of the boundary equilibrium $X_1(1)$ ($X_2(1)$), and the same holds for the intersections (if any) with the other invariant edge $s = 0$. However, it may happen that the two branches intersect with $s = 1$ but have no intersections with $s = 0$, because they may merge together for $s > 1$.

We do not further develop the dynamic analysis of system (19) as we leave it for future work, according to the aim of this paper. We just remark that by replacing the standard replicator (1) with the exponential replicator (2) we are able to consider some new trajectories characterized by temporary negative profits (due to heavy punishments) and account for either some inertia (low β) or overshooting (high β) effects. Having a model able to represent a wide range of scenarios allows us to capture a greater number of real life situations and to design more appropriate policy interventions.

4. DISCRETE-TIME FISHERY WITH MARINE PROTECTED AREAS

In this section we consider a discrete-time dynamic model to study the time evolution of a fish stock in an aquatic environment that is (virtually) divided into two neighboring zones characterized by different fishing strategies. Indeed, often the fishing legislation partitions a fishery into regions, and in each region a different harvesting policy is imposed. This is particularly true at international level. Nevertheless, the fish moves between regions, so that the stock of one region depends not only on the harvesting effort and biological growth of that region, but also on the stock and catch of the neighboring areas. Fish mobility gives rise to an interdependence between the two areas such that, at each time period, it can increase the population of the zone where fish stock is scarcer, and thus the division of the water basin may act as a device to restore resources in the overexploited region. So, an authority in charge of a sustainable resources man-

agement may subdivide a common pool resource into regions where different fishing rules are applied, including the case of regions where fishing is forbidden, see e.g. Dubey et al. (2003). An increasing interest for this kind of regulation is witnessed by the rich literature, conferences and congresses devoted to it (see e.g. Bohnsack, 1993; Clark, 1996; Holland, 2002; Sumaila, 1998; Sumaila and Charles, 2002).

The model proposed in Bischi et al. (2009) has been motivated by the project to create a Marine Protected Area (MPA) in an international sea zone in the Adriatic Sea called “Fossa di Pomo”, see e.g. Antonelli et al. (2006). In particular, this MPA is not intended as a reserve area (i.e. a no-fishing zone) but as a region with a carefully regulated fishing activity like, for example, one where a limited fishing effort is allowed. Clearly, this level, together with other profit considerations, affects fishers’ choice to work in that area or harvest somewhere else.

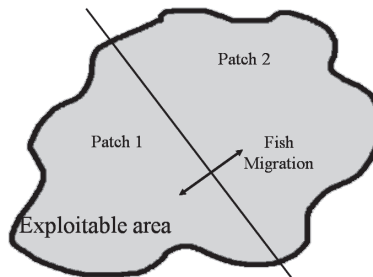
The model in Bischi et al. (2009) accounts for two fishing areas with different fishing policies and fishers allowed to choose where to harvest. In particular, it is assumed that an aquatic environment is divided into two adjacent patches, where in region 1 a *constant fishing effort* policy is allowed and in region 2 fishers engage into an *oligopolistic competition*. Furthermore, the fraction of fishers that, at each time period, decides to operate in one of the two regions updates endogenously via an adaptive switching mechanism based on a particular kind of imitation dynamics (see e.g. Bischi et al., 2003; Vega-Redondo, 1996). Instead, in this paper we propose to use the exponential replicator switching mechanism (2).

Let $X(t)$ be the total quantity of fish biomass in a water basin at time period t , and let $X_1(t)$ and $X_2(t)$ be the quantity of biomass in the patch 1 and patch 2 respectively, with $X(t) = X_1(t) + X_2(t)$. Assuming that fish can migrate between the different patches according to a linear diffusion mechanism, the biological evolution of the resource in each patch is modelled by the following two-dimensional dynamical system:

$$\begin{cases} X_1(t+1) = X_1(t) G(X(t)) - \sigma[X_1(t) - X_2(t)] - H_1(t) \\ X_2(t+1) = X_2(t) G(X(t)) + \sigma[X_1(t) - X_2(t)] - H_2(t) \end{cases} \quad (23)$$

where the function $G(X)$ represents the growth function, $\sigma > 0$ is a diffusion coefficient, and $H_i(t)$ represents the quantity of fish harvested in time period t in region i . Figure 1 provides a graphical representation of the situation considered.

FIGURE 1



A subdivision of a fishing ground into patches, with similar fishing policies, is also considered in Bischi and Lamantia (2007). However, in that paper, two independent growth functions govern the evolution of the fish stock, implying that the fish population of each region is assumed to interact only with the individuals of the same region. Instead, in this case the growth function in each patch involves interactions with both subpopulations, i.e. summing up the two equations (23) gives

$$X_1(t+1) + X_2(t+1) = (X_1(t) + X_2(t))G(X_1(t) + X_2(t)) - H(t)$$

where $H(t) = H_1(t) + H_2(t)$ is the total harvested quantity.

In the following, the logistic growth function (3) is considered, i.e.

$$X(t+1) = X(t)G(X(t)) = X(t)(1 + g - mX(t))$$

and it is assumed that the docked fish is sold in a single market no matter where it is harvested, so that the price is given by the linear inverse demand function

$$p(t) = a - bH(t) \quad (24)$$

where a and b are positive parameters and $H(t)$ is the total harvested quantity from both patches.

Denote by N the total number of fishers in the industry, by n_1 the total number of fishers active in patch 1 and by $n_2 = N - n_1$ the number of fishers operating in patch 2. Let $r = n_2/N$ be the fraction of fishers outside the MPA and $(1 - r)$ be the fraction of agents fishing inside the MPA.

In patch 1, the Marine Protected Area, the n_1 fishers are allowed to harvest with an imposed constant fishing effort. If E is the individual fishing effort imposed by a regulator, then the total harvest in that region is given by

$$H_1(t) = qEN(1 - r)X_1$$

where q is the ‘‘catchability’’ coefficient, linked to the adopted technology. The idea of a constant fishing effort is very common in the literature on mathematical bioeconomic models of the fisheries (see e.g. Clark, 1990, and references therein) and constitutes the most employed method for harvesting control.

The profit at time t of a representative fisher operating in zone 1 is given by

$$\begin{aligned} \pi_1(t) &= qEX_1(t)(a - bH(t)) - \delta E \\ &= qEX_1(t) \{a - b[ENq(1 - r(t))X_1(t) + Nr(t)x(t)]\} - \delta E \end{aligned} \quad (25)$$

where δ is the unitary cost of effort, $H_1(t) = N[1 - r(t)]qEX_1(t)$ and $H_2(t) = Nr(t)x(t)$ represent the total harvest in patch 1 and 2 respectively, $r(t)$ is the fraction of agents operating outside the MPA in period t , $x(t)$ is their individual catch, whose determination is given below.

In patch 2, the n_2 bounded rational fishers engage in a Cournot oligopolistic competition. The cost function of a representative fisher in this zone, for harvesting a quantity x when the fish stock is x_2 , is given by

$$C(x, X_2) = \gamma \frac{x^2}{X_2}$$

where γ reflects the harvesting technology outside the MPA. The quantity of resource harvested by a single oligopolist, $x(t)$, is determined in the following way. If the fishers in each patch were homogeneous, then we could write the time t profit of a representative fisher working in zone 2 as

$$\pi_2(t) = x(t) (a - bH) - \gamma \frac{x(t)^2}{X_2(t)} \quad (26)$$

However, solving the profit maximizing problem could be too costly or difficult. For this reason, it is assumed that oligopolists act in a “boundedly rational” way, in the sense that they update their harvesting plans in the direction of increasing profits, as specified by the so called profit-gradient dynamics (see Flam, 1993; Furth, 1986; Bischi and Naimzada, 2000). Thus, the quantity $x(t + 1)$ of a representative oligopolist as a function of $r(t)$, $x(t)$, $X_1(t)$ and $X_2(t)$ can be written as

$$\begin{aligned} x(t + 1) &= x(t) + k \cdot \frac{\partial \pi_2(t)}{\partial x(t)} = \\ &= x(t) + k \cdot \left\{ a - bN [2r(t)x(t) + Eq(1 - r(t))X_1(t)] - \frac{2x(t)\gamma}{X_2(t)} \right\} \end{aligned} \quad (27)$$

where $k > 0$ is the speed of adjustment to the direction of the profit gradient.

At any time period, agents are allowed to change the fishing zone in which they operate thus changing their harvesting strategy, but differently from [Bischi et al., 2009], here we assume that the switching between the two different harvesting strategies is modelled by the exponential replicator equation (2) because profits (26) and (25) may become negative. So the complete model is represented by the following map in \mathbb{R}^4 with dynamic variables X_1 , X_2 , x and r :

$$T : \begin{cases} X_1(t + 1) = X_1(t) G(X_1(t) + X_2(t)) - \sigma[X_1(t) - X_2(t)] - N[1 - r(t)] qEX_1(t) \\ X_2(t + 1) = X_2(t) G(X_1(t) + X_2(t)) + \sigma[X_1(t) - X_2(t)] - Nr(t)x(t) \\ x(t + 1) = \max \left[0, x(t) + k \cdot \left\{ a - bN [2r(t)x(t) + qE(1 - r(t))X_1(t)] - \frac{2x(t)\gamma}{X_2(t)} \right\} \right] \\ r(t + 1) = r(t) \frac{r(t)}{s(t) + (1 - s(t)) e^{\beta \Delta \pi(x(t))}} \end{cases} \quad (28)$$

where $\Delta \pi(x) = \pi_1(x) - \pi_2(x)$, $H_1(t) = N[1 - r(t)] qEX_1(t)$ and $H_2(t) = Nr(t)x(t)$ with the individual oligopolistic harvest given in (27). As usual, the sets $r = 0$ and $r = 1$ are dynamic invariant in the sense that if $r(t) = 0$ (1) then also $r(t + 1) = 0$ (1). This corresponds to the obvious statement that when a strategy is not performed at a given time it can not be imitated.

A steady state of the dynamical system (28) is a point $S = (X_1^*, X_2^*, x^*, r^*)$ satisfying the following nonlinear system of equations

$$\begin{cases} X_1 [1 + g - m(X_1 + X_2)] - \sigma(X_1 - X_2) - N(1-r)qEX_1 = 0 & (1) \\ X_2 [1 + g - m(X_1 + X_2)] + \sigma[X_1 - X_2] - Nrx = 0 & (2) \\ aX_2 - bNX_2[2rx + qE(1-r)X_1] - 2x\gamma = 0 & (3) \\ r(1-r)(\pi_1 - \pi_2) = 0 & (4) \end{cases} \quad (29)$$

where the logistic growth (3) has been assumed and π_1, π_2 are given in (25) and (26) respectively.

When no biomass is present in each patch, i.e. $X_1 = X_2 = 0$, from (29, 2) we get $x = 0$ and (29, 4) reduces to $r(1-r)(c - \delta E) = 0$. In this case, the equilibria are given by $S_1^0 = (0, 0, 0, 0)$ and $S_2^0 = (0, 0, 0, 1)$. When $c = \delta E$, any level $r^* \in [0, 1]$ guarantees an equilibrium of the type $S_3^0 = (0, 0, 0, r^*)$. All these cases represent extinction equilibria and it is easy to prove that all these boundary equilibria S_1^0, S_2^0, S_3^0 are unstable for any parameters' values.

Let us now consider the case $r = 0$, i.e. all agents work inside the MPA. Obviously equation (29, 4) is satisfied and from (29, 3) we get that the harvesting equilibrium level of each representative agent is given by $x_0^* = \frac{(a - bNqEX_1)X_2}{2\gamma}$. Hence, the system (29) reduces now to

$$\begin{cases} X_1 [1 + g - m(X_1 + X_2)] - \sigma(X_1 - X_2) - NqEX_1 = 0 \\ X_2 [1 + g - m(X_1 + X_2)] + \sigma[X_1 - X_2] = 0 \end{cases} .$$

Starting from the second equation, for $X_2 \neq \frac{\sigma}{m}$, we obtain $X_1 = g(X_2) = \frac{X_2(1 + g - \sigma - mX_2)}{mX_2 - \sigma} > 0$ that, substituted back into the first equation, gives the equilibrium level in the variable X_2 as a zero of the single variable function

$$f(X_2) = \frac{X_2 [(2X_2m - 1 - g)(1 + g - 2\sigma)\sigma + ENq(\sigma - X_2m)(1 + g - \sigma - X_2m)]}{(\sigma - X_2m)^2}$$

It is immediate to notice that $f(0) = 0$ and that $\lim_{X_2 \rightarrow +\infty} f(X_2) = +\infty$; moreover, f has a vertical asymptote at $X_2 = \frac{\sigma}{m}$, so that $\lim_{X_2 \rightarrow \frac{\sigma}{m}^-} f(X_2) = -\infty$. By continuity of f in the interval $(\frac{\sigma}{m}, +\infty)$, at least one $X_2^* > \frac{\sigma}{m}$ exists such that $f(X_2^*) = 0$, hence we can define an equilibrium point $S_0^* = (g(X_2^*), X_2^*, x^*, 0)$. Note that the condition $f'(0) > 0$, equivalent to $\frac{1+g}{\sigma} < 1 + \frac{1}{1+ENq}$, implies the existence of another equilibrium point such that $X_2^* \in (0, \frac{\sigma}{m})$.

The analysis of the case $r = 1$, i.e. with all agents working outside the MPA, leads to a similar exercise. Again equation (29, 4) is satisfied and the harvesting equilibrium level of each representative agent is $x_1^* = \frac{aX_2}{2(bNX)}$. The existence of equilibria can be studied similarly to the previous case. Under the likely assumption

that $1 + g > 2\sigma$, it is possible to prove analytically that no equilibrium exists with $X_1^* < \frac{\sigma}{m}$ and that at least one equilibrium $S_1^* = (X_1^*, g(X_1^*), x_1^*, 1)$ always exists with $X_1^* > \frac{\sigma}{m}$. Moreover, it is easy to verify that condition $a < \frac{4b\gamma\sigma(1+g-2)}{m}$ ensures the uniqueness of the equilibrium S_1^* under the case $r = 1$.

We finally turn to the problem of the existence of interior equilibria with $r \in (0, 1)$ and with $X_1, X_2 > 0$. As before, we can find those equilibria in terms of a two-dimensional system of equations. In fact, from (29, 1), we can write $r(X_1, X_2) = \frac{ENq - 1 - g + m(X_1 + X_2) - \sigma(X_1 - X_2)}{ENq}$. By adding together (29, 1) and (29, 2) and substituting back the expressions for r and $1 - r$, an expression for $x(X_1, X_2) = \frac{[(1 + g - m(X_1 + X_2))X_2 + \sigma(X_1 - X_2)]EqX_1}{X_1[ENq - 1 - g + m(X_1 + X_2)] - \sigma(X_1 - X_2)}$ is obtained. By employing these two expressions for $r(X_1, X_2)$ and $x(X_1, X_2)$ in (29, 3) and (29, 4), the following system of equations in X_1 and X_2 is derived to characterize the interior equilibria with $r \in (0, 1)$ and $X_1, X_2 > 0$:

$$\begin{cases} a - bNqE(1-r)X_1 - 2\left(bNr + \frac{\gamma}{X_2}\right)x = 0 \\ qEX_1[a - b(qENX_1(1-r))] - \delta E + c = x\left[a + bNqEX_1(2r-1) - \left(bNr + \frac{\gamma}{X_2}\right)x\right] \end{cases}$$

Note that any solution (X_1^*, X_2^*) only defines a feasible equilibrium point when $r(X_1^*, X_2^*) \in [0, 1]$ and $x(X_1^*, X_2^*) > 0$. This system can be solved numerically, but this is beyond the scope of this paper.

As in the previous section, a deeper study of (29) will produce some new results due to the possibility of accounting for temporary negative profits and fishers' inertia in switching to the best performing strategy.

5. A HYBRID MODEL OF FISHERY WITH TWO-SPECIES AND DISCRETE-TIME STRATEGY SWITCHING

In this section we revisit the fishery model proposed in Bischi et al. (2013) but with more general cost functions (that include fixed costs) and replacing the standard replicator dynamics with the exponential one. The original model was motivated by the introduction of a fishing policy imposed by the Italian authorities in the Adriatic Sea, see e.g. the book Antonelli et al. (2005), that allowed a single fisher to harvest only one fish species at the time, with the possibility to change to another one at (discrete) time periods. As harvesting costs are inversely proportional to the available fish biomass, at each switching time, more fishers are attracted towards the more abundant species, thus giving the possibility to the scarcer resource to restore in biomass.

³ The second equation of the system defines the locus of points where profits inside and outside the MPA are equal.

In Bischi et al. (2013) two non-interacting species are exploited by a population of N identical fishers. At time t , $n_1(t)$ fishers harvest only fish population 1, whose biomass density is $X_1(t)$, and $n_2(t)$ players harvest only fish population 2, whose biomass density is $X_2(t)$, with $n_1(t) + n_2(t) = N$. Let $r(t) = r_1(t) = n_1(t)/N$ be the fraction of fishers that at time t are allowed to harvest species 1 and $r_2(t) = 1 - r(t)$ the complementary fraction interested in species 2.

The time setting adopted in the model assumes that the natural growth and the harvesting of the two species occurs in continuous time, whereas the switching happens at discrete times. In particular, agents update their harvesting strategy period by period, according to an evolutionary mechanism based on a profit-driven replicator dynamics. Denoting by $G_i(X_i)$ the specific growth function of species i and by H_i its harvesting rate, the model can be formulated as:

$$\begin{cases} \dot{X}_1(t) = X_1(t)G_1(X_1(t)) - H_1(X_1(t), X_2(t)) \\ \dot{X}_2(t) = X_2(t)G_2(X_2(t)) - H_2(X_1(t), X_2(t)) \\ r(t) = \begin{cases} r(t-s) \frac{e^{\beta \bar{\pi}_1^*(t)}}{r(t-s)e^{\beta \bar{\pi}_1^*(t)} + [1-r(t-s)]e^{\beta \bar{\pi}_2^*(t)}} & \text{if } \frac{t}{s} = \left\lfloor \frac{t}{s} \right\rfloor \\ r\left(\left\lfloor \frac{t}{s} \right\rfloor s\right) & \text{otherwise} \end{cases} \end{cases} \quad (30)$$

where \dot{X}_i denotes the derivatives of biomass with respect to time, $[x]$ is the largest integer not greater than x (i.e. the floor of x), and

$$\bar{\pi}_i^*(t) = \frac{\int_{t-s}^t \pi_i^*(\tau) d\tau}{s}; \quad i = 1, 2 \text{ and } 0 < s \leq t \quad (31)$$

is the average profit observed by each agent at the end of every switching time of length s .

Note that, in the third equation in (30), at each switching time, each fisher is assumed to know the average profits of both groups during the previous period and be able to compare them over the last fishing period. The average profit of each strategy is taken as a fitness measure of that strategy, according to the paradigms of evolutionary game theory (see Hofbauer and Sigmund, 1998; Weibull, 1995): if $\bar{\pi}_1^*(t) > \bar{\pi}_2^*(t)$ then $r(t)$ increases, i.e. a fraction of fishers harvesting species 2 switch to harvest species 1, otherwise $r(t)$ decreases.

Following Szidarovszky and Okuguchi (1998), the harvesting rates $H_1(t)$ and $H_2(t)$ are computed according to the profit maximization problem of a Cournot oligopoly game. Assuming that the current total harvest is wholly supplied to the market, prices are determined according to the following horizontal differentiated linear inverse demand functions (see Singh and Vives, 1984, for details)

$$\begin{cases} p_1 = f_1(H_1, H_2) = a_1 - b_1(H_1 + \sigma H_2) \\ p_2 = f_2(H_1, H_2) = a_2 - b_2(\sigma H_1 + H_2) \end{cases} \quad (32)$$

where a_i is the reservation price for species i , b_i represents the slope of the demand for fish i and $\sigma \in [0, 1]$ is the symmetric degree of substitutability between the two fish varieties. In particular, if $\sigma = 0$ then the two varieties are independent in demand. On the other hand, for $\sigma = 1$ they are perfect substitutes. As a standard practice in fishery oligopoly models, quadratic harvesting costs are assumed for both species, i.e. for harvesting h_i units of species i an agent faces a cost given by

$$C_i(X_i, h_i) = \gamma_i \frac{h_i^2}{X_i} + c_i \quad (33)$$

where $\gamma_i > 0$ represents a cost coefficient related to the fishing technology for catching species i and c_i is a fixed cost also related to switching species. Given these specifications of the cost and demand functions, the profit of fisher q belonging to group i ($i = 1, 2$) when harvesting is $h_{i,q}$ reads as

$$\pi_{i,q} = p_i h_{i,q} - \gamma_i \frac{h_{i,q}^2}{X_i} - c_i, \quad i = 1, 2 \quad (34)$$

In order to decide the quantity of species i to be harvested, the representative fisher q in group i solves the problem $\max_{h_{i,q}} \pi_{i,q}$. From the first order conditions and employing the symmetry property that players within each group are homogeneous (i.e. $h_{i,q} = h_{i,u}$, $i = 1, 2$;) we obtain the following Nash equilibrium (see Bischi et al., 2013, for details)

$$h_i^*(X_i, X_j) = \frac{a_i X_i (b_j X_j [1 + N r_j] + 2\gamma_j) - a_j b_i N r_j X_i X_j \sigma}{(b_i X_i (1 + N r) + 2\gamma_i)(b_j X_j (1 + N(1-r)) + 2\gamma_j) - b_i b_j N^2 (1-r) r X_i X_j \sigma^2}, \quad i, j = 1, 2, i \neq j \quad (35)$$

where $r_1 = r = \frac{n_1}{N}$ and $r_2 = (1-r) = \frac{n_2}{N}$ are the fractions of fishers that harvest species 1 and 2, respectively. Hence, the harvesting terms in (30) become $H_1 = N r h_1^*$ and $H_2 = N(1-r) h_2^*$ respectively. Moreover, by inserting (35) into (34), we get the optimal individual profits as

$$\pi_i^* = \left(b_i + \frac{\gamma_i}{X_i} \right) (h_i^*)^2 - c_i. \quad (36)$$

In Bischi et al. (2013), it is assumed that the two populations of (shell)fish follow a logistic natural growth rate of the form

$$G_i = \rho_i \left(1 - \frac{X_i}{k_i} \right), \quad i = 1, 2 \quad (37)$$

where ρ_i and k_i are, respectively, the intrinsic rate of growth and the carrying capacity of species i . The dynamic model is then investigated by using both analytical and numerical methods, also focusing on some benchmark cases. For example, in the limiting case $s \rightarrow 0$, i.e. with fishers changing their fishing strategy continuously, one gets $\bar{\pi}_i^*(t) = \pi_i^*(t)$, and the last equation in (30) can be replaced by the simpler

replicator equation in continuous time (see e.g. Maynard Smith (1982), Hofbauer and Sigmund (1998), Weibull (1995))

$$\dot{r}(t) = r(t) [\pi_1^*(t) - (r(t)\pi_1^*(t) + (1-r(t))\pi_2^*(t))] = r(t)(1-r(t)) [\pi_1^*(t) - \pi_2^*(t)] \quad (38)$$

which states that $\dot{r}(t) > 0$ if $\pi_1^*(t) > \pi_2^*(t)$. In this case, the model assumes the simpler form of a nonlinear 3-dimensional system of ordinary differential equations. In order to obtain analytical results, the case of constant prices is considered (i.e. $b_1 = b_2 = 0$). The dynamical model (30) reduces to the simplified form:

$$\begin{cases} \dot{X}_1 = X_1 G_1(X_1) - Nr \frac{a_1 X_1}{2\gamma_1} \\ \dot{X}_2 = X_2 G_2(X_2) - N(1-r) \frac{a_2 X_2}{2\gamma_2} \\ \dot{r} = r(1-r) \left[\frac{a_1^2 X_1}{4\gamma_1} - \frac{a_2^2 X_2}{4\gamma_2} \right] \end{cases} \quad (39)$$

Note that in the invariant subspaces defined by $r = 0$ and $r = 1$, the first two differential equations in (39) are uncoupled. This simplified form of the model constitutes a useful benchmark for a deeper understanding of the hybrid model (30). In fact, an equilibrium point for the system (39) is also a fixed point for the hybrid system (30), although the converse is not necessarily true. This follows from the fact that the first and the second dynamic equations in the two specifications are identical, and the replicator dynamics in discrete and in continuous time share the same equilibrium conditions: $r(t) = r(t-s)$ for $r = 0$, $r = 1$ or $\pi_1^*(t) = \pi_2^*(t)$. Indeed, if instantaneous profits are identical in equilibrium, then the average profits of the two strategies over the non-switching time interval of length s are also identical. Nonetheless, we can have an equilibrium point such that the average profits of the two strategies over the interval s are equal, even though instantaneous profits are not equal over the interval. In the case of the hybrid model (30), $r(t)$ becomes a piecewise-constant function, like an endogenously driven bang-bang variable, whose discontinuous jumps occur at discrete times and lead to sudden switches among different dynamic scenarios, which is a typical behavior of hybrid systems. Numerical simulations performed in Bischi et al. (2013) allow us to investigate the role played by s (the switching time) as well as the effects of non-constant prices, i.e. demand functions (32) with slope $b_i \neq 0$, $i = 1, 2$, on the dynamics of the more realistic hybrid model (30). Such analysis gives evidence of possible advantages of profit-driven self-regulated harvesting strategy choices over other practices, both from the point of view of the biomass levels (i.e. biological sustainability) and wealth (economic profitability). Indeed, for certain sets of parameters, numerical simulations show that this kind of myopic and adaptive self-regulation may ensure a virtuous trade-off between profit maximization and resource conservation, driven by cost externalities and market pressures.

It is then interesting to understand whether the introduction of a different switching mechanism confirms the original results, or how, eventually, those results are

modified and what is the fundamental contribution of allowing for negative profits and different kinds of players inertia in the overall system dynamics.

6. A GENERAL MODEL OF EVOLUTIONARY COMPARISON BETWEEN TWO HARVESTING RULES

In this section we first propose, following Bischi et al. (2018), a general framework to address dynamic models where players can follow different behavioral rules and periodically revisit their choice on the basis of relative performances, so that the long-run spread of any rule in the agents' population can be seen as a proxy for the long-run profitability of the two behaviors. The particular switching mechanism considered here is the exponential replicator dynamics (2). We then suggest some fundamental questions that can drive the analysis of these models and some new ideas that can extend the current work. We finally implement this general setup to the context of natural resources exploitation, where a limited number of harvesting strategies is allowed and exploiters can freely choose among them according to profit maximization arguments.

Let us consider an oligopoly market where N ex-ante identical agents can choose their production quantity by following different behavioral rules. For the sake of the argument, let us deal with only two different behavioral rules and denote by $h_i(t)$ the production quantity at time t by a generic firm adopting rule $i = 1, 2$. At time t , the behavioral rule 1 is distributed in the population with frequency $r(t) \in [0,1]$ and, obviously, behavioral rule 2 with the complementary frequency $1 - r(t)$. A behavioral rule (or strategy) can be defined as a map that specifies next period production level $h_i(t+1)$ as a function of the current quantities $h_j(t)$, $i, j = 1, 2$ and agents' partition in the population $r(t)$

$$h_i(t+1) = H_i(h_1(t), h_2(t), r(t)); \quad i = 1, 2 \quad (40)$$

We then assume that profits $\pi_i(t)$ (or more generally the payoffs or utility functions) are observable by all firms, i.e. each agent knows his/her own payoff as well as the ones of the other actors, and that, at each time period, each agent can switch to the more profitable behavioral rule according to the *exponential replicator* (2). By coupling the behavioral rules in (40) with the evolutionary dynamics in (2), a three-dimensional map T is defined in the phase space $(h_1, h_2, r) \in A \subseteq \mathbb{R}_+^2 \times [0, 1]$, where $\mathbb{R}_+ = [0, +\infty)$:

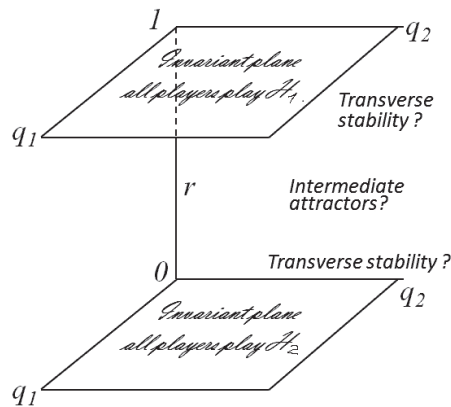
$$T : \begin{cases} h_1(t+1) = H_1(h_1(t), h_2(t), r(t)) \\ h_2(t+1) = H_2(h_1(t), h_2(t), r(t)) \\ r(t+1) = \frac{r(t)}{r(t) + [1 - r(t)] e^{\beta[\pi_2(t) - \pi_1(t)]}} \end{cases} \quad (41)$$

One important point to consider in the analysis of models like (41) is the role of the invariant planes $r = 0$ and $r = 1$, where all the agents choose the same strategy H_2 or H_1 respectively (see Figure 2). On these invariant planes, the dynamics are governed by the two-dimensional restrictions of T , that assume the form of triangu-

lar maps, i.e. unidirectionally coupled dynamical systems or systems with a skew-product structure (see Bischi et al., 2015, 2018; Cerboni-Baiardi et al., 2015). From an economic point of view, this fact has the obvious interpretation that absent behaviors remain absent. However, a second interesting aspect to address in the analysis of these models is the introduction of a mutation in agents' behavior: this may spread over the population or may be reabsorbed. This phenomenon can be ascertained through the study of the transverse stability of the attractors on the invariant planes. In general, an attractor on one of these two-dimensional restrictions of the phase space may be *transversely stable*, so that it attracts trajectories starting outside the restriction, i.e. from $r(0) \in (0,1)$; in this case, the attractor on the restriction is also an attractor of the three-dimensional map T . Thus, many properties of the attractors that characterize the long-run evolution in the case of homogeneous behavior (i.e. all agents using the same strategy) can often be separately studied in the two-dimensional restriction. However, in the case of cyclic and chaotic dynamics on the invariant planes, even more complex situations can be obtained with respect to their transverse stability properties. Indeed, these chaotic two-dimensional invariant sets inside a three-dimensional phase space can be transversely stable on average, thus giving rise to weaker attractors in Milnor sense and, consequently, to on-off intermittency phenomena or riddled basins. The interested reader is referred to Milnor (1985), Alexander et al. (1992), Bischi and Gardini (2000), Bischi et al. (2015).

The study of the transverse stability of these “pure strategy” attractors provides useful information about the fate of small mutations. In other words, this kind of analysis helps us to understand whether the introduction of a different heuristic by just one firm (or a few firms) in an oligopoly market spreads within the population or it dies out spontaneously, see e.g. Cerboni-Baiardi et al. (2015) and Bischi et al. (2015).

FIGURE 2



An important extension of the model (41) can be obtained by introducing some form of memory, i.e. agents consider a discounted average of past payoffs in the switching decision instead of comparing only the last observed values. In other words, the fitness associated with a behavioral rule may be measured according to

the accumulated payoffs instead of just current ones (see e.g. Bischi et al., 2020, for details). Following this argument, one can assume that in the evolutionary model the fitness measure $U_i(t)$ of behavioral rule i at each time step also involves a portion of the profits accumulated in the past, that is

$$U_i(t) = (1 - \omega) \pi_i(t) + \omega U_i(t - 1) \quad (42)$$

where $\omega \in [0, 1]$ represents a *memory parameter* that takes into account a convex combination of the current expected profit and the accumulated ones. This specification is denoted as “normalized memory” in Hommes et al. (2012). From the recursive formula (42), reasoning by backward induction, it is easy to get the expression of the accumulated profit

$$U_i(t) = (1 - \omega) \sum_{k=0}^{t-1} \omega^k \pi_i(t - k) + \omega^t U_i(0), \quad i = 1, 2 \quad (43)$$

as a measure of fitness expressed in terms of a discounted weighted sum with exponentially fading weights. As before, the parameter $\omega \in [0, 1]$ gives an indication of the memory, with $U_i(t) = \pi_i(t)$ for $\omega = 0$, whereas a uniform arithmetic mean of all the past payoffs is obtained in the other limiting case $\omega = 1$.

If we use $U_i(t)$ as the fitness associated with behavioral rule i in the *exponential replicator*, the third equation in model (41) becomes

$$r(t + 1) = \frac{r(t)}{r(t) + [1 - r(t)] e^{\beta[U_2(t) - U_1(t)]}} \quad (44)$$

together with the linear iterative utility dynamics (42). We refer to Bischi et al. (2020) for a deeper analysis of this evolutionary model with memory.

As we have seen throughout the paper, this general framework can be implemented to approach the literature on the exploitation of common pool renewable natural resources, like the fishery, where different harvesting strategies are simultaneously applied by different fishers on the same renewable resource. Understanding how different fishing strategies spread among fishers is a fundamental step to understand the long-run evolution of fish stocks and perform some form of welfare analysis.

A first change in (41) would then be to add the time evolution of the natural resource. Starting from the very general expression

$$X(t + 1) = X(t) G(X(t))$$

in the specific application of (41) to bioeconomic models, several growth functions can be used according to what type of fish stock is in consideration. In the models described in the previous sections we always worked with the logistic function (37), but different kinds of growth functions can be considered, according to the biological properties of the fish population and the marine environment at hand. A commonly used growth function is

$$G(X) = \rho e^{\left(1 - \frac{X}{k_i}\right)}, \quad i = 1, 2$$

known as Ricker function from Ricker (1954), or

$$G(X) = \frac{\rho}{1 + \alpha X}, \quad i = 1, 2$$

known as Beverton-Holt function (see e.g. Getz and Haight, 1989).

As a consequence of the specific application, the next period harvesting $h_i(t+1)$ becomes a function of the current quantities, fishers' partition $r(t)$ as well as resource stock:

$$h_i(t+1) = H_i(h_1(t), h_2(t), r(t), X(t)); \quad i = 1, 2.$$

hence the overall system is described by a four-dimensional map T is defined in the phase space (X, h_1, h_2, r)

$$T : \begin{cases} X(t+1) = X(t)G(X(t)) - H(h_1(t), h_2(t), r(t), X(t)) \\ h_1(t+1) = H_1(h_1(t), h_2(t), r(t), X(t)) \\ h_2(t+1) = H_2(h_1(t), h_2(t), r(t), X(t)) \\ r(t+1) = \frac{r(t)}{r(t) + [1 - r(t)] e^{\beta[\pi_2(t) - \pi_1(t)]}} \end{cases}$$

where the first equation describes the time evolution of the natural resource subjected to total harvest $H = h_1 + h_2$; the second and third equations update the harvest according to the different behavioral rules adopted; and the last equation describes the time evolution of the agents' population composition given by (2).

This general map can be further adjusted by modifying its individual elements. One example could be the profits entering into equation (2): those could be modelled to account for different forms of punishment related to a common norm; or they can be adjusted to include the presence of green consumers willing to pay a premium price for a fish certified harvested adopting a more sustainable technique. Finally, further kinds of behavioral rules can be examined by considering alternative forms of command and control measures or market based incentives.

7. CONCLUSIONS

In this paper we have considered oligopolistic markets for the exploitation of a common pool natural resource (in particular the fisheries) where the population of exploiters can employ different strategies for deciding their next-period harvest. This is a common situation when authorities adopt a more "libertarian paternalism" approach (Thaler and Sunstein, 2003) leaving the possibility to freely choose among a limited number of alternative harvesting strategies according to profit maximization considerations to the exploiters.

Furthermore, we have included a dynamic evolutionary component, by assuming that harvesting strategy choices can be repeated at discrete time periods, and, at these

times, players are allowed to change their mind in terms of their action, i.e. switching their strategy. In this paper such a dynamic mechanism has been mathematically represented by the profit-driven exponential replicator dynamics.

Instead of proposing a new particular model and developing a deep mathematical analysis and economic interpretation of its results, we reviewed three dynamic fishery models proposed in the literature and modified them with the discrete time exponential replicator. The analysis of these new models is intentionally short, as this task is left for future studies. Here we only highlighted what novelty aspects this alternative switching mechanism can bring and possible directions of analysis.

Finally, the last section offers a general template that can be adapted to develop several different models according to the kind of oligopolistic competition and/or natural growth functions and/or the presence or not of memory. Examples of these possibilities, not taken from natural resource modeling, can be found in the literature, see e.g. Cerboni-Baiardi et al. (2015), Bischi et al. (2015, 2018, 2020), Lamantia and Radi (2015), Naimzada and Sbragia (2006), Radi (2017). In this section, we indicated two important points of interest in the analysis of the dynamics. Those are related to the presence of two invariant planes representing homogeneous behaviors (all agents adopting the same strategy) on which complex dynamics may occur associated with possible occurrence of riddling and synchronization phenomena when stability with respect to perturbations transverse to the invariant planes is considered, see e.g. Alexander et al. (1992), Bischi and Cerboni-Baiardi (2017). In our case transverse perturbations means possible mutations with respect to the universally adopted strategies represented by dynamics on the invariant planes.

As a final note, we stress that the models proposed in this paper require an interdisciplinary approach to be studied, as they involve competencies from economics, ecology, social sciences and, of course, mathematics. This is the recommended approach to study complex systems. Moreover, even the mathematical methods implemented here arise themselves from an interdisciplinary trade off between the mathematical, economic and biological literature: the mathematical theory of games, initially developed in the framework of economic problems to describe rational strategic interactions, has then been applied to the context of biological evolution theory (in term of selective reproduction) and then such evolutionary game theory has been used to represent social and economic situations where the imitation of the most successful strategies occurs. This is an important lesson to learn and a guide for future research developments.

REFERENCES

- Antonelli G., Bischi G.I., Viganò E. (2005). *La sostenibilità nel settore della pesca*. Franco Angeli, Milano
- Antonelli G., Bischi G.I., Lamantia F., Viganò E. (2006). Il ruolo delle riserve marine nella gestione sostenibile della pesca. Costi, benefici, modelli. In Castellini A., Ragazzoni A., Trevisan G. (eds.). *L'apporto degli economisti agrari alla ricerca scientifica nel settore ittico*. Franco Angeli, Milano
- Alexander J.C., Yorke J.A., You Z., Kan I. (1992). Riddled basins. *International Journal of Bifurcation & Chaos*, **2**, 795-813
- Bischi G.I., Cerboni-Baiardi L. (2017). Bubbling, Riddling, Blowout and Critical Curves. *Journal of Difference Equations and Applications*, **23**, 939-964
- Bischi G.I., Cerboni-Baiardi L., Radi D. (2015). On a discrete-time model with replicator dynamics in renewable resource exploitation. *Journal of Difference Equations and Applications*, **21**, 954-973
- Bischi G.I., Dawid H., Kopel M. (2003). Spillover Effects and the Evolution of Firm Clusters. *Journal of Economic Behavior & Organization*, **50**, 47-75
- Bischi G.I., Gardini L. (2000). Global Properties of Symmetric Competition Models with Riddling and Blowout Phenomena. *Discrete Dynamics in Nature and Society*, **5**, 149-160
- Bischi G.I., Kopel M. (2002). The Role of Competition, Expectations and Harvesting Costs in Commercial Fishing. In Puu T., Sushko I. (eds.). *Oligopoly Dynamics: Models & Tools*, Springer Verlag, Berlin
- Bischi G.I., Kopel M., Szidarovszky F. (2005). Expectation-Stock Dynamics in Multi-Agent Fisheries. *Annals of Operations Research*, **137**, 299-329
- Bischi G.I., Lamantia F. (2007). Harvesting dynamics with protected and unprotected areas. *Journal of Economic Behavior & Organization*, **62**, 348-370
- Bischi G.I., Lamantia F., Radi D. (2013). Multi-species exploitation with evolutionary switching of harvesting strategies. *Natural Resource Modeling*, **26**, 546-571
- Bischi G.I., Lamantia F., Radi D. (2015). An evolutionary Cournot model with limited market knowledge. *Journal of Economic Behavior & Organization*, **116**, 219-238
- Bischi G.I., Lamantia F., Radi D. (2018). Evolutionary oligopoly games with heterogeneous adaptive players. In Corchón L., Marini M. (eds.). *Handbook of Game Theory and Industrial Organization*. Volume I. Edward Elgar Publishing, Cheltenham
- Bischi G.I., Lamantia F., Sbragia L. (2004). Competition and cooperation in natural resources exploitation: An evolutionary game approach. In Carraro C., Fragnelli V. (eds.). *Game Practice and the Environment*, Edward Elgar Publishing, Cheltenham

- Bischi G.I., Lamantia F., Sbragia L. (2009). Strategic interaction and imitation dynamics in patch differentiated exploitation of fisheries. *Ecological Complexity*, **6**, 353-362
- Bischi G.I., Lamantia F., Scardamaglia B. (2020). On the influence of memory on complex dynamics of evolutionary oligopoly models. *Nonlinear Dynamics*, **102**, 1097-1110, (open access) DOI:10.1007/s11071-020-05827-9
- Bischi G.I., Naimzada A. (2000). Global Analysis of a Duopoly Game with Bounded Rationality. *Advances in Dynamic Games and Applications*, **5**, 361-385
- Bischi G.I., Merlone U. (2017) Evolutionary minority games with memory. *Journal of Evolutionary Economics*, **27**, 859-875, DOI:10.1007/s00191-017-0526-4
- Bischi G.I., Merlone U., Pruscini E. (2018). Evolutionary dynamics in club goods binary games. *Journal of Economic Dynamics & Control*, **91**, 104-119
- Bohnsack J.A. (1993). Marine Reserves: They Enhance Fisheries, Reduce Conflicts, and Protect Resources. *Oceanus*, **36**, 63-71
- Brock W.A., Hommes C.H. (1997). A rational route to randomness. *Econometrica*, **65**, 1059-1095
- Cabrales A., Sobel J. (1992). On the limit points of discrete selection dynamics. *Journal of Economic Theory*, **57**, 407-419
- Carboni-Baiardi L., Lamantia F., Radi D. (2015). Evolutionary competition between boundedly rational behavioral rules in oligopoly games. *Chaos, Solitons & Fractals*, **79**, 204-225
- Clark C.W. (1990). *Mathematical Bioeconomics*, 2nd edition. John Wiley & Sons, New York
- Clark C.W. (1996). Marine Reserves and the Precautionary Management of Fisheries. *Ecological Applications*, **6**, 369-370
- Cushing J.M., Leverage S., Chitnis N., Henson S.M. (2004). Some Discrete Competition Models and the Competitive Exclusion Principle. *Journal of Difference Equations and Applications*, **10**, 1139-1151
- Dawkins R. (1976). *The Selfish Gene*. Oxford University Press, Oxford
- Droste E., Hommes C.H., Tuinstra J. (2002). Endogenous fluctuations under evolutionary pressure in Cournot competition. *Games and Economic Behavior*, **40**, 232-269
- Dubey B., Peeyush, C., Sinha, P. (2003). A model for fishery resource with reserve area. *Nonlinear Analysis*, **4**, 625-637
- Eriaee G.H., Okuguchi K. (2006). Bifurcation and stability in imperfectly competitive international commercial fishing. *Keio Economic Studies*, **41**, 61-71
- Flam S.D. (1993). Oligopolistic Competition: from Stability to Chaos. In Gori F., Geronazzo L., Galeotti M. (eds.). *Nonlinear Dynamics in Economics and Social Sciences, Lecture Notes in Economics and Mathematical Systems*, Springer, Wien-New York
- Furth D. (1986). Stability and Instability in Oligopoly. *Journal of Economic Theory*, **40**, 197-228

- Geritz S.A.H., Kisdi E. (2004). On the mechanistic underpinning of discrete-time population models with complex dynamics. *Journal of Theoretical Biology*, **228**, 261-269
- Getz W.M., Haight R.G. (1989). *Population Harvesting: Demographic Models of Fish, Forest, and Animal Resources*. Princeton University Press, Princeton
- Hardin G. (1968). The Tragedy of the Commons. *Science*, **162**, 1243-1248
- Hofbauer, J., Sigmund K. (1998). *Evolutionary Games and Population Dynamics*. Cambridge University Press, Cambridge
- Hofbauer J., Sigmund K. (2003). Evolutionary Game Dynamics. *Bulletin of The American Mathematical Society*, **40**, 479-519
- Hofbauer J., Weibull J.W. (1996). Evolutionary selection against dominated strategies. *Journal of Economic Theory*, **71**, 558-573
- Holland D.S. (2002). Integrating Marine Protected Areas Into Models For Fishery Assessment and Management. *Natural Resource Modeling*, **15**, 369-386
- Hommes C.H. (2013). *Behavioral rationality and heterogeneous expectations in complex economic systems*. Cambridge University Press, Cambridge
- Hommes C.H., Kiseleva T., Kuznetsov Y., Verbic M. (2012). Is more memory in evolutionary selection (de)stabilizing?. *Macroeconomic Dynamics*, **16**, 335-357
- Kopel M., Lamantia F., Szidarovszky F. (2014). Evolutionary competition in a mixed market with socially concerned firms. *Journal of Economic Dynamics & Control*, **48**, 394-409
- Lamantia F., Radi D. (2015). Exploitation of renewable resources with differentiated technologies: an evolutionary analysis. *Mathematics and Computers in Simulation*, **108**, 155-174
- Levhari, D., Mirman L.J. (1982). The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution. In Mirman L.J., Spulber D.F. (eds.). *Essays in the Economics of Renewable Resources*, North-Holland, Amsterdam
- May R.M. (1975). Biological populations obeying difference equations: Stable points, stable cycles, and chaos. *Journal of Theoretical Biology*, **51**, 511-524
- Maynard Smith J. (1982). *Evolution and the Theory of Games*. Cambridge University Press, Cambridge
- Mesterton-Gibbons, M. (1993). Game-theoretic resource modeling. *Natural Resource Modeling*, **7**, 93-147
- Milnor J. (1985). On the concept of attractor: Correction and remarks. *Communications in Mathematical Physics*, **102**, 517-519
- Naimzada A.K., Sbragia L. (2006). Oligopoly games with nonlinear demand and cost functions: Two boundedly rational adjustment processes. *Chaos, Solitons & Fractals*, **29**, 707-722

- Radi D. (2017). Walrasian versus Cournot behavior in an oligopoly of bounded rational firms. *Journal of Evolutionary Economics*, **27**, 933-961
- Ricker, W.E. (1954). Stock and Recruitment. *Journal of the Fisheries Research Board of Canada*, **11**, 559-623
- Sethi R., Somanathan E. (1996). The evolution of social norms in common property resource use. *American Economic Review*, **86**, 766-788
- Singh N., Vives X. (1984). Price and Quantity Competition in a Differentiated Duopoly. *Rand Journal of Economics*, **15**, 546-554
- Sumaila U.R. (1998). Protected Marine Reserves as Fisheries Management Tools: a Bioeconomic Analysis. *Fisheries Resources*, **37**, 287-296
- Sumaila U.R., Charles A.T. (2002). Economic Models of Marine Protected Areas: an Introduction. *Natural Resource Modeling*, **15**, 261-272
- Szidarovszky F., Okuguchi K. (1998). An oligopoly model of commercial fishing. *Seoul Journal of Economics*, **11**, 321-330
- Szidarovszky F., Okuguchi K. (2000). A Dynamic Model of International Fishing. *Seoul Journal of Economics*, **13**, 471-476
- Taylor P., Jonker L. (1978). Evolutionarily stable strategies and game dynamics. *Mathematical Biosciences*, **40**, 145-156
- Thaler R.H., Sunstein C.R. (2003). Libertarian Paternalism. *American Economic Review*, **93**, 175-179
- Vega-Redondo F. (1996). *Evolution, Games and Economic Behaviour*. Oxford University Press, Oxford
- von Neumann J., Morgenstern O. (1944). *Theory of Games and Economic Behavior*. Princeton University Press, Princeton
- Weibull J.W. (1995). *Evolutionary Game Theory*. The MIT Press, Cambridge